Strategies For Aluminum Recycling: Insights From Material System Optimization

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Submitted to the Department of Material Science and Engineering
In Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in Materials Science and Engineering
at the
Massachusetts Institute of Technology
September 2005

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Abstract

The dramatic increase in aluminum consumption over the past decades necessitates a societal effort to recycle and reuse these materials to promote true sustainability and energy savings in aluminum production. However, the path towards this goal is full of challenges which lead to inefficiencies in the usage of secondary materials. These frictions are due to (i) rapidly changing consumption patterns, (ii) compositional incompatibility in scrap streams and (iii) incomplete information in the decision making process around scrap consumption.

This thesis tackles these inefficiencies by developing optimization-based decision tools and modeling techniques for the assessment of sorting technologies and scrap management from procurement to production. In the course of managing and accounting for the aforementioned variability and uncertainties in the material system inputs, the goal is to present cost-effective strategies to increase scrap consumption under applicable context of different operating environment in aluminum production. These decision tools also aim to foster a fundamental shift in decision-making behavior to factor in uncertainties into the scrap management process.

A sorting algorithm with an arbitrary number of output streams is created as a guide to quantify the effects of wrought and cast recovery rates, sorting cost, scrap content, and product mix on sorting technologies application and development. In collaboration with Norsk Hydro Aluminum, an evaluation of wrought-versus-cast sorting technology is undertaken. For a reasonable range of sorter recovery rates and costs, the process leads to overall cost savings and increase in scrap consumption. Unlike cost savings and scrap consumption, however, the sorter utilization rate does not increase monotonically with improvements in recovery rates. Furthermore, under limited scrap supplies, not all products benefited in increased scrap consumption with sorting.

Stochastic optimization techniques are introduced to address demand and scrap compositions uncertainties faced by different decision-makers along the aluminum production chain. With the idea of recourse and scrap net residual value, increased scrap purchase and usage were determined to be an effective hedge against adverse demand swings. Traditional forecast-based deterministic decision tools were found to be too costly and conservative in scrap usage on average. At the operator level, stochastic modeling draws relevance in its ability to link production tolerance level for compositional variance to the underlying compositional uncertainties in scrap materials. The technique also supports diversification in scrap sources as a way to mitigate compositional variance in product scrap usage.

Overall these models and methodologies target various scrap usage inefficiencies in the aluminum production chain. Their application and associated insights can bring society one step closer towards sustainable development, not only in aluminum, but potentially for other light metals as well.

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Acknowledgements

A doctoral degree is as much an academic pursuit as it is a self-discovery process, neither of which can be accomplished without significant feedback and support from other individuals and organizations. During these years at MIT, I have had the fortune to learn from people who are truly passionate about their research. The energy, intense curiosity and inquisitiveness that exemplify people here are inspirations that will stay with me long after I leave.

First and foremost I want to thank my advisors, Professor Randy Kirchain and Professor Lionel Kimerling, for their friendships and for providing profound intellectual and personal guidance throughout these years. They taught me that the ultimate joy of research is in the discovery of new knowledge. I wish them continued success in redefining excellence in teaching and research. I also want to thank Professor Joel Clark for all his encouragement and for sharing his knowledge and experience in multi-disciplinary research. On stochastic modeling, Professor Richard de Neufville, Professor Richard Larson, and Professor Olivier de Weck provided invaluable insights into the applicability and limitation of various methods. And thanks to Eun Suk Suh and Harn Wei Kua who shared their findings on the application of uncertainty modeling in automotive manufacturing and building technology industries.

A number of individuals at Hydro Aluminum provided major impetus for my research work in addition to financial support. In particular I want to thank Hans Ole Riddervold and Sigrid Guldberg for working closely with me during my stay in Oslo, as well as introducing me to the healthy Norwegian lunch diet of bread and salad. The West Side Story in Norwegian was also unforgettable. Deep in the tranquil fjords of Sunndalsora, Norway, Geir Oyen and Kolbjurn Hasle were instrumental in giving me an understanding of the difficulties in working with compositional uncertainty. Equally important, however, were the skills I picked up from Geir on how to reel in four fish simultaneously!

Back in Boston, I owe the joy of my earlier years at MIT to my EMAT teammates, without whom my transition from work back to an academic environment would have been much less stimulating and rewarding. To my fellow or soon-to-be MSL alumni, best of luck with life after MIT and thanks for making it fun. To those MSLers continuing to shoot for the stars at MIT, may the force be with you whatever your aspirations are. Special thanks to Dr. Frank Fields and Dr. Richard Roth in helping me with research directions and elucidating complex ideas.

My years at MIT are also incomplete without a group of friends to help me balance work and play. To all of you, Louis, Patrick, Game Master Albert, Sally, Ricky, Henry, Black Hole Albert, Doc Albert, Tin Chung, PMY and Ah Yeung, I wish you the best of luck wherever your minds and hearts take you. Special thanks to my girlfriend, Maisie, for her unwavering encouragement and selflessness when it meant the most.

Last but certainly not least, deepest thanks to my mom Linda, dad Raymond, Grandma, sister Leona and brother-in-law Jason for their unconditional love and belief in me. Mom, those books you read with me when I was a kid has certainly come a long way! And dad, you are truly a guiding light. Grandma, thanks for always praying for my well-being. Sis, thanks for always being such a good listener and congratulations on baby Adrian! What a joy he is. I will be the best uncle I can be. And Jason, it is so good to have a big brother. Everyone, a supportive family is truly priceless.

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Chapter 1 : Introduction

1.1 Aluminum, Society & Environment

Aluminum is a critical component of modern day materials use. It is used in a large number of industries including transportation, packaging, construction, consumer durables and other engineering applications[1]. In fact, by the end of the twentieth century, 32 million tons of aluminum was consumed worldwide, making it the second most utilized metal in the world. Within the US, the three biggest individual markets for aluminum are transportation, building and construction, and packaging which represent 33%, 20% and 13% respectively of all annual shipments (Aluminum Association 2004). While the abundance of bauxite ore around the world does not indicate any imminent shortage of aluminum (in fact, aluminum is the third most abundant element), there are certainly severe environmental consequences for heavy reliance on primary production versus recycling this metal. Recycling aluminum only requires 5% of the energy and emits 5% of the CO_2, a greenhouse gas, compared to primary production of hot metal (Keoleian 1997, Stodolsky 1995). Even though 55% of the energy source for primary aluminum production is currently from renewable energy[2], the remaining 45% are still polluting and environmentally harmful (International Aluminum Institute 2002). While the aluminum industry continues to raise the energy efficiency of primary production (International Aluminum Institute 2002), it still carries one of the highest primary-versus-secondary energy production requirement differences compared to many other prevalent engineering materials. Hence, aluminum stands out as a leading material for recycling efforts.

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[1] These include electrical wiring, electrical towers, consumer durables, furniture, machinery and sports equipment (International Aluminum Institute).

[2] The sources are mostly hydroelectric power.

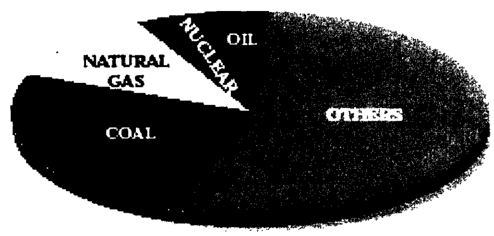


Figure 1.1 Energy source for primary aluminum production (International Aluminum Institute 2002)

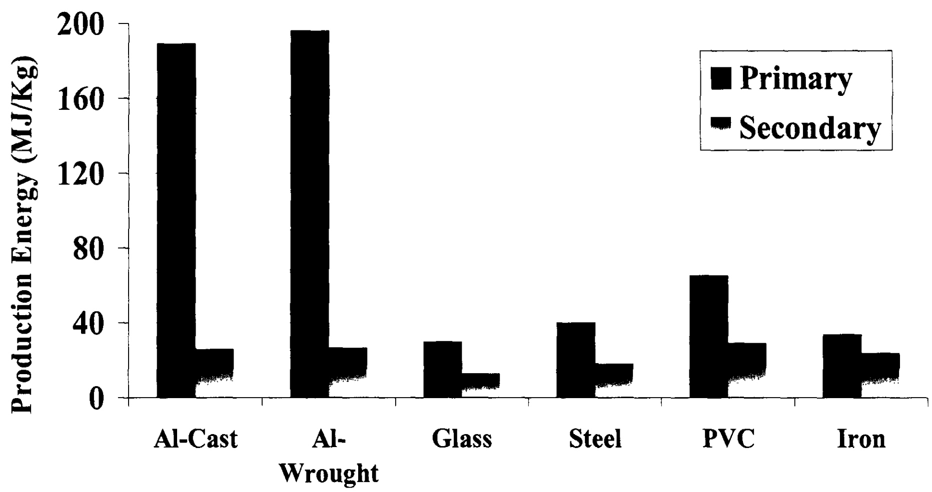


Figure 1.2 Primary versus secondary material production energy requirements. (Keoleian 1997)

1.2 Aluminum Usage & Recycling Challenges

Perhaps in part due to being a highly valuable component in the consumer waste stream, aluminum is often perceived to be a sustainable material and one that is highly recycled. In fact, of the major metals, post-consumer aluminum and steel scraps represent 20-25% of domestic apparent consumption in recent years, compared to only 9% for copper (USGS Mineral Commodities Summary 2005). Yet, a more detailed view of the specific aluminum markets would reveal hidden issues in this simple statement. While statistics regarding the recovery and usage of secondary aluminum in all the major markets are not available, some observations can be made in particular of the packaging and transportation industries. Some obstacles in aluminum recycling efforts relate to the actual rate of recycling, and the infrastructures and technologies available to promote such effort. Others involve inherent problems and challenges due to the constantly evolving nature of aluminum consumption and the chemical incompatibility of different aluminum alloys and products in the recycling chain.

According to the Institute of Scrap Recycling Industries, the aluminum beverage can (mostly 3105) recycling rate in the United States has been averaging around 60-65% recently. Close to 54 billion used cans (1.6 billion pounds of aluminum alloy) were collected in the US in 2002. However, this rate is far too low for the aluminum can to be conceived as a highly recycled product; especially when it has the potential to be completely close-loop recycled (the same material goes back into making new cans). Since it is rare for used aluminum beverage cans to be mixed with other aluminum alloys during the collection, recovery and recycling processes, this low rate of aluminum recycling from beverage cans can be mostly attributed to low participation rates from consumers resulting in leakage from the material system.

While the volume of aluminum cans in the market today is undoubtedly significant, packaging still ranks second in total tonnage of aluminum consumption compared to the transportation industry. Regarding aluminum consumption in the transportation sector, it has not always been the case that aluminum was used extensively. In fact it was only in 1994 that transportation first became the largest single sector of consumption with passenger cars accounting for the majority of the subsequent growth. The impetus for much of this rise in consumption in the transportation sector is the desire to reduce vehicle weight and to meet government fuel economy requirements (Ostroff 2004). In 2002, aluminum was the third most used material in cars and trucks, according to the Aluminum Association. In fact, between 1973 and 2002, the average vehicle aluminum content has risen steadily from 91 pounds to 274 pounds (Benedyk 2002). With a global annual production of 50 million new vehicles and average aluminum content per vehicle of 274 pounds, the *potential* annual automotive contribution to the secondary aluminum scrap pile is over 13 billion pounds. Table 1-I lists examples of recent vehicles with high aluminum content.

Vehicle Model	Aluminum Content (lbs)
Lincoln LS	500
Oldsmobile Aurora	500
Chevy Trailblazer	390
Nissan Altima	360
Peugeot 607	418
Citroen C5	330

Table 1-I Aluminum content of selected vehicles

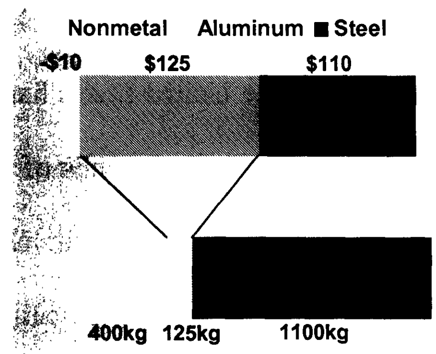


Figure 1.3 Average material content and end-of-life material value of year 2000 vehicle (Ducker 1999)

Even though the aluminum content only represents 7%[3] of the average motor vehicle by weight, it typically accounts for over 50% of the vehicle's value as scrap (compare Figure 1.3). Because of this value, up to 85% of the aluminum in cars are recovered and recycled at end-of-life. The environmental and monetary benefits are some of the reasons Ford Motor Company and Alcan jointly launched North America's first (October, 2002) closed-loop recycling program for automotive aluminum sheet scrap. This static picture, however, belies the underlying issues in aluminum recycling for automotive applications.

The applicability and economic value of scrap materials are delicately balanced with their inherent handling difficulty during processing. This difficulty stems from the widely varying chemistry and properties of the incoming scraps. In additional to the variability in chemical content due to the mixed nature of scrap materials, scrap types exhibit trends due to changes in

[3] Based on average aluminum content of 274 pounds for passenger cars and light trucks in year 2002 and total car weight of about 4,000 (Ducker Research Company). Each pound of aluminum replaces two pounds of iron or steel.

the material choices in associated applications. For instance, the estimated combined aluminum content of all vehicles currently on the road is 36 million tons worldwide and growing on the order of 10% per year (Gesing 2001). This growth involves two types of aluminum alloys differentiated by the way they are formed as well as in chemistry – cast and wrought. A wrought product is one that has been subjected to mechanical working by processes such as rolling, extrusion, and forging. A cast product is one in which the shape has been produced by introducing molten aluminum into a mold, and includes processes such as sand casting, permanent mold casting, and die casting, etc. Castings have less stringent compositional targets and a high tolerance for impurities, and therefore can easily absorb current mixed alloy scrap. Wrought on the other hand is much less tolerant and current practice indicates that less than 10% scrap can be used (Cosquer 2003). With silicon as a proxy for alloying content, Figure 1.4 indicates the generally higher alloying content for cast versus wrought alloys.

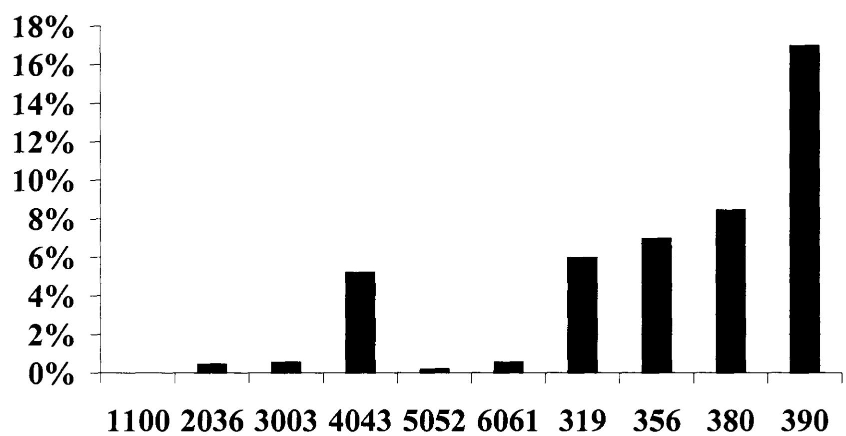


Figure 1.4 Silicon content of various wrought (xxxx) and cast (xxx) products. (Datta 2002)

Historically, cast products represented the bulk of aluminum alloy produced and were thus able to absorb a significant amount of post-consumer scrap materials (USGS Minerals Yearbook 2003). Independent studies indicate that the growth in demand for wrought aluminum will exceed that of cast aluminum in the near future (Cosquer 2003, Schultz 1999). Some are even as pessimistic as to quote that the market for secondary castings is not likely to grow (Gesing 2001).

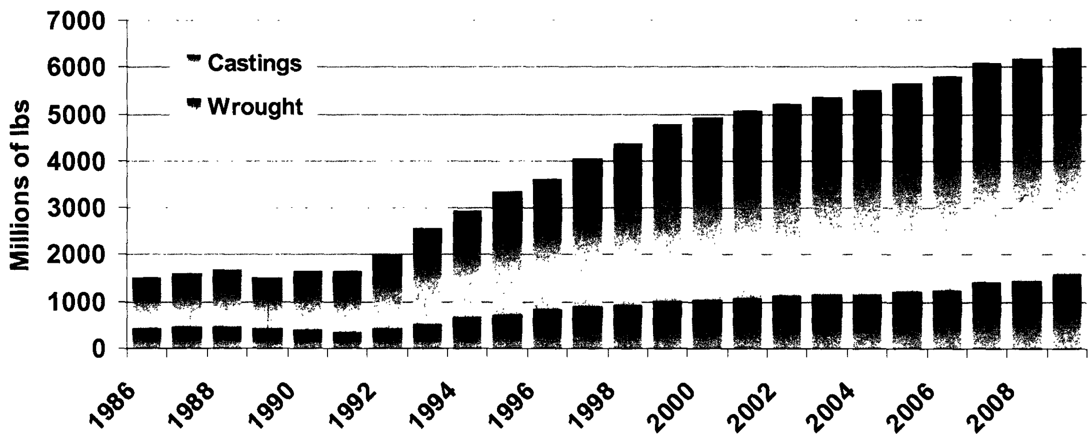


Figure 1.5 US passenger & light trucks aluminum consumption split between castings and wrought (Ducker 2002)

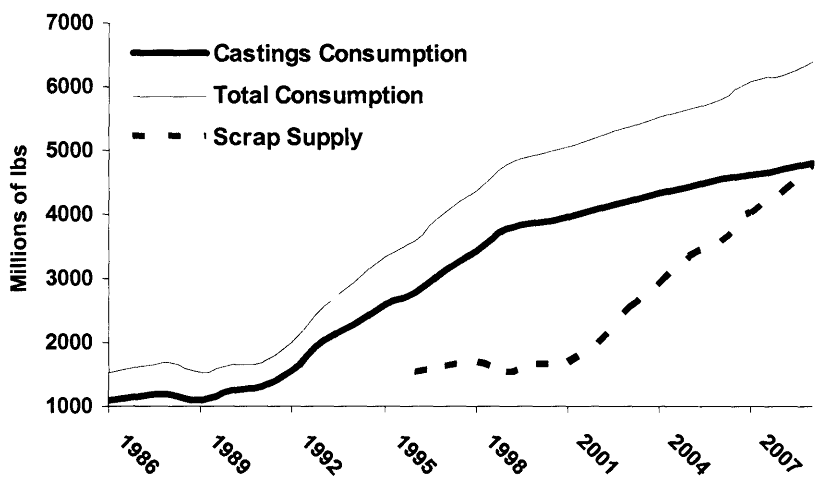


Figure 1.6 Potential issues arising from changing patterns of consumption between automotive wrought and cast content (Ducker 2002)

Figure 1.5 indicates the source of issues with regard to the changing patterns of consumption between wrought and cast content. The projected compounded annual growth rate for wrought content in vehicles is more than double that of cast material at 5.5% versus 2.3%. This is in contrast with the historical growth rate of 5.0% for wrought versus 6.7% for castings. Today, even though castings for engines and transmission still account for a significant portion of the automotive aluminum content, wrought products such as aluminum closure panels is rapidly catching up. The number of aluminum closure panels went from 2.2 million parts to 3.8 million parts from 1999 to 2002. This represents a compounded annual increase of 20%! On top of this there are smaller-scaled but more radical concepts in materials design for cars such as aluminum body in the Chrysler Plymouth Prowler or aluminum space frame on the Audi A8 and A2. With the assumption that vehicles on average has a lifespan of 10 years and taking into considerations for the above mentioned changes, Figure 1.6 shows a near-term crossing point between the automotive casting consumption and the historical scrap supply stream. This intersection and subsequent overshooting of the scrap supply over the castings consumption level means that the rest of the scraps above this castings consumption level will have to be absorbed elsewhere, presumably in the wrought content of the vehicle. However, as discussed earlier, wrought products have less tolerance for variability and amount of impurities. This is the essence of the risk involved of not being able to repurpose the mixed scrap streams. This is highly undesirable given the current regulatory trends toward more stringent recycling requirements, not to mention the negative environmental impact.

1.3 Thesis Relationship to Prior Work

This thesis is a relay on the early works of Professor Randolph Kirchain and Alex Cosquer in applying optimization techniques in the study of aluminum recycling (Cosquer 2003, Kirchain 2003). It was previously demonstrated that through materials selection and design choices, scrap consumption can be dramatically enhanced in automotive applications. Their framework also introduced modeling elements for the evaluation of single-stage, two-stream output sorting technologies. Mindful of the environmental and economic significance of aluminum sustainability, other researchers are addressing this topic from many different angles. Some are focusing mostly on factors and technologies that are influential in the automotive sector (van Schaik and Reuter 2003). Others are expending effort in specific classes of sorting technologies development (Gesing 2002). The current research is complementary towards these development and research efforts.

The need for continuing research in this area stems from several fronts. The challenges facing aluminum recycling are not limited to those mentioned in the previous section. Inefficiencies in scrap consumption due to chemistry mismatch cannot be addressed by materials selection and design choices alone. Although such strategies might be promising under certain operational constraints, they might not apply in situations where the chemical and physical properties of the products require certain materials design. While these levers for scrap usage improvement are appreciated in this thesis, other sources of inefficiencies are tackled as well. The scope of sorting technology modeling capabilities is expanded beyond two-stream output to allow for a general N-stream output. Furthermore, the mathematical framework and logic for multiple-stage sorting is presented. In previous work, while not fully implemented, it was suggested that the two-stream output framework can be readily adapted for multiple-stream output by cascading the two-stream output modeling elements downstream. However, this suggested model structure is unreflective of actual materials flow and furthermore introduces computational inefficiencies in that the variable space grows exponentially with the number of output streams. With the current proposed model structure, the growth in variable space is only linear in the number of output streams.

Prior decision tools available to aluminum producers and other participants in the aluminum production chain are often deterministic. They typically fail to capture one critical aspect of any real engineering system – uncertainty. Variability is only studied in the context of sensitivity analyses performed on operational parameters. This is an incomplete solution since it does not provide actionable decisions that directly incorporate information on operational uncertainties. In the current work, through the introduction of stochastic programming techniques (Shih 1993, Dempster 1980), various uncertainties faced by different stakeholders in this industry are modeled and accounted for in new decision framework. Specific techniques that will be explored include chance constraint programming and recourse modeling. Through critical examination of uncertainties in the aluminum production and scrap management processes, this thesis provides industry with a set of user-friendly decision-making tools that better reflect real operating environment and reasons for enhancing scrap usage.

The compilation of this thesis is ultimately a journey down the road of questions and answers towards the betterment of scrap utilization in the aluminum production environment. To what extent should scrap sources be sorted to be value added? At what costs are sorting technologies economically sensible? How does the recovery rate of a sorter affect its utilization rate and is this affected by the product mix as well? Do the considerations of operational uncertainties such as demand and compositional uncertainties enhance or reduce scrap usage? What is the economically optimal way to utilize scraps in light of uncertain operational conditions? What are the driving forces by which operational uncertainties affect scrap usage? Under limited scrap supplies conditions[4], how do considerations for operational uncertainties affect the relative attractiveness of various scrap types? Can product compositional design changes benefit scrap

[4] Limitation in scrap supplies is interpreted as the inability to source an extra unit of scrap at a price at or below what is implied by the marginal benefit of having that extra unit of scrap.

usage even in uncertain operational environments? Through critical thinking and methodologies development on questions such as these, the goal of this thesis is to contribute to aluminum sustainability.

1.4 Thesis Overview

This thesis focuses on developing tools and insights to support decision-making around the aforementioned challenges within the aluminum production and recycling industry. The aim is to characterize modeling techniques that identify cost-effective strategies to alleviate the issues through either technology adoption or modified industry behavior with regard to scrap consumption and management. The materials flow pertinent to scrap consumption in aluminum production is illustrated schematically in Figure 1.7 along with the associated problems. The details of the algorithms that deal with decision making at each step will depend on the exact nature of the problem at hand.

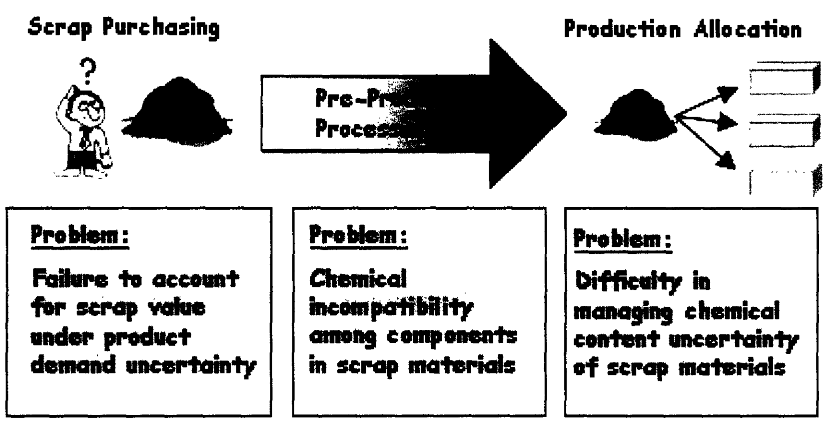


Figure 1.7 Sources of inefficiencies in aluminum scrap consumption.

Chapter 2 begins with an introduction on relevant optimization techniques that are used through out this thesis as aids to derive insights into scrap consumption and management. For completeness, one other technique often employed in stochastic optimization called Monte Carlo method, will be discussed as to why it was not used. The focus will be on understanding the construction and logic behind these mathematical programming frameworks. The details of their relevance on aluminum recycling will be made explicit in later chapters. Following this, sample sorting technologies will be briefly introduced in Chapter 3 to give the reader a sense of the physical means of scrap sorting. A comprehensive discussion of the modeling framework for a two-stage N-output stream sorter is the goal of Chapter 4. In particular, the chapter will build up the model framework step by step from the point of view of a single source of mixed scrap. The benefit of doing so is not to confuse the reader with the simultaneous considerations of multiple scrap sources; the overall model of multiple scrap sources is then an array of the resulting picture. It also serves as a conceptual link between single and multi-stage sorting models. At this juncture, the groundwork has been laid to apply the knowledge towards critical examination of inefficiencies in aluminum scrap consumption. In Chapter 5 marks the beginning of a trail to tackle such inefficiencies starting with scrap purchasing. By explicit considerations of demand uncertainty through a stochastic optimization technique introduced earlier known as recourse modeling, it will be demonstrated that robust scrap usage and purchasing decisions should account for the many sources of variability in the operating environment. By doing so, scrap usage can often be enhanced. The idea of scrap option value will be introduced and intimately tied to the need for hedging strategies in light of uncertain demand. In Chapter 6, the sorting model previously established will be adapted for a single-stage three-stream output sorting study in collaboration with Hydro Aluminum. A set of cast/wrought mixed scraps will be considered together with a range of cast/wrought products. This processing technology is proven to circumvent the difficulty in handling scrap materials in production environment due to their mixed nature. The cost effectiveness and applicability of such technology is uncovered under different operating assumptions. The economics of adopting sorting technologies will be illustrated from the viewpoint of rent-for-service as well as direct investment. Then the concept of another stochastic optimization technique, namely chance constraint method, will be applied in Chapter 7 to account for compositional uncertainties in the production environment. At this final stage of materials flow prior to market, diversification among scrap material choices is presented as a key to control the risks of compositional variances. This control becomes a natural driver for scrap tolerance. At the same time, chance constraint rigorously ties the compositional diligence of scrap suppliers to their desirability as raw material sources. Chapter 8 puts together some of the modeling building blocks to address multiple sources of uncertainties and variability. The mathematics involved is discussed together with the challenges both from a modeling as well as data requirement perspective. In the conclusions chapter, the relative significance of the various sources of inefficiencies and their suggested model-driven remedies will be weighted against each other in terms of potential for scrap consumption improvement and cost savings. Finally, ideas for future work will be presented as reflections and critique of work from this thesis.

Chapter 2 : Introduction on Mathematical Programming

Optimization is also known in modern terminology as mathematical programming. In the most general terms, it is the act of minimizing or maximizing a quantifiable goal under mathematically defined constraints in reaching that goal. It is a vast subject with origins in the works of Dantzig, Lagrange, Euler and many others who followed. However, the concept of optimization was slow to gain acceptance in many practical applications until the availability of high-performance computing power. Since then, with the development of computationally tractable algorithms to solve such problems, the field gained acceptance as a scientific approach to decision making in applications as varied as engineering science and economics. Today, optimization modeling techniques are well suited for studying a wide range of designs and operations of materials recovery system (Lund 1994, Stuart 2000).

This chapter is not a comprehensive review of optimization theory. Instead, it will introduce key concepts that arise in linear and nonlinear, deterministic and stochastic mathematical programs. Basic solution techniques will also be introduced primarily to provide a general sense for how mathematical programs are solved. Readers who are interested in major techniques such as simplex algorithm and interior-point method, which also make use of ideas discussed in this chapter, can refer to many excellent texts on these subjects (Luenberger 2003, Chong 2001, Bertsekas 1999, Bertsimas 1997). The ideas introduced in this chapter are important to understand the results and discussions in the chapters to follow.

2.1 Deterministic Linear Optimization

A mathematical program attempts to identify the best values for a set of decision variables in order to guarantee an extremum of a function of those decision variables $(x_1,\ldots,x_n)$, called an objective function, subject to a set of constraints. The general form of a linear program (LP) is as follow. Minimize or maximize:

Eq 2.1 $f(x_1, x_2, ..., x_n) = c^T x, c = (c_1, c_2, ..., c_n), x = (x_1, x_2, ..., x_n)$

Subject to:

Eq 2.2 $a_{i,1}x_1 + a_{i,2}x_2 + ... + a_{i,n}x_n \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_i, i = 1,2,....,m$

or in compact vector notation, $Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b$

The *x's* are the decision variables, the *c's*, *a's* and *b's* are constants. With all constraints being equality constraints, the condition $m < n$, where m is the number of constraints and n is the number of decision variables, is a necessary condition for an optimization problem to exists. If $m = n$, the problem has a unique solution. Otherwise if $m > n$ the problem might be over-determined and not have a solution at all unless if some of the constraints are degenerate. In the special case where all the constraints are of the type of equality, the problem statement is known as a standard form, and a common technique to solve for the extremum is by the Lagrange multiplier method. In fact this method is applicable also with inequality constraints with additional considerations. It is also at the heart of many nonlinear optimization techniques. To illustrate the idea of Lagrange multipliers, the objective function must be reformulated as an augmented objective function, also known as a Lagrangian:

Eq 2.3 $L(x) = f(x_1, x_2, ..., x_n) - \lambda_1 g_1(x) - \lambda_2 g_2(x) - ... - \lambda_m g_m(x)$

where

$g_1(x) = a_{1,1}x_1 + a_{1,2}x_2 + ... + a_{1,n}x_n - b_1$

...

$g_m(x) = a_{m,1}x_1 + a_{m,2}x_2 + ... + a_{m,n}x_n - b_m$

are the individual constraints. The λ's represent the Lagrange multipliers and can have physical meanings depending on the problem. They are often known as shadow prices because they represent, at the optimum, the change in the objective function with respect to a marginal change in the right-hand side of the associated constraint, b_i. With the standard form restated with an augmented objective function, the solution technique of Lagrange multipliers proceeds as follow:

Eq 2.4 $$\nabla L(x) = \left(\frac{\partial L}{\partial x_1}, \ldots, \frac{\partial L}{\partial x_n}, \frac{\partial L}{\partial \lambda_1}, \ldots, \frac{\partial L}{\partial \lambda_m} \right) = (0, \ldots, 0, 0, \ldots, 0)$$

This provides a system of equations to solve for the optimal solution. In solving this set of equations, the Lagrange multipliers are also found for the optimum, which then can be used for sensitivity analyses. As mentioned earlier, the Lagrange multiplier method can also be used with inequality constraints. The key is to introduce *slack* variables into the constraints and to treat the slack variables as part of the variables to be determined for the optimal solution. A simple numerical example will illustrate this.

Max: $f(x, y) = x + y$

s.t.: $3x + y = 9 \rightarrow g_0(x, y) = 3x + y - 9 = 0$

as well as non-negativity constraints on the decision variables (ie, x, $y \geq 0$). To restate this problem statement in an augmented standard form:

Eq 2.5 $$L(x, y, \lambda_0, \lambda_x, \lambda_y, S_x, S_y) = f(x, y) - \lambda_0 g_0(x, y) - \lambda_x (x - S_x^2) - \lambda_y (y - S_y^2)$$

Note that slack variables S_x and S_y have been introduced to convert the two non-negativity inequality constraints on the decision variables x and y into equality constraints. The slack variables are squared to ensure that the slack effects are positive since each of the original inequality constraints are of positive-definite type. A special relationship exists between the slack variables and the associated Lagrange multiplier of each constraint. If the associated

Lagrange multiplier is non-zero, then the slack variable must be zero meaning that the constraint is binding or strict (equality holds). The inverse of this logic statement is also true. Each of the Lagrange multipliers in the Lagrangian corresponds to one equality constraint. Take derivatives with respect to the decision variables, Lagrange multipliers, and slack variables to reveal the system of equation that will determine the optimal solution.

(1) $\frac{\partial L}{\partial x} = 1 - 3\lambda_0 - \lambda_x = 0$

(2) $\frac{\partial L}{\partial y} = 1 - \lambda_0 - \lambda_y = 0$

(3) $\frac{\partial L}{\partial \lambda_0} = 3x + y - 9 = 0$

(4) $\frac{\partial L}{\partial \lambda_x} = x - S_x^2 = 0$

(5) $\frac{\partial L}{\partial \lambda_y} = y - S_y^2 = 0$

(6) $\frac{\partial L}{\partial S_x} = 2\lambda_x S_x = 0$

(7) $\frac{\partial L}{\partial S_y} = 2\lambda_y S_y = 0$

One particular difficulty with the above system of equations is in system equation (6) and (7). In particular due to the special relationship between the Lagrange multipliers and the slack variables, there are four cases of possibilities: (I) S_x=0, S_y=0, λ_y?0, λ_x?0, (II) S_x?0, S_y?0, λ_y=0, λ_x=0, (III) S_x=0, S_y?0, λ_y?0, λ_x=0, (IV) S_x?0, S_y=0, λ_y=0, λ_x?0. Inspection of the system of equations show that cases (I) and (II) are impossible. Comparison between the solutions for cases (III) and (IV) indicates that the optimal (maximum) solution is given by:

$x = 0, y = 9, \lambda_0 = 1, \lambda_x = -2, \lambda_y = 0, S_x = 0, S_y^2 = 9; f(x, y) = 9$

According to the Lagrange multiplier λ_x if the non-negativity constraint for decision variable x is instead $x \geq 1$, the overall objective value will decrease by 2.

The above discussions centered on the application of the method of Lagrange multipliers in solving deterministic optimization problems. In the following sections the underlying concepts just discussed, including those of shadow prices, still apply but the attentions are focused on techniques for stochastic optimizations. In particular, the theories and applications for major stochastic optimization techniques such as recourse and chance constraints will be developed. These concepts are fundamental to the results presented in later chapters.

Some or all of the constraints in a real world optimization problem are often stochastic. Rigorous mathematical techniques have been developed to deal with engineering systems modeling under uncertainty in the optimization constraints. These techniques, generally known as stochastic programming, have been utilized to address uncertainty in a number of environmental and resource management scenarios (Ellis 1985, ReVelle 1969), amongst other applications (Dupacova 2002, Kira 1997, Martel 1981, Growe 1995). The following sections describe the major types of stochastic programming techniques.

2.2 Chance Constrained Programming

Chance Constrained Programming (CCP) was developed by Charnes and Cooper in 1963 (Charnes 1963). Since then this technique has been applied in contexts as varied as resource planning and traffic control (Li 2002, Waller 2001, Shih 1993). Among stochastic optimizations, there is a class of problems where violations of constraints cannot be avoided completely and that compensation actions for such violations cannot be clearly formulated with associated costs[5]. Nevertheless, decisions have to be made under these circumstances without a clear view of what is likely to happen. In such cases, it makes sense to think of the optimization problem as guaranteeing some level of constraint feasibility. Central to CCP are deterministic reliability factors α with values between 0 and 1. A reliability factor specifies the probability that a certain stochastic constraint is to be met. It can be an operational observation based on history or a tolerance level. In particular, in the context of materials production, α can be interpreted as the observed likelihood that the compositions of finished goods pass the compositional specifications. It can also be related to the tolerance level of the production manager. The goal of CCP is to translate non-deterministic constraints into deterministic equivalents such that standard mathematical programming (Lingo, GAMS, etc.) can be used to model the stochastic nature of the problem. Through the introduction of a variable that relates the reliability factor to the choice of optimization decision variables, the deterministic equivalents adequately capture the uncertainty factors in the underlying problem. The idea behind CCP is best illustrated through an example. In the following example, a full optimization problem is not shown rather the emphasis will be on deriving the formulation for chance constraints deterministic equivalents. A full application will be presented in a later chapter. While the detailed derivations are slightly abstract, the final outcome of translating the stochastic constraints into deterministic constraints has a very elegant and intuitive interpretation.

Suppose a person is creating a salt bath with some maximum allowed salt content by mixing n number of salt solution each with a stochastic level of salt content. This person must satisfy, with reliability α, the constraint that the overall mixture has a lesser amount of salt than the

[5] Either the costs associated with such compensations cannot be known ahead of time, or that the enumeration of the compensations is prohibitive from a computational standpoint, or both.

maximum allowed[6]. Suppose this person has attempted this before. Then his α can simply be the average of his past success rate. The non-deterministic constraint is:

Eq 2.6 $$\Pr\{(a' = \sum_i D_i x_i) \leq D^*\} \geq \alpha$$

where
i = Index for solutions (1, 2, …., n)
D_i = Random variable of salt content in solution i
x_i = Decision variables on mass fraction of solution i in the final salt bath
D^* = Deterministic maximum amount of salt allowed in the final salt bath
a' = Actual level of salt content in the final salt bath
a = Amount of salt in the final salt bath implied by α (more explanation below)
α = Required probability by which the stochastic constraint must be met

An assumption will be made that the random variables D_i are Gaussian variables[7]. The large number of material sources involved warrants this practice. Given that D_i are Gaussian variables, the following definitions emerge:

$D = (D_i, i = 1, 2, \ldots, n)$ is the vector of random variables D_i
$x = (x_i, i = 1, 2, \ldots. n)$ is the vector of decision variables x_i
$E(D_i)$ = The expected level of salt content in solution i
V_D = Covariance matrix of D
$$\mu_a = \sum_i E(D_i)x_i$$
$$\sigma_a = \sqrt{x^T V_D x}$$

So far, the constraint remains non-deterministic and cannot be modeled based on standard mathematical programming techniques. The key to formulating the deterministic equivalent is to introduce a new variable $X(\alpha)$ that will relate the probability specified by the reliability factor α to the decision variables x_i and the stochastic constraint. The new variable is introduced as:

Eq 2.7 $$X(\alpha) = \frac{a - \mu_a}{\sigma_a}$$

[6] The premise might be that salt solution with higher salt content is less expensive, but since a full cost optimization problem is not the emphasis here, cost implications will not be discussed at this point.
[7] The Central Limit Theorem states that the resultant distribution from combinations of a large number of systems with various distributions (not necessarily Gaussian) is always Gaussian. It should be noted, however, that normality is not a requirement in the development of the chance constraints method.

where

Eq 2.8 $$F_X(X(\alpha)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X(\alpha)} e^{-t^2/2} dt = \alpha$$

One should recognize that $X(\alpha)$ is a normalized Gaussian variable[8] and $F_X(.)$ is the cumulative normal distribution. Since the variables a, μ_a and σ_a are all x_i -dependent, the explicit dependence on α in the expression $X(\alpha)$ implies a relationship between the x_i and the reliability factor α. This implicit relationship removes the Pr{.} in the stochastic constraint in order to arrive at the deterministic equivalent. To make this implicit relationship between a, μ_a, σ_a and α more explicit, a change of variable can be carried out as $t \rightarrow a'$ where:

Eq 2.9
$$t = \frac{a' - \mu_a}{\sigma_a} \rightarrow a' = \mu_a + \sigma_a t \rightarrow \frac{da'}{dt} = \sigma_a \rightarrow dt = \frac{da'}{\sigma_a}$$
$$F_X(X(\alpha)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X(\alpha)} e^{-t/2} dt = \frac{1}{\sigma_a \sqrt{2\pi}} \int_{-\infty}^{a} e^{-(a'-\mu_a)^2/2\sigma_a^2} da' = \alpha$$

which is simply a regular cumulative distribution function in the variable a' which can also be expressed mathematically as:

Eq 2.10 $$\Pr\{a' \leq a = \mu_a + X(\alpha)\sigma_a\} = \alpha$$

If this expression for α is substituted into Eq 2.6 then

Eq 2.11 $$\Pr\{(a' = \sum_i D_i x_i) \leq D^*\} \geq \Pr\{a' \leq a = \mu_a + X(\alpha)\sigma_a\}$$

In Eq 2.11, the variable a' is simultaneously present on the left-hand side of the $\leq$ sign inside both Pr{.} expressions. This allows the upper bound D^* to be directly compared to the upper bound $\mu_a + X(\alpha)\sigma_a$ according to the $\geq$ sign in between the two Pr{.} expressions. Because the

[8] A normally distributed variable with mean zero and standard deviation of one, ie, a standard normal distribution

left-hand side Pr{.} expression is $\geq$ the right-hand side Pr{.} expression, the derivation for the deterministic equivalent is complete. It can be expressed as:

Eq 2.12 $$D^* \geq \mu_a + X(\alpha)\sigma_a$$

where $X(\alpha)$ is simply the inverse of the cumulative normal distribution, $F_X^{-1}(F_x(X(\alpha)))$. It should be noted that D^* is deterministic and the right-side is dependent on x_i which are in turn dependent on the choice of a deterministic reliability factor α. It should also be noted that the original linear chance constraint has become non-linear. Eq 2.12 has a graphical interpretation that will reveal the underlying meaning of this result. In Figure 2.1, it is apparent that the final form of the chance constraint is simply a deterministic constraint with a "safety margin" provided by the term $X(\alpha)\sigma_a$. The greater the desired safety margin (ie, lower tolerance for error), the larger the chosen value for α and hence larger $X(\alpha)$.

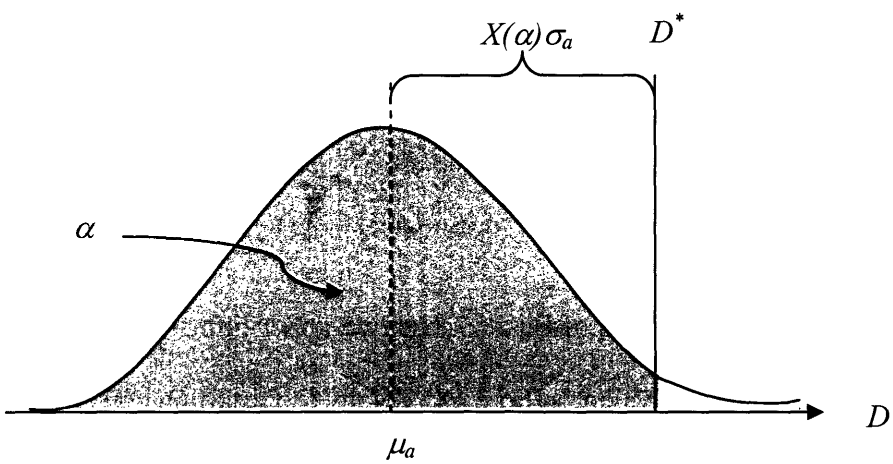


Figure 2.1 Graphical interpretation of chance constraint formulation result.

The above development has been for an upper bound constraint. However, suppose there is also a lower bound, D_L, on the level of salt required in the finished solution such that

Eq 2.13 $$\Pr\{(a' = \sum_i D_i x_i) \geq D_L\} \geq \alpha'$$

The deterministic equivalent for Eq 2.13 is not simply to change the sign for Eq 2.12 from ? to ?. However, the ideas applied above can also be applied to find the deterministic equivalent of Eq 2.13. The following statement is the resulting deterministic equivalent to the statement in Eq 2.13:

Eq 2.14 $$\Pr\{(a' = \sum_i D_i x_i) \le D_L\} \le (1-\alpha')$$

Similar to the derivations in the case of an upper bound constraint, a variable that relates α' to the decision variables x_i is introduced next. Again, $X(1\text{-}\alpha)$ is a Gaussian variable.

Eq 2.15 $$X(1-\alpha') = \frac{a-\mu_a}{\sigma_a}$$

Using Eq 2.15 and performing a transformation similar to Eq 2.9 and Eq 2.10, one can express Eq 2.15 as:

Eq 2.16 $$\Pr\{a' \le a = \mu_a + X(1-\alpha')\sigma_a\} = 1-\alpha'$$

Substituting Eq 2.16 into Eq 2.14 results in:

Eq 2.17 $$\Pr\{(a' = \sum_i D_i x_i) \le D_L\} \le \Pr\{a' \le a = \mu_a + X(1-\alpha')\sigma_a\}$$

Now since a' is present on both sides of Eq 2.17, the two upper limits can be directly compared, resulting in the deterministic equivalent in Eq 2.18. Again, the resulting constraint is non-linear.

Eq 2.18 $$D_L' \le \mu_a + X(1-\alpha')\sigma_a$$

2.2.1 A Note on Nonlinearity

To clarify the above discussions on CCP and to highlight the implications of non-linearity on computation, a specific instance of the salt solution problem will be investigated. In the following example, there are four available salt solution with individual mean levels of salt (μ_1

..μ_4) and standard deviations ($\sigma_1 .. \sigma_4$). Suppose, also for the time being, that there is only an upper bound constraint on the level of overall salt content in the final bath.

To be complete, the correlations, ρ_{ij}[9], between the salt contents of the four types of solutions are also needed in order to formulate the constraint. Following the format of Eq 2.15, the deterministic equivalent of this particular small example is:

Eq 2.19
$$D^* \geq \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 + \mu_4 x_4 + X(\alpha)\sqrt{\begin{array}{l}\sigma_1^2 x_1^2 + \sigma_2^2 x_2^2 + \sigma_3^2 x_3^2 + \sigma_4^2 x_4^2 + 2x_1 x_2 \rho_{12}\sigma_1\sigma_2 + 2x_1 x_3 \rho_{13}\sigma_1\sigma_3 \\ + 2x_1 x_4 \rho_{14}\sigma_1\sigma_4 + 2x_2 x_3 \rho_{23}\sigma_2\sigma_3 + 2x_2 x_4 \rho_{24}\sigma_2\sigma_4 + 2x_3 x_4 \rho_{34}\sigma_3\sigma_4\end{array}}$$

Noticeably, Eq 2.19 is non-linear with all of the terms within the square-root containing decision-variable products. There are special cases when the constraint statement can revert back to linearity. The cases are (1) when there is perfect correlation amongst the salt content of the four types, $\rho_{ij} = +1$ for all *i* and *j*, and (2) when ½ of the correlations are +1 and ½ are -1. While the ability to linearize the deterministic equivalent can be a tremendous time saver for computational purposes, cases where this matches with the physical reality are rare. For most optimization routines, non-linear solving capabilities are much slower in comparison to linear capabilities. These issues scale exponentially with the dimension of the problem. It will be shown in later chapters that non-linearity does not pose a significant computational issue when a small number of alloys are produced. For larger dimensions, the challenge remains for chance constraint to be an efficient decision tool for uncertainty modeling.

For completeness, in the case when all correlations are perfectly positive, Eq. 6.14 becomes:

Eq 2.20
$$D^* \geq \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 + \mu_4 x_4 + X(\alpha)[\sigma_1 x_1 + \sigma_2 x_2 + \sigma_3 x_3 + \sigma_4 x_4]$$

---

[9] Correlation $\rho_{ij} = \sigma_{ij}/\sigma_i\sigma_j$. In other words it is the covariance between the salt content of solution *i* and *j* divided by the production of the standard deviation of the salt content of solution *i* and *j*.

On the other hand when 50% of the correlations are perfectly positive and 50% perfectly negative, Eq. 6.14 becomes (with half +/-):

Eq 2.21 $$D^* \geq \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 + \mu_4 x_4 + X(\alpha)[\pm\sigma_1 x_1 \mp \sigma_2 x_2 \pm \sigma_3 x_3 \mp \sigma_4 x_4]$$

2.2.2 About α

One might wonder what will happen when $\alpha = 1$. Mathematically this means that $X(\alpha) = \infty$! However, practically speaking this is an impossible case. As in the previous salt bath example, the supply of salt content is uncertain. To demand a 100% reliability of meeting the salt content requirement is to ignore reality (ie, ignoring uncertainty)! If uncertainty is part of the problem, there are no absolute guarantees, and therefore practically speaking this reliability factor or tolerance level α must fall strictly within $0 < \alpha < 1$.

2.3 Recourse Model

With some stochastic problems, the actions to compensate for constraint violations can be specified and quantified in terms of costs. Such cases gave rise to multiple-stage decision making schemes generally known as recourse modeling. This technique inherently relies on an "action-reaction" principle. The action is a decision to be made prior to knowledge of a stochastic. The reaction is the recourse to be taken given that decision and a particular outcome in order to satisfy all the constraints. Given such information, the stochastic optimization problem can be formulated as an LP. Similar to the chance constraint programming technique, the deterministic equivalents of the stochastic constraints are derived based on knowledge about the probabilistic distribution of the chance outcomes. However, rather than seeking to meet the stochastic constraints at a certain reliability level, the recourse model seeks to minimized the expected cost of the overall problem by probabilistically weighing the consequences of deviations from the expected outcomes of the stochastic elements, ie, the cost of the first-stage decision plus the expected costs associated with recourse decisions.

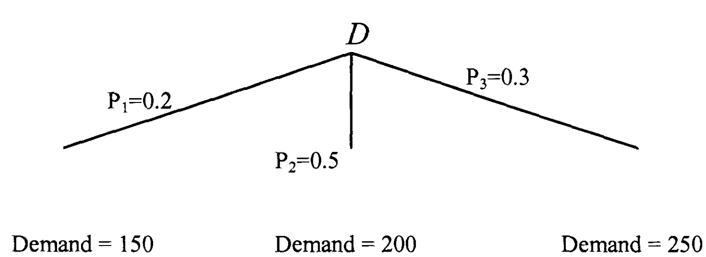
The objective of a recourse problem, which is intended to be either minimized or maximized, can be abstractly stated as follows:

Eq 2.22 $$f(C^1, D^1) + g(C^{>1}, p^{>1}, D^{>1})$$

In Eq 2.22, the contribution from the first stage to the objective function is given by the function *f*(.). D^1 is the vector of stage-one, a-priori decision variables – the attributes which characterize mathematically the state of the decision. The contribution from later stages to the objective function is given by the function *g*(.). $D^{>1}$ is the vector of later-stage recourse variables over all possible outcomes and $p^{>1}$ is the vector of the probabilities of those outcomes. The overall cost impact of the recourse decisions to the overall objective are weighted by those probabilities. The relative impacts of the various constraints are also weighted by those probabilities. This way the formulation allows for volatility in the various inputs to the model. In other words, the objective is an expected objective rather than a deterministic objective. C^1 and $C^{>1}$ are the cost vector whose aggregate contribution to the objective function is being maximized or minimized in an optimization problem.

Explicit formulation of the recourse model is a natural fit for problem statements where clear recourse actions can be specified. This methodology can be and has been applied towards a wide variety of problems including supply chain and inventory management, resource planning, financial planning and even communication networks (Cattani 2003, Petruzzi 2001, Dupacova 2002, Kira 1997, Martel 1981). Typically the first stage of a recourse model involves a decision to be made without knowing the outcome of some future stochastic event(s). However, the decision maker has an opportunity for recourse. This means that although the initial decision might not meet the stochastic constraints, the model allows for an adjustment in the subsequent stages of decision-making (recourse). This allowance for recourse adds to the overall cost of the optimization problem. In a two-stage recourse model, there is only one decision vector for stage one, while there are as many decision vectors to be determined as there are potential outcomes. In this sense, the first-stage decision accounts for the later stage costs but does not bias towards any specific outcome. This is known as the non-anticipative property of recourse model. Again, the idea is best illustrated through an example.

An application of the recourse model methodology in aluminum recycling will be provided in detail in a later chapter. To illustrate the basic concepts, consider the following simplified but classic supply chain problem. A vendor needs to decide the number of product X to manufacture in-house prior to knowing how many he can sell because manufacturing takes time. Suppose that the cost for him to make one is \$1 while the cost for him to purchase the finished product from another supplier is \$2. Suppose this vendor is adamant about meeting all his demands for the period. If he did not make enough, he can purchase the rest demanded by his customers on demand for \$2 each. This is his recourse and \$2 is his cost for the recourse. Suppose also in this simplified world that there are only three possible demand scenarios of 150, 200 or 250 product X with probabilities 0.2, 0.5 and 0.3. The overall situation can be summarized as follow:



D is the number of product X to be made that has to be decided prior to knowing the outcome of the demand for the day. R_i is the number of finished product X that the vendor will have to purchase from his friendly supplier in order to meet the demands under scenario i. As stated above, the recourse model minimizes the overall expected cost of the strategy taken. It is formulated as follow:

Minimize: $1D + 2E\{R_1 + R_2 + R_3\} \rightarrow 1D + 2(0.2)R_1 + 2(0.5)R_2 + 2(0.3)R_3$

Subject to: $D + R_1 \geq 150$

$D + R_2 \geq 200$

$D + R_3 \geq 250$

Knowledge of the probabilities of the different possible demand outcomes allow us to write the above in a deterministic form. The solution to this problem statement will tell the vendor what the optimal production plan is irrespective of the demand scenario as well as the amount to purchase from the other supplier once demand is apparent. Of course, only one of the solutions matters when the stochastic demands materialize.

The above simple example was illustrated for discrete outcomes. Theoretically the recourse model is capable of solving problems with continuous distribution of uncertain outcomes. However, even with current numerical techniques, such continuous random variable problems are generally feasible only for questions with random vectors of small dimensionality (Louveaux 1997). Discretization can help to reduce the size of the variable space, and the optimization can still capture a certain degree of reality and render useful information. With the aluminum recycling studies, for instance, discretized recourse modeling will be demonstrated in later chapters to yield useful insights.

Another method that is often mentioned in stochastic studies is the Monte Carlo Method. While not being used in this thesis, its basic concepts and relationship to recourse will be briefly mentioned and reasons for it not being used will also be discussed.

2.4 Monte Carlo Method

This method statistically simulates processes that depend on random variables. At the heart of the Monte Carlo method is the reliance on pseudo-random numbers[10] to generate a random population of states, which are tested against some model. In theory, as long as there is an available random number generator and knowledge of the probability distribution of the stochastic states, the Monte Carlo method can be applied to an optimization problem.

It has been established that using random numbers uniformly distributed between 0 and 1, one could generate random numbers of other probability distributions (Manno 1999). Suppose the uniformly distributed random number is u. Using u, another random number q can be generated with a probability density function of $f(x)$. If the random number q can have any value from $-\infty$ to $+\infty$, then the relationship between the cumulative probability density function $F(x)$ and $f(x)$ is as follow:

Eq 2.23 $$F(x) = \int_{-\infty}^{x} f(x')dx'$$

The relationship between $F(x)$ and x is exactly the relationship between u and q. In other words, the values for q can be generated by

Eq 2.24 $$q = x = F^{-1}(F(x)) = F^{-1}(u)$$

The values for q generated this way follow the probability density function $f(x)$. Relating this discussion to the aluminum recycling studies, q can be composition with some probability

[10] These are numbers that are generated with numerical algorithms and behave similarly to true random numbers

distribution *f(x)*. As hinted at in section 2.3, optimization modeling with the aid of Monte Carlo methods can capture the underlying probability distribution of the stochastic elements while providing for implicit recourse. Since the modeling framework is aimed at optimizing an objective function, when statistical fluctuation causes a particular solution to fail a constraint, the model will react by readjusting the solution. In doing so, the original solution with a total cost of C_0 will be discarded and a new solution with total cost of C_1 will be selected. The difference in strategy (e.g., using different amounts of raw material for different finished goods) between the new solution and the old solution is the implicit recourse chosen by the model, while C_1-C_0 is the cost of recourse.

However, the difficulties with practical usage Monte Carlo simulation are in its implementation and interpretation of simulation results. First of all, depending on the dimensionality of the stochastic variables space, a large number of sampling (i.e., simulation runs) would be needed to deliver a dense enough representation of the joint probability distribution of all stochastic elements. This computational demand translates into inefficiency and often impracticality for quick decisions. For instance, with just two stochastic variables and discretized outcomes of ten possibilities each, 10?10 = 100 simulation runs will be required just to cover this space. At the same time, a Monte Carlo simulation on an optimization problem provides as many solutions as there are the number of simulation runs. In practice, it might be difficult to synthesize an actionable strategy based on all the solutions provided, unlike the case for recourse model or chance constraint method. The average of all the possible solutions might not be an optimal stochastic solution after all; even worst, it might not be a feasible solution at all. For these reasons, the Monte Carlo method is not used in this study. The method is more applicable in risk analyses where the objective is to determine the range of possible outcomes given stochastic input states, and thereby ascertain the likelihood of failure.

2.5 Summary

This chapter gave a very quick overview of some of the key concepts in optimization. Ideas applicable to both linear and non-linear techniques, including Lagrange multipliers and shadow prices, were discussed. The mathematics and logic behind several popular stochastic optimization techniques were also exposed. A hint on their applications was presented using simple problems. More detailed applications will follow after a brief discussion on sorting technologies and the associated model development central to the discussion on sorting technologies.

Chapter 3 : The Significance of Demand Uncertainty

The focus of this chapter and the next is on the role that demand uncertainty should play in formulating raw material purchasing strategies. It is important to realize that demand uncertainty is just one of many uncertainties that are present along the chain of aluminum scrap flow. Other sources of uncertainties include, but are not limited to, raw materials pricing, supply, compositions, etc. Compositional uncertainties are studied in a later chapter. However, given that the primary control of the purchasing function is over the *quantity* of raw materials and not any particular property of those raw materials, uncertainty in the quantities of products demand is a natural starting point in studying the role uncertainties play in scrap consumption.

This chapter provides several thought-provoking examples on the interesting effects demand uncertainty have over scrap materials purchasing. They will prompt questions regarding the appropriateness of making scrap materials purchasing based purely on expected demand. Alternative scrap purchasing schemes will be introduced with minimal explanations on why they should work better from both an economic as well as scrap consumption standpoints. Justifications for the alternative and methodologies on generating the associated scrap purchasing strategies under uncertain demand will be provided in detail in the next chapter with the aid of a case study.

3.1 Does Demand Uncertainty Matter in Scrap Purchasing?

The following are three motivating and conceptually fundamental examples to make the case for the importance of uncertainty in scrap management decision making.

Consider an abstract demand cycle for some materials production. Suppose there are only two raw materials available, S and P. Assume that ton for ton each can produce the same amount of final product, but due to some constraints they are priced differently. S costs \$1/unit and P costs \$2/unit. However, P is always readily available while S must be procured ahead of time. The imminent decision is therefore how much S to purchase. Inventory cost is \$0.1/unit. The green line in Figure 3.1 represents the fluctuation in actual demand and the red-line represents the mean demand around which purchasing strategy for S is formed.

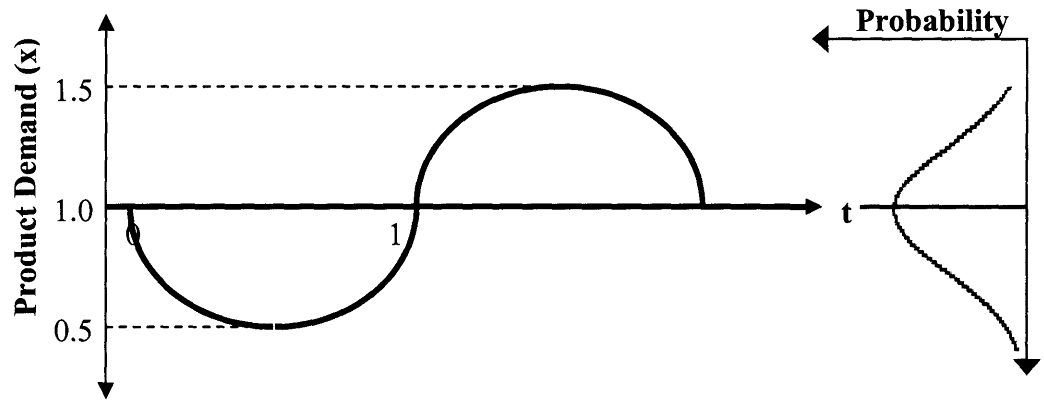


Figure 3.1 Abstract demand cycle for materials production.

Consider the costs involved in each period following the mean demand-based purchasing strategy. At time 0 there are no raw materials in inventory. The mean demand is apparently 1 unit from Figure 3.1. Behaving according to a mean-demand-based strategy will cost \$2.55. But consider a different strategy according to Table 3-I. According to this other strategy, more scraps were purchased and used, and the costs over these periods were less! The extra units of S purchased under this other strategy carried cost savings that outweigh the inventory costs, leading to less reliance on more expensive primary materials.

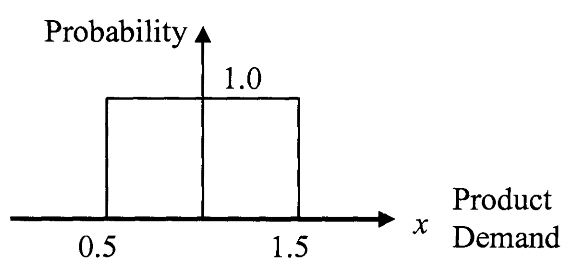
The question of course is how can this new strategy be justified and what conditions made it possible to be less costly? What was done differently and why? Is there something particular about the pricing conditions on raw materials S and P that made this new strategy better? How about the inventory costs, does that not make a difference in the overall costs between the two strategies? What about the compositional differences between raw materials? These are all valid questions and they all matter when it comes to decisions regarding scrap materials purchasing and usage. A framework is needed to incorporate these factors and more into a decision tool that can help material processors come to better scrap purchasing and usage strategies beyond the mean demand-based ones. That framework will be presented in the next chapter with a focus of demand uncertainty.

Table 3-I. Comparison between mean-based scrap purchasing strategy versus another strategy.

	Mean-Based Strategy		A Different Strategy	
	Period 0	Period 1	Period 0	Period 1
Expected Demand	1.0	1.0	1.0	1.0
Actual Demand	0.5	1.5	0.5	1.5
S in inventory (a)	0.0	0.5	0.0	0.6
S Purchased (b)	1.0	0.5	1.1	0.5
S on hand (a+b)	1.0	1.0	1.1	1.1
S Used	0.5	1.0	0.5	1.1
P Used	0.0	0.5	0.0	0.4
Cost of *S*	$1.00	$0.50	$1.10	$0.50
Cost of *P*	0.00	1.00	0.00	0.80
Inventory Cost	0.05	0.00	0.06	0.00
Costs	$1.05	$1.50	$1.16	$1.30

The second example makes the relationship between the various factors hinted at above even more explicit and formulaic.

Let product demand distribution be uniformly distributed [0.5,1.5] as follow:



Suppose again there are two raw materials *S*, *P* that cost differently, but can produce the same amount of product. Demand is denoted by x and must be strictly satisfied. *S* has a longer lead time and as such must be pre-purchased. The cost of S, P and inventory on a per unit basis are C_S, C_P and C_I respectively. The expected cost of a raw material purchasing strategy of a units of S in light of the above demand distribution is:

$$F(a) = aC_S + \int_{0.5}^{a} (C_I - C_s)(a-x)dx + \int_{a}^{1.5} C_P(x-a)dx$$
$$= aC_S + (C_I - C_S)[ax - \frac{x^2}{2}]_{0.5}^{a} + C_P[\frac{x^2}{2} - ax]_{a}^{1.5}$$
$$= \frac{1}{2}(C_I - C_S + C_P)a^2 + (C_S - \frac{C_I - C_S}{2} - \frac{3C_P}{2})a + \frac{C_I - C_S}{8} + \frac{9C_P}{8}$$

If $a > x$, inventory costs will be incurred, while allowing for salvage value of unused raw materials. Otherwise if, $a \le x$, raw material P will be acquired to fill the gap. To find the raw material purchasing strategy that will minimize this cost:

$$\frac{\partial F}{\partial a} = (C_I - C_S + C_P)a - \frac{3C_P}{2} - \frac{C_I - C_S}{2} + C_S = 0$$
$$\therefore a^* = \frac{(\frac{C_I - C_S}{2} + \frac{3C_P}{2} - C_S)}{C_I - C_S + C_P}$$

This optimal solution illustrates the trade-off between cost savings from using scrap versus primary and the carrying cost, represented here simply by the inventory cost which can also include the effects of depreciation in scrap value and time value of money. Of course, the optimal decision also critically depends on the underlying probability distribution. Clearly these parameters can be chosen to give an optimal scrap purchasing decision that is identical to a mean-based strategy (a = 1 unit). However, the point is that they need not match up depending on what the various parameters are. Based on recent prices for primary and secondary aluminum, the optimal solution[11], a^* will be greater than 1 when the carrying cost is less than \$260 per ton.

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[11] Primary price assumed to be \$1,300 per ton and scrap materials at 80% of that level.

Finally, in this last example the questions of whether the concepts introduced in the previous examples can be carried on to multiple-period planning is laid to rest. Once again demand is uniformly distributed between 0.5 and 1.5 units as in the prior example. Given this uncertain demand, various amounts of scrap material *S* are considered to be appropriate to hold on hand. The optimal amount is the amount defined as the one that will result in the lowest average production cost. Average is taken over 1,000 periods with the actual time-frame for each period left as arbitrary. The inventory cost is fixed at 10% of the scrap material cost as in the first example. Figure 3.2 illustrates the evolution of the optimal amount of *S* to be held on hand for various primaries and scrap material unit prices. It is clear that depending on the raw material prices, the optimal amount of scrap to be held on hand is not necessarily at the mean demand level.

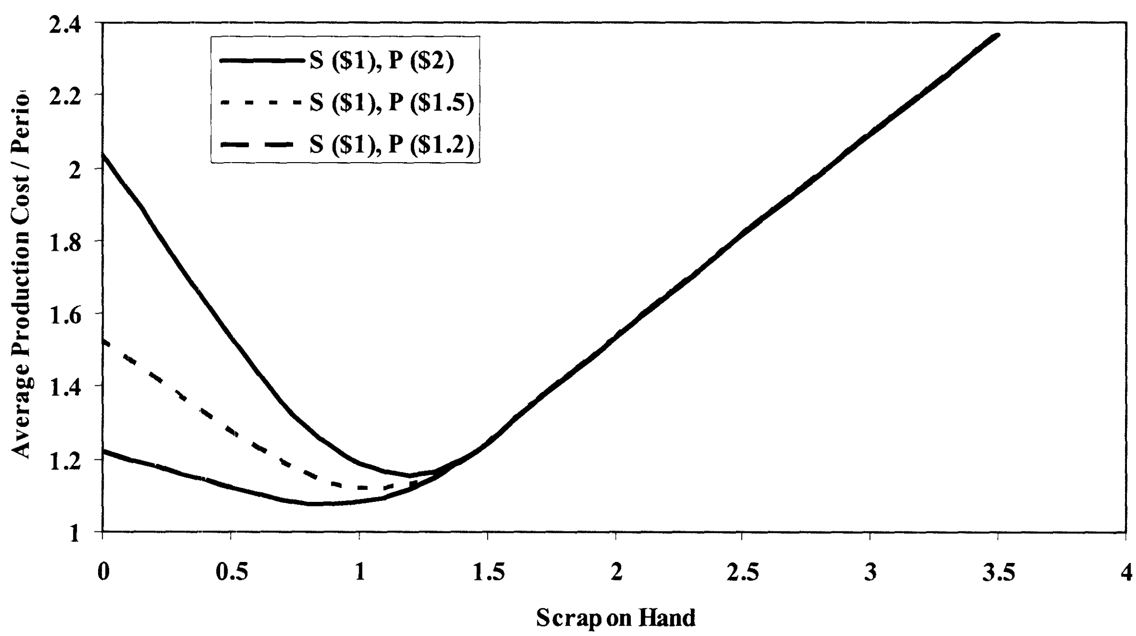

Figure 3.2 Average production cost per period versus optimal amount of scrap kept on hand.

3.2 How should demand uncertainty be addressed?

Given that these examples have shown scrap purchasing based on expected demand are prone to be sub-optimal, the need is to have some way to systematically arrive at better alternatives. That decision framework must be versatile enough to consider simultaneous production of multiple products with multiple raw materials. It must also capture the effects of raw materials pricing and inventory costs as well as the collective degree of uncertainty of the various products. This framework is the topic of the next chapter.

Chapter 4 : Managing Demand Uncertainty – A Recourse Model Framework

Once aluminum scraps are available in the market, the drive towards scrap consumption is always initiated by the purchasing of such raw materials. As such, the examination of sources of inefficiencies in aluminum scrap usage begins with raw materials purchasing. This chapter is devoted to a critical examination of this initial decision with the hope of uncovering driving forces for improvement in scrap consumption.

Traditional scrap purchasing behavior based on point forecasts on demand will be contrasted with an alternative method that takes into account the underlying demand uncertainty. The goal of this chapter is to show that the benefits of such scrap purchasing behavioral changes can be quantified and justified from both a scrap usage and economic perspectives. In particular, this benefit is directly attributed to explicit considerations for demand uncertainty in scrap management decision-making. Such uncertainties are considered within a two-stage recourse optimization framework. A brief conceptual review of the recourse framework and its relationship specifically to demand uncertainty is followed by hypothetical yet realistic case studies. Case results will demonstrate that, although intuitive, alloy production planning based solely on expected outcomes leads to more costly production on average than planning derived from more explicit treatment of uncertainty. By factoring in the penalties associated with different possible outcomes, the new scrap purchasing decisions better positioned the alloy producer to weather uncertain outcomes and promoted greater scrap purchases.[12]

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[12] Part of this chapter is based upon a paper (Li, P.P. and Kirchain, R.E., "Quantifying Economic and Scrap Usage Impacts of Operational Uncertainty Within Alloy Production Planning") presented at the TMS 2005 Annual Meeting Cast Shop Technology: Aluminum Melting: Strategies and Sourcing Symposium, San Francisco February, 2005.

4.1 Scrap Management Challenges in the Treatment of Demand Uncertainties

Materials demand is undoubtedly difficult to predict with certainty (Holland, 2001, Lee 1997). An immediate appreciation of this can be gained by examining the historical volatility in aggregate US demand for a number of metals. Figure 4.1 illustrates the annual change in apparent consumption from 1970 to 2000 (Buckingham 2002).

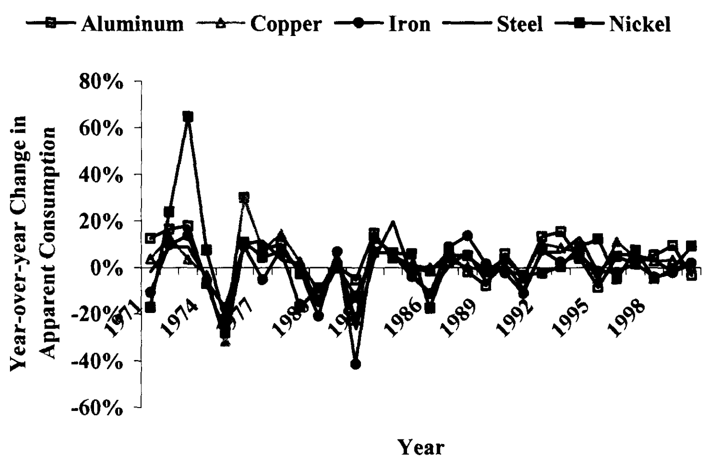

Figure 4.1. Normalized historical US apparent consumption of aluminum, copper, iron, steel and nickel

The lack of clear predictability presents a challenge for raw material purchasing whose goal is to have raw materials ready for production needs. Even when long-term prospects are promising failure to account for such variations can become a source of inefficiency in secondary material usage and creates sometimes unrecoverable cash flow problems for any operation. In fact, lessons from other resource and environmental management scenarios can be drawn in which explicit treatment of uncertainties in decision-making led to more efficient usage of resources: capital, natural and financial. For instance, applications in conservation biology have ranged from qualitative scenario planning to more quantitative techniques such as simulations, hypothesis-testings and Bayesian statistics for improvements in policy-making, population growth studies and genetic models (Peterson 2003, Ralls 2000). Monte Carlo simulations were effective in providing a range of likely optimal designs in urban water management and more specifically, flood-risk management projects selection (Al-Futaisi 1999, Geldof 1997). Others attempted to quantify the economic impact of uncertainties among considerations for other system factors. De Weck et al. applied real options analyses to improve the economic performance of communication satellite deployment by the order of 30% (de Weck 2004, de Neufville 2004). Leotard borrowed from the finance literature and assumed energy demand to follow geometric Brownian motion in devising guidelines and derivatives contracts for power generation and transmission capacity investments (Leotard 1999). Another example from the energy sector relied on chance constraints to model the uncertain characteristics of coal properties in coal blending operations in power plants, resulting in optimal tradeoffs between emissions and costs (Shih 1993).

Despite the uncertainties in aluminum production environment, definite business-critical decisions must be made on a regular basis. Modeling tools are available to help support these decisions, improving decisions not just about raw materials purchasing, but also mixing, upgrading and sorting of secondary materials (van Schaik 2003, van Schaik 2002, Cosquer 2003, Kirchain 2003). Analytical approaches may be used within such tools to embed consideration of uncertainty in the decision-making, but generally this occurs through the use of statistical analyses that are used to forecast expected outcomes. Combined with expert opinion these expected outcomes are used within inherently deterministic models. This combination of statistical analysis and modeling suffers from two fundamental limitations. First of all, implicitly assessments based on mean expected conditions assume that deviation from that value has symmetric consequences. It also assumes risk neutrality. For many production related decisions, the repercussion of missing a forecast are inherently non-symmetrical. Secondly, such models generally provide static single scenario strategies accompanied by only implicit guidance regarding how to adjust strategies when confronted with changing conditions.

This chapter introduces an analytical approach, a linear recourse-based optimization model, which accommodates a richer set of probabilistic information and thereby attempts to address these two shortcomings. Although the case which is presented examines only one relevant form of uncertainty – variable demand – the method is readily extensible to address uncertainty in raw material availability and factor prices[13]. The key ideas behind recourse modeling are reviewed in the next section.

4.2 A Brief Review of Recourse Modeling

A recourse model is an optimization model that simultaneously considers multiple stages of related decision making with the goal of satisfying both current needs as well as planning for uncertain future events (Petruzzi 2001, Cattani 2003). In a two-stage model, a set of stage-one decisions are to be made immediately based upon what is known at the present in combination with a dependent second set of recourse plans which will be implemented in the second stage depending upon how future conditions unfold. While there is only one set of optimal stage one decisions, for every possible outcome in stage two, there will be a set of recourse decisions (plans). The power of this method is that it is able to embed expectations about later events into the decisions taken at the present. In essence, a single best set of stage-one decisions are made with respect to the magnitude and likelihood of all possible outcomes in the later stage. This decision making scheme for a two-stage model on scrap management and usage is illustrated in Figure 4.2 in which references for the specific decisions to be made in each stage for a case to be described below are in brackets. The potential outcomes have been discretized for computational tractability. A single set of decisions (scrap-prepurchases) correspond to all possible outcomes

[13] Considerations of raw material compositional uncertainty require other, non-linear modeling methods.

(different product demands) at a later stage. For each potential outcome is a second stage plan (recourse: primaries and alloying element purchases). The second stage plan is the recourse action — when the products cannot be made with pre-purchased scrap materials, primaries and alloying elements will have to be purchased.

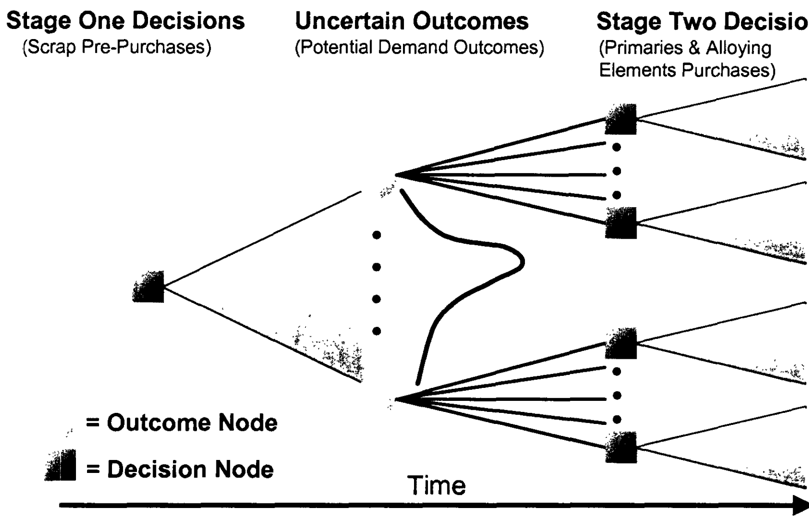

Figure 4.2. Schematic representation of a two-stage recourse model (specific decisions for case to be described below are in brackets).

4.3 Demand Uncertainty Base Case

To demonstrate the value of recourse modeling, a Base case examining demand uncertainty is considered. The case deals with the raw material sourcing decisions that confront an aluminum alloy producer who is planning for an uncertain demand one specific time period from today. To produce these finished goods, raw materials — both scrap materials and primary materials — must be acquired. For the purposes of this case, it is assumed that while primary materials can be obtained on demand as needed, scrap materials must be procured ahead of time (pre-purchased) before actual production[14]. For instance, scrap materials will have to be contracted today for delivery for later production needs. This represents a two-stage recourse decision setting whereby a decision needs to be made today to enter into a contract for scrap supplies

[14] An assumption verified with industry participants.

while primary material needs can be deferred until actual production in the future. Since primary materials are generally more expensive than scrap materials, when a suboptimal set of scrap materials were pre-purchased, the producer will have to pay the penalty of having to use more primary materials than optimally needed. The notion of optimality will be further developed below, but basically the aim is to minimize overall expected production costs. Although this construction is an oversimplification of actual purchasing practices, the model presented is readily adaptable to more accurately reflect specific sourcing constraints. In particular, both scrap and primary raw materials likely must be contracted with each specific type having typical necessary lead times.

For the purposes of this case, production is assumed to be distributed across four alloys – two cast alloys (380, 390) and two wrought alloys (6061, 3003). These alloys were chosen because of their prevalence within overall industry production and should be illustrative of results for similar alloys. In addition to a full complement of primary and alloying elements, the producer has available seven post consumer scraps from which to choose. Prices and compositions used within the model for both input materials and the finished alloy products are summarized in Table 4-I, II and III, respectively. Average prices on primaries and scrap materials as well as recent prices on alloying elements were taken from the London Metals Exchange. The particular scraps and product types chosen are based on studies by Gorban (Gorban 1994) reflecting some of the major alloys used among automotive wrought and cast products and the scrap materials which would be expected to derive from those products. Finished good compositional specifications are based on international industry specifications (Datta 2002). Scrap compositional information is also taken from Gorban. In order to ensure that results are not biased towards any particular product type, all products were modeled using the same average demand and probability distribution. Furthermore, all raw materials were initially assumed to be unlimited in availability in order to avoid the potential effects of limited raw materials supplies. The model framework presented herein can be used for cases of non-uniform demand and constrained scrap supply without modification.

Table 4-I. Prices of Raw Materials

Primary & Elements	Cost / T	Scrap Materials	Cost / T
P1020	$1,360	Brake	$1,000
Silicon	1,880	Transmission	1,000
Manganese	2,020	Media Scrap	1,000
Iron	320	Heat Exchange	1,000
Copper	266	Bumper	1,000
Zinc	980	Body Sheet	1,000
Magnesium	2,270	All Al Eng. & Trans.	1,000

Table 4-II. Compositions of Scrap Materials

Raw Material	Average Compositions (wt. %)					
	Si	Mg	Fe	Cu	Mn	Zn
Brake	1.54	1.23	0.40	0.62	0.14	0.12
Transmission	10.30	0.21	0.90	3.79	0.28	2.17
Media	4.88	0.64	0.53	1.00	0.11	1.00
Heat Exchange	2.88	0.21	0.44	0.68	0.59	0.20
Bumper	0.39	0.78	0.38	0.32	0.09	0.75
Body Sheet	0.47	1.34	0.21	0.57	0.19	0.07
All Al Eng. & Trans.	8.61	0.30	0.68	2.69	0.27	1.26

Table 4-III. Finished Goods Chemical Specifications

Finished Alloys	Average Compositions (wt. %)					
	Si	Mg	Fe	Cu	Mn	Zn
380	8.50	0.10	1.00	3.50	0.25	1.50
390	17.00	0.88	0.65	4.50	0.05	0.05
3003	0.30	0.03	0.35	0.13	1.25	0.05
6061	0.60	1.60	0.35	0.28	0.08	0.13

Figure 4.3 illustrates the probability distribution function assumed for all the finished goods demand outcomes. The mean demands for alloys 380, 390, 3003 and 6061 were all modeled at

20kT each. The coefficient of variation[15] in demand for all four finished products is roughly 10%. Although finished good demand may be more accurately represented by a continuous probability distribution function, in order to leverage the computational efficiency and power of linear optimization methods, the probably distribution must be discretized. Furthermore, it is expected that production planners in real life will not have a continuous probabilistic view of demand outcomes (Choobineh, 2004). For the purposes of the case, each finished good has five possible demand outcomes, symmetric around the mean (a symmetric discrete probability distribution function). All together they represent 625 (i.e., 5^4) demand scenarios (5 possibilities $\times$ four finished products). The model formulation can be executed with finer probability resolution, but at the expense of greater computational intensity.

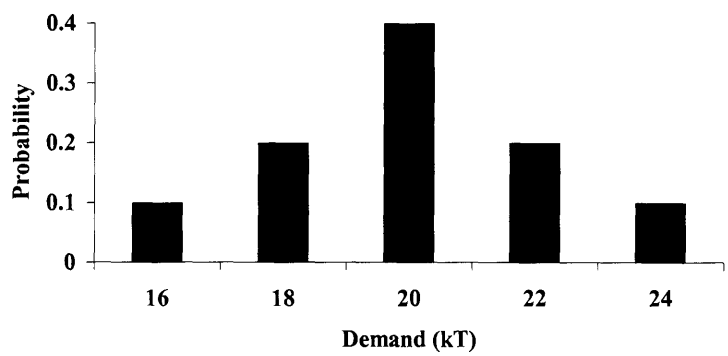

Figure 4.3. Probability distribution for all products demand under Base Case.

4.4 Scrap Purchasing Recourse Model

The recourse model for this case is formulated as a linear optimization model (Chong 2001). The mathematical definition of the model is given in Eq 4.2 to Eq 4.7. The goal of this model is to minimize the overall expected production costs of meeting various finished goods demand through an optimal choice of raw material purchases and allocations. By accounting for the probabilities and magnitude of demand variations, the model optimizes the cost of every possible

[15] Defined as σ/μ where σ is the standard deviation and μ is the mean

demand scenario weighted by the likelihood of those scenarios. The primary outcome from such a model will define both a scrap pre-purchasing strategy as well as a set of production plans (including primary and alloying element purchasing schedules) for each demand scenario. Effectively, this provides an initial strategy and a dynamic plan for all known events. The variables to solve for are $D^1{}_s$, $D^1{}_{sfz}$ and $D^2{}_{pfz}$ which will be defined subsequently together with other notations.

Minimize:

Eq 4.1 $$\sum_{s} C_s D_s^1 + \sum_{p,f,z} C_p P_z D_{pfz}^2 - \sum_{s,z} (0.95) C_s P_z R_{sz}$$

subject to

Eq 4.2 $$D_s^1 \le A_s$$

The amount of residual scrap for each scenario is calculated as:

Eq 4.3 $$R_{sz} = D_s^1 - \sum_{f} D_{sfz}^1$$

For each demand scenario z there are scrap supplies constraints as determined by the amount of scrap pre-purchased,

Eq 4.4 $$\sum_{f} D_{sfz}^1 \le D_s^1$$

Eq 4.4 enforces the aforementioned condition that scrap materials must be ordered before final production. As such, at production time, no more scrap can be used than was ordered. Similarly, a production constraint exists for each scenario, quantifying how much of what alloy must be produced:

Eq 4.5 $$\sum_s D^1_{sfz} + \sum_p D^2_{pfz} = B_{fz} \geq M_{fz}$$

For each alloying element *c,* the composition of each alloy produced must meet production specifications (Datta 2002):

Eq 4.6 $$\sum_s D^1_{sfz} U_{sc} + \sum_p D^2_{pfz} U_{pc} \leq B_{fz} U_{fc}$$

Eq 4.7 $$\sum_s D^1_{sfz} L_{sc} + \sum_p D^2_{pfz} L_{pc} \geq B_{fz} L_{fc}$$

All other variables are defined below:

R_{sz} = Residual amount of scrap *s* unused in scenario *z*
C_s = unit cost ($/t) of scrap material *s*
C_p = unit cost of primary material *p*
$D^1{}_s$ = amount (kt) of pre-purchased scrap material *s*
P_z = probability of occurrence for demand scenario *z*
$D^2{}_{pfz}$ = amount of primary material *p* to be acquired on demand for the production of finished good *f* under demand scenario *z*
A_s = amount of scrap material *s* available for pre-purchasing
$D^1{}_{sfz}$ = amount of scrap material *s* used in making finished good *f* under demand scenario *z*
B_{fz} = amount of finished good *f* produced under demand scenario *z*
M_{fz} = amount of finished good *f* demanded under demand scenario *z*
U_{sc} = max. amount (wt. %) of element *c* in scrap material *s*
L_{sc} = min. amount of element *c* in scrap material *s*
U_{pc} = max. amount of element *c* in primary material *p*
L_{pc} = min. amount of element *c* in primary material *p*
U_{fc} = max. amount of element *c* in finished good *f*
L_{fc} = min. amount of element *c* in finished good *f*

Within this problem formulation, the objective function (Eq 4.1) includes cost contributions from not only the purchase of scrap and primary materials, but also the salvage value of unused scrap materials. Unused scrap occurs for scenarios where stage-two demand was insufficient to consume all of the scrap which was pre-purchased in stage one. It is critical to note that unused scrap that was pre-purchased has embodied value. It can be resold or used for future production. In deterministic analyses, no unused scrap will ever be purchased since any unneeded scrap will simply drive up costs, making its existence irrational. In the stochastic environment, some extra scrap might be pre-purchased that will be useful on average but will lead to unused scrap in certain scenarios. To be conservative, an assumption has been made that the salvage value will be at a discount to the cost of acquiring that scrap material. The discount is assumed to be 5%. One interpretation of this discount is time value of money. Another is the cost of storage of this unused material. In future work the impact of this parameter should be quantified separately and more precisely. To be complete, it should also be noted that the salvage value is not always at a discount to the original cost of acquisition. In a rising scrap price environment or tight supply market (Gesing 2002), the rise in price can more than offset factors such as time value of money or cost of storage. The objective function also factors in the probabilistic nature of the demand outcomes. This modifies the effects of expected primary usage as well as the salvage value of unused scraps.

4.5 Basic Comparison of Mean-Based vs. Recourse-Based Scrap Purchasing Strategies

One of the goals of this study is to examine the implications on scrap purchasing with and without explicitly accounting for uncertainties in product demands. These two strategies are first defined below.

<u>Mean-Based Scrap Purchasing Strategy</u>

The mean-based strategy was formulated based on knowledge of only the mean of the finished goods demand. The results of this strategy are intended to reflect those of common industry practice, using forecasting and deterministic analytical tools to support purchasing and batch mixing decisions. The operating constraints are the same as those of the Base Case with the exception of ignoring the possible variations in finished goods demand (as described in Figure 4.3). In other words, all finished good production quantities were set at 20 kt each. The results of this strategy are shown in the column "Mean-Based Strategy" in Table 4-IV. This strategy was accommodated in the formulation presented previously by setting the probability of 20 kt demand to one, with all other demand levels at a probability of zero. Mathematically this can be concisely stated as:

Eq 4.8 $$P_z = \begin{matrix} 1, z = m \\ 0. z \neq m \end{matrix}$$

The symbol m denotes the mean demand scenario in which all product demands are 20kt. Once this stage one decision is made, the optimization problem is changed to reflect the fact that $D^1{}_s$ are no longer variables. In other words, the scrap pre-purchasing stage is done and the pre-purchased amounts are fixed.

Recourse-Based Scrap Purchasing Strategy

This alternative strategy is based upon full consideration of the probability distribution of demand outcomes using the two-stage recourse model. With the assumption of independence among product demands,

Eq 4.9 $$P_z = \prod_f P_z^f$$

The symbol P_z^f represents the probability of product f having a demand amount according to scenario z. For instance, in scenario 1, all the product demands are 16kt and each of them has a probability of occurrence of 0.1 (Figure 4.3). As a result, $P_1 = (0.1)^4$. It should be emphasized that the formulation of probability calculation in Eq 4.9 is not a defining feature of recourse modeling. In fact, the recourse strategy can be formulated with any probability distribution as long as it conforms to the norms of probability theory. Under this alternative strategy, the model establishes a stage 1 purchasing plan to best accommodate all of the 625 possible production scenarios.

Table 4-IV. Base Case: Scrap purchasing mean-based strategy (decision based only on mean demand) & recourse-based strategy (decision based on probability distribution of demand)

Scrap Material	Mean-Based Strategy (kt)	Recourse-Based Strategy (kt)	Δ (kt)	Δ %
Brake	14.0	15.4	1.4	10.0%
Transmission	18.2	18.7	0.5	2.8%
Media Scrap	-	-	-	-
Heat Exchange	4.4	6.4	2.0	45.5%
Bumper	6.4	6.9	0.5	7.8%
Body Sheet	10.9	10.9	-	-
All Al Eng. & Trans.	-	-	-	-
Total Scrap	**53.9**	**58.3**	**4.4**	**8.2%**
Exp. Costs	**$92.7M**	**$92.4M**		

In Table 4-IV, a comparison is made between the scrap purchasing decisions implied in stage one under these two strategies. Even with only 10% coefficient of variation, sizeable increases in the purchasing decisions of certain scrap types can be seen with the recourse-based strategy. In aggregate, recourse-based strategy drives scrap purchasing up by more than 8%. The difference in pre-purchasing strategy between the two strategies is essentially a hedge against adverse movements in product demands. This hedge is the difference in scrap purchasing implied between the mean-based and recourse-based strategies. Its origin will be further explained below. From here on, the extra scrap purchased in recourse-based strategy relative to mean-based will be termed the hedge basket. In the case presented, this hedging operation took the form of greater scrap purchase. In the absence of this hedge, more costly primary material and alloying elements will have to be used in certain scenarios. The expected cost savings stemming from such hedging operations can be attributed to an asymmetry between the economic benefits of having cheaper scrap to use when needed compared against the net costs involved in acquiring and storing added scrap material in those cases when it is unnecessary.

On an individual scrap basis, recourse-based strategy did not drive up the purchases uniformly. Notably, while Heat Exchanger scrap purchase increased by only 5%, Transmission scrap purchase grew by nearly triple that at almost 15%. In contrast to both, Body Sheet purchasing was unchanged in the recourse-based strategy. In order to understand why, the usage of these scrap types among the four products must be examined. In the results presented in Figure 4.4, the evolution of scrap usage are examined for five particular demand scenarios, ranging from all low (16kt for all products) to all high (24kt for all products). The scrap usages for the Base Case are represented in the solution space under recourse-based strategy and the cost effect of these solutions are weighted by their respective probabilities of occurrence in the overall objective function. Excluded from Figure 4.4 are the contributions from media and all Al engine scraps since there were none. As evident from these results, a number of scraps are predominantly used by a single product. For instance, brake scrap is used exclusively for the production of 390, while body sheet and bumper scraps are heavily drawn by 6061. In these cases, it is also clear that hedging amounts for these particular scrap types in light of stochastic demands are driven by uncertainties in these specific products and not others. For heat exchange scrap, the contributions by the various products are more mixed. In fact the usage of this scrap type among the wrought and cast products is roughly split. Therefore the hedge on heat exchange scrap is driven by uncertainties in demand of more than one product, most notably 3003 and 380. While scrap usage generally increase for a certain product as demand rises, this trend is not always monotonic when there are substitution effects, ie, same scrap being used by different products.

The versatility of a particular scrap in its application towards the production of the various finished products turns out to be a deciding factor in whether or not it is represented in the hedging basket. The fact that Body Sheet scrap was not represented can be explained as follow.

Body Sheet was primarily used in the production of 6061. However, 6061 also made use of Heat Exchange and Bumper scraps. In contrast to Body Sheet scrap, Heat Exchange and Bumper scraps are used in all four products. Under uncertain demand for all products and given that the scraps do not cost differently from each other, the system will automatically favor having more versatile scraps over less versatile ones. As a result, Body Sheet scrap was not part of the hedge.

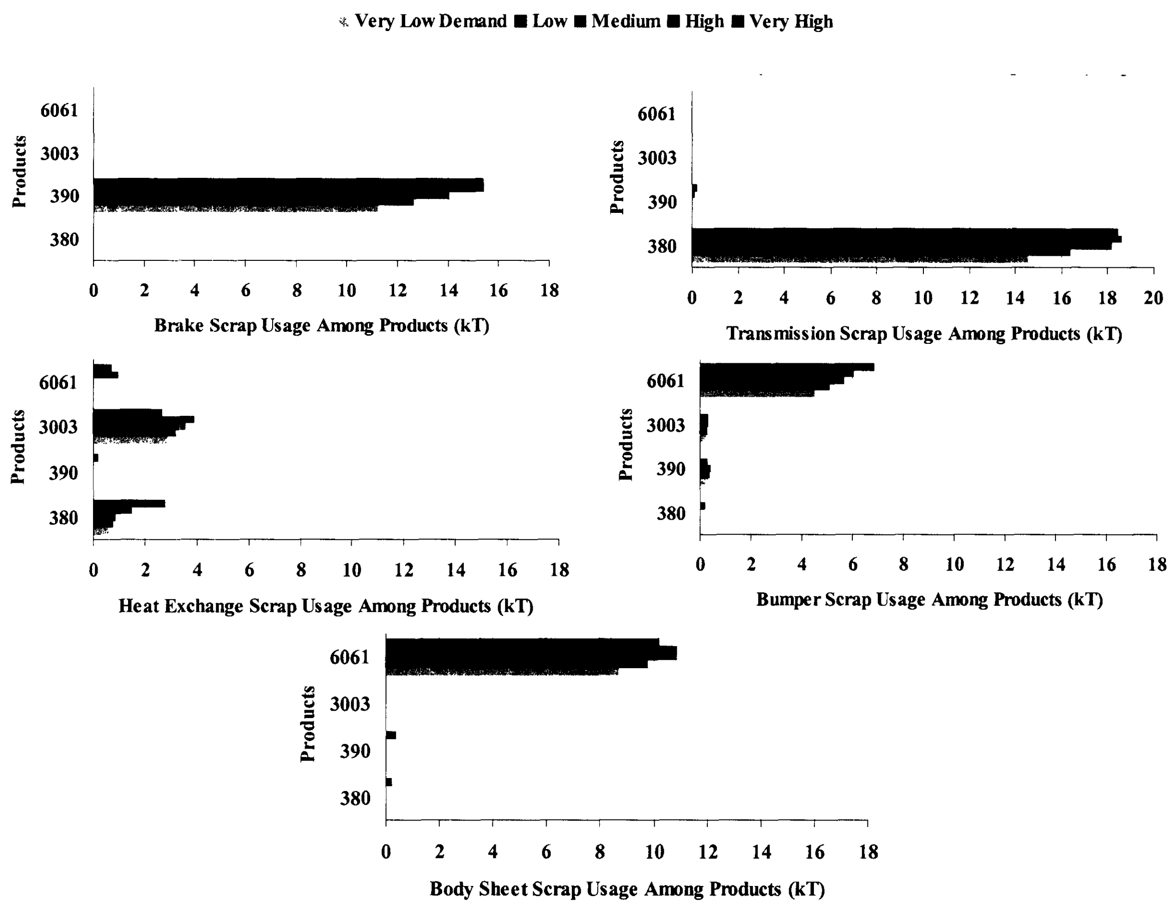

Figure 4.4. Scrap usage among products under the recourse model framework for a range of demand scenarios.

For the Base Case, the expected cost savings derived from the recourse-based strategy was \$0.3M compared with the more traditional mean-based approach. This methodology of taking into account the probabilistic nature of product demand will never on average lead to a strategy that results in a higher expected production cost. Given the same ability to make forecasts, incorporating more information into the analysis simply cannot make the analyses worst.

Associated with this cost savings in the Base Case is an increase in scrap consumption of over 1kt. This is a natural consequence of the extra scraps available from the hedging operation.

The greater scrap pre-purchases were driven by the potential for higher product demands. Therefore given greater amounts of scraps available, more scraps will be consumed in the scenarios with higher product demands. On the other hand, a certain amount of scraps were used for the lower demand scenarios under the relatively lower amount of scraps pre-purchased for the mean-based strategy. Since these scraps were strictly a subset (smaller amounts) of the scrap pre-purchased under recourse-based strategy, there is no reason why they will not be used under the production environment implicated by this alternative strategy. As a result, scrap usage has reasons to increase under recourse-based strategy for the higher demand scenarios and no reason to decrease for the lower demand scenarios. This leads to the observation that for the Base Case extra scraps from the hedge automatically led to greater expected scrap usage.

There are several deciding factors as to whether a scrap material will be represented in the hedge basket. Three types of scrap versus products behavior in the hedge are readily identifiable from Figure 4.5 to 4.9. Focusing first on Figure 4.5 to 4.8, it can be noticed that when the alloys are produced exclusively of one another, recourse-based decisions will always lead to a hedge that is proportional to the mean-based purchase. In this case, the proportionality happens to be 10%. Then, if these individual hedges were summed and compared to the aggregate hedge when a portfolio of these products was produced, three modifications to this hedge formation are observed. First, if the scrap is exclusively consumed in a product and that product only make use of that one scrap material, the aforementioned relationship of proportionality strictly holds. This was the case for the consumption of Brake scrap in alloy 380. Then, if a scrap material is consumed predominantly (but not necessarily exclusively) in an alloy that makes relatively substantial use of other scrap materials as well, the aggregate hedge will be diminished relative to the sum of the individual hedges. This was the case for Bumper, Transmission and Body Sheet scraps. Finally, the scrap materials that will enjoy the greatest percentage increase in forming the aggregate hedge are the ones that are the most versatile and evenly distributable across products. The Heat Exchange scrap is a prime example of this in the Base Case.

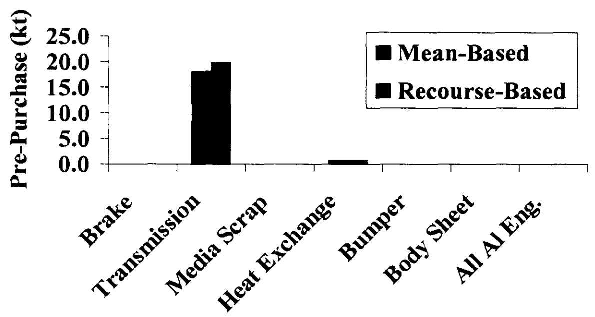

Figure 4.5 Hedge on the exclusive production of 380

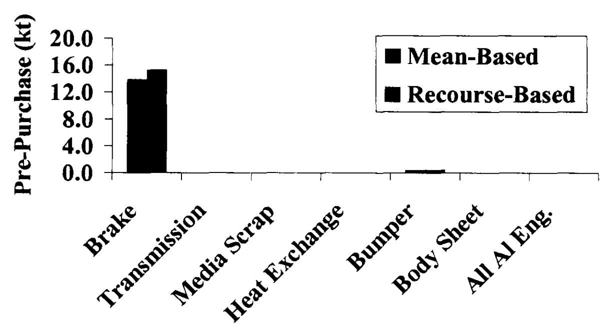

Figure 4.6. Hedge on the exclusive production of 390

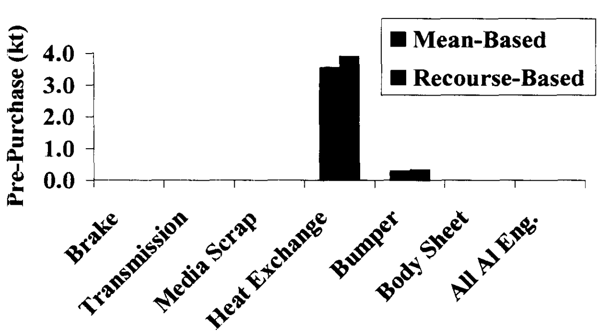

Figure 4.7. Hedge on the exclusive production of alloy 3003

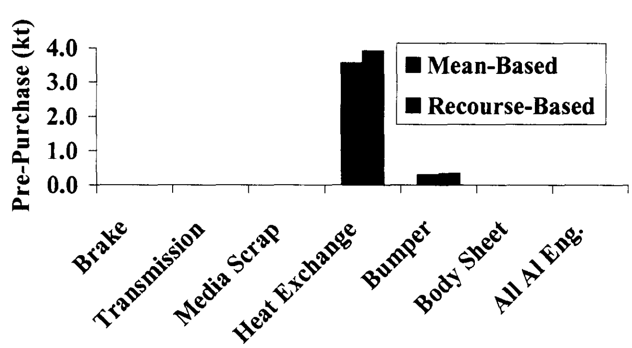

Figure 4.8. Hedge on the exclusive production 6061

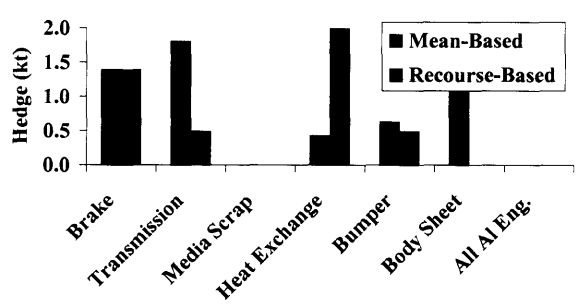

Figure 4.9. Comparison between the sum of isolated hedge in the exclusive production of individual alloys with the hedge on aggregate production of a portfolio of those products.

It was observed that in none of the cases where all alloy demands were at or below 20kt were there scrap usage difference between the two strategies. The greater scrap usages were also driven to different degrees by the four alloys. As will be confirmed by demand shadow prices studies in a later section, alloys that have greater tendencies to consume more scrap materials tend to be more effective in this respect. One way to illustrate this is by a cluster map as shown in Figure 4.10. In this figure, the horizontal axis represents the 625 demand scenarios in increasing order of scrap usage increase (recourse- vs mean-based). The horizontal bars corresponding to each alloy indicates the extent of scenarios where that particular alloy had a demand > 20kt while all other alloys carried demand ≤ 20kt. The solid bars are representative of the range spanned by these scenarios, but not every scenario within that range is such a case. Nevertheless, these ranges are all inclusive of such cases for all four alloys. As implied earlier, alloys that tend to use more scrap (ie, 380) occupy a higher range than those that do not (ie, 3003).

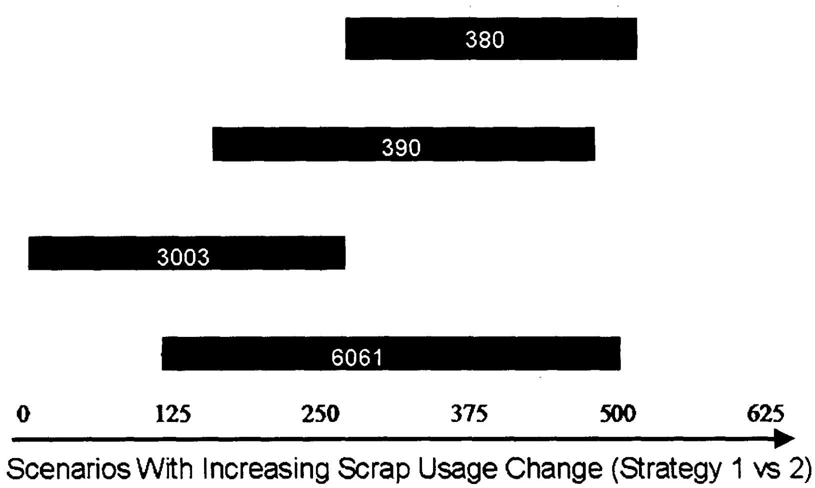

Figure 4.10 Cluster map for alloys produced (each bar represent the span of scenarios where corresponding alloy demand > 20kt holding all other alloy demands ≤ 20kt

Ultimately, the appropriateness of hedging through greater scrap purchases is sensitive towards many factors. The most obvious ones include degree of demand uncertainty, primary/secondary price spread, the ability to resell unused scrap, and storage cost. Some of these effects were already hinted at by the motivating examples in the beginning of this chapter. The following sub-sections will examine the effects these factors have in more detail on the need and form of hedging, while still assuming unlimited scrap supplies.

4.5.1 Impact of Magnitude of Demand Uncertainty on Hedging

In the Base Case, at approximately 10% demand uncertainty, the benefits derived from the recourse-based strategy was $0.3M in cost savings and slightly over 1kt increase in average scrap usage. These benefits are expected to rise with increasing product demand uncertainty. Recall that the extra scraps made available through the hedging operation was a direct result of the potential for favorable demand swings. As this swing extends into greater demand territory, given the Base Case pricing assumptions, a larger hedge basket will be required to service the extra demand. Therefore, it is clear that with greater demand uncertainty, the greater the hedging amount will be. From the prior section, it was argued that the logic follows that with the extra scraps available, the average scrap usage will increase. Therefore, as the magnitude of demand uncertainty increases, the average scrap usage benefit will increase based on the recourse strategy. With this increase in scrap usage, the expected cost savings will also rise due to the reduction in dependence on more expensive primary materials.

4.5.2 Impact of Salvage Value on Hedging

In the results presented thus far, an assumption has been made that unused scrap materials have salvage value equal to 95% of their original costs. Naturally deviations from this assumption will have an impact upon how one should approach scrap pre-purchasing. Figure 4.11 illustrates the sensitivity of the hedge on scrap salvage value. It is important to be clear that the hedge is not always positive. Ultimately in light of uncertain demand, there are two factors for wanting more or less scrap materials. One is the potential cost savings that can be derived from having cheaper scrap materials to use when needed (price differential advantage). The other is the *net* cost of carrying that scrap material (carrying cost) until it leaves inventory. The carrying cost can be defined as the acquisition price of the raw material less the salvage value of the raw material. If the salvage value of the scrap is too low, the carrying cost will more than offset the price differential advantage such that less scrap will be desirable, leading to negative hedge. The hedge was positive under the Base Case because the price differential advantage outweighs the carrying cost of those scraps. As Figure 4.11 illustrates, below approximately 65% salvage value, it no longer pays to have those extra scrap on hand. In fact, below this point, it is better to have less scrap on hand than implied by the mean-based strategy. At 65% salvage value, the cost of carrying an extra unit of scrap is perfectly balanced by the price differential advantage from having that extra unit. The 0kt line is a reference for the mean-based strategy. The difference between the two driving forces for hedging, namely cost savings from the price differential less the carrying cost will be termed the *option value* of scrap. The hedge will be positive (negative) when the option value is positive (negative).

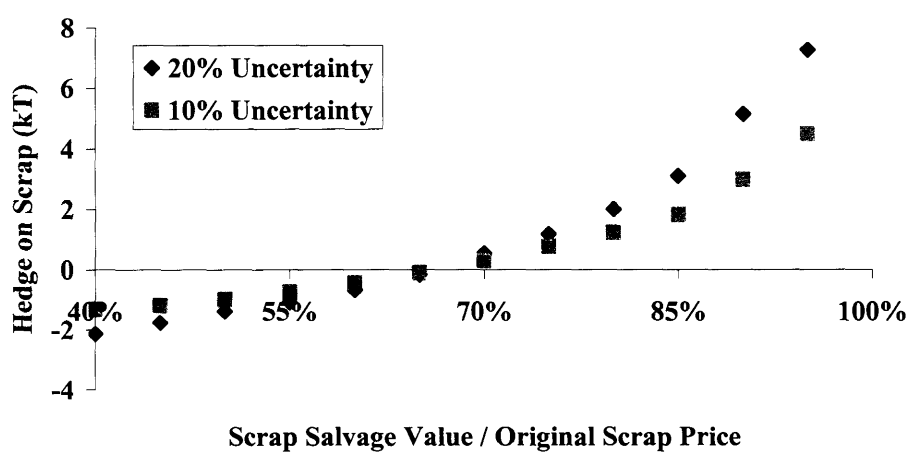

Figure 4.11. Effects of scrap salvage value on scrap pre-purchase hedging strategy.

From Figure 4.11 it is apparent that the hedge as a function of increasing salvage value is convex. This can be understood by considering the effects of the two option value driving forces separately. As the salvage value drops, there is a tendency to purchase less scrap according to recourse-based strategy because the carrying cost is increasing. But while lower salvage value implies higher carrying cost, having less scrap material also denies the material system of the price differential advantage stemming from the price difference between scraps and primaries. This price differential advantage is independent of the salvage value. These two effects oppose each other resulting in a rather slow rate of decrease in the hedging amount in low salvage value environment. On the other hand, when the salvage value is high the price differential advantage remains while the cost of carry is also reduced. This double positive in higher salvage value environment is the momentum behind the convexity observed in Figure 4.11.

The option value is also intimately tied to the magnitude of the underlying demand uncertainty. Larger uncertainties imply higher option value and results in greater driving forces for hedging. When the price differential advantage more than offsets the carrying cost, greater demand uncertainty will translate this effect into more positive hedging. Similarly, when the carrying cost dominates, greater demand uncertainty will exacerbate the situation by pushing for less scrap purchasing, ie, more negative hedging. Hence it is observed in Figure 4.11 that with greater demand uncertainty, the curve rotates inward (counter-clockwise).

4.5.3 Impact of Secondary/Primary Price Gap on Hedging

Variations in the price differential advantage affect the option value of scrap which in turn affects the degree of hedging. Figure 4.12 studies the effects of different secondary versus primary price gap on the hedge under the Base Case. At 95% price differential, it no longer pays to purchase more scrap than implied by the mean-based method. As discussed previously, the option value of scrap increases with demand uncertainty. This effect is manifested in Figure 4.12 in that with greater uncertainty, the net offsetting effects of the carrying cost and the price differential advantage is magnified, leading to a clockwise rotation of the curve. Specifically, above a price ratio of 95%, the carrying cost dominates over the price differential advantage.

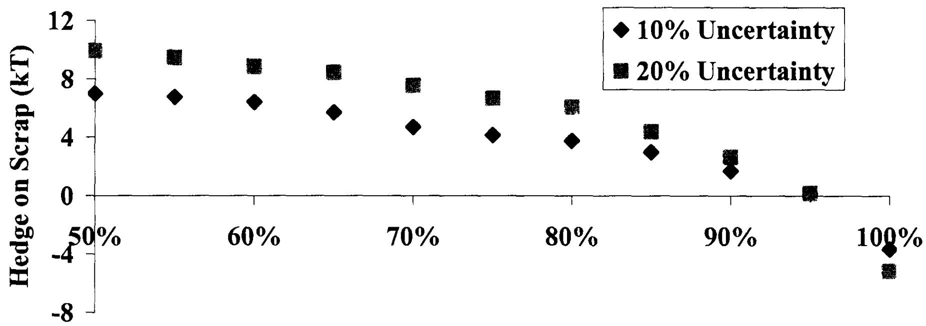

Figure 4.12. Effects of scrap-to-primaries price ratio on scrap pre-purchase hedging strategy.

As the price between scrap and primary materials converge, the price differential advantage of scrap goes to zero. Therefore the downward trend with increasing scrap-to-primaries price ratio is no surprise. The observed concavity is due to different system constraints on either end of the price ratio spectrum. When the price ratio is close to one, there is no barrier against the drop in the hedge amount except of course the overall scrap purchase cannot go below zero. As long as this point is not reached, the hedge will continue to dive. When the price ratio is low, the price differential advantage is large. However, even if the ratio goes to zero (scrap is free), the increase in the hedging amount will not accelerate. The mismatch in the compositions between scrap materials and the products sets a scrap consumption limit. Only so much scrap can be used by the production before which it makes economic sense to use primaries instead.

The discerning reader might also notice some choppiness in the response of the hedge on the price ratio. While the overall trend is down with higher price ratio, there seems to be inflection points and regions that tend to flatten out before dipping again. Interestingly, such effects were not apparent in Figure 4.11. Flatness in the hedging response to changes in price ratio is a sign of insensitivity. The relative insensitivity versus that of the hedging amount towards the salvage value is apparent from the formulation of the objective function. In Figure 4.11, as the salvage ratio varies only the carrying cost of scrap is changing; the price differential advantage is constant. Therefore the sensitivity of the hedge towards the salvage ratio is entirely driven by the change in the carry cost. However, in Figure 4.12 as the price ratio varies *both* the carrying cost and the price differential advantage are changing. Nevertheless, the carrying cost is changing very slowly. When the price differential between scrap and primaries rises by 5%, the carrying cost only goes up by 5% × (1 - 95%) = 0.15%. The choppiness in Figure 4.12 is attributable to the slowly varying carrying cost effect which leads to the "step-like" features, while the overall trend is due to the changing price differential advantage.

The convexity and concavity observed in Figure 4.11 and Figure 4.12 gives the planner a sense of how frequently the hedge should be adjusted by buying and selling scraps. The absolute distance between these curves and the zero hedge reference line can be taken as a measure of potential for cost savings. For instance, when the salvage ratio is low, a small change in the ratio does not change this potential significantly. However, in high salvage ratio regions, the hedge is much more sensitive and as such should be monitored and adjusted more frequently. Similarly when the price ratio between scraps and primaries is large, the hedge should be adjusted more frequently than when the price ratio is low.

4.6 Limitations in Scrap Supplies and Shadow Prices

So far the discussions have been with unlimited scrap supplies. While that served to focus the attention on specifically the implications of using recourse- versus mean-based scrap purchasing strategy, the constraint on scrap supplies must be considered. At the minimum, it should be demonstrated that hedging is important and beneficial as well under limited scrap supplies. In doing so, additional insights will be drawn from studies of the various shadow prices.

4.6.1 Impact of Scrap Supplies Limit on Hedging and Scrap Supplies Shadow Prices

Under the recourse-based method, the scrap purchasing strategy can be decomposed into two parts. One part is a response to the mean product demand. This is given by the mean-based scrap purchasing solution. The other is a response to the potential deviations from this expectation given by the shape of the probabilistic distribution. This response corresponds to the hedge basket — an optimal mix of extra scraps to have in response to a given profile of demand uncertainty. With unlimited scrap supplies, as the degree of uncertainty increases without changing the skewness, more of this basket will be needed but the mix proportions will not change significantly even as the kurtosis evolves[16]. This means that when a particular scrap type was not needed in low volatility environment, under the assumptions of unlimited scrap supplies that scrap will not be a significant part of the hedging basket regardless of how much volatility changes. This was the case for Body Sheet scrap in the Base Case.

In the event of limited scrap supplies, the hedging strategy will change with volatility increase. Not only will the amount of scrap purchased for hedging increase, the mix of scrap that forms the hedge will also change as shown in Figure 4.14. For instance, under unlimited scrap supplies, Body Sheet scraps never became part of the hedge basket. However, with scrap supplies capped at 15kt, Body Sheet scrap became a meaningful component of the hedge. The hedging basket was altered because the limitation in scrap supplies fundamentally changed the optimization

[16] Towards the far right-hand side of Figure 4.13 the hedge basket changes slightly due to changes in demand kurtosis, defined broadly as the flatness of the probability distribution.

solution space of this material system. More intuitively, as some scraps such as Transmissions were exhausted, other scraps will take its place in the hedge basket.

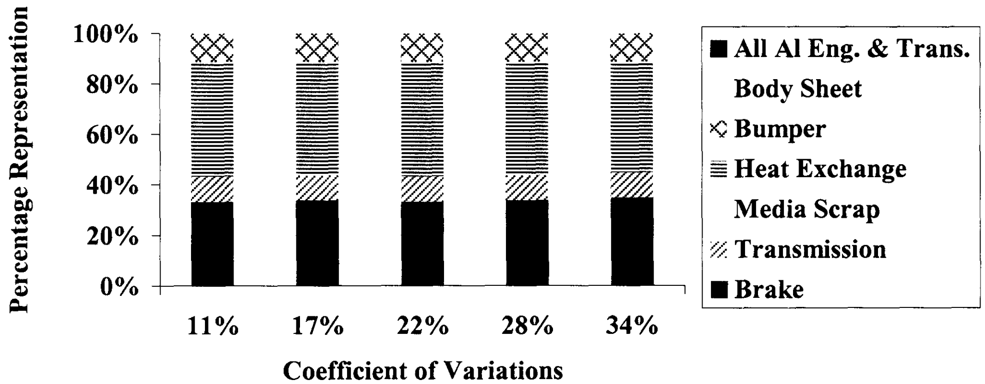

Figure 4.13. Hedge basket scraps content with unlimited scrap supplies.

Of course one clear implication of limitation in scrap supplies is that one cannot simply ramp up an existing strategy for low volatility into a high volatility environment. Both the amount as well as the mix of scraps will have to be altered. Most importantly, however, the need for hedging remains even with limitations in scrap supplies, although the hedge amount dropped slight from 4.5kt to 4.3kt (at 10% uncertainty) and with that the corresponding expected cost savings. The slight reduction in the hedging potential is caused by limitations in the scrap supplies which reduces the degree of freedom in which scraps can be used with each other and with primaries.

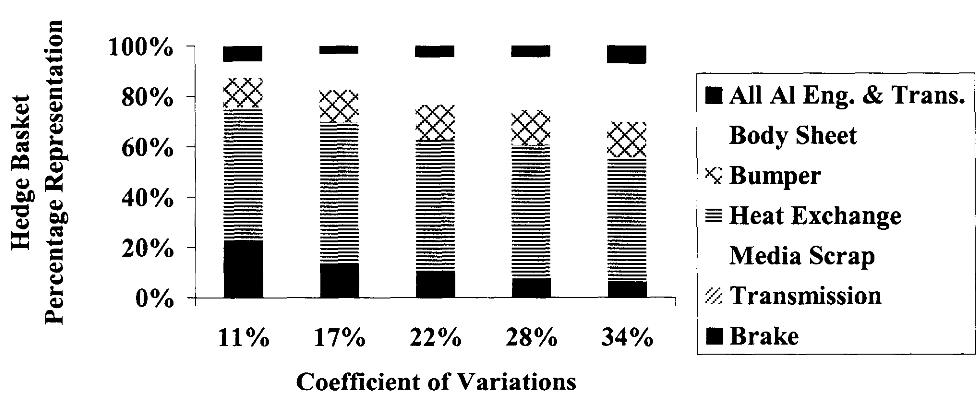

Figure 4.14. Hedge basket scraps content with limited scrap supplies (15kt each).

In the event of limited scrap supplies, accounting for demand uncertainty versus not accounting for it can also have dramatic effects on the marginal value of scraps. This translates into the extra dollar amount above the current price of the scrap the planner should be willing to pay for an extra unit of that scrap material. Figure 4.15 shows the evolution of the shadow prices of various scraps as the underlying demand uncertainty increases. Not shown are Media, Bumper or All Al Engine scraps since their supplies are not strictly limiting (their purchases have not reached the availability limit).

Under the Base Case, as demand uncertainty becomes greater, positive deviations from the mean demand warrant more scrap materials. As a result, the benefits (shadow prices) of having more scraps increase. This increase manifests itself in the generally upward sloping behavior of the shadow prices with demand uncertainty in Figure 4.15. As the size of the hedge grows additional scrap types are fully consumed, triggering a shadow price value (cf. Body Sheet at 34% coefficient of variation). As each scrap reaches its availability limit, the ways by which an arbitrary compositional specification can be attained is further limited. The model must then identify another combination of raw materials to meet specification at lowest costs. This phenomenon is demonstrated roughly in the ranking changes in Figure 4.15. Each major change in ranking occurs as the solution space becomes more constrained and the model identifies another set of raw materials from which to compose the products. In particular, one observes three regions of distinct ranks among the shadow prices on availability in Figure 4.15. The first transition occurred at around 30% demand uncertainty at which point Heat Exchange scraps became a limited supply in addition to Brake and Transmission scraps which were already limited prior to this point. Similarly, around 40% demand uncertainty, Body Sheet became an additional limited scrap. These break points correspond to the transitions in the ranks among these scrap supply shadow prices on availability.

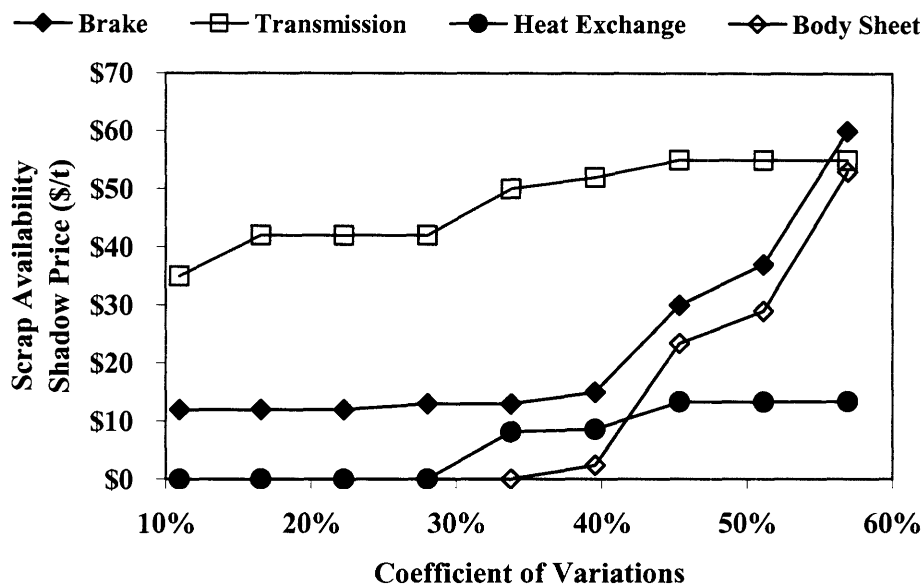

Figure 4.15. Shadow prices of scrap availability as a function of demand uncertainty.

4.6.2 Interpreting Demand Shadow Prices

Based on studies on demand shadow prices, prior research has suggested alloy substitution as a way to increase scrap consumption (Cosquer and Kirchain 2003), albeit only for deterministic demand. By definition, the demand shadow price represents how much it will cost to produce an extra unit of product[17]. However, in light of uncertain demand the interpretation is slightly trickier since a shadow price exists for each product and for each potential demand scenario. Of course, the shadow price for each product in each scenario is still the change in the objective function with respect to a unit change in the demand of that specific product in that scenario. Since the cost effects of any particular scenario on the overall objective function is weighted according its probability of occurrence, the shadow price for a product demand in a scenario is a

[17] Demand shadow prices are negative since an extra unit of product demand incurs costs associated with its production.

probability-weighted impact on the objective function of a unit change in that product's demand. Therefore in analogy to a resultant vector from a vector sum, if these product-specific and probability-weighted shadow prices for all the demand scenarios are summed together, the final sum is the shadow price on an expected unit change in demand for that particular product. Another possible interpretation for this sum is a composite shadow price SP':

Eq 4.10 $$\sum_z P_z(SP') = \sum_z (SP_z) \rightarrow SP' = \sum_z (SP_z)$$

The last expression in Eq 4.10 follows because the sum of the probabilities for all scenarios equal to one. Products that tend to consume more scrap under deterministic demand exhibits lower demand shadow prices. With limitation in scrap supplies, the demand shadow prices are always higher than under unlimited scrap supply situations due to reduction in degrees of freedom in scrap usage. An expected unit change in product demand does not alter the size of potential demand uncertainty. Instead, it serves only as a lateral shift in the whole demand distribution. When the system has unlimited scrap supplies, an extra unit of average demand can be made from the same set of scraps and primaries. Therefore, demand shadow prices are largely invariant with demand uncertainty when scrap supplies are unlimited. However, even under limited scrap supplies, the shadow prices on demand do not vary dramatically. The shadow prices do trend up with greater demand uncertainty, but the order of change is not significant enough to alter the ranking amongst the products. Therefore any cost savings and scrap consumption improvement schemes (Cosquer and Kirchain 2003) by substituting one alloy for another in these product applications in a low demand volatility environment will also be relevant in a higher demand volatility environment.

Care should be given to note that these alloys are all significantly different from each other compositionally. It is conceivable that with alloys that are within the same family, for instance the 6000 series, their compositions will be relatively closer to each other and will carry much closer demand shadow prices. After all, the amount it costs to produce an alloy is largely driven by its chemical content. In such cases, the change in demand shadow prices with greater demand uncertainty may lead to actual changes in the ranking. If that is indeed the case, the relevance of strategies involving substituting one alloy for another in a product application can depend on the underlying demand uncertainty.

4.6.3 Products Compositional Shadow Prices

The chemical composition of a product is designed to carefully reflect the desired properties of the alloy. Yet these fixed compositional ranges demanded by the products are often the source of mismatch that results in less efficient scrap usage. As such, in additional to other efforts to promote secondary material usage, compositional design changes should be considered. Given that the desired properties of the alloys are strict, only small degrees of compositional alterations are likely to be tolerable. This section examines the cost impact (shadow prices) of changing the compositional specification by 1% (ie, if the original specification was a maximum of 2 wt%, the new specification would be 2.02 wt%). The compositional shadow prices derived directly from the model has both the effects of production amount as well as composition. In order to isolate the effect of compositional change on the overall objective function, the model reported shadow price is normalized[18] in the following manner as shown in Eq 4.11 for an upper compositional limit shadow price. Notice that the effect of probability weighting is already reflected in the reported shadow prices. First by multiplying this reported shadow price by the batch amount made for that scenario, the amount of product made in that particular scenario is fixed. The compositional specification is then varied by some Δ as a percentage of the original composition.

[18] Normalization is carried out assuming that the production amount is fixed and that a 1% change in compositional specification was incurred.

Finally, the division by the probability-weighted batch produced, which is effectively the average batch produced, serves to present the adjusted compositional shadow price on a per unit weight basis.

Eq 4.11 $$SP_{fc} = \left(\frac{\sum_{z} (\text{Reported } SP) \times B_{fz}}{\sum_{z} P_z B_{fz}} \right) \times U_{fc} \times \Delta$$

As is customary to do so in industry, the cost impact will be examined on a per ton basis. The operating assumptions are the same as the Base Case. Similar to the interpretation of the demand shadow prices, there are a large number of scenarios to be considered and each of them will have an associated shadow price. Since each one of them is already a probability-weighted impact, their summation will provide a probability-weighted shadow price impact of a 1% compositional change holding product demand fixed at each scenario. Alloys compositional changes that reflect cost benefits are direct indications that such changes can bring about greater secondary material usage. Since primary materials and alloying elements are generally more expensive than scrap, the cost benefits tend to come from greater scrap employment.

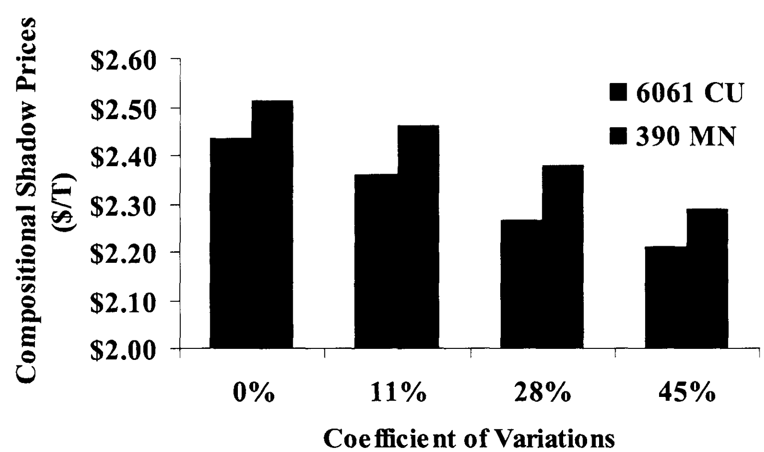

Figure 4.16. Compositional shadow price ($/t) for a 1% change in maximum chemical specification.

Compositional shadow prices are selectively shown for maximum constraints in Figure 4.16 for alloys and compositions based on ones that have cost impact of >$1/t for a Δ=1% compositional change. While this threshold seems arbitrary, it is difficult to expect any real interest from industry to effect compositional design alterations based on lesser amounts of cost benefits. This figure relates these compositional shadow prices to the degree of uncertainty around demand. The results corresponding to 0% coefficient of variation is the mean-based strategy. As a reference the maximum Cu content of 6061 and Mn content of 390 were 0.4wt% and 0.1wt% respectively. In this Base Case, it was demonstrated earlier with the recourse-based strategies that there are greater scrap purchase versus the mean-based strategies. Since shadow prices measure the benefit of relaxing a strictly limiting constraint, the more constrained a particular system, the higher the related shadow prices. In particular, for the compositional shadow price, the cost benefit is a measure of the marginal cost savings by using an extra bit of scrap material rather than having to use some higher cost alternative (more costly scraps, primaries, and/or alloying elements). Given that under the recourse-based strategies more scraps were purchased with greater coefficient of variations, there is a greater pool of cheap scrap material to choose from with increasing demand uncertainty. This directly translates into a less compositionally constrained production environment. As such, the corresponding compositional shadow prices decrease as the coefficient of variation increase. However, extrapolation on Figure 4.16 indicates that even with greater demand uncertainty, the shadow prices do not change by an order of magnitude. This means that efforts spent on altering alloy chemical specifications for the benefit of materials recycling will be meaningful even in uncertain demand environments.

4.7 Recourse Method and Demand Skewness

By now it is clear that the impact on scrap usage and costs from deviations around the expected demand is high asymmetric. This was the case even when the underlying demand uncertainty was itself symmetric. However, there is no particular reason why demand distribution must be a mirror image around the mean. In today's environment, for instance, it is more than likely that the demand distribution will be weighted towards the higher demand side. Fortunately, the discretized recourse method presented in this chapter can easily handle such skewness in the underlying uncertainty. An example of this is illustrated in Figure 4.17.

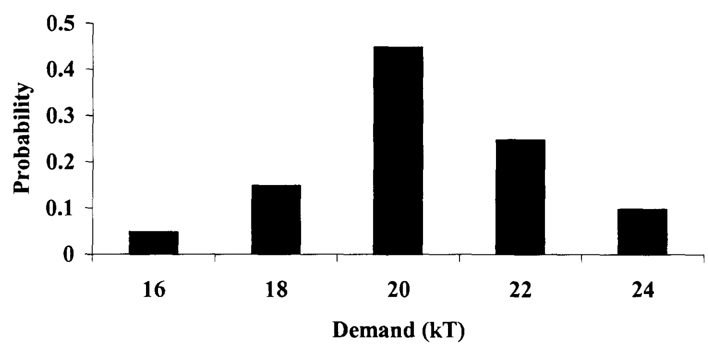

Figure 4.17. Probability distribution function for all products demand with skewness.

Table 4-V. Case with skewness in demand probability: Scrap purchasing mean-based strategy (decision based only on mean demand) and recourse-based strategy (decision based on probability distribution of demand)

Scrap Material	**Mean-Based Strategy (kt)**	**Recourse-Based Strategy (kt)**	**Δ (kt)**	**Δ %**
Brake	14.3	15.4	1.1	7.7%
Transmission	18.6	18.8	0.2	1.1%
Media Scrap	-	-	-	-
Heat Exchange	4.5	6.4	0.9	20.0%
Bumper	6.5	6.9	0.4	6.2%
Body Sheet	11.1	11.2	0.1	0.9%
All Al Eng. & Trans.	-	-	-	-
Total	**55.0**	**58.7**	**3.7**	**6.7%**

In Table 4-V, it is assumed for the mean-based strategy that the modeler assumed a deterministic demand outlook of 20.4kt (mean) for the purpose of analyses. Recourse-based strategy takes into account the full demand probability distribution with skewness. Overall for both strategies, Table 4-IV lists the total amount of scrap purchased increased relative to the Base Case (symmetric distribution). This is to be expected since on average the demand for all finished products are higher. As a result, more scrap material will be purchased to service this extra demand. However, several other interesting effects took place due specifically to skewness in demand. Without skewness, Body Sheet was never introduced as a hedging component, but it is included now that the demand distribution is skewed. These changes in the hedge are indicative of the importance skewness plays in determining the hedge basket.

4.8 Summary on Recourse Modeling of Demand Uncertainty

This chapter provided an analytical framework for materials producers to handle demand uncertainties in their planning and execution of scrap consumption in their production environment. The burden of making forecasts still rests with the planners themselves, as the goal here does not involve *how* to make predictions regarding product demand. However, armed with such an analytical tool, decision-makers can better employ their forecasting prowess in promoting greater economic and scrap usage efficiency. It should be stressed that the recourse method introduced in this chapter is not only restricted in the study of demand uncertainty, but supply-side uncertainties as well.

Case results demonstrate that alloy production planning based solely on expected outcomes leads to more costly production on average than planning derived from more explicit treatment of uncertainty. By introducing the concept of scrap option value and decomposing its drivers into carry cost and price differential advantage, the advantage of hedging in scrap purchasing is established. Separate considerations for the two driving forces also served to explain the sensitivity of the hedge on the salvage value and price differential between scraps and primaries. It was determined that in order to adequately capture the benefits of hedging, the hedge should be adjusted more frequently as the salvage value increase and the price between primaries and scraps converge. Not all circumstances under which hedging is performed will result in greater scrap consumption, and the turning points were marked at approximately 65% salvage value and 95% price differential in the Base Case. Various insights were then drawn from shadow prices studies. Alloy substitutions were found to be fruitful even under uncertain demand in effecting greater scrap usage and cost savings. The case was made for substitution among alloys from different families, whereas arguments for substitution within families are less clear. Of course these are baring other technological constraints. The degree of demand uncertainty also had a deciding impact on the relative marginal value of scrap. It also alters the scrap mix in the hedge basket, and diminishes but not eliminates the need for hedging under limited scrap supplies. Regardless of whether scrap supplies are limited, however, the hedging strategies implicated by the recourse model provide greater expected cost savings as the magnitude of uncertainty rises. In percentage terms, the corresponding scrap purchase increase, usage increase and expected cost savings for the Base Case were approximately 8%, 3% and 0.3% respectively.

Ultimately, the aluminum remelting business faces many sources of efficiencies and uncertainties surrounding scrap consumption, which make production planning and execution a challenge. This chapter focused on the impact of demand uncertainty and potential resolutions. The next several chapters will examine other inefficiencies and present analytical methods to address them.

Chapter 5 : Sorting Model Development

Before raw materials are sent to the production floor, they can be preprocessed to enhance their usefulness. One of these preprocessing routes is via sorting. The reader can refer to the Appendix for brief discussions on what sorting technologies are in general and what specific types are being researched and developed. Generally speaking, they are techniques that promote homogeneity in the compositions of scrap streams.

Many questions ultimately surround the applicability of a sorting technology. Is it cost effective? Which scraps should be sorted? The answers are closely tied to the operating conditions such as product mix, scrap supply mix and sorting performance. Often times the interactions between these factors are complex and a decision framework is needed to guide sorting choices in light of such constraints. A sorting model for aluminum production and recycling is an optimization framework that takes into account the supply/demand, compositional and sorting technology constraints and attempts to minimize costs[19]. As sorting technologies continue to develop and mature, it is conceivable that the sorting processes will proceed in stages, first between wrought and cast fractions and then into finer levels of segregations. Therefore it is important in the future to have a theoretical framework and algorithm to model multiple-stage sorting. This chapter develops this framework in detail and lays the modeling groundwork for a case study on sorting technology in the next chapter.

[19] The framework can be readily adapted for profit maximization.

5.1 First-Stage Sorting or Not?

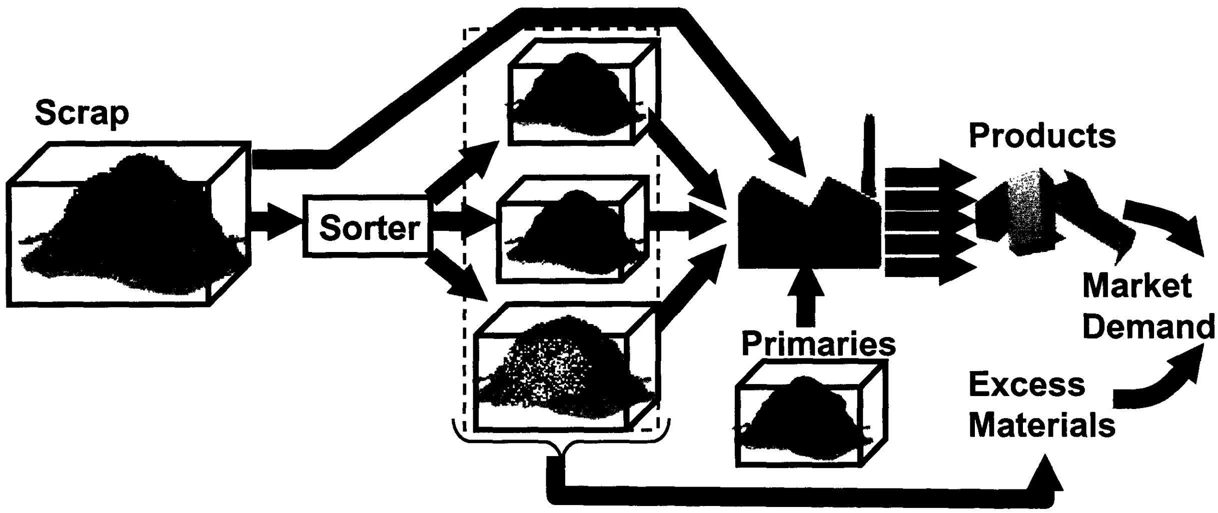

Figure 5.1 Sorting as a choice and the way it fits into the bigger picture of materials production.

Even though sorting is the main topic in this chapter, it is important to note that in the model to be developed, it is a choice rather than a necessity. As shown in Figure 5.1, mixed scrap materials do not always pass through the sorter. In fact, absent the availability of sorting technologies, on an abstract level the operation of materials production is a matching and allocation of raw materials (scraps, primaries and alloying elements) to finished products. With sorting, however, the set of available raw materials is transformed into a larger group with better compositional control.

In the following, the logic of the sorting model development will begin by tracing the fate and path taken by a single piece of scrap material. The overall model will ultimately consist of all the constraints and paths that can be taken by all scrap materials as well as primary materials. A summary of the overall model and explanations for the various subscripts and variables are given at the end of this chapter.

The first stage (which might be wrought versus cast) can be represented by the following box diagram Figure 5.2. More than two output streams are possible from the sorter to allow for generality:

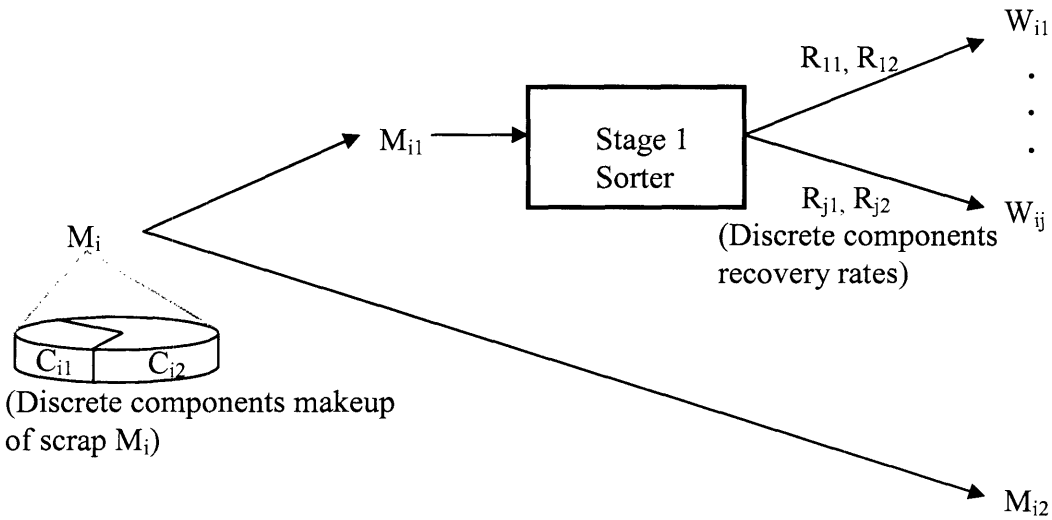

Figure 5.2 First stage sorting schematic representation.

As scrap material M_i enters the system, the first decision to be addressed by the model is whether to sort this material and if so how much of it should be sorted. By definition, primary materials are uniform and, therefore, are not sorted. As such, the discussions here are restricted to scrap materials only. From the point of view of conservation of mass, the following must hold:

$$M_{i1} + M_{i2} = M_i$$

Unsorted stream M_{i2} will then pass through to the next stage unchanged. For the passage of M_{i1} through the first stage sorter, conservation of mass dictates that:

$$W_{ij} = M_{i1} \sum_m C_{im} R_{jm}$$

$$\sum_j W_{ij} \le M_{i1}$$

For a materials production facility, not only is the quantity of material flowing through the system a concern, but also the compositions of the resulting streams. The compositions of the sorted and unsorted streams from the first stage of sorting can be determined as follows. For element (*e*) content within the unsorted stream, we have the following.

$$M_{i2}^{e,\max} = M_{i}^{e,\max}$$

$$M_{i2}^{e,\min} = M_{i}^{e,\min}$$

This is intuitive since an unsorted stream must retain its original composition. For the sorted streams W_{ij}, their compositional content can be determined as a function of the original materials discrete component makeup C_{im} of the input material M_{i1} and the recovery rate of the sorter on these various materials component R_{jm} for each output stream *j*. For these sorted streams, the compositional content relationships are:

$$W_{ij}^{e,\max} = \frac{M_{i1}\sum_{m} C_{im} R_{jm} M_{m}^{e,\max}}{M_{i1}\sum_{m} C_{im} R_{jm}} = \frac{\sum_{m} C_{im} R_{jm} M_{m}^{e,\max}}{\sum_{m} C_{im} R_{jm}}$$

$$W_{ij}^{e,\min} = \frac{M_{i1}\sum_{m} C_{im} R_{jm} M_{m}^{e,\min}}{M_{i1}\sum_{m} C_{im} R_{jm}} = \frac{\sum_{m} C_{im} R_{jm} M_{m}^{e,\min}}{\sum_{m} C_{im} R_{jm}}$$

5.2 Multiple-Stage Sorting Model Development

At the end of the first stage of sorting, single scrap material M_i has been transformed into *j*+1 potential streams. The quantity and composition of these streams are known based on the formulae above and on knowledge of the quantity and composition of the input material M_i. Armed with such information, one can proceed onto stage two sorting. Again, to allow for generality, more than two streams of output are possible from stage two. The logic and model representations for this two-stage model are readily extensible for additional stages.

As a recap for the first stage, in order to model the quantity of materials that come out of stage one as well as what their compositions are, the following information is required for the input material:

? Quantity of input material

? Discrete component makeup of input material

? Elemental composition of individual components of discrete component

? Recovery efficiency of sorter for each material component

Similarly, with sorting in stage two, these four pieces of information are required. The quantities of material are straightforward since they are the W_{ij} and the M_{i2} that were outputs from stage one. The elemental compositions of individual components do not change, since they are the same components! However, the discrete component *makeup* of input materials in stage two *will* change due to the effects of materials segregation in stage one. Fortunately, there is enough information from stage one to determine *a priori* what the makeup is for each input material into stage two. Before proceeding on to determine this makeup, a schematic of the stage two sorting procedure is helpful.

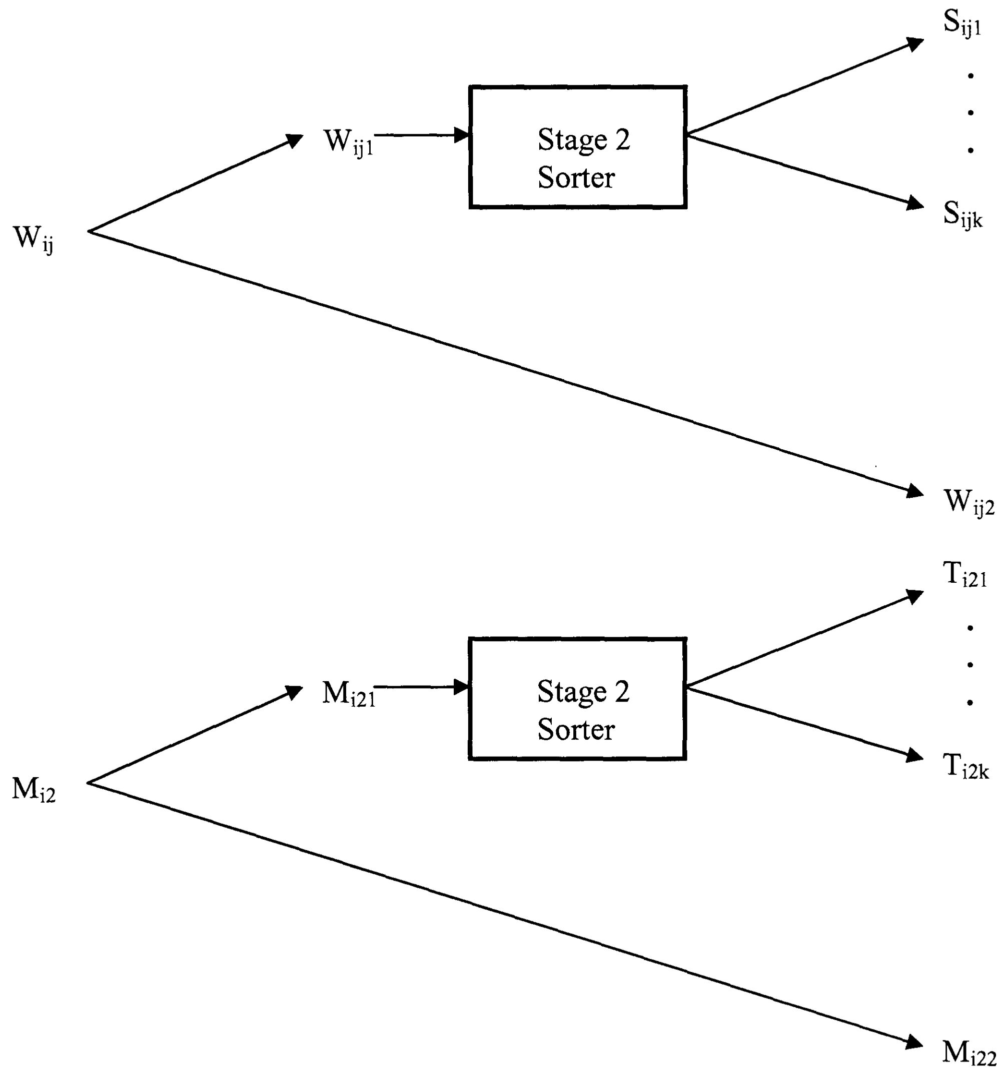

Figure 5.3 Schematic for stage two sorting

In Figure 5.3 sorting is differentiated between material that went through stage one sorting (W_{ij}) and those that did *not* go through stage one sorting (M_{i2}). Similar to the arguments in stage one, conservation of mass implies that:

$$M_{i21} + M_{i22} = M_{i2}$$

$$W_{ij1} + W_{ij2} = W_{ij}$$

$$S_{ijk} = W_{ij1} \sum_{m} C_{ijm} R_{km}$$

$$\sum_k S_{ijk} \le W_{ij1}$$

$$T_{i2k} = M_{i21} \sum_m \overline{C}_{i2m} R_{km}$$

$$\sum_k T_{i2k} \le M_{i21}$$

Amongst the four key pieces of information that are needed to determine the quantity and chemical composition of the output materials from stage two, three of them are already known. The quantity of input material are exactly the quantity of output from stage one (W_{ij}, M_{i2}). The elemental compositions of the individual component materials do not change. The recovery efficiencies of the sorter are a function of the sorter capabilities and are assumed to be given. Therefore the only information that remains to be calculated are the component makeups ($\overline{C}_{i2m}$, C_{ijm}) of the two possible types of input materials into stage two. One type has been sorted in stage one while the other has not. For $\overline{C}_{i2m}$ it is very simple and should be equal to C_{im}, since there was no sorting done on this output stream in stage one. The C_{ijm} are functions of both the recovery efficiencies of the stage one sorter (R_{jm}) and the component makeup (C_{im}) of the stage one input materials. Considering both of these effects yields a relationship of the form:

$$C_{ijm} = \frac{M_{i1} C_{im} R_{jm}}{M_{i1} \sum_m C_{im} R_{jm}} = \frac{M_{i1} C_{im} R_{jm}}{W_{ij}} = \frac{C_{im} R_{jm}}{\sum_m C_{im} R_{jm}}$$

With the component makeup of the input materials to stage two sorting determined, the chemical composition of the output streams from stage two can be determined in a similar logic to stage one output compositions. For the maximum compositional constraints:

$$S_{ijk}^{e,\max} = \frac{W_{ij1} \sum_m C_{ijm} R_{km} M_m^{e,\max}}{W_{ij1} \sum_m C_{ijm} R_{km}} = \frac{\sum_m C_{ijm} R_{km} M_m^{e,\max}}{\sum_m C_{ijm} R_{km}}$$

$$T_{i2k}^{e,\max} = \frac{M_{i21}\sum_m \overline{C}_{i2m} R_{km} M_m^{e,\max}}{M_{i21}\sum_m \overline{C}_{i2m} R_{km}} = \frac{\sum_m \overline{C}_{i2m} R_{km} M_m^{e,\max}}{\sum_m \overline{C}_{i2m} R_{km}}$$

$$W_{ij2}^{e,\max} = W_{ij}^{e,\max}$$

$$M_{i22}^{e,\max} = M_i^{e,\max}$$

Similar equations can be written for the minimum compositional constraints. At the end of stage two, the original input material in stage one has been split into $(j+1)(k+1)$ output streams. Ultimately, the terminal material streams from stage two together with available primary materials will be allocated for the production of the various alloys desired. Section 4.3 summarizes of the overall optimization objective and constraints with detailed explanations for all the notations encountered in the discussions. Figure 5.4 illustrates the overall two stage sorting model schematic for a single input material stream with allocation of terminal material streams in the final production of alloys. For clarity purposes, only two output streams are illustrated for each sorting step and not all arrows are shown. However, the reader can infer and generalize the schematic for more output streams per sorting step.

The above discussions serve to explicate the logic behind the development of the sorting model. Sorting technologies are potential strategies to resolve scrap usage inefficiencies stemming from compositional incompatibilities. However, as mentioned earlier, sorting technologies can only be widely adopted when their cost-effectiveness can be demonstrated and the impact of operating conditions understood. The sorting and mixing model summarized in the next section provides a framework to answer many of the questions surrounding the applicability of sorting technologies.

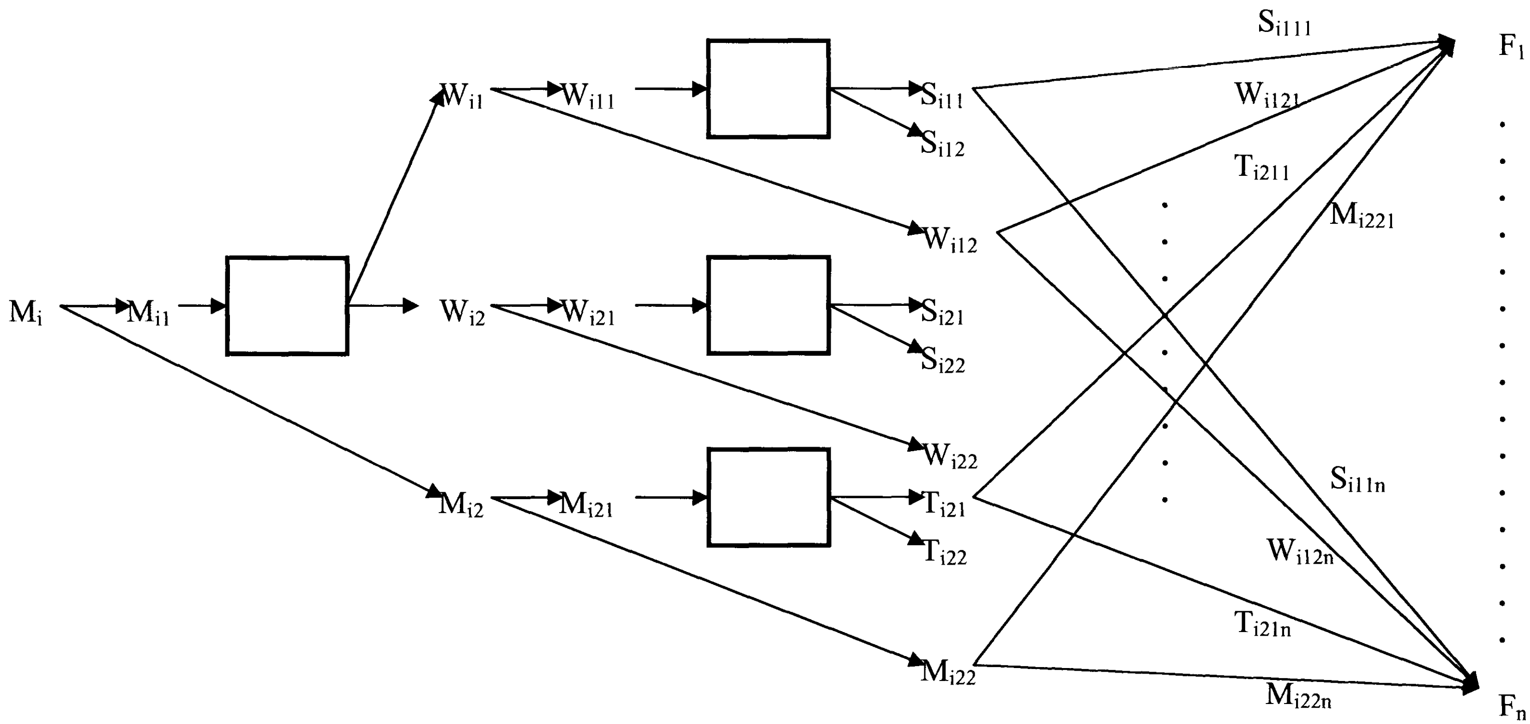

Figure 5.4 Overall schematic of two-stage sorting with two output streams per sorting step

5.3 Two-Stage Sorting Optimization Mathematical Formulation

Suppose the objective is to minimize the cost relating to alloy production.

Min: $\sum_i (C_i M_i + Z_1 M_{i1}) + \sum_p C_p M_p + Z_2 \sum_i (M_{i21} + \sum_j W_{ij1})$

Subject to the following constraints

<u>*Materials Input Stage*</u>

Materials availability constraint:

$$\sum_i M_i \leq A_i$$

$$\sum_p M_p \leq A_p$$

<u>*Stage 1 Sort*</u>

Mass flow constraints:

$$M_{i1} + M_{i2} = M_i$$

$$W_{ij} = M_{i1} \sum_m C_{im} R_{jm}$$

$$\sum_j W_{ij} \leq M_{i1}$$

Compositional determinants:

$$M_{i2}^{e,\max} = M_i^{e,\max}$$

$$M_{i2}^{e,\min} = M_i^{e,\min}$$

$$W_{ij}^{e,\max} = \frac{M_{i1} \sum_m C_{im} R_{jm} M_m^{e,\max}}{M_{i1} \sum_m C_{im} R_{jm}} = \frac{\sum_m C_{im} R_{jm} M_m^{e,\max}}{\sum_m C_{im} R_{jm}}$$

$$W_{ij}^{e,\min} = \frac{M_{i1} \sum_m C_{im} R_{jm} M_m^{e,\min}}{M_{i1} \sum_m C_{im} R_{jm}} = \frac{\sum_m C_{im} R_{jm} M_m^{e,\min}}{\sum_m C_{im} R_{jm}}$$

Stage 2 Sort

Mass flow constraints:

$$M_{i21} + M_{i22} = M_{i2}$$

$$W_{ij1} + W_{ij2} = W_{ij}$$

$$S_{ijk} = W_{ij1} \sum_m C_{ijm} R_{km}$$

$$\sum_k S_{ijk} \leq W_{ij1}$$

$$T_{i2k} = M_{i21} \sum_m \overline{C}_{i2m} R_{km}$$

$$\sum_k T_{i2k} \leq M_{i21}$$

Compositional determinants:

$$\overline{C}_{i2m} = C_{im}$$

$$C_{ijm} = \frac{M_{i1} C_{im} R_{jm}}{M_{i1} \sum_m C_{im} R_{jm}} = \frac{M_{i1} C_{im} R_{jm}}{W_{ij}} = \frac{C_{im} R_{jm}}{\sum_m C_{im} R_{jm}}$$

$$S_{ijk}^{e,\max} = \frac{W_{ij1} \sum_m C_{ijm} R_{km} M_m^{e,\max}}{W_{ij1} \sum_m C_{ijm} R_{km}} = \frac{\sum_m C_{ijm} R_{km} M_m^{e,\max}}{\sum_m C_{ijm} R_{km}}$$

$$T_{i2k}^{e,\max} = \frac{M_{i21} \sum_m \overline{C}_{i2m} R_{km} M_m^{e,\max}}{M_{i21} \sum_m \overline{C}_{i2m} R_{km}} = \frac{\sum_m \overline{C}_{i2m} R_{km} M_m^{e,\max}}{\sum_m \overline{C}_{i2m} R_{km}}$$

$$W_{ij2}^{e,\max} = W_{ij}^{e,\max}$$

$$M_{i22}^{e,\max} = M_i^{e,\max}$$

$$S_{ijk}^{e,\min} = \frac{W_{ij1}\sum_m C_{ijm} R_{km} M_m^{e,\min}}{W_{ij1}\sum_m C_{ijm} R_{km}} = \frac{\sum_m C_{ijm} R_{km} M_m^{e,\min}}{\sum_m C_{ijm} R_{km}}$$

$$T_{i2k}^{e,\min} = \frac{M_{i21}\sum_m \overline{C}_{i2m} R_{km} M_m^{e,\min}}{M_{i21}\sum_m \overline{C}_{i2m} R_{km}} = \frac{\sum_m \overline{C}_{i2m} R_{km} M_m^{e,\min}}{\sum_m \overline{C}_{i2m} R_{km}}$$

$$W_{ij2}^{e,\min} = W_{ij}^{e,\min}$$

$$M_{i22}^{e,\min} = M_i^{e,\min}$$

Output Materials Allocation

Finished alloy demand quantity constraint:

$$\sum_i \sum_n (M_{i22n} + \sum_k T_{i2kn} + \sum_j (W_{ij2n} + \sum_k S_{ijkn})) + \sum_p \sum_n M_{pn} \geq F_n$$

Conservation of mass:

$$\sum_n M_{pn} \leq M_p$$

$$\sum_n M_{i22n} \leq M_{i22}$$

$$\sum_n T_{i2kn} \leq T_{i2k}$$

$$\sum_n W_{ij2n} \leq W_{ij2}$$

$$\sum_n S_{ijkn} \leq S_{ijk}$$

Compositional constraint on finished alloy:

$$\sum_i \sum_n (M_{i22}^{e,\min} M_{i22n} + \sum_k T_{i22}^{e,\min} T_{i2kn} + \sum_j (W_{ij2}^{e,\min} W_{ij2n} + \sum_k S_{ijk}^{e,\min} S_{ijkn})) +$$
$$\sum_p \sum_n M_p^{e,\min} M_{pn} \geq F_n^{e,\min} F_n$$

$$\sum_i \sum_n (M_{i22}^{e,\max} M_{i22n} + \sum_k T_{i22}^{e,\max} T_{i2kn} + \sum_j (W_{ij2}^{e,\max} W_{ij2n} + \sum_k S_{ijk}^{e,\max} S_{ijkn})) +$$
$$\sum_p \sum_n M_p^{e,\max} M_{pn} \le F_n^{e,\max} F_n$$

All variables are non-negative. The notations and variables are detailed below.

i = Input scrap material index
n = Finished alloy index
m = Material (makeup) component index
p = Primary material index
q = Sort stage index
j = Stage one sort output stream index
k = Stage two sort output stream index
C_i = Cost (per unit wt.) of scrap material i
C_p = Cost (per unit wt.) of primary material p
Z_q = Cost of sorting (per unit wt.) for sort stage q
M_p = Quantity of input primary material p
M_i = Quantity of input scrap material i
M_{i1} = Quantity of input scrap material i that went through stage one sorting
M_{i2} = Quantity of input scrap material i that did not go through stage one sorting
$M_i^{e,max}$ = Maximum wt. % content of element e in stream M_i
$M_i^{e,min}$ = Minimum wt. % content of element e in stream M_i
$M_{i2}^{e,max}$ = Maximum wt. % content of element e in stream M_{i2}
$M_{i2}^{e,min}$ = Minimum wt. % content of element e in stream M_{i2}
$M_m^{e,max}$ = Maximum wt. % content of element e in material component m
$M_m^{e,min}$ = Minimum wt. % content of element e in material component m
A_p = Quantity of availability primary material p
A_i = Quantity of availability scrap material i
W_{ij} = Quantity of output into stream j from stage one sorting with input M_i
C_{im} = Wt. % representation of material component m in raw material i
R_{jm} = Recovery efficiency (%) of material component m in stage one sort output stream j
$W_{ij}^{e,max}$ = Maximum wt. % content of element e in stream W_{ij}
$W_{ij}^{e,min}$ = Minimum wt. % content of element e in stream W_{ij}
M_{i21} = Quantity of input scrap material i that did not go through stage one sorting *but* went through stage two sorting
M_{i22} = Quantity of scrap material i that went through *neither* stage one nor stage two sorting
W_{ij1} = Quantity of scrap material i that went through *both* stage one and stage two sorting
W_{ij2} = Quantity of scrap material i that went through stage one *but not* stage two sorting
S_{ijk} = Quantity of material output from stage two output stream k that traces its origin to stage one output stream j and scrap material i

T_{i2k} = Quantity of scrap material *i* that *only* went through stage two sorting and *not* stage one sorting

R_{km} = Recovery efficiency (%) of material component *m* in stage two sort output stream *k*

$\overline{C}_{i2m}$ = Wt. % representation of material component *m* in scrap material *i*

C_{ijm} = Wt. % representation of material component *m* in material stream W_{ij}

$M_p^{e,max}$ = Maximum wt. % content of element *e* in primary material *p*

$S_{ijk}^{e,max}$ = Maximum wt. % content of element *e* in stream S_{ijk}

$T_{i2k}^{e,max}$ = Maximum wt. % content of element *e* in stream T_{2kj}

$W_{ij2}^{e,max}$ = Maximum wt. % content of element *e* in stream W_{ij2}

$M_{i22}^{e,max}$ = Maximum wt. % content of element *e* in stream M_{i22}

$M_p^{e,min}$ = Minimum wt. % content of element *e* in primary material *p*

$S_{ijk}^{e,min}$ = Minimum wt. % content of element *e* in stream S_{ijk}

$T_{i2k}^{e,min}$ = Minimum wt. % content of element *e* in stream T_{2kj}

$W_{ij2}^{e,min}$ = Minimum wt. % content of element *e* in stream W_{ij2}

$M_{i22}^{e,min}$ = Minimum wt. % content of element *e* in stream M_{i22}

$F_n^{e,max}$ = Maximum wt. % content of element *e* allowed in product F_n

$F_n^{e,min}$ = Minimum wt. % content of element *e* allowed in product F_n

M_{pn} = Quantity of primary material *p* allocated towards production of F_n

M_{i22n} = Quantity of scrap material *i* that went through *neither* stage one nor stage two sorting and allocated towards production of F_n

W_{ij2n} = Quantity of scrap material *i* that went through stage one *but not* stage two sorting and allocated towards production of F_n

S_{ijkn} = Quantity of material output from stage two output stream *k* that traces its origin to stage one output stream *j* and scrap material *i* and allocated towards the production of F_n

T_{i2kn} = Quantity of scrap material *i* that *only* went through stage two sorting and *not* stage one sorting and allocated towards the production of F_n

Chapter 6 : Economic and Recycling Impact Assessment of Sorting Technologies

When it comes to environmental concerns, there is no doubt that using more aluminum scrap materials in production is beneficial. However, when it comes to technologies being developed to promote such a cause, the associated economic benefits are harder to grasp at a glance. One such technologies aiming to address inefficiencies in scrap consumption is broadly termed light metals sorting. The idea is to segregate mixed (aluminum) scrap streams into more compositionally homogeneous scrap output. The finer the separation, the lesser the scrap content uncertainty and the more control and confidence material producers will have over their compositions and thus usage. Of course, with the associated benefits over scrap consumption and depending on scrap pricing conditions, price differential advantage, there will be added costs from the development and deployment of such a technology. In fact, depending on the actual expected performance of the sorter, the scrap pricing environment, and products being produced, sorting technologies may or may not prove to be economical. Even if it is economically viable, these factors will certainly affect the way sorting is utilized. For instance, not all scrap types will necessarily be sorted. To answer many of these critical operational questions and to address investment decisions, a systematic framework that account for such driving forces will be needed. This chapter is entirely devoted to filling in this gap and to bridge sorting technologies development with potential deployment. The methodology to be presented is an optimization model with which a feasibility case study on one type of sorting technology has been carried out in collaboration with Norsk

Hydro[20]. With the built-in flexibility of the working model, it can be applied towards the study of sorting technologies with an arbitrary number of output streams. The model serves to support industry decision-making regarding investments in and application of sorting technologies to increase scrap use and lower production costs.

6.1 The Double-Sided Nature of Scraps

There are both economic as well as environmental drivers for scrap consumption. Scraps are typically cheaper than primary materials. However, this relationship can be fickle as evident by the historical volatility on the price differential. The environmental driver is more stable and in some sense more dramatic. In fact, the explosive worldwide growth in aluminum consumption prompts concerns not with mineral scarcity, but rather with among other things, the energy consumption inherent with primary aluminum production. Using secondary sources can reduce the energy consumption by over 90%. Yet this friendly attribute of scraps is met with the difficulty in working with scrap compositions. In particular, the components that make up scrap streams are often compositionally distinct in many ways and cannot be used directly in production when mixed. Several authors have raised concerns about maintaining high levels of aluminum scrap reuse in the face of changing patterns of aluminum consumption (Drucker Research 1999, Aluminum Association 1999, Gorban 1994). Although a significant surplus of aluminum scrap is not necessarily imminent, the concerns do point to current or emerging inefficiencies in scrap reuse (Gorban 1994). Economic inefficiencies occur when high value alloys are repurposed into compositionally tolerant alloys. These actions are direct

[20] Part of this chapter is based upon a paper (Li, P.P., Guldberg, S., Riddervold, H.O., Kirchain, R.E., "Identifying Economic and Scrap Reuse Benefits of Light Metals Sorting Technologies") presented at the TMS 2005 Annual Meeting Post-Consumer Recycling Symposium, San Francisco February, 2005.

consequences of the difficulty in handling scrap compositions. In the absence of technological changes, current usage trends would suggest an increase in this practice.

To avoid this erosion of value, several firms and institutions have been developing light metals sorting technologies (Reuter 2004, Mesina 2004, Gesing 2002, Maurice 2000). Of course, just like any emerging technology, they need to provide definitive economic benefits to the industry as an incentive for common deployment. Short of completing a commercial-scale pilot test of a sorting technology, most industry participants would like some way of gauging the economic and scrap usage impact of such capabilities beforehand. Prior to investing in or purchasing this capability, it will be important to get a sense of how much the technology is worth and how the applicability will change with different product mix, scrap mix, etc. Once the choice has been made to adopt this technology, the operators will then need a decision framework for which scraps to sort, how much to sort, as well as how to allocate these processed materials in production. In the following case study with Hydro Aluminum, these questions and more are addressed on a wrought-versus-cast sorting technology. The results should be of interest to people who are developing or considering using such a technology.

6.2 Sorting Case Study: Base Case

This case study is reflective of a large-scale European Union aluminum producer with an annual 100kt production capacity (Table 6-II). The split between cast and wrought products, at 70%/30% is representative of the split observed in the secondary market (Buckingham 2002). The amounts scheduled for the individual alloys are based on expert opinion and their ratios are illustrative of prevailing trends in the European market. Scrap supplies represent 60% of production capacity at 60kt. This availability is estimated based on the sourcing needs of a producer focused on cast products. There are four scrap types sourced from across Europe. These include i) old rolled, ii) Al-ELV scrap, iii) shredded extrusion and (iv) co-mingled respectively. These are listed as Base Sheets, Base Casts, Base Extrusions and Co-Mingled respectively in Table 6-I, which also indicates their pricing and basic content. The prices on these raw materials were taken recently from the London Metals Exchange. Unit prices shown throughout this chapter have been normalized to emphasize economic trends rather than absolute dollar amount. For the purpose of clarity, the compositional data on the raw materials, products and primary elements are provided in Appendix B. The compositions on the scrap materials were sampled and determined by spectrographic analysis. Compositions on alloys are compliant with international standards. Metal yields have been estimated at 93% for scraps and 98% for primaries.

6.2.1 Sorting Technology Description

The sorting technology considered in this chapter was modeled as a single-stage sort with three output streams ? bins 1, 2 and 3. The Base Case assumed that the remelter will be renting this service from a third party (rent-for-service), and therefore no fixed costs will be incurred. The sorting costs will be based on a per ton usage basis. Later studies in this chapter will also examine the implications of sorting as an investment. Bin 1 receives 95% of the wrought alloys from the incoming scrap stream and 5% of incoming cast constituent. Bin 2, receives 95% of the cast alloys and 5% of the wrought. The final Bin 3 receives 100% of all the other remaining scrap components. These values approximate the sorting recovery rates reported for the "hot crush" technique with prior separation of the "other" fraction (DeGaspari 1999). The Base Case sorting cost was estimated at $30/ton. This figure is rather conservative compared to contemporary sorting techniques such those for stainless steel and iron[21]. As light metal sorting technologies develop, this cost is likely to come down. The impact of this assumption is explored in subsequent analysis.

Table 6-I. Quantity and Makeup of Available Scrap Types

<table>
<tr><th rowspan="3">Scrap Type</th><th rowspan="3">Normalized Price / Ton[22]</th><th rowspan="3">kt</th><th colspan="4">Alloy mass fraction wt.%</th></tr>
<tr><th colspan="2">Wroughts</th><th rowspan="2">Casts</th><th rowspan="2">Others*</th></tr>
<tr><th>Sheets</th><th>Extrusions</th></tr>
<tr><td>Base Casts</td><td>1.04</td><td>30</td><td colspan="2">30%</td><td>56%</td><td>14%</td></tr>
<tr><td>Base Extrusions</td><td>1.24</td><td>10</td><td>15%</td><td>70%</td><td>-</td><td>15%</td></tr>
<tr><td>Base Sheets</td><td>1.09</td><td>10</td><td>75%</td><td>15%</td><td>4%</td><td>6%</td></tr>
<tr><td>Co-Mingled</td><td>1.00</td><td>10</td><td>30%</td><td>30%</td><td>16%</td><td>24%</td></tr>
</table>

*Include tubes, wires, etc.

Table 6-II. Modeled Alloy Products Demand

Alloy	Qty. Demanded (kt)
230	20
226	20
239	30
6111	10
6082	2
6060	14
3104	2
3105	2

6.3 Sorting and Mixing Model Description

The optimization model is at the heart of the framework that assesses the applicability of sorting in the greater context of remelt operations. In Chapter 4, a two-stage version of the optimization model was presented. Here a single-stage version is adopted for this Base Case study. The two-stage model, instead, would be applicable when a finer level of sorting beyond wrought versus cast separation is carried out after wrought/cast sort. The model helps to quantify the value which sorting brings to a remelter. At a somewhat

[21] Industry estimates the sorting cost for stainless steel and iron to be approximately $20/t.

[22] Price per ton is normalized to the price per ton of co-mingled scraps.

abstract level, the primary decisions for a remelter involve composing alloy batches by carefully selecting and mixing various amounts of scrap and primary materials. Sorting provides added flexibility as well as complexity to these decisions by affording the secondary processor the opportunity to upgrade the materials which they have at hand. With sorting, the set of operational decisions include what raw materials to use, which to sort and how much of sorted and unsorted materials should be allocated towards which product.

In practice, sorting may not occur at the remelting facility, but rather at the scrap supplier. Furthermore, sorting technologies might be developed in-house in which case it can be interpreted as incurring fixed costs, amortized over the useful life of the sorter. From an analytical perspective, the methods and results presented here are equally applicable to these arrangements. A remelter will need to decide whether to rent the service from a third-party, buy it or to develop sorting capabilities in-house. If the latter route is taken, how much should be spent? For a scrap supplier, it is critical to identify those markets for whom sorting provides added value and which technologies are able to deliver that most effectively. Both parties will need to know which scraps to sort if sorting is deemed appropriate.

6.3.1 Single-Stage Sorting and Mixing Model

The model presented below is an extension of one developed to examine strategic raw material allocation decisions (Cosquer and Kirchain 2003). The model developed here assumes that sorting occurs as a single stage 1-to-j stream operation. There is no theoretical limit on j. In a physical sense, this means that for any scrap stream entering the sorter, j possible output streams can be modeled where the characteristics of the j

output streams are determined by both the constituents within the incoming stream and the performance of the technology of interest. Figure 6.1 shows this graphically, identifying both the key variables and indices that will be detailed subsequently.

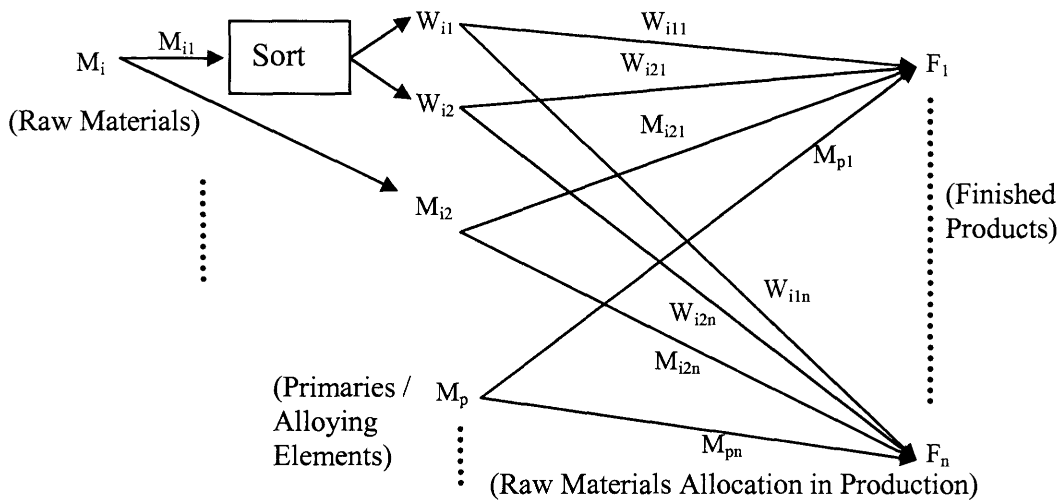

Figure 6.1. Schematic of materials sorting and allocation of sorted and unsorted material streams towards production.

In order to handle both rent-for-service as well as investment decisions surrounding sorting deployment, the model must be formulated as a mixed-integer optimization problem (Chong 2001). This method is widely applied in operational batch production decisions throughout the aluminum industry. The model presented here differs from those operational tools primarily in its simultaneous assessment of multiple production goals and its extension to explicit sorting decisions. Other related optimization studies on sorting have focused on the optimization of processes, technologies, and overall resource cycles (Mesina 2004, Reuter 2004, van Schaik 2004, Dalmijn 2003, van Schaik 2002, Xiao 2002, Gesing 2002, Maurice 2000). Current work examines sorting technologies from the point of view of the economic value – cost savings and scrap utilization – provided to key stakeholders. The following set of equations (Eq 6.1 –

Eq **6.14**) describes the various elements of the model, including the decision-making objective and constraints with explanations of the variables and indices to follow. Operational decisions are made with an objective to minimize operating costs. The model could be readily adapted to accommodate other objectives, such as profit or scrap use maximization.

Minimize:

Eq 6.1

$$\underbrace{(\sum_i C_i M_i + \sum_p C_p M_p)}_{\text{(Raw Material Costs)}} + \underbrace{\sum_i Z_1 M_{i1}}_{\text{(Sorting)}} - \underbrace{\sum_i R_i (M_i - \sum_{j,n} W_{ijn} - \sum_n M_{i2n})}_{\text{(Residual Scrap Salvage Value)}} + \underbrace{\{if\ (\sum_i M_{i1} > 0)\ then\ C_F\ else\ 0\}}_{\text{(Fixed Costs Consideration)}}$$

There are four specific items included in the objective function (Eq 6.1). There are cost contributions from raw materials purchase and sorting operation. Since not all sorted scrap materials are necessarily allocated in final production, there is salvage value associated with unallocated sorted scraps. The assumption is that these scraps will carry a value at their original costs, although admittedly depending on the nature of the nature scrap the value can be higher or lower. To capture the physical realities of batch construction and sorting performance, the objective function is subject to the following constraints on materials supply, demand, compositions, conservation of mass and sorting recovery rates.

Raw materials supply constraints:

Eq 6.2 $$M_p \le A_p$$

Eq 6.3 $$M_i \le A_i$$

Pre-sorting and post-sorting mass conservation:

Eq 6.4 $$M_{i1} + M_{i2} = M_i$$

Eq 6.5 $$\sum_j W_{ij} \le M_{i1}$$

Sorted and unsorted material streams allocation for production:

Eq 6.6 $$\sum_n M_{pn} \le M_p$$

Eq 6.7 $$\sum_n M_{i2n} \le M_{i2}$$

Eq 6.8 $$\sum_n W_{ijn} \le W_{ij}$$

Batch production requirements:

Eq 6.9 $$\sum_i \sum_n \sum_e (M_{i2n} M_{i2}^{e,ave} Y_{i2}^e + \sum_j W_{ijn} W_{ij}^{e,ave} Y_{ij}^e) + \sum_p \sum_n \sum_e M_{pn} M_p^{e,\min} Y_p^e \ge F_n$$

Compositional specifications requirements:

Eq 6.10 $$\sum_i \sum_n (M_{i2n} M_{i2}^{e,ave} Y_{i2}^e + \sum_j W_{ij}^{e,ave} W_{ijn} Y_{ij}^e) + \sum_p \sum_n M_p^{e,\min} M_{pn} Y_p^e \ge F_n^{e,\min} F_n$$

Eq 6.11 $$\sum_i \sum_n (M_{i2}^{e,ave} M_{i2n} Y_{i2}^e + \sum_j W_{ij}^{e,ave} W_{ijn} Y_{ij}^e) + \sum_p \sum_n M_p^{e,\max} M_{pn} Y_p^e \le F_n^{e,\max} F_n$$

Quantities of materials recovered through sorter:

Eq 6.12 $$W_{ij} = M_{i1} \sum_m C_{im} R_{jm}$$

Compositional determinants for unsorted material streams:

Eq 6.13 $$M_{i2}^{e,ave} = M_i^{e,ave}$$

Compositional determinants for sorted material streams:

Eq 6.14 $$W_{ij}^{e,ave} = \frac{\sum_m C_{im} R_{jm} M_m^{e,ave}}{\sum_m C_{im} R_{jm}}$$

All variables are non-negative. The indices and variables used above are shown in Figure 6.1 and defined as:

i,n,m = Input scrap, finished alloy, and materials component indices
p,q,j = Primary material and alloying element index, sort stage index, stage one sort output stream index
C_i = Cost (per unit wt.) of scrap material i
C_p = Cost (per unit wt.) of primary material p
C_F = Annual fixed costs of sorter

R_i = Residual salvage value (per unit wt.) of scrap material i
Z_q = Cost of sorting (per unit wt.) for sort stage q
M_p = Qty. of input primary material or alloying element p acquired
M_i = Qty. of input scrap material i acquired
M_{i1} = Qty. of input scrap material i that went through stage one sorting
M_{i2} = Qty. of input scrap material i that did not go through stage one sorting
$M_i^{e,ave}$ = Average wt. % content of element e in stream M_i
$M_{i2}^{e,ave}$ = Average wt. % content of element e in stream M_{i2}
$M_m^{e,ave}$ = Average wt. % content of element e in material component m
Y_{i2}^e = Metal yield (%) for scrap material i that did not go through sorting
Y_{ij}^e = Metal yield (%) for sorted scrap material stream W_{ij}
Y_p^e = Metal yield (%) for primary or alloying element p
A_p = Qty. of availability primary material or alloying element p
A_i = Qty. of availability scrap material i
W_{ij} = Qty. of output into stream j from stage one sorting with input M_i
C_{im} = Wt. % representation of material component m in scrap material i
R_{jm} = Recovery rate (%) of material component m in sort output stream j
$W_{ij}^{e,ave}$ = Average wt. % content of element e in stream W_{ij}
$M_p^{e,max}$ = Maximum wt. % content of element e in primary material p
$M_p^{e,min}$ = Minimum wt. % content of element e in primary material p
$F_n^{e,max}$ = Maximum wt. % content of element e allowed in product F_n
$F_n^{e,min}$ = Minimum wt. % content of element e allowed in product F_n
M_{pn} = Qty. of primary material or alloying element p allocated towards production of F_n
M_{i2n} = Qty. of unsorted scrap material i allocated towards production of F_n
W_{ijn} = Qty. of scrap material i that went through stage one sorting and ended up in stream j that was allocated towards production of F_n

6.3.2 Computation and Output

As with other mathematical models in this thesis, the bulk of the optimization programs were written and executed in the mathematical modeling language LINGO. The solution to this material system optimization problem provides the most economical set of decisions involving (i) how much of each raw material to acquire, (ii) how much of each acquired raw material to sort, (iii) and how much of the sorted and unsorted material streams should be allocated towards production. The driving forces of supply demand and compositional constraints are embedding into an integrated model, with the advantage that decisions and insights that would otherwise seem non-intuitive can be reflected. In fact, sometimes the material system constraints are too entangled for intuition to be of any value. For instance, while one will not intuit that cast-like material will be used in the production of wrought products, with sorting that might be the case. While important, human intuitions can sometimes be an under-informed impediment to technological changes[23].

Beyond these decision aids, shadow prices studies are also critical because sorting technologies are in many ways a disruptive technology. For instance, it can completely alter the way a remelter can work with scrap materials and as such it should be expected that the scrap supply shadow prices will change. Whether actually this leads to changes in the relative marginal benefits of scraps will be discovered.

6.4 Impact of Sorting on Scrap Consumption

Table 6-III indicates the optimal production allocation of sorted and unsorted scraps and sorting decisions as determined by the model for the Base Case inputs. The balance of production raw materials were made up by appropriate primaries and alloying elements. Due to their prompt scrap-like content, Base Extrusions and Base Sheets were never sorted at 95% recovery rates. At this recovery rate, sorted scraps were used in all alloys. However, the degree of usage differed by product by nature of compositional differences. For instance, the majority of materials consumption by the 6000 series products is with the relatively "clean" scraps (Base Extrusions and Base Sheets) and primaries. Note that a meaningful amount of Bin 1 materials (wrought-like) were consumed by the cast products. These were used presumably as cheaper dilution agents relative to primaries.

[23] "Everything that can be invented has been invented." — *Charles H. Duell, Commissioner, US Patent Office, 1899*

Table 6-III. Raw Materials Consumption (t) with Sorting (Base Case)

Alloy	Base Casts				Base Extrusions				
	Bin 1	Bin 2	Bin 3	Un-sorted	Bin 1	Bin 2	Bin 3	Un-sorted	
230	-	-	-	-	-	-	-	3,792	
226	-	11,624	2,965	4,229	-	-	-	-	
239	3,688	129	-	-	-	-	-	2,602	
6111	569	-	-	1,332	-	-	-	-	
6082	260	-	-	-	-	-	-	1,688	
6060	1,727	25	-	-	-	-	-	1,918	
3104	237	-	-	14	-	-	-	-	
3105	260	-	-	-	-	-	-	-	
	Base Sheets				**Co-Mingled**				**Primary & Alloying**
	Bin 1	**Bin 2**	**Bin 3**	**Un-sorted**	**Bin 1**	**Bin 2**	**Bin 3**	**Un-sorted**	
230	-	-	-	-	3,122	-	492	-	13,380
226	-	-	-	-	-	1,441	1,048	-	188
239	-	-	-	5,469	-	-	-	-	19,331
6111	-	-	-	1,194	-	-	-	768	6,539
6082	-	-	-	-	-	-	76	108	18
6060	-	-	-	2,791	-	-	-	843	7,353
3104	-	-	-	546	-	-	-	375	929
3105	-	-	-	-	1,486	-	241	-	156

By comparison, Table 6-IV indicates the materials consumption when sorting is not available; the pattern of scrap material consumption changes markedly. In fact, not only the amount of scrap used changed, but also the *types* of scrap used changed. Some of these changes are not entirely intuitive. Product 6082 swapped less expensive scraps for a more expensive one! Product 239 eliminated its dependence on Co-Mingled when sorting was available. These curiosities and others will be explained in more detail in the next section in conjunction with cost impact discussions. In aggregate, scrap consumption increased from 85% to 95% when sorting was available.

Table 6-IV. Raw Materials Consumptions (t) in Production Without Sorting

Alloy	Base Casts	Base Extrusion	Base Sheet	Co-Mingled	Primaries & Alloying Elements
230	-	-	-	1,599	18,891
226	19,217	-	-	1,647	608
239	243	6,879	6,385	1,749	16,135
6111	1,413	-	-	1,413	7,522
6082	-	956	1,147	31	16
6060	90	2,164	2,468	1,990	7,917
3104	51	-	-	655	1,371
3105	-	-	-	917	1,171

The fact that overall scrap consumption increased with sorting is not a surprise. After all, the purpose of sorting is to allow material processors to cope with scrap input compositions such that they can be used in otherwise unaccommodating circumstances. Mainly due to the price differential between scrap materials and primaries on average, one would also expect reduction in production cost as a result of sorting. While these statements are generally correct on an aggregate level, there are significant variations when the effects are examined by product.

Figure 6.2 examines the degree of scrap usage with and without sorting for each of the individual alloys which were investigated. Scrap-to-batch ratio represents the fraction of the batch production that was made from scrap materials, sorted or not. Scrap consumption for wrought products increased from 42% to 52% of batch production while the corresponding increase for cast products was from 51% to 55%. The relatively stronger increase in scrap consumption by the wrought products is indicative of their relative tolerance for alloying content in comparison with cast products. While most of the products increased their consumption of scraps, product 239 notably had a decrease. Detailed look at the change in raw material consumption by 239 indicates that a significant amount of Base Extrusion that was used by this product under the no sorting scenario transferred to the production of 230 with sorting. For product 6082, even though there was no significant change in the percentage of scrap consumption, the types of scraps consumed changed from using large amounts of Base Sheets and Base Extrusions under no sorting to zero Base Sheets consumption with sorting. The major reason for these changes is limitation in scrap supply. The supplies of Base Sheets and Base Extrusions are completely consumed. On the far right of Figure 6.2, the relaxation of this constraint led to a dramatic rise in scrap consumption via sorting. This jump underscores the potential of sorting in effecting greater scrap consumption. At the same time, it also points to one of the key reasons for the slow adoption of sorting technology deployment — limitations in scrap availability in parts of the world.

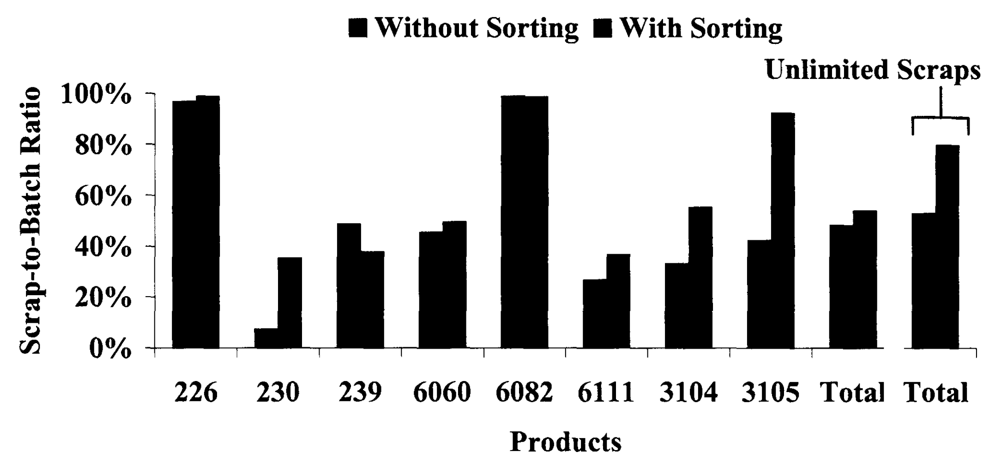

Figure 6.2 Base Case scrap consumption for individual products with sorting.

6.5 Impact of Sorting on Production Costs

The changes in scrap consumption observed in the prior section are entirely driven by economics since they are the solutions from a cost minimization. Overall the cost savings from sorting was \$0.8M or approximately 0.6% (Table 6-V). On an individual product basis, many of the peculiarities in scrap usage can be explained from a cost perspective.

Table 6-V. Scrap Consumption Aggregated By Scrap Type.

Scrap type	**With Sorting**		**Without Sorting**	
	% Consumed	**Qt. (kt)**	**% Consumed**	**Qt. (kt)**
Base Casts	90.2%	27	70.0%	21
Base Extrusions	100.0	10	100.0	10
Base Sheets	100.0	10	100.0	10
Co-Mingled	100.0	10	100.0	10
Overall Total	**95.1**	**57**	85.0	**51**
Total Costs	**$129,241,000**		**$130,014,906**	

Figure 6.3 correlates the changes in scrap usage patterns for the individual products to cost savings/increases associated with such changes. For 6111, 3104 and 3105, the increase in scrap consumption directly led to cost savings due to the price gap difference between secondary and primary materials. With 6082, even though there was practically no change in the scrap consumption, the replacement of less expensive scraps with more expensive ones led to an increase in costs. All of the Base Sheet that it consumed under no sorting was instead allocated towards the production of 6060, 6111 and 3104. Taking a system's point of view, this benefited the overall production costs. It is apparent from product 226 that increase in scrap consumption does *not* guarantee cost savings. Even though it made use of more scrap, and cheaper ones no less, the associated cost benefit was not enough to outweigh the sorting costs involved. Similarly, a decrease in scrap consumption does not automatically lead to increase in costs, as long as the scrap types changed to reflect cheaper scraps that more than offset the increase in costs due to increased primary usage. Nevertheless, the argument for limitation in scrap supplies once again applies here. In particular, the cost savings would have been 2.9% instead of 0.6%

had scrap supplies be unlimited. These are significant dollar differences considering the fact that the overall production costs run in the hundred of millions.

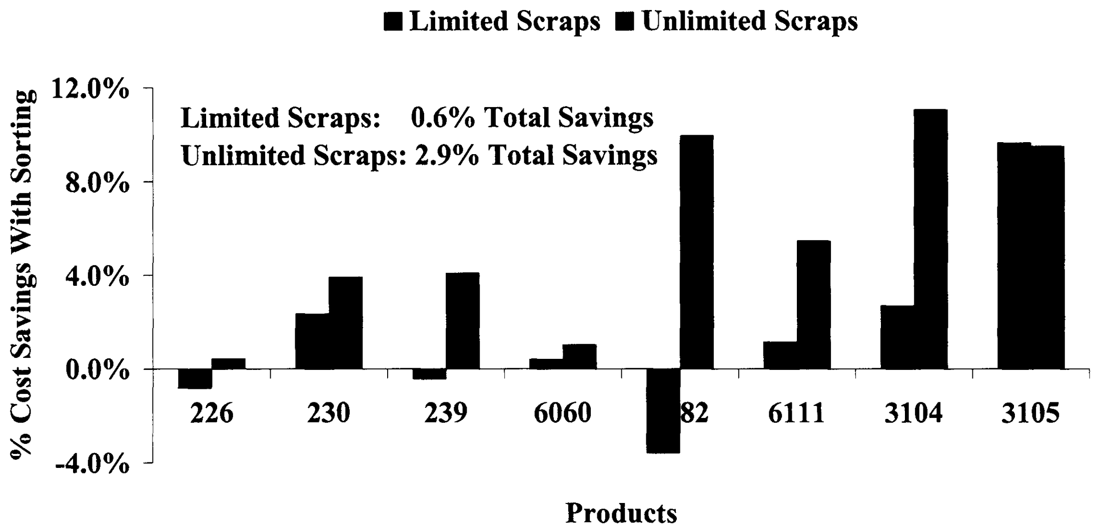

Figure 6.3 Base Case scrap usage cost impact with sorting.

6.6 Insights on Sorting From Shadow Prices

In the previous chapter, a methodology was presented that enables demand uncertainty to be factored into the decision-making process for scrap purchasing. In doing so, it was shown that scrap consumption can be increased. That strategy involved no upfront investment or added costs in its implementation. It is purely a behavioral change. Sorting technologies, on the other hand, is much more than just a behavioral change. It is more disruptive in the sense that it costs money to implement and it can alter the way scraps are allocated in production. In some ways, this relative disruptiveness is reflected in the demand shadow prices (Figure 6.4).

Without exception, the demand shadow prices dropped when sorting was made available. Shadow prices are responses to constraints; the more constrained a system, the greater the shadow prices. When sorting is available, the system had more ways to utilize scraps, therefore the demand shadow prices decreased. However, unlike those observed in the previous chapter, the ranking among the product demand shadow prices *do*es change. It was remarked earlier on that, due to closeness in compositions, ranking changes among alloys within the same family might be expected when demand uncertainties were considered. Indeed, such ranking changes are seen as well with sorting. However, beyond that, ranking changes between alloys from *different* families were also observed. The bottom line is that sorting can dramatically alter the marginal cost of production. Therefore, if strategies were devised to increase scrap consumption and lower production costs by alloy substitutions in product applications, those strategies should be re-examined with sorting.

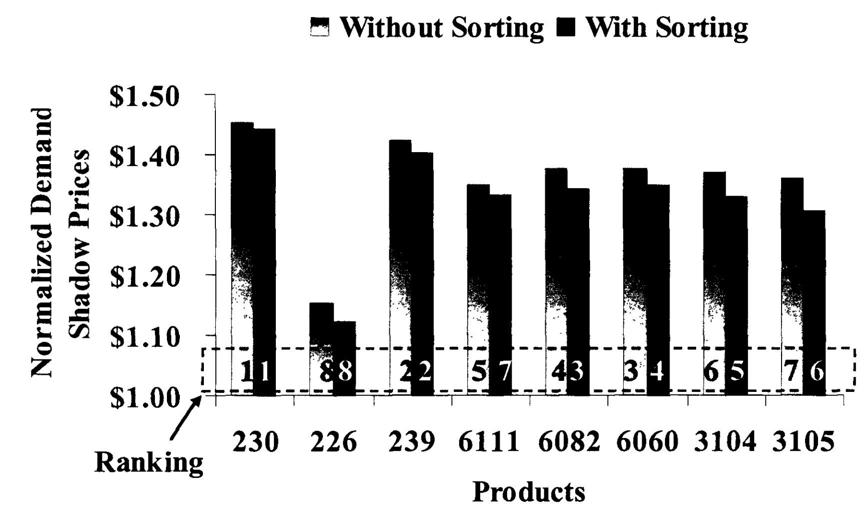

Figure 6.4 Demand shadow prices with and without sorting (numbers at the bottom of the bars represent ranking).

Sorting can alter the marginal benefits of scraps as well. The scrap supply shadow prices represent the degree to which the overall production cost will be reduced when one extra unit of the corresponding scrap is made available, assuming that the scrap is entirely consumed. The greater the scrap supply shadow price, the larger the reduction in production cost. The degree by which production cost is reduced can vary depending on the basis for comparison. All else being equal, a system that is currently making use of less scrap material will benefit more from the availability of this extra scrap. The reason is that there is a higher likelihood that the extra scrap will be used to replace dependence on more expensive primary materials. When sorting is available, the system is more flexible in making use of scrap materials and can already make use of more scrap. Therefore, with sorting, the scrap supply shadow prices tend to be lower relative to no sorting (Figure 6.5). Since Base Casts scraps were not entirely consumed, there is no corresponding shadow price.

With sorting, the ranking among the scrap supply shadow prices was shuffled relative to no sorting. The ranking correlated positively to the percentage of scrap consumed, for each scrap type, by wrought products under sorting and no sorting. When scrap is used in products with less alloying content (ie, wrought), the shadow price will be greater since these products tend to use more primary material in their production when appropriate scrap is not available. For the scrap types in Figure 6.5, however, Base Extrusions costs more than Base Sheets which in turn costs more than Co-Mingled. Given that the scrap supply shadow prices with sorting no longer follows this unit price trend, the implication is that when sorting is available, a more expensive scrap is not necessarily less attractive than a less expensive one. The technology enables scrap material to be used in a more flexible manner across products. Therefore, even though prior discussions pitched the value of sorting partly at the mercy of limitations in scrap supplies, sorting can also remove a remelter's dependence on cheap scrap materials.

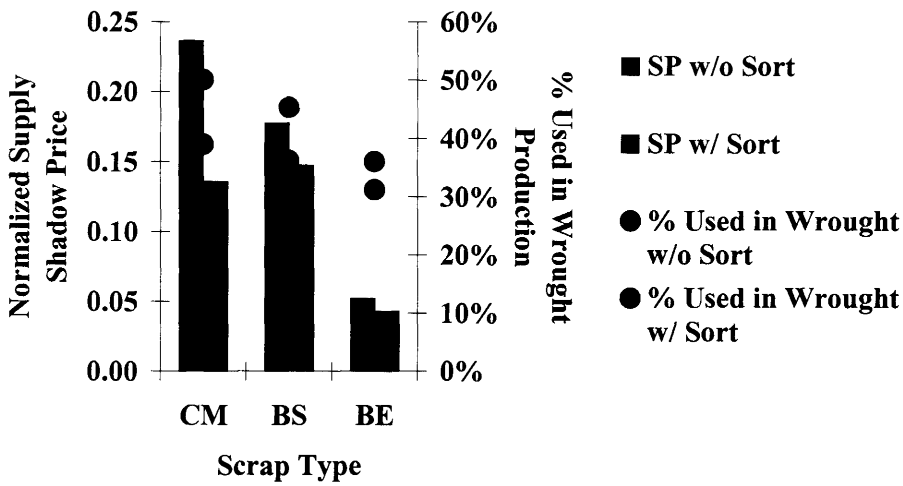

Figure 6.5 Scrap supply shadow prices with and without sorting.

6.7 Sensitivities of Sorting

In the following, sorting is assessed in the context of variations in sorting costs, recovery rates, and raw material pricing. Both their impact on scrap consumption as well as production costs will be studied. Besides employing sorting under rent-for-service and bearing only variable sorting costs, it will also be studied as an investment.

6.7.1 Sensitivity of Scrap Consumption on Recovery Rates

The effects of recovery rates on the percentage change in scrap consumption are shown for the Base Case in Figure 6.6. In the following discussions, Bin 3 is treated as an invariant such that it always collects 100% of the "other" fraction. The improvement in scrap consumption with better recovery rates appeals to intuition — sorting overcomes barriers for scrap consumption. If this figure is viewed as a contour map, the gradient along the cast recovery rate axis is steeper than that along the wrought recovery rate axis. For this Base Case, scrap consumption is more sensitive towards fluctuation in cast recovery rate than wrought recovery rate. The reason can be traced to the different effects that the two recovery rates have on the amount and grade[24] of the sorted materials.

Figure 6.6 Percentage change in scrap consumption, sensitivity on recovery rates (Base Case).

The cross sections AA' and BB' in Figure 6.6 refers to Figure 6.7 and Figure 6.8 respectively. In these figures the amount and grade of the sorted materials in the Base Cast Bin 1 and 2 are shown. The widths of the bins are roughly to scale relative to each other. Base Cast is chosen here for illustration, bearing in mind that Co-Mingled are also sorted. Following the wrought recovery rate from A to A', it is noted that Bin 1 is trending away from wrought grade as Bin 2 becomes less cast-like. However, neither the material grade nor the amount in the individual bins is changing drastically. For Bin 2, as the wrought recovery rate decreased, the material is still usable in cast production provided that small amounts of alloying elements are added on average. The cost impact of this requirement is not dramatic. The scrap consumption of this bin will therefore not drop rapidly as long as there is a large enough pool of cast products to absorb this material. As for Bin 1, when the wrought recovery dropped, the applicability of this

[24] Grade is defined as the weight (concentration) of the desired product in the output stream.

material stream in wrought production is definitely compromised. However, they can still act as relatively cheap dilution agent as long as there are enough cast products to make use of it. This is another contributing factor to offset the decrease in scrap consumption due to lower wrought recovery rate. Turning the attention now to Figure 6.8, the story is a bit different. Following the cast recovery rate from B to B', the amount in these bins are changing very quickly. While the grade in Bin 2 remains decidedly cast-like throughout this range of cast recovery rates, as this recovery rate deteriorated, a significant amount of desirable material is removed for cast production. This is a strong driving force for the drop in observed scrap consumption. At the same time, the Bin 1 material is rapidly becoming contaminated with cast fractions. Even before the extreme of 50% wrought and cast recovery rate, the sorted stream adds no significant value to the system relative to no sorting since the material grade is approaching that of unsorted Base Cast material. These factors taken together contribute to the relative sensitivity of scrap consumption on cast recovery rates relative to wrought recovery rates for this cast oriented Base Case production scenario.

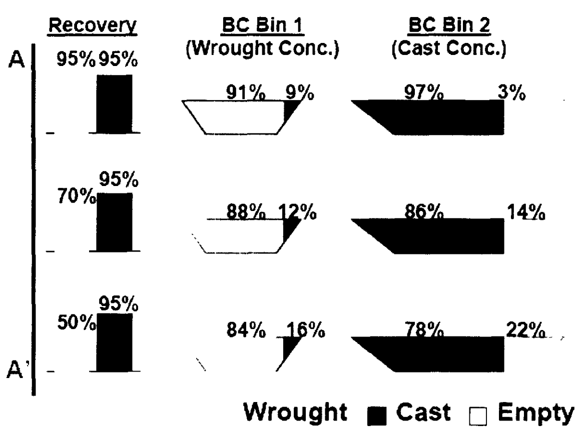

Figure 6.7. Recovery and grades variation holding cast recovery at 95%.

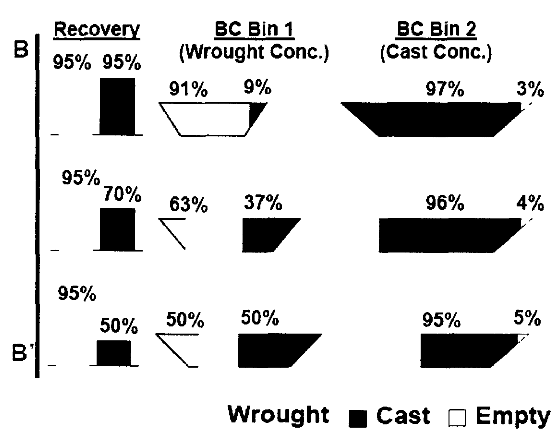

Figure 6.8. Recovery and grades variation holding wrought recovery at 95%.

The inference from the previous paragraph is that as the product mix shift away from cast products to wrought products, the sensitivity of scrap consumption towards wrought recovery rate will increase. This hypothesis is tested in Figure 6.9 in which a wrought-heavy case was constructed in which the demands in the Base Case were altered from 70% cast/ 30% wrought to 70% wrought/ 30% cast (Wrought-Heavy Case). The relative amounts demanded within cast products and within wrought products have been kept the same. Once again the "others" components of the scraps were assumed to be 100% segregated into Bin 3 and the sorting cost was maintained at $30/t.

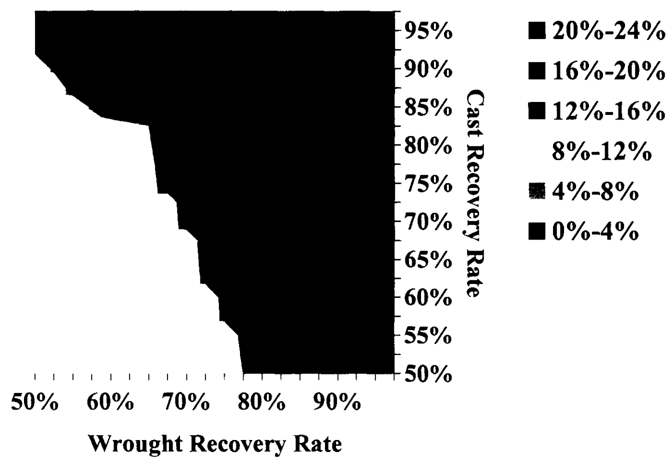

Figure 6.9 Percentage change in scrap consumption, sensitivity on recovery rate (Wrought-heavy Case).

It is verified from the gradient in this figure that the sensitivity towards the two recovery rates are now more even relative to the Base Case. This change in the gradient is a direct result of the product mix shift. Obviously the amount and grade of the various bins illustrated in Figure 6.7 and Figure 6.8 remain valid since they have nothing to do with the actual product mix itself. However, their applicability towards the products is of course critically dependent on this. Recall that in the prior paragraph, the moderate sensitivity of scrap consumption on wrought recovery rate was contingent upon having a large amount of cast products to absorb the sorted scraps. In this wrought-heavy case, this pool has shrunk and instead been replaced with less scrap tolerant wrought products. As a result, scrap consumption is much more sensitive towards the wrought recovery rate. For wrought-heavy production, the wrought recovery rate must be high such that there are enough materials from Bin 1 for allocation towards wrought products. Secondly, materials that are in Bin 1 must not be contaminated with cast fractions which will only make it "dirtier". This is not a luxury that wrought products can tolerate.

One also observes that in this wrought-heavy production, the benefit on scrap consumption from sorting is more significant (in terms of percentage) than for cast-heavy production. This is intuitive from the understanding that wrought products have lower tolerance for alloying content. For sorting technology research and aluminum sustainability, the implications of the differential sensitivity towards the cast and wrought recovery rate relative the product mix are the following. If the product mix is cast-heavy, a 1% improve in cast recovery rate is more beneficial towards scrap consumption than an equal amount in wrought recovery rate increase. Furthermore, its means that controlling for volatility in cast recovery rate is more critical than for wrought recovery rate in cast-heavy production.

In the above discussions, whether the product mix was cast- or wrought-oriented, the results from Figure 6.6 and Figure 6.9 indicate that as the recovery rates improved, the scrap consumption improved. A natural tendency would be to assume that with improved recovery rates, the sorter can be used more intensely to generate more applicable sorted material streams for consumption. However, this is not necessarily the case as shown in

Figure 6.10. For a cast recovery rate of roughly 94%, as wrought recovery rate improved from 90% to 100%, the amount of scrap sorted actually *decreased.*

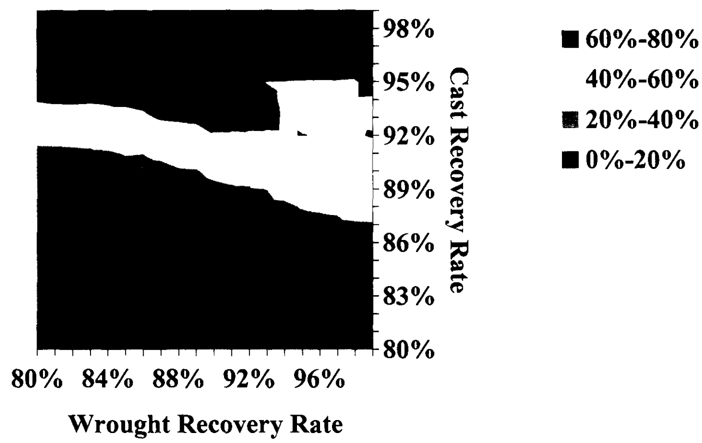

Figure 6.10. Percentage of mixed scrap sorted (Base Case).

In fact, the intensity of sorter application does not always have a direct correlation with the recovery rates performance. Sorting is an enabling technology that allows the material system greater flexibility in employing scraps, whether sorted or not. Whether to sort more or less scrap ultimately depends on the tradeoff between several factors: (1) Useful materials one can get via sorting and the associated cost benefit, (2) Cost of bringing the composition of this material inline with production specification, (3) Salvage value of excess materials, and (4) Cost of sorting. Given a certain product mix and scrap supply mix, the recovery rates will impact the materials content of the sorted fractions and dictate the usefulness of these processed materials. The cost of bringing these materials inline compositionally with production specification also stems directly from this. Ultimately, for sorting to be cost effective, the combined effects of (1) and (3) must outweigh (2) and (4). Typically if sorted scraps are more economical than unsorted or primary materials, the amount of materials sorted will be capped by one of the elements from the output stream reaching a maximum allowance in the production specification. From this point, as the recovery rates deteriorate or improve, whether the fraction of scrap sorted increase or decrease will depend on the relative grade between the sorted scraps and the products for which they are intended. At the elemental level of detail, if the recovery rate improvement results in a *decrease* in the percentage representation of a critical element[25] in the sorted output while increasing (but not exceeding the spec) it for other elements, the amount of this scrap sorted will *increase*. More of this scrap now needs to be sorted to get the same *amount* of this critical element in the output stream. The converse of this argument is also true. If the recovery rate improvement results in an *increase* in the percentage representation of a critical element in the sorted stream while decreasing it for other elements, the amount of this scrap sorted will *decrease*. With a high number of products and scrap types, multiple "peaks" in the Figure 6.10 contour can be observed. However, the basic underlying mechanism for sorting or not is always driven by these factors.

Of course, this discussion on the relationship between intensity of sorting to sorting performance is purely academic if sorting is employed as a third-party service. In that case, one would mostly be paying for what is used; cost is variable. However, remelters can develop sorting internally or make a purchase of the necessary machinery and software outright. Either case represents an investment. In those cases, associated fixed costs will dominate sorting deployment. If the sorting intensity is expected to be low, there is a clear danger of running up the unit cost of sorting. The following brief exposition attempts to clarify the admittedly abstract discussion behind the drivers for the

[25] Defined as an element whose percentage representation in a sorted scrap is at or above the maximum allowed in a product specification.

lack of correlation between sorting intensity and sorting performance. It is followed naturally by a discussion on the impact of sorting costs on the applicability of sorting, both from a rent-for-service and investment perspective. In doing so, the intensity of sorting will be tied back into the decisions for investment in sorting technologies.

6.7.2 Sorting Intensity and Performance

The fact that the utilization rate[26] of the sorter does not necessarily increase with better sorting performance is somewhat unexpected but real. A simple example can illustrate the reasons for this manifestation. In the following, consider only two elemental compositions, A and B. A single product of 100 units is being produced with x% A and y% B. Available raw materials include unlimited pure forms of A and B ($2 per unit) as well as a mixed scrap ($1 per unit) composed of 50% A and 50% B. The sorter separates the mixed scrap into two streams, stream 1 collects most of the A material while stream 2 collects most of the B material. Sorting cost is $0.05 per unit sorted. In the following studies, the recovery rates refer to the recovery of material A into stream1 and material B into stream 2. For instance, an 80% recovery rate means that 80% of material A from the mixed scrap ends up in stream 1, while 80% of material B from mixed scrap ends up in stream 2.

Figure 6.11 and Figure 6.12 presents the optimal amounts of mixed scrap to be sorted under different sorter recovery rates. This can be interpreted as a sorter utilization rate. The greater the amount of mixed scrap sorted, the higher the utilization rate. Following the evolution of the grade of the sorted streams and relating them to the desired composition of the product will help in understanding the relationship between utilization rate and sorter performance. For scenario 1 (product: 70% A, 30% B), below the 70%

[26] Defined as the percentage of available mixed scrap sorted.

recovery rate, the grade of stream 1 (closer of the two to the desired product composition) is too low on A and too high on B. Typically in such compositional violations, the dominant effect in limiting the applicability of the sorter stream is the element that is too high. The element that is too high in content incurs dilution costs, which due to the order of magnitude, is much more significant than the additive costs required to correct for the element that is too low. Therefore for recovery rates below 70% in scenario 1, the violation on B is controlling the applicability of the sorted streams. As the recovery rate improved from 50% to 70%, this violation is reduced leading to higher applicability of the sorted material which in turn leads to greater amount of mixed scrap sorted. To be even more explicit, as the recovery rate improved from 50% to 70%, lesser *amount* of B is being recovered in stream 1, and therefore a *greater* amount of scrap must be sorted in order to gather the amount of B required for production. At 70% recovery rate, the sorted material from stream 1 has a perfect match in grade to that required by the product. Beyond 70% recovery rate, violation on A becomes the dominant effect. In particular, there is now too much A in stream 1 and too little B in stream 2. As the recovery rate continued to improve above 70%, this violation is be exacerbated leading to lesser amounts of mixed scrap sorted driven by lower applicability. Once again, to be more explicit, as the recovery rates improved, greater *amount* of A is being recovered in stream 1, and therefore a *lesser* amount of scrap needs to be sorted to gather the amount of A required for production. Notice that the peak utilization rate corresponds exact to the grade of the desired product (70% A, 30% B). Similar observations and arguments can be made for Figure 6.12 in the production scenario 2 (product: 90% A, 10% B). When there are many more compositions, products and scraps, the interactions become more complex but the resulting relationship between utilization rate and recovery rates are just combination and superposition of the basic ideas discussed above.

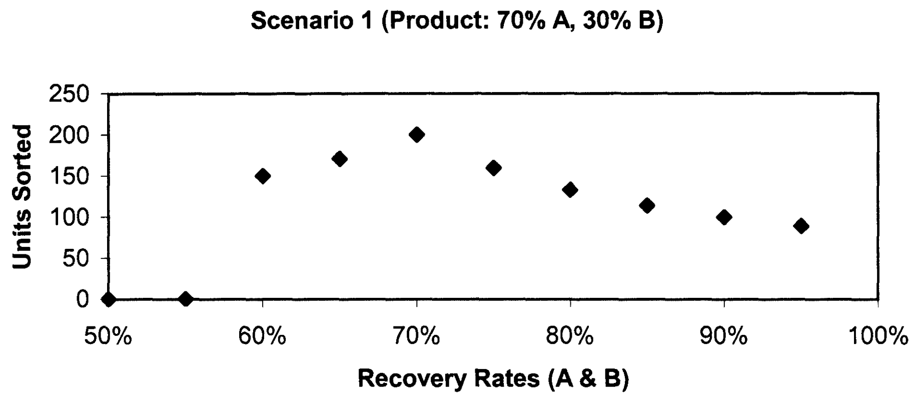

Figure 6.11 Units of mixed scrap sorted for scenario 1 (product 70%A, 30% B).

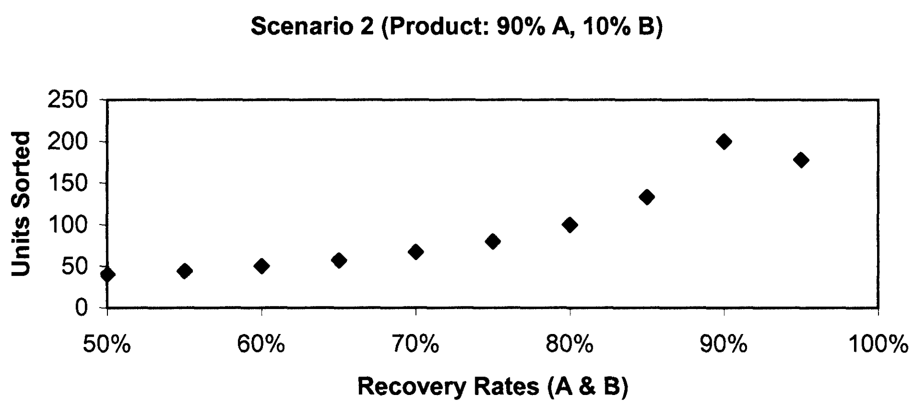

Figure 6.12 Units of mixed scrap sorted under production scenario 2 (product 90%A, 10% B)

6.7.3 Sensitivity of Scrap Consumption and Value on Sorting Costs

Whether sorting is employed as a rented service or as an investment, the cost of deployment is undoubtedly a critical determining factor in its adoption. Figure 6.13 illustrates the percentage of scrap consumption increase when sorting is rented at various sorting cost, holding recovery rates (wrought and cast) at 95%. From this figure it seems that this sorting technology holds promise for improving scrap consumption even as far out as \$125/t. However, realistically industry would want a certain level of cost savings before considering its implementation. While the percentage cost savings from Figure 6.14 seems small, when the overall production cost is in the hundred of millions, a 1% saving can translate into over a million dollars of savings. Therefore in order to be employed, sorting technologies will have to be within the darkest region in Figure 6.14 in terms of costs as well as performance. At a performance of 95% recovery rate, the critical sorting cost per ton seems to be around $17-18/t. Since the absolute amount of savings scale with the size of the production, a larger producer will naturally have an easier time adopting sorting technologies.

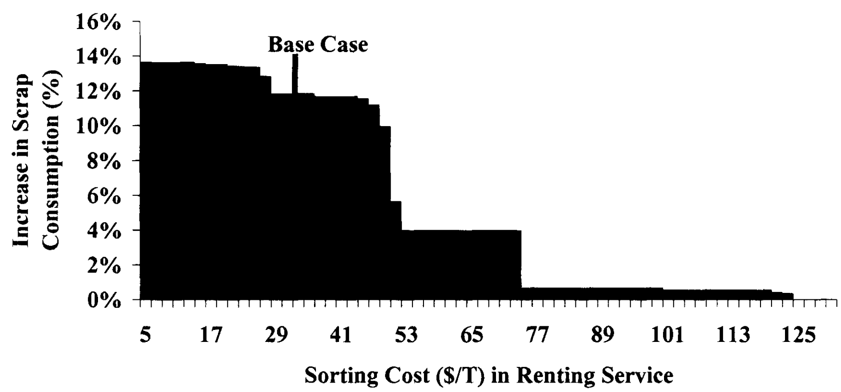

Figure 6.13. Percentage increase in scrap consumption with sorting at 95% recovery rates under rent-for-service model.

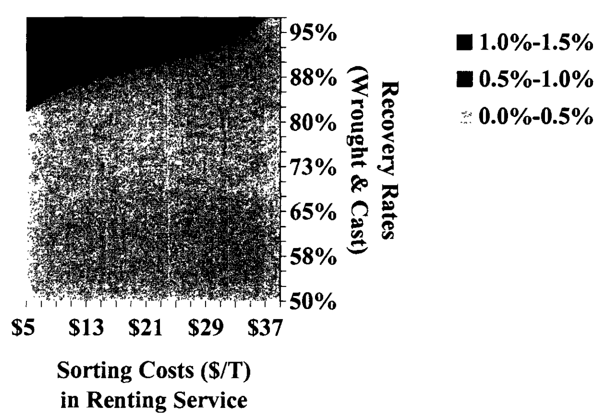

Figure 6.14. Percentage cost savings with respect to different recovery rates (wrought and cast) and sorting costs ($/t) under rent-for-service model.

When sorting is an investment, the utilization rate matters as discussed earlier. As an investment, on top of variable costs a sorter carries with it an associated fixed cost. However number of years over which this fixed cost is amortized, there will be a portion of the total costs involved that does not vary with the level of utilization. This means that if the utilization rate is low, the implied per ton cost of owning that sorter can skyrocket. As such, one would expect a critical level of sorter utilization rate, below which ownership or investment in the sorting technology will be uneconomical.

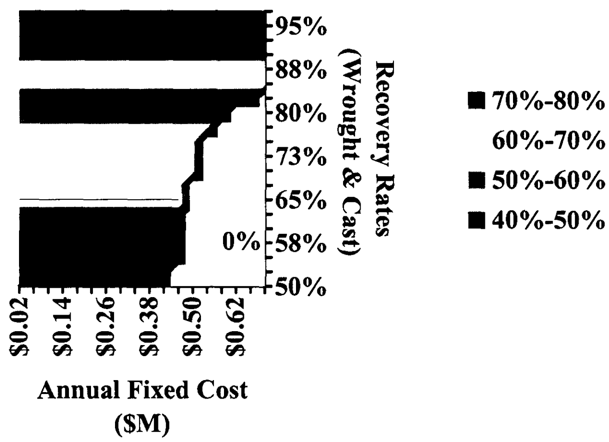

Figure 6.15 Percentage mixed scrap sorted (sorter utilization rate) under Base Case. White area indicates no sorting.

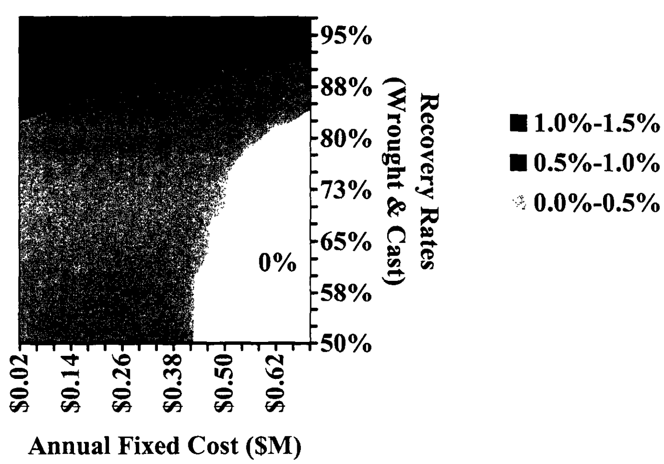

Figure 6.16 Percentage cost savings with sorting available under Base Case.

Figure 6.15 and Figure 6.16 illustrates the percentage of mixed scrap sorted (utilization rate) and the percentage cost savings given a range of sorting performance and annual fixed cost. A variable cost of $5/t has been levied to account for certain operating costs such as administration. The annual fixed cost is interpreted as the portion of investment in the sorting technology that is amortized over one year. The total investment is roughly[27] the annual fixed cost multiplied by the number of years of service of the equipment. Of course, as the annual fixed cost goes up, the unit cost of sorting goes up. However, it is important to realize as mentioned above that greater sorting performance does not necessarily lead to greater utilization rate. Therefore the unit cost can also be on the rise as sorting performance improves. As Figure 6.15 indicates, however, this effect is insufficient to offset the cost benefits from sorting as sorting performance improved. The critical rate for sorting intensity was observed at 48%, below which point there is a sharp drop to 0% cost savings.

Once again, in order for sorting to provide a sufficient level of cost savings to be of interest to industry, performance and costs will have to stay within the darkness region in Figure 6.16. Accounting for the percentage of mixed scrap sorting in this region, at 95% recovery rate the critical fixed cost plus variable cost translates into $17-18/t unit sorting cost. This agrees with the critical sorting cost per ton observed in the renting model.

6.7.4 Sensitivity of Sorting Value on Raw Material Pricing

As with the hedging considerations of demand uncertainty, the value of sorting and indeed its ability to improve scrap consumption are dependent upon the pricing condition on primaries and secondary materials. Scrap consumption will no longer improve with

[27] In practice one should also account for the time value of money as well.

sorting beyond a critical price differential. For practical purposes, that price differential will be dictated by the cost savings potential from sorting.

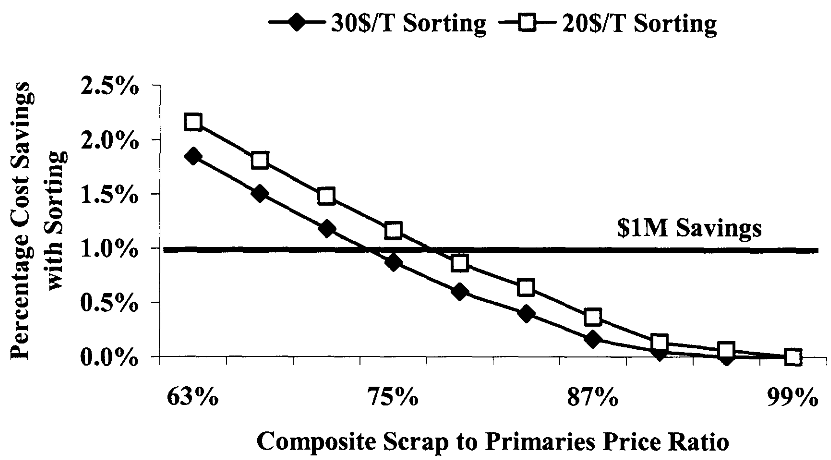

Figure 6.17 Percentage cost savings from sorting given price differential between scraps and primaries.

Figure 6.17 indicates improvement in cost savings as the price differential widens while holding recovery rates at 95%. Since there are four scraps of interest, a composite scrap price weighted by supply was compared against the price of P1020 or P0508. As a point of reference the Base Case was carried out at 79% price ratio at $30/t cost of sorting. The $1M savings mark would suggest that scrap prices should stay below 75-80% of the price of primary in order for sorting to be value-added to industry participants. However, since this figure assumes scrap supplies to stay constant throughout, it is probably a conservative estimate. For the supply of scrap to stay constant while there is a structural drop in scrap prices, the demand curve should move to lower demand. But the drop in scrap prices is more likely to prompt greater demand for scraps. Therefore with the drop in scrap prices there should be an associated increase in equilibrium scrap supplies, further increasing the cost savings from sorting. In any case, Figure 6.17 provides a rough estimate of the relationship between the cost savings potential from sorting and the prevailing price differential between scraps and primary materials.

6.8 Summary on Optimization Study of Sorting Technology

This study of wrought versus cast sorting based on EU scraps and production demands suggests that sorting technologies can be of value for a range of operating conditions. The results are conservative in the sense that industry specifications for alloy chemical content were used. In reality, remelters themselves are expected to have even more stringent compositional requirements, which should translate into a greater need for sorting. At 95% recovery rates, definite cost savings and scrap consumption improvement were observed at a sorting cost of $30/t. However, it is more likely that this cost will have to be lowered to the $20/t range in order to stimulate real interest among industry participants. This is regardless of whether this technology is rented, purchased outright or developed in-house.

Because sorting effectively alters the way scraps can be used in production environment, it is a disruptive technology in the sense that it can alter the economics of scraps and production. Sorting can change the relative marginal benefits of scraps and the relative cost of production. While limitations in scrap supplies clearly reduce the potential benefits of this technology, sorting can also break the dependence of remelters on cheap scraps. With sorting, cheaper scraps no longer always have the higher marginal benefits. On the other hand, schemes that attempt to increase scrap consumption and reduce costs by substituting one alloy for another must be revisited when sorting is made available. As the product mix lean towards cast products, the sensitivity of scrap consumption on cast recovery rate relative to wrought recovery becomes more pronounced. The implication is that during operations or in research and development, it is more important to control cast recovery variations as product mix becomes more cast-oriented. It was revealed that sorting performance improvement does not necessarily lead to more intensive usage of the technology. This is critical when sorting is an investment rather than a third-party service. In that case, the utilization rate has to stay above 40% in order for sorting to be employed.

With the decision tool presented in this chapter, remelters now have a systematic way of assessing sorting as an investment or as a technology option. The decision algorithm can be used with other types of sorting technologies as well. Furthermore, the methodology can be applied not only towards aluminum, but also other kinds of light metals.

Chapter 7 : Making Use of Chemical Compositional Variations in Batch Production

7.1 Principle of Raw Material Diversification

In the previous chapters, the emphasis has been on methodologies to enhance scrap usage through upgrading scrap materials, and better scrap purchasing management. At the operator level, where decisions have to be made regarding which raw materials to use for a batch production, chemical compositional variation stands out as the major uncertainty challenge. This is problematic because materials production often has strict compositional specifications. From the perspective of operators, there is no uncertainty in supply or demand because they knows exactly what needs to be made and what raw materials are available for use. However, due to the inherently mixed and often poorly defined content of secondary material streams, the chemical compositions of scrap materials carry a certain degree of uncertainty. For instance, while two individual piles of scrap materials might have very similar compositions on average, they can exhibit entirely different and unrelated levels of variability. Under a deterministic optimization framework for raw material choice and assuming that the pricing difference between these piles are insignificant, the operator will be indifferent between using any one of those piles. In other words, if 1 ton of such scrap material is needed, 1 ton will be taken from *one* of those piles. However, this logic is flawed under a more robust decision framework that accounts for compositional uncertainty. Under this new stochastic optimization framework, 1/2 ton will be taken from each pile in order to reduce the overall level of produced compositional uncertainty. This is the principle of raw material diversification. It is not limited only to raw materials of similar average compositions and will be applicable whenever scrap piles are not perfectly correlated in compositions. This logic and its benefits will be explained below. In doing so, its relationship to the chance constraint stochastic optimization technique will be demonstrated.

7.2 The Operator's Challenge

In day-to-day alloy production, the goal is often to find an optimal (ie, lowest cost) way of mixing various, on-hand raw materials in the production such that the finished products have the desired chemical characteristics. To realize this, modern cast house operators make use of linear programming based batch mixed algorithms. One of the challenges in these production decisions is to adequately account for and manage the inherent chemical compositional uncertainties in each batch of raw material. Given that dilution practices in real-time production can be time-consuming and costly, it is highly desirable to be as accurate as possible in the first place about the chemical nature of the melt. To accommodate this uncertainty, conventional cast house practice is to constrict the alloy production specification to levels more narrow than the actual compositional specifications. However, there are various issues with this practice. First of all how much narrowing in the specification window is enough? What is the implied margin of error given a certain window? Also, this kind of practice is static and can penalize the system's ability to use scrap. A better practice would be to relate the margin of error to the underlying uncertainty in chemical compositions of the raw materials that are actually used.

Realistically, there is no *absolute* way to prevent any error. Even careful sampling cannot determine the exact composition of every single batch. The statistical knowledge derived from sampling can only provide statistical indicators of the chemical compositions. These provide the operator with a sense of the magnitude of uncertainty involved but cannot provide a guarantee on the exact compositions. Nevertheless, these are information that should be made used of. They can provide more robust and realistic insights into the production environment and lead to more cost-effective raw material allocation.

This chapter explores methods which allow remelters to introduce explicit treatment of uncertainty into their decision making. A small example can illustrate the incentives for doing so. There are two 10kt piles of scrap type A with the same average compositions. For the time being assume for simplicity that the chemical composition of interest is silicon at 5wt% (500t) on average. The degree of compositional uncertainty is σ_{Si} = 1wt% (100t) and is the same among the two piles of scraps and they are uncorrelated. Suppose a batch production requires 4kt of scrap type A. In aggregate there are 20kt of scrap type A to choose from. Ignoring the compositional uncertainty, the operator will be faced with a seemingly degenerate set of solutions regarding which of the two piles to use. This is because all four piles have the same average content of silicon. However, let us consider both the mean *and* standard deviation of the resulting product of two different decisions: (Case 1) x_1=4kt from pile 1 versus (Case 2) x_1=x_2=2kt. For Case 1, the average silicon content is $Si_{ave} = x_1(5wt\%) = (4kT)(5wt\%) = 200t$ and the standard deviation of the final product will be $\sigma_P = \sqrt{x_1^2(1wt\%)^2} = 40t$. For Case 2, the average silicon content is exactly the same as Case 1. However, the standard deviation is $\sigma_P = \sqrt{x_1^2(1wt\%)^2 + x_2^2(1wt\%)^2} = 28t$. Of course, no one would want Case 1 given the possibility of Case 2. A compliant average specification with lower degree of produced uncertainty provides a greater comfort zone for error. This is not a statistical trickery, but rather a very logical result. Because the four piles of scrap type A have uncorrelated fluctuations in compositions, an increase in one pile could have been offset by an uncorrelated decrease in another. In the traditional deterministic framework, this information is lost. In fact, in the traditional deterministic framework, depending on the actual operational practice, the system might be over-penalized and under-utilizing scraps.

The concepts just discussed are in fact reflected directly in the chance constraint optimization framework. Furthermore, it is applicable whenever the compositions of the various scrap materials are less than perfectly correlated. Recall from Eq 2.12 that $D^* \geq \mu_a + X(\alpha)\sigma_a$ is the deterministic equivalent of a chance compositional constraint. This can be verbalized as "requirement ≥ mean + deviation". The mean is given by:

$$\mu_a = \sum_i x_i \mu_i$$

and the deviation term is given by:

$$X(\alpha)\sigma_a = X(\alpha)\sqrt{\sum_i \sum_j x_j x_i \sigma_{ij}} \text{ where } \sigma_{ij} = \sigma_i^2,\ i{=}j$$

In the discussions above, it was demonstrated that choices for x_i can lead to the same mean but dramatically different deviation term. Since the requirement D^* is fixed, it is a simple mental exercise to show that if simply by choice of x_i that the deviation can be lowered, then the mean term can be made greater while still satisfying Eq 2.12. The only way the mean term can be made greater is to have greater x_i. This means that the chance constraint methodology has the potential to increase scrap usage compared to traditional practices with excessive static buffer or the lack of raw materials diversification practice. In any case, the chance constraint method eliminates a lot of the problems aforementioned that are associated with traditional static treatment of compositional uncertainty. The following case study demonstrates these benefits.

7.3 An Application of Chance Constraints

The following production scenario is an adaptation of the case presented in Chapter 6. All the case information, including scrap supply and pricing is the same except for the following. First of all, scrap materials are now stochastic in compositions. The standard deviation is 10% of the mean. For each scrap material there are now two compositionally identical but uncorrelated piles (this was done to demonstrate raw materials diversification benefits). The uncertainty surrounding scrap compositions leads to chance constraints in the optimization formulation.

Minimize:

Eq 7.1 $$\sum_{s} C_s D_s + \sum_{p,f} C_p D_{pf}$$

This is the overall cost of batch production excluding reworking costs[28] and subject to the following constraints. The scrap usage must not exceed the amount available:

Eq 7.2 $$D_s \le A_s$$

Eq 7.3 $$\sum_{f} D_{sf} \le D_s$$

The amount of products produced must be equal to or more than what is required:

Eq 7.4 $$\sum_{s} D_{sf} + \sum_{p} D_{pf} = B_f \ge M_f$$

For each alloying element *c*, the composition of each alloy produced must meet production specifications:

[28] Future work should attempt to quantify the cost of rework.

Eq 7.5 $$\Pr\left\{\sum_s D_{sf}U_{sc} + \sum_p D_{pf}U_{pc} \le B_f U_{fc}\right\} \ge \alpha_{fc}$$

? $$\sum_s D_{sf}\overline{U}_{sc} + \sum_p D_{pf}\overline{U}_{pc} + X(\alpha_{fc})\left(\sum_s\sum_{s'}\rho^U_{ss'c}\sigma^U_{sc}\sigma^U_{s'c}D_{sf}D_{s'f}\right)^{1/2} \le B_f U_{fc}$$

Eq 7.6 $$\Pr\left\{\sum_s D_{sf}L_{sc} + \sum_p D_{pf}L_{pc} \ge B_f L_{fc}\right\} \ge \beta_{fc}$$

? $$\sum_s D_{sf}\overline{L}_{sc} + \sum_p D_{pf}\overline{L}_{pc} + X(1-\beta_{fc})\left(\sum_s\sum_{s'}\rho^L_{ss'c}\sigma^L_{sc}\sigma^L_{s'c}D_{sf}D_{s'f}\right)^{1/2} \ge B_f L_{fc}$$

All variables are defined below:

C_s = unit cost (\$/t) of scrap material *s*
C_p = unit cost of primary material *p*
D_s = amount (kt) of scrap material *s* used
D_{pf} = amount of primary material *p* to be used for the production of finished good *f*
A_s = amount of scrap material *s* available for usage
D_{sf} = amount of scrap material *s* used in making finished good *f*
B_f = amount of finished good *f* produced
M_f = amount of finished good *f* demanded
U_{sc} = max. amount (wt. %) of element *c* in scrap material *s*
$\overline{U}_{sc}$ = average max. amount (wt. %) of element *c* in scrap material *s*
σ^U_{sc} = standard deviation of the max. amount (wt%) of element *c* in scrap material *s*
L_{sc} = min. amount of element *c* in scrap material *s*
$\overline{L}_{sc}$ = average min. amount (wt. %) of element *c* in scrap material *s*
σ^L_{sc} = standard deviation of the min. amount (wt%) of element *c* in scrap material *s*
α_{fc} = confidence level (%) for compositional constraint with respect to the maximum amount of element *c* in product *f*
β_{fc} = confidence level (%) for compositional constraint with respect to the minimum amount of element *c* in product *f*
$\rho^U_{ss'c}$ = correlation coefficient between max. composition *c* of scrap materials *s* and *s'*
$\rho^L_{ss'c}$ = correlation coefficient between min. composition *c* of scrap materials *s* and *s'*
U_{pc} = max. amount of element *c* in primary material *p*
L_{pc} = min. amount of element *c* in primary material *p*
U_{fc} = max. amount of element *c* in product *f*
L_{fc} = min. amount of element *c* in product *f*

The statements Pr{.} state that those constraints which were required to be strictly satisfied under deterministic modeling are now only satisfied α and β percent of the time. Thus α and β are desired levels of confidence factors by which the operator can use to adjust his or her sense of importance for that particular elemental composition to be within specs. The function *X(.)* is the inverse of a normalized cumulative Gaussian distribution function which relates the underlying raw material composition standard deviations to the desired level of confidence. The symbols $\rho_{ss'c}$ represents the correlation between the fluctuations in composition c of raw material s and s'. By definition $\rho_{ss'c} = 1$ when $s = s'$. In the special case when there are absolutely no correlation among raw material compositions, only squared terms will remain in the standard deviation calculations in Eq 7.5 and Eq 7.6. In most cases when considering compositional uncertainties, the optimization framework will become nonlinear. Notice that only in two special cases when Eq 7.5 and Eq 7.6 will revert back to linearity with average compositions replacing actual compositions. The bases for these two cases, however, are entirely different. The first case is if there are *truly* no statistical fluctuations in the raw material compositions, ie, all $\sigma = 0$. In this case, average composition is just the same as the actual composition. The second case is that in limit of α and β ? 50% while the compositions of raw materials are uncertain (ie, σ ? 0). This later case is when *X(50%)* = 0. In this second case, the operator has essentially chosen to flip a fair coin as to whether the compositional constraint will be satisfied.

In the results presented below using chance constraint, 99% confidence level was chosen for all compositions with 10% compositional standard deviation and zero compositional correlation between non-similar scraps. The comparison in Figure 7.1 is between two styles of managing scrap compositional uncertainty. The solid line represents the total amount of scrap usage under the chance constraint method. The vertical bars indicate the total amount of scrap used if the original compositional specification (max/min "window") would be reduced in percentage. For instance a 50% window for 9-11% Si would mean 9.5%-10.5% Si.

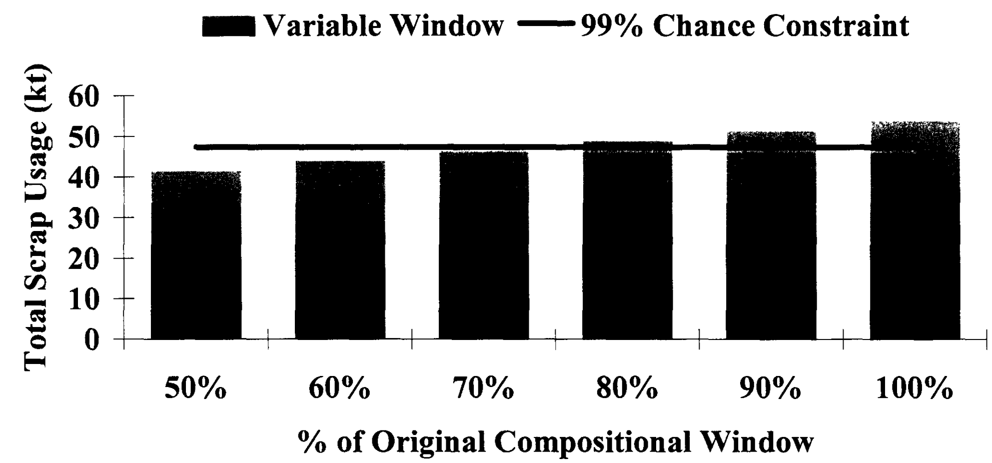

Figure 7.1 Comparison of total scrap usage between 99% confidence chance constraint method versus varying compositional specification window

Depending on the actual practice, Figure 7.1 shows that it is possible to induce greater overall scrap usage with the chance constraint method, even at 99% confidence level. It should be noted that in this figure, and the other results presented in this chapter, the compositions on the scrap materials have been averaged rather than taken as a range, as such the scrap consumption and cost benefits of chance constraint over the "window" narrowing method is conservative as presented here. Of course, with greater confidence requirement and higher inherent scrap compositional uncertainty, the likelihood of the chance constraint method inducing greater scrap consumption will be lessened. Nevertheless, with the chance constraint method, there is a clear tolerance level for error and the magnitude of the underlying compositional uncertainty is directly tied to the decisions for amount of scrap use.

If the actual usage of scrap materials is examined by pile (recall that there are 2 piles of each of the 7 scrap types), a critical result that is missing in deterministic and linear optimization tools is the importance of the principle of raw materials diversification. Figure 7.2 shows the usage of scrap materials by pile. The "1" and "2" following the scrap name refer to the duplicate piles. In reality, on the production floor there might be many more than just two. However, the arguments to follow still hold. What this figure shows is that for any given scrap type, the usage of scrap for duplicate piles are the same, evenly split between them. In contrast, in a linear deterministic optimization framework, there will be many degenerate solutions involving one pile or the other or some random combination of the two. In other words, one possible flawed solution would involve selecting all from Brake 1 or all from Brake 2. The reason this is flawed and that the correct choice is to have split amounts from both piles is a key result of the principle of raw material diversification.

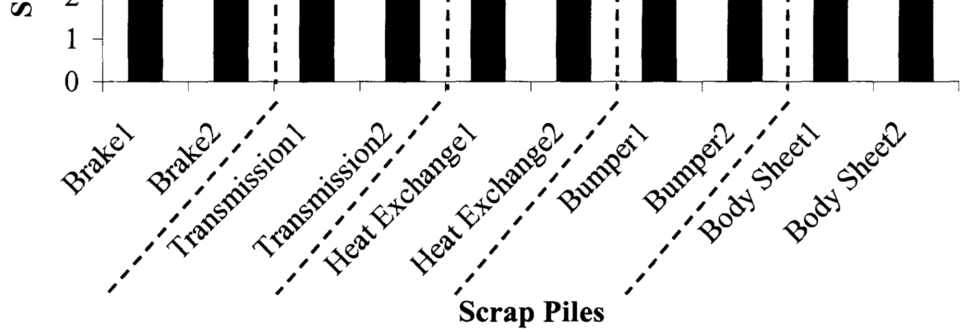

Figure 7.2 Under chance constraint method the amount of scrap usage by pile.

The principle of raw material diversification states that by spreading scrap usage amongst piles of scrap materials that are uncorrelated or weakly correlated, the overall compositional uncertainty can be reduced. The reduction in compositional uncertainty is important because as seen earlier, it leads directly to an increase in the system's ability to use scrap, not to the mention the peace of mind for the operator! A simple derivation will illustrate this important concept and why in this particular case the solution leads to even amounts of scrap use from duplicate piles.

Suppose one has to choose an amount c from two piles of identical and compositionally uncorrelated scrap material. This amount can be, for example, an optimal amount that has been determined as the overall amount of usage for that particular scrap type in a production scenario. Let the decision variables be x_1 and x_2. Furthermore let the compositional uncertainty be abstractly referred to as σ_1 and σ_2. Since the two piles are compositionally identical but uncorrelated, $\rho_{12}=0$, and $\sigma_1=\sigma_2=\sigma$. Then the choices of x_1 and x_2 that will yield the lowest overall compositional uncertainty and therefore the least constrained system are calculated as follow.

Min: $\sigma_T = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_1 x_2 \rho_{12}} = \sqrt{x_1^2\sigma^2 + x_2^2\sigma^2}$

s.t.: $x_1 + x_2 = c$; non-negative x_1 and x_2

Therefore the overall compositional variation can be rewritten as:

$$\sigma_T = \sqrt{x_1^2\sigma^2 + (c - x_1)^2\sigma^2} = \sqrt{x_1^2\sigma^2 + x_1^2\sigma^2 + c^2\sigma^2 - 2cx_1\sigma^2}$$

To find the x_1 and x_2 that produces the lowest σ_T, this expression is differentiated with respect to σ_T and set to zero.

$$\frac{\partial \sigma_T}{\partial x_1} = \frac{1}{2}\frac{4x_1\sigma^2 - 2c\sigma^2}{\sqrt{x_1^2\sigma^2 + (c-x_1)^2\sigma^2}} = 0$$

$$\rightarrow x_1 = x_2 = c/2$$

Had the correlation between the two piles been non-zero, the solution would have been different and not evenly half-half. However, as long as the correlation is not perfect, there should be a split in usage between the two piles. This is not captured in linear deterministic formulations.

Although materials diversification can lead to greater scrap usage while managing the aggregate compositional uncertainty in production, it will not benefit all scrap types consumption equally even though overall scrap usage increased from 43kt to 47kt. Figure 7.3 compares scrap consumption with and without raw material diversification practice among similar scrap piles. Not having this practice means forcing the operator to use only one of the two piles of available scrap for each scrap type. Bear in mind that even when forcing the operator to use just one of the two piles, the system is being optimized (albeit with its hands tied behind its back!). This optimization naturally leads some of the compositions to be maxed out in the production specification since scrap is cheap. When the system attempts to allow for the flexibility of raw materials diversification and re-optimize, some of the scrap usage that are already at the maximum compositional allowances must decrease in order to allow other scrap usage to increase. The increase (or decrease) in a certain scrap's usage will affect all compositional requirements since all the elements in a scrap type are coupled.

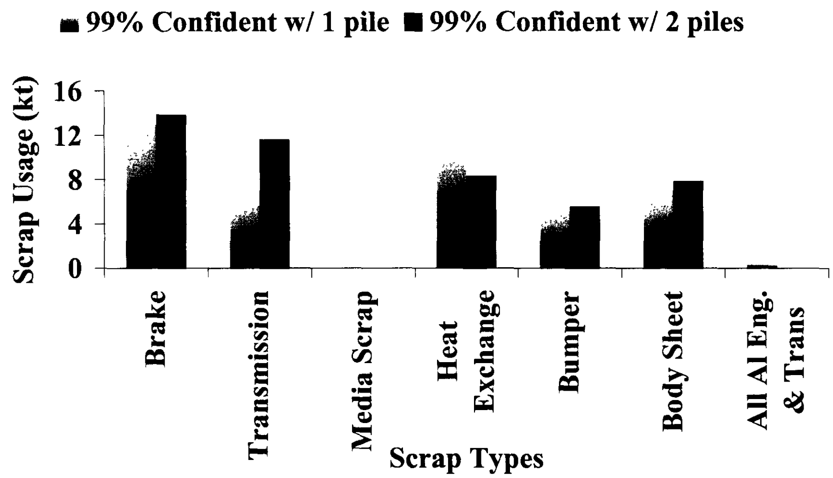

Figure 7.3 Scrap consumption under chance constraints with and without raw materials diversification practice among similar scrap piles.

The principle of raw material diversification is also apparent from another perspective. In Figure 7.4, a comparison is made between the amount of different types of scraps used in the production under chance constraint method and compositional window shrinkage method. The choice was made to compare the 70% window because this leads to the total amount of scrap use closest to that for 99% confident chance constraints. While Figure 7.1 indicates that at this window size (70%), the chance constraint method leads to a slightly greater but similar amount of scrap use, the distribution of usage between the various scrap types changed quite a bit. It is noticeable that the amount of scrap usage among scraps tended to “even out” with the chance constraint method versus the linear and deterministic window shrinkage method. It is apparent, for example, that the gap between the maximum and minimum amount of scrap usage among the used scrap types became smaller under the chance constraint method. Again, this is a direct result of the principle of raw material diversification. There is a benefit to overall compositional variation reduction to spread out the usage of scrap materials with associated economic benefit, especially when the correlations among compositions are weak.

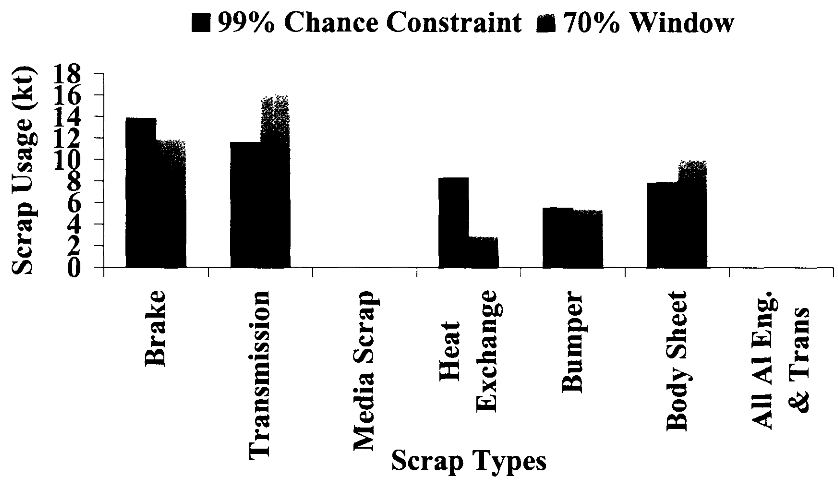

Figure 7.4 Comparison in scrap type usage between chance constraint (99%) method and compositional window (70%) method.

7.4 Summary on Compositional Variations in Batch Production

Compositional uncertainty with scrap materials is probably one of the biggest headaches that the operator has to deal with. This chapter presented the chance constraint method as a robust methodology that ties this uncertainty directly to an operational tolerance level for compositional misspecification. In doing so, scrap usage can be tuned according to the evolution of variations in scrap compositions and the operator will have a good sense of the likelihood of the need for reworking. This technique also eliminates the dilemma of which scrap piles to use when faced with compositionally similar but uncorrelated scrap supplies. It also brought forth the concept of raw material diversification as a natural way of controlling scrap compositional variations. Nevertheless, question remains as to whether this technique can actually improve scrap consumption or produce cost savings in practice. The answer depends on knowledge of actual industry practice, frequency of reworking and the associated costs. It is clear, however, that with lower tolerance for misspecification and higher scrap compositional uncertainties, the benefits from this technique is reduced. In any case, the application of this technique towards production level compositional uncertainty is promising but still preliminary based on current studies.

Chapter 8 : Modeling Multiple Sources of Uncertainties and Scrap Management Inefficiencies

The previous chapters have presented detailed discussions of methods for dealing with different sources of uncertainties in decision-making and provided detailed results evaluating production strategies to address inefficiencies in scrap usage. In particular, demand uncertainties, compositional uncertainties and the mixed nature of scrap materials have been considered as causes of inefficient scrap usage. While some of these sources of uncertainties affect a single level of operation within the aluminum production chain, others confound decisions simultaneously throughout corporation. For instance, at the plant operator level, there are no uncertainties regarding supply or demand, the only source of uncertainty at the time of production is compositional uncertainty regarding the scrap materials. However, while the plant bears the unavoidable responsibility to deal with scrap compositional variations, should the purchasing department also consider compositional uncertainties besides demand uncertainties, and how should this be modeled? Furthermore, without a doubt, sorting is an operation that is plagued with uncertainties. What are the sources of such uncertainties and how should one deal with them? This chapter attempts to answer questions such as these. In doing so, basic modeling framework will be presented and some of the data requirements as well as obstacles to such efforts will be discussed.

8.1 Simultaneous Considerations for Demand & Compositional Uncertainties

The uncertainties surrounding demand and compositions are orthogonal to each other in terms of real interactions as well as from a modeling perspective. As such it is rather straight forward to incorporate the two into a unified model framework. In fact, referring back to Chapter 6, the mathematical formulation does not change for Eq 4.1 to Eq 4.5. However, the compositional constraints will become stochastic and have to be transformed into their deterministic equivalent. The transformation is realized through none other than a chance constraints formulation. The result is a hybrid model with both recourse and chance constraint elements. The transformations are presented below with new terminologies explained at the end of this section.

With the simultaneous considerations for demand and compositional uncertainties, hedging operations are more complex to monitor since there are now two sources of variability. However, in return for slightly more complexity and computational resources, this practice allows purchasing to differentiate scrap suppliers not only by price, but also in their abilities to deliver consistent compositional specifications.

$$\sum_s D^1_{sfz} U_{sc} + \sum_p D^2_{pfz} U_{pc} \le B_{fz} U_{fc} \rightarrow$$

Eq 8.1 $$\sum_s D^1_{sfz} \overline{U}_{sc} + \sum_s D^2_{pfz} U_{pc} + X(\alpha_{fc})(\sum_s \sum_{s'} \rho^U_{ss'c} \sigma^U_{sc} \sigma^U_{s'c} D_{sfz} D_{s'fz})^{1/2} \le B_{fz} U_{fc}$$

$$\sum_s D^1_{sfz} L_{sc} + \sum_p D^2_{pfz} L_{pc} \ge B_{fz} L_{fc} \rightarrow$$

Eq 8.2 $$\sum_s D^1_{sfz} \overline{L}_{sc} + \sum_s D^2_{pfz} L_{pc} + X(1-\beta_{fc})(\sum_s \sum_{s'} \rho^L_{ss'c} \sigma^L_{sc} \sigma^L_{s'c} D_{sfz} D_{s'fz})^{1/2} \ge B_{fz} L_{fc}$$

D^2_{pfz} = amount of primary material p to be used for the production of finished good f under scenario z
D^1_{sfz} = amount of scrap material s used in making finished good f under scenario z
B_{fz} = amount of finished good f produced under scenario z
U_{sc} = max. amount (wt. %) of element c in scrap material s
$\overline{U}_{sc}$ = average max. amount (wt. %) of element c in scrap material s
σ^U_{sc} = standard deviation of the max. amount (wt%) of element c in scrap material s
L_{sc} = min. amount of element c in scrap material s
$\overline{L}_{sc}$ = average min. amount (wt. %) of element c in scrap material s
σ^L_{sc} = standard deviation of the min. amount (wt%) of element c in scrap material s

α_{fc} = confidence level (%) for compositional constraint with respect to the maximum amount of element c in product f
β_{fc} = confidence level (%) for compositional constraint with respect to the minimum amount of element c in product f
$\rho^{U}_{ss'c}$ = correlation coefficient between max. composition c of scrap materials s and s'
$\rho^{L}_{ss'c}$ = correlation coefficient between min. composition c of scrap materials s and s'
U_{pc} = max. amount of element c in primary material p
L_{pc} = min. amount of element c in primary material p
U_{fc} = max. amount of element c in product f
L_{fc} = min. amount of element c in product f

8.2 The Roots of Uncertainties in Sorting Operations

The modeling of uncertainties in sorting operations is more challenging both from a mathematical formulation perspective as well as from an informational requirement standpoint. First of all, what kinds of uncertainties are realistic in sorting? Baring liberation issues[29], interactions with industry experts[30] reveal that there are primarily two kinds of uncertainties. They are distinct and independent of each other. One such source of uncertainty relates to the sorter itself. While on average a sorter might sort out R% of cast fractions, there can be deviation from this performance during actual operations. The second source of uncertainty comes from the scrap materials themselves. Anyone who has ever handled scraps in a production environment knows that they are compositionally uncertain. In order to account for such uncertainties in the modeling framework, statistical information regarding these two sources of deviations are required. In the case of the sorter, it is sufficient that the modeler has access to the expected performance of the sorter as well as likely variations quantified as standard deviations or variances. Therefore it is tempting to assume that simply knowing the mean and standard deviation of the compositional content of input scrap streams is enough to account for

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[29] One liberation issue in aluminum recycling is the attachment of non-aluminum content to aluminum scrap pieces. Liberation in general refers to the clean break of discrete components from others.
[30] Industry experts involved in aluminum recycling studies and technology assessment.

compositional uncertainties in a sorting model. However, this is not the case. In fact, it is the characterization of the accuracy in determining what constitutes a piece of scrap and the fluctuation in performance of the sorting recovery process that are the key requirements.

As a counter-example, take a simple scrap stream with two independent components x and y (eg, x is 6061 and y represents 380). And for the sake of simplicity suppose there is only one element of interest. For the moment also assume that it is known for certain that the scrap stream consists of exactly 40% x and 60% y. This scrap proceeds with a single stage sort between components x and y with perfect sorting, as depicted in Figure 8.1.

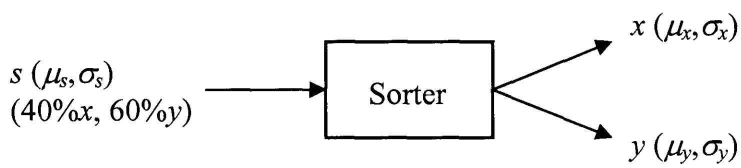

Figure 8.1 A simple scrap stream with two distinct components under perfect sorting

Consider now the case where the mean and standard deviation of this input scrap stream, ie μ_s, σ_s, are known. Based on this information, is it possible to determine μ_x, σ_x, μ_y and σ_y? These are absolutely necessary in order to fully characterize the statistical nature of the sorted output scrap streams. They are also indispensable to formulate probabilistic constraints on produced compositions. The following relationships exist between these sorted and unsorted scrap streams:

$$\sigma_s^2 = (40\%)^2 \sigma_x^2 + (60\%)^2 \sigma_x^2$$

Clearly with one equation and two unknown, it is impossible to uniquely determine the variability of both scrap streams. In fact, it is even impossible to pinpoint the average compositions of the resulting scrap streams. However, this begs the question: are the uncertainties surrounding compositions really associated with the composition of individual components of the scrap streams? After all, suppose one can perfectly characterize the individual components of the scrap streams as 40% 6061 and 60% 380, are the compositional ranges not already deterministic? Certainly with products that have come off the production lines of major aluminum manufacturers, those specifications will by and large be strictly met. In that case, there are *no* uncertainties around the compositional maximum and minimum values of the scrap components. If so, current deterministic treatment of sorting can already handle the modeling requirements. But anyone who deals with scraps will disagree on the basis that scrap streams do not have absolute maximum and minimum range of compositions within which they will fall. So what causes this jitter in scrap compositions? The answer lies exactly with the characterization of scrap components makeup. In fact, materials processors do *not* know exactly that a certain scrap stream is 40% 6061 and 60% 380. They might know on average, but there are uncertainties surrounding this estimate.

Therefore in order to model sorting under uncertainties, material processors have to perform statistical metrology on their sorting operations. In particular, the sorter needs to be characterized and the people or machinery that performs the materials identification process must also be characterized. These characterizations will then yield the expected accuracy and standard deviations of such operations. The modeling efforts, among other things, will then be a way to quantify the associated benefits associated with the reduction of such variability.

8.3 Modeling Uncertainties in Sorting Operations

At the moment, statistical metrology is certainly not readily available with all sorting operations. However, assuming such information becomes available in the future, the modeling of the uncertainties associated with the discussions in the previous section can be carried out with chance constraint methods. The formulation is a bit complex but can be reasoned through basic statistical arguments. As with other parts of this thesis, details of the statistical concepts referenced in this section can be found in Appendix C. In the following the ideas will be conveyed with a single scrap stream, a single product, and a one-step sorter with two output streams. Primaries and additive elements are not considered as they do not contribute to the sorting framework; including them here will serve no purpose other than to mutter the conceptual clarity. Implicitly this means the following simplified problem assumes the finished good can be made entirely from scrap. The input scrap stream has two components[31], *A* and *B*, with uncertain percentage representation. For the purpose of consistency, the variables and terminologies in this section resemble those used in Chapter 4. The formulation will be discussed and explanations for the variables are presented below.

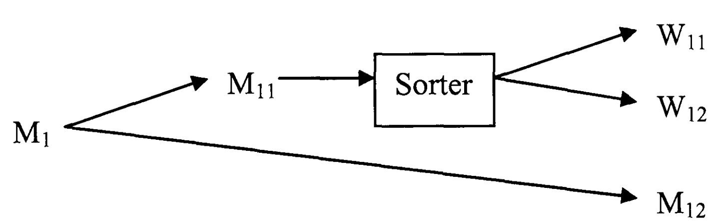

Figure 8.2 A single scrap stream going through sorting with two sorted streams

[31] A component can be an alloy like 6061

Table 8-I Stochastic settings of sorting operations

	A	B
Scrap Component Representation (%)	C_{1A}	C_{1B}
Average scrap component representations	μ_{1A}	μ_{1B}
Standard deviation of scrap components representation	σ_{1A}	σ_{1B}
Recovery rate (%) of components in output stream W_{11}	R_A	R_B
Average recovery rate (%) of components in output stream W_{11}	μ_A	μ_B
Standard deviation of recovery rates	σ_A	σ_B

The objective function is:

$$Min: \quad C_1(M_{11}+M_{12})+Z_1M_{11}$$

Scrap availability is one constraint:

$$M_{11}+M_{12}\le A_1$$

The recovery of individual components from the input scrap streams into the two output streams are determined by the makeup of the input scraps as well as the recovery rate of the sorter on the individual components:

$$E[M_{11}C_{1A}R_A]=E[W_{11}^A] \qquad E[M_{11}C_{1A}(1-R_A)]=E[W_{12}^A] \qquad E[M_{12}C_{1A}]=E[M_{12}^A]$$

$$E[M_{11}C_{1B}R_B]=E[W_{11}^B] \qquad E[M_{11}C_{1B}(1-R_B)]=E[W_{12}^B] \qquad E[M_{12}C_{1B}]=E[M_{12}^B]$$

$$E[W_{11}^A+W_{11}^B]=E[W_{11}] \qquad E[W_{12}^A+W_{12}^B]=E[W_{12}]$$

Notice in the above the differentiation between $E[.]$ terms and other variables such as M_{11} and M_{12}. While they are all variables to be determined in the calculations, the difference is that the $E[.]$ terms are stochastic variables (they have elements of uncertainties associated), while M_{11} and M_{12} are deterministic decision variables. For materials allocation:

$$\Pr\{P_{M12}M_{12}+P_{W11}W_{11}+P_{W12}W_{12}\ge F_1\}\ge\varpi_1$$

Similarly for the compositional constraint:

$$\Pr\{P_{M12}(M_{12}^{A}A_{\max}^{e}+M_{12}^{B}B_{\max}^{e})+P_{W11}(W_{11}^{A}A_{\max}^{e}+W_{11}^{B}B_{\max}^{e})+$$
$$P_{W12}(W_{12}^{A}A_{\max}^{e}+W_{12}^{B}B_{\max}^{e})\leq F_{1}F_{1}^{e,\max}\}\geq\alpha_{1}^{e}$$

$$\Pr\{P_{M12}(M_{12}^{A}A_{\min}^{e}+M_{12}^{B}B_{\min}^{e})+P_{W11}(W_{11}^{A}A_{\min}^{e}+W_{11}^{B}B_{\min}^{e})+$$
$$P_{W12}(W_{12}^{A}A_{\min}^{e}+W_{12}^{B}B_{\min}^{e})\geq F_{1}F_{1}^{e,\min}\}\geq\beta_{1}^{e}$$

All variables are non-negative and $1\geq P_{M12},P_{W11},P_{W12}\geq 0$.

Once again in order to work with these stochastic constraints, one must differentiate between which terms are the real stochastic elements. The component compositions are deterministic, at least their range is known, therefore terms like $A_{\max}^{e},B_{\min}^{e}$ are constants. The terms like P_{W11},P_{M12} are variables, but they are decision variables that are chosen as part of the optimization. The only stochastic variables are the following:

$$M_{12}^{A},M_{12}^{B},W_{11}^{A},W_{12}^{A},W_{11}^{B},W_{12}^{B}$$

In order to transform the formulae above into their deterministic equivalents, the expected value, variances and covariance of these terms are required. Therefore before proceeding with the deterministic equivalent transformation one must calculate these terms. The basis for the details of the following expressions can be found in Appendix C.

$$E[W_{11}^{A}]=M_{11}\mu_{1A}\mu_{A} \qquad E[W_{11}^{B}]=M_{11}\mu_{1B}\mu_{B}$$

$$E[W_{12}^{A}]=M_{11}(\mu_{1A}-\mu_{1A}\mu_{A}) \qquad E[W_{12}^{B}]=M_{11}(\mu_{1B}-\mu_{1B}\mu_{B})$$

$$E[M_{12}^{A}]=M_{12}\mu_{1A} \qquad E[M_{12}^{B}]=M_{12}\mu_{1B}$$

$$Var[W_{11}^{A}]=M_{11}^{2}(\sigma_{1A}^{2}\mu_{A}^{2}+\sigma_{A}^{2}\mu_{1A}^{2}+\sigma_{1A}^{2}\sigma_{A}^{2})$$

$$Var[W_{11}^{B}]=M_{11}^{2}(\sigma_{1B}^{2}\mu_{B}^{2}+\sigma_{B}^{2}\mu_{1B}^{2}+\sigma_{1B}^{2}\sigma_{B}^{2})$$

$$Var[W_{12}^{A}]=M_{11}^{2}[\sigma_{1A}^{2}+(\sigma_{1A}^{2}\mu_{A}^{2}+\sigma_{A}^{2}\mu_{1A}^{2}+\sigma_{1A}^{2}\sigma_{A}^{2})-2\mu_{A}(\mu_{1A}^{2}+\sigma_{1A}^{2})+2\mu_{1A}^{2}\mu_{A}^{2}]$$

$$Var[W_{12}^B] = M_{11}^2[\sigma_{1B}^2 + (\sigma_{1B}^2\mu_B^2 + \sigma_B^2\mu_{1B}^2 + \sigma_{1B}^2\sigma_B^2) - 2\mu_B(\mu_{1B}^2 + \sigma_{1B}^2) + 2\mu_{1B}^2\mu_B^2]$$

$$Var[M_{12}^A] = M_{12}^2\sigma_{1A}^2$$

$$Var[M_{12}^B] = M_{12}^2\sigma_{1B}^2$$

In considering the covariance, without considering the actual correlation between the six stochastic variables, there can be $\binom{6}{2} = \frac{6!}{(6-2)!2!} = 15$ such covariance terms. However, if one examines the actual formulae for the six stochastic variables, only those variables with common terms in the formulae will be correlated. This puts a bound on the number of covariance to be determined at just six.

$$\begin{aligned}
Cov[W_{11}^A, W_{12}^A] &= M_{11}^2E[(C_{1A}R_A - \mu_A\mu_{1A})(C_{1A} - C_{1A}R_A - \mu_{1A} + \mu_A\mu_{1A})] \\
&= M_{11}^2(E[C_{1A}^2]\mu_A - E[C_{1A}^2R_A^2] - \mu_{1A}^2\mu_A + \mu_{1A}^2\mu_A^2) \\
&= M_{11}^2((\mu_{1A}^2 + \sigma_{1A}^2)\mu_A - (\mu_{1A}^2\mu_A^2 + \sigma_{1A}^2\mu_A^2 + \sigma_A^2\mu_{1A}^2 + \sigma_{1A}^2\sigma_A^2) - \mu_{1A}^2\mu_A + \mu_{1A}^2\mu_A^2) \\
&= M_{11}^2(\sigma_{1A}^2\mu_A - \sigma_{1A}^2\mu_A^2 - \sigma_A^2\mu_{1A}^2 - \sigma_{1A}^2\sigma_A^2)
\end{aligned}$$

$$Cov[W_{11}^B, W_{12}^B] = M_{11}^2E[(C_{1B}R_B - \mu_B\mu_{1B})(C_{1B} - C_{1B}R_B - \mu_{1B} + \mu_B\mu_{1B})]$$

$$\begin{aligned}
Cov[W_{11}^A, M_{12}^A] &= M_{11}M_{12}E[(C_{1A}R_A - \mu_A\mu_{1A})(C_{1A} - \mu_{1A})] \\
&= M_{11}M_{12}E[C_{1A}^2\mu_A - \mu_{1A}C_{1A}R_A - \mu_A\mu_{1A}C_{1A} + \mu_A^2\mu_{1A}) \\
&= M_{11}M_{12}((\mu_{1A}^2 + \sigma_{1A}^2)\mu_A - \mu_{1A}^2\mu_A - \mu_{1A}^2\mu_A + \mu_A^2\mu_{1A}) \\
&= M_{11}M_{12}(\sigma_{1A}^2\mu_A - \mu_{1A}^2\mu_A + \mu_A^2\mu_{1A})
\end{aligned}$$

$$Cov[W_{11}^B, M_{12}^B] = M_{11}M_{12}(\sigma_{1B}^2\mu_B - \mu_{1B}^2\mu_B + \mu_B^2\mu_{1B})$$

$$\begin{aligned}
Cov[W_{11}^A, M_{12}^A] &= M_{11}M_{12}E[(C_{1A}R_A - \mu_A\mu_{1A})(C_{1A} - \mu_{1A})] \\
&= M_{11}M_{12}E[C_{1A}^2\mu_A - \mu_{1A}C_{1A}R_A - \mu_A\mu_{1A}C_{1A} + \mu_A^2\mu_{1A}) \\
&= M_{11}M_{12}((\mu_{1A}^2 + \sigma_{1A}^2)\mu_A - \mu_{1A}^2\mu_A - \mu_{1A}^2\mu_A + \mu_A^2\mu_{1A}) \\
&= M_{11}M_{12}(\sigma_{1A}^2\mu_A - \mu_{1A}^2\mu_A + \mu_A^2\mu_{1A})
\end{aligned}$$

$$Cov[W_{11}^B, M_{12}^B] = M_{11}M_{12}(\sigma_{1B}^2\mu_B - \mu_{1B}^2\mu_B + \mu_B^2\mu_{1B})$$

$$Cov[W_{12}^A, M_{12}^A] = M_{11}M_{12}E[(C_{1A} - C_{1A}R_A - \mu_{1A} + \mu_A\mu_{1A})(C_{1A} - \mu_{1A})]$$
$$= M_{11}M_{12}(\sigma_{1A}^2 - \sigma_{1A}^2\mu_A)$$

$$Cov[W_{12}^B, M_{12}^B] = M_{11}M_{12}(\sigma_{1B}^2 - \sigma_{1B}^2\mu_B)$$

With these expected values, variances and covariance established, the deterministic equivalent problem formulation can be stated. In particular note that the only information needed regarding the stochastic elements is their individual expected value and variance (or standard deviation). The objective function does not change since there is no explicit dependence on any stochastic term.

$$Min: \quad C_1(M_{11} + M_{12}) + Z_1M_{11}$$

The scrap supply constraint also does not change.

$$M_{11} + M_{12} \leq A_1$$

The scrap streams recovery is restated as:

$$E[W_{11}^A] = M_{11}\mu_{1A}\mu_A \qquad E[W_{11}^B] = M_{11}\mu_{1B}\mu_B$$

$$E[W_{12}^A] = M_{11}(\mu_{1A} - \mu_{1A}\mu_A) \qquad E[W_{12}^B] = M_{11}(\mu_{1B} - \mu_{1B}\mu_B)$$

$$E[M_{12}^A] = M_{12}\mu_{1A} \qquad E[M_{12}^B] = M_{12}\mu_{1B}$$

$$E[W_{11}] = M_{11}(\mu_{1A}\mu_A + \mu_{1B}\mu_B) \qquad E[W_{12}] = M_{11}(\mu_{1A} + \mu_{1B} - \mu_A\mu_{1A} - \mu_B\mu_{1B})$$

The materials allocation toward finished product will then make use of these terms according to the chance constraint method:

$$P_{M12}(E[M_{12}^A] + E[M_{12}^B]) + P_{W11}(E[W_{11}^A] + E[W_{11}^B]) + P_{W12}(E[W_{12}^A] + E[W_{12}^B])$$
$$+ X(1 - \varpi_1)(P_{M12}^2(Var[M_{12}^A] + Var[M_{12}^B]) + P_{W11}^2(Var[W_{11}^A] + Var[W_{11}^B])$$
$$+ P_{W12}^2(Var[W_{12}^A] + Var[W_{12}^B]) + 2P_{W11}P_{W12}(Cov[W_{11}^A, W_{12}^A] + Cov[W_{11}^B, W_{12}^B])$$
$$+ 2P_{W11}P_{M12}(Cov[W_{11}^A, M_{12}^A] + Cov[W_{11}^B, M_{12}^B])$$
$$+ 2P_{W12}P_{M12}(Cov[W_{12}^A, M_{12}^A] + Cov[W_{12}^B, M_{12}^B]))^{1/2} \geq F_1$$

Similarly the compositional constraints will also appeal to the chance constraint method:

$$
\begin{aligned}
&P_{M12}(A^{e}_{\min}E[M^{A}_{12}]+B^{B}_{\min}E[M^{B}_{12}])+P_{W11}(A^{e}_{\min}E[W^{A}_{11}]+B^{e}_{\min}E[W^{B}_{11}])+P_{W12}(A^{e}_{\min}E[W^{A}_{12}]+B^{e}_{\min}E[W^{B}_{12}])\\
&+X(1-\beta^{e}_{1})[P^{2}_{M12}((A^{e}_{\min})^{2}Var[M^{A}_{12}]+(B^{e}_{\min})^{2}Var[M^{B}_{12}])\\
&+P^{2}_{W11}((A^{e}_{\min})^{2}Var[W^{A}_{11}]+(B^{e}_{\min})^{2}Var[W^{B}_{11}])+P^{2}_{W12}((A^{e}_{\min})^{2}Var[W^{A}_{12}]+(B^{e}_{\min})^{2}Var[W^{B}_{12}])\\
&+2P_{W11}P_{W12}((A^{e}_{\min})^{2}Cov[W^{A}_{11},W^{A}_{12}]+(B^{e}_{\min})^{2}Cov[W^{B}_{11},W^{B}_{12}])\\
&+2P_{W11}P_{M12}((A^{e}_{\min})^{2}Cov[W^{A}_{11},M^{A}_{12}]+(B^{e}_{\min})^{2}Cov[W^{B}_{11},M^{B}_{12}])\\
&+2P_{W12}P_{M12}((A^{e}_{\min})^{2}Cov[W^{A}_{12},M^{A}_{12}]+(B^{e}_{\min})^{2}Cov[W^{B}_{12},M^{B}_{12}])]^{1/2}\geq F_{1}F_{1}^{e,\min}
\end{aligned}
$$

$$
\begin{aligned}
&P_{M12}(A^{e}_{\max}E[M^{A}_{12}]+B^{e}_{\max}E[M^{B}_{12}])+P_{W11}(A^{e}_{\max}E[W^{A}_{11}]+B^{e}_{\max}E[W^{B}_{11}])\\
&+P_{W12}(A^{e}_{\max}E[W^{A}_{12}]+B^{e}_{\max}E[W^{B}_{12}])\\
&+X(\alpha^{e}_{1})[P^{2}_{M12}((A^{e}_{\max})^{2}Var[M^{A}_{12}]+(B^{e}_{\max})^{2}Var[M^{B}_{12}])\\
&+P^{2}_{W11}((A^{e}_{\max})^{2}Var[W^{A}_{11}]+(B^{e}_{\max})^{2}Var[W^{B}_{11}])\\
&+P^{2}_{W12}((A^{e}_{\max})^{2}Var[W^{A}_{12}]+(B^{e}_{\max})^{2}Var[W^{B}_{12}])\\
&+2P_{W11}P_{W12}((A^{e}_{\max})^{2}Cov[W^{A}_{11},W^{A}_{12}]+(B^{e}_{\max})^{2}Cov[W^{B}_{11},W^{B}_{12}])\\
&+2P_{W11}P_{M12}((A^{e}_{\max})^{2}Cov[W^{A}_{11},M^{A}_{12}]+(B^{e}_{\max})^{2}Cov[W^{B}_{11},M^{B}_{12}])\\
&+2P_{W12}P_{M12}((A^{e}_{\max})^{2}Cov[W^{A}_{12},M^{A}_{12}]+(B^{e}_{\max})^{2}Cov[W^{B}_{12},M^{B}_{12}])]^{1/2}\leq F_{1}F_{1}^{e,\max}
\end{aligned}
$$

After substituting for the *E*[.], *Var*[.] and *Cov*[.] terms, the problem statement will be stated entirely in terms of deterministic decision variables, constants, mean values of stochastic elements and variances of stochastic elements. The stochastic optimization has thus been transformed into a deterministic set of equations and constraints. The following legend explains the various terms, most of which should be familiar from Chapter 4 and Chapter 7. All variables are non-negative.

i = Input scrap material index
n = Finished alloy index
m = Material (makeup) component index
q = Sort stage index
j = Output stream index
k = Stage two sort output stream index
C_i = Cost (per unit wt.) of scrap material i
Z_q = Cost of sorting (per unit wt.) for sort stage q
M_i = Quantity of input scrap material i
M_{i1} = Quantity of input scrap material i that went through sorting
M_{i2} = Quantity of input scrap material i that did not go through sorting

$M^{m}{}_{i2}$ = Quantity of component m in M_{i2}
$m_{max}{}^{e}$ = Maximum wt. % content of element e in component m
$m_{min}{}^{e}$ = Minimum wt. % content of element e in component m
A_i = Quantity of availability scrap material i
W_{ij} = Quantity of output into stream j from stage one sorting with input M_i
$W^{m}{}_{ij}$ = Quantity of component m in W_{ij}
C_{im} = Wt. % representation of material component m in raw material i
R_m = Recovery rate (%) of material component m in output stream index 1
$F_n^{e,max}$ = Maximum wt. % content of element e allowed in product F_n
$F_n^{e,min}$ = Minimum wt. % content of element e allowed in product F_n
P_{xxx} = % of material stream xxx allocated towards finished good
$\omega^{e}{}_{n}$ = Confidence level for meeting batch amount in finished good n
$\alpha^{e}{}_{n}$ = Confidence level for max. content of element e in finished good n
$\beta^{e}{}_{n}$ = Confidence level for min. content of element e in finished good n
R_m = Recovery rate of component m in output stream index 1
μ_m = Expected recovery rate of component m in output stream index 1
σ_m = Std. dev. of recovery rate of component m in output stream index 1
C_{im} = % representation of component m in input scrap stream i
μ_{im} = Expected % representation of component m in input scrap stream i
σ_{im} = Std. dev. of % representation of component m in input scrap stream i

8.4 Summary on Modeling Multiple Uncertainties

Simultaneous considerations of multiple sources of uncertainties might be important for certain stages of aluminum recycling operations. This chapter considered two candidates — demand and compositional uncertainties in scrap purchasing, and scrap content and recovery performance uncertainties during sorting. As demonstrated, stochastic optimization techniques such as recourse modeling and chance constraints can be applied for these studies. Beyond the mathematical framework, equally important are the collection of statistical information that are required as input for such studies. By accounting for both demand and compositional uncertainty, material purchasers can rank scrap suppliers not only in terms of price, but compositional diligence as well. All else being equal, scraps exhibiting poor compositional control will likely constitute less of any hedge basket. Another potentially interesting question is what level of compositional uncertainty will alter the relative marginal benefits of various scrap supplies. The modeling of multiple sources of uncertainties in sorting operations can provide guidance on the economic and scrap usage impact of reduction in such inaccuracy pitfalls. For instance, if the effects of the accuracy in characterizing scrap content are relatively insignificant compared to sorter performance then more energy can be spent on improving and controlling the recovery rates.

Chapter 9 : Conclusions

Key ideas that were explored in this thesis include hedging operations in scrap purchasing, sorting technologies and raw materials diversification. The common thread that ties them together is the emphasis on unlocking traditionally hidden value in scrap materials. The goal is to promote greater scrap consumption. While there is a common theme, each idea and strategy tackles a different kind of inefficiency in the current state of scrap usage.

9.1 Recourse-based Scrap Purchasing Strategies

> *The hedge amount, and the associated economic and scrap consumption impact, increases with demand uncertainties and price differential advantage of secondary over primary materials, and decreases with the carry cost of the hedging operations.*

Hedging in the context of this thesis is the action of buying a Δ basket of scrap materials on top of a set implied by expected production requirements. This action is the result of recourse-based modeling that brings out the option values of scraps and intimately ties them to the underlying demand uncertainty, salvage value and price differential between primary and secondary materials. Under favorable conditions of high demand uncertainty, high salvage value and large price differential between primary and secondary materials, the intensity of hedging increases as do the option values (Figure 9.1 and Figure 9.2). Hedging operations capitalizes on this hidden value and provides cost savings as well as scrap consumption benefits. When the option value is positive, it pays to purchase extra scrap beyond what is typically implied by deterministic analyses, thereby also directly leading to greater scrap consumption. The driving forces for deriving this option value led to explanations for the sensitivity of this value on salvage value and price differentials, as well as guidance on the frequency of and need for hedge rebalancing.

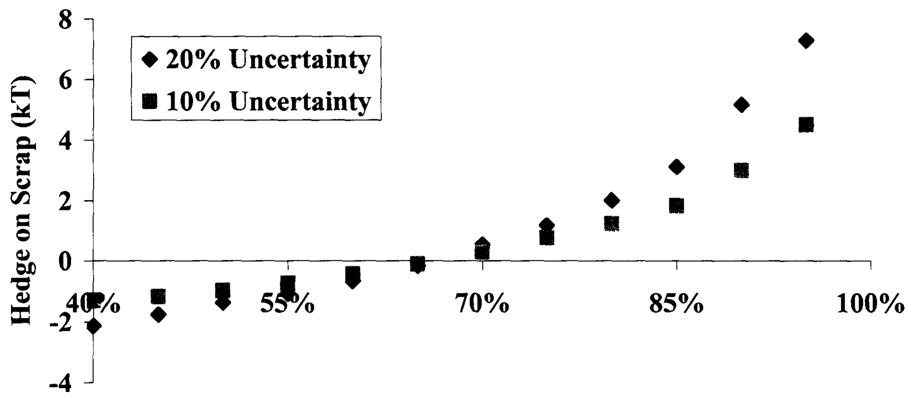

Figure 9.1. Effects of scrap salvage value on scrap pre-purchase hedging strategy.

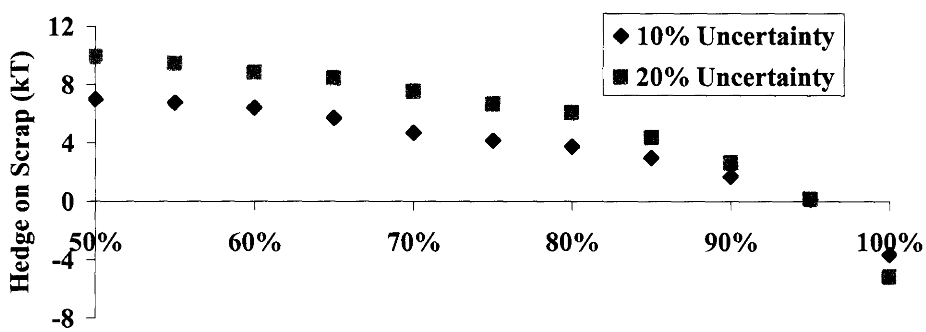

Figure 9.2. Effects of scrap-to-primaries price ratio on scrap pre-purchase hedging

9.2 Light Metals Sorting Strategies

> *The relative importance of wrought versus cast recovery rates depends significantly on the product mix. For a proven performance of 95% recovery, scrap consumption benefits can be economically justified for an implementation cost below \$20/t in the case of a large cast-oriented remelter.*

The concept of salvage value was carried over to the study of sorting technologies, in which the question is not whether sorting can promote greater scrap consumption, but under what circumstances. The answer lies within the control of the recovery performance, associated costs, scrap types and products being made. For instance, a cast-oriented operation will be more sensitive towards fluctuations in cast recovery rates (Figure 9.3). The EU case study showed at 95% recovery rates for cast and wrought fractions, sorting cost should stay below $20/t in order to be economical (figure reference). This finding did not change regardless of whether sorting is developed in-house, purchased or rented from a third-party service provider. When sorting technology is conceived as an investment, the utilization rate cannot fall below roughly 40%, at which point it will become too costly. Of course, actual deployment of sorting technologies is still at the mercy of scrap availability. However, it is also recognized that sorting technologies are disruptive in the sense that they have the potential to alter the economics of scrap usage. In particular, with sorting capabilities, it was shown that cheaper scraps no longer always lead to higher marginal benefits. Furthermore, attempts for increasing scrap consumption by alloy substitutions should be reexamined under sorting.

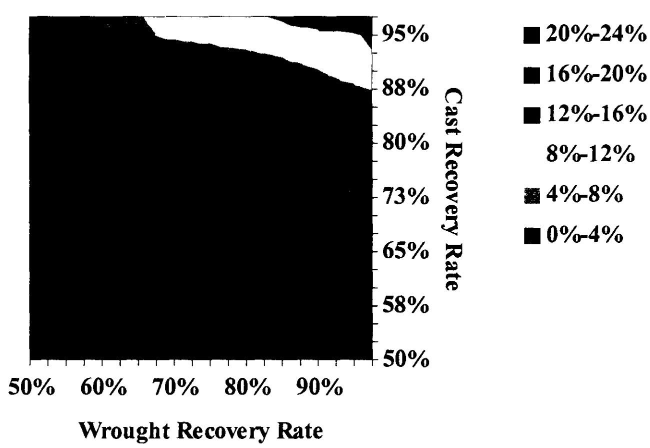

Figure 9.3 Percentage change in scrap consumption, sensitivity on recovery rates (Cast-Oriented Case).

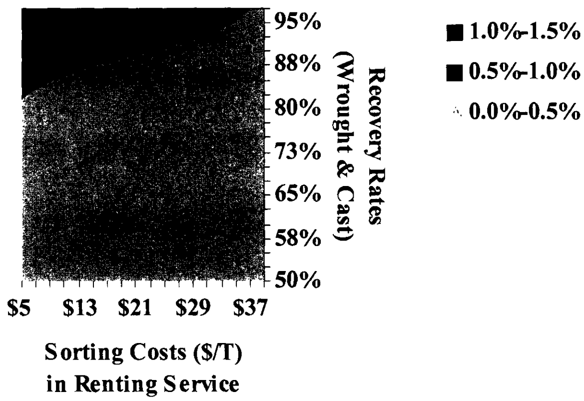

Figure 9.4. Percentage cost savings with respect to different recovery rates (wrought and cast) and sorting costs ($/t) under rent-for-service model.

9.3 Raw Materials Diversification Strategies in Materials Production

> *Tying tolerance for chemical mis-specifications to the statistical nature of scraps content promotes compositional diligence amongst scrap suppliers and raw materials diversification practice in remelting operations. Potential exists for economic and scrap usage benefits.*

Finally the idea of raw material diversification as a way to control the compositional uncertainty of scrap materials was born out of chance constraint arguments. Although information such as the likelihood of reworking and the associated costs have not been factored into the analysis, and should be in future work, this technique is arguably relevant towards promoting scrap consumption even without such considerations. Traditionally industry attempts to control products compositional variances by narrowing the chemical specifications. For a similar level of scrap consumption, chance constraints will even out the distribution of scrap usage (Figure 9.6), directly leading to a reduction in the overall produced compositional variance versus traditional practice. This has direct implications for promoting scrap consumption as well as the potential for cost savings (Figure 9.5). At the very least, chance constraint provides remelters a rigorous handle on tolerance for compositional variability. It also clearly provides incentives for scrap suppliers to do a better job with tracking, classifying and perhaps standardizing the compositions of their materials. Depending on actual practice by industry, chance constraint methods might improve scrap consumption.

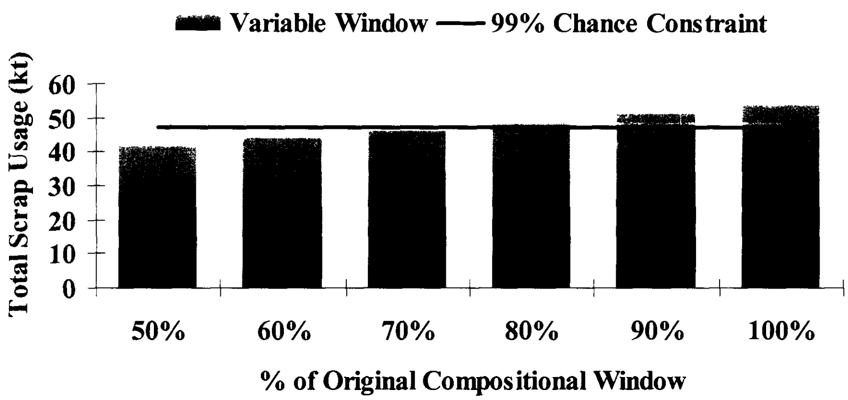

Figure 9.5 Comparison of total scrap usage between 99% confidence chance constraint method versus varying compositional specification window

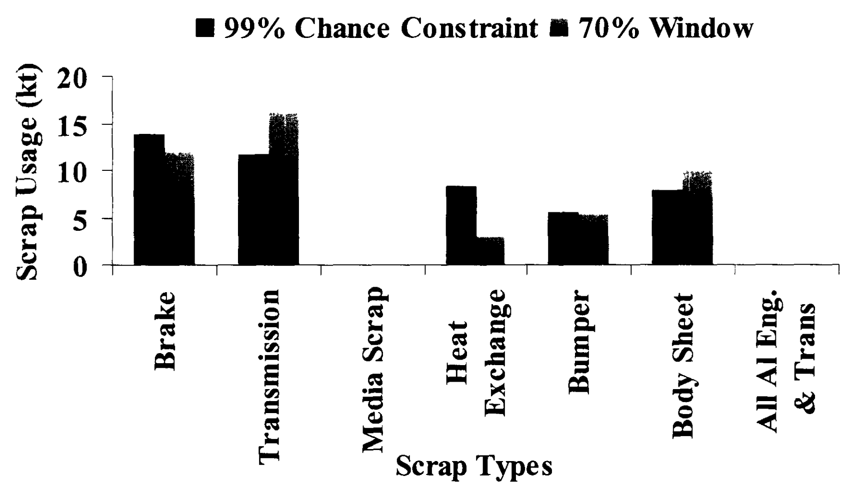

Figure 9.6 Comparison in scrap type usage between chance constraint (99%) method and compositional window (70%) method.

9.4 Comparative Summary of Light Metals Recycling Strategies

In order to place hedging, sorting and raw materials diversification on equal footing for comparison of their relative impact on scrap consumption and costs, these techniques are individually applied on the 100kt EU production case presented for the sorting case study in Chapter 6. Among the various factors that affect the results, only the sorting recovery rate is held constant at 95% (DeGaspari 1999). All other factors including demand uncertainty, primary vs. secondary price gap, salvage value, cost of sorting, and compositional uncertainty are examined for a range of values. These ranges are justified based on historical market and industry observations as well as best educated guess when factual observations are not available. Actual sources for ranges are clearly stated subsequently.

Scrap composition uncertainty ranges from 5% to 90% based on data from industry scrap suppliers. Historically over the last 10 years or so[32], the price ratio between primaries and average scraps has been bounded by 55% to 85% (Plunkert 2003). Furthermore, for the same period, the domestic apparent consumption of aluminum carried a demand uncertainty of up to 20% annually (Plunkert 2003). The average annual scrap price volatility, which can be used as a gauge of the direction for salvage value, assuming the scrap is held in inventory, is up to 40% (Kelly 2005, Plunkert 2003). For sorting cost, experts opined that it is currently at \$30/t. A lower bound has been estimated at \$10/t, leading to an average of \$20/t which is roughly how much it takes to sort steel and iron today.

[32] The period is from 1992 to 2003 post dissolution of the Soviet Union when the country flooded the market with aluminum partly in exchange for hard currency. This is arguably the last single extraordinary world event that affected aluminum pricing.

Based on these observations and estimations, Table 9-I summarizes the operational parameters that bound the high and low impact on scrap consumption and costs. The scrap consumption and cost impact given these operational parameters, and with respect to status quo, are given in Figure 9.7 and Figure 9.8 for the various strategies discussed in this thesis. For recourse-based hedging, the status quo is to treat demand as a point-forecast without regard for the implications of potential deviations. For wrought/cast sorting, the status quo is the lack of sorting capabilities. For the raw materials diversification practice via chance constraint, the reference state is to ignore the benefits of weak compositional correlation amongst scrap piles. Another possible comparison for the chance constraint method is relative to the practice of shrinking the compositional "window". However, without actual knowledge of industry practice, reworking rate and associated costs, a fair comparison cannot be made and will at best be a wild guess.

Table 9-I Most optimistic and pessimistic operating parameters for aluminum recycling strategies comparison.

	Recourse-Hedging		**Wrought/Cast Sorting**		**Raw Materials Diversification (99% Confidence)**	
	Optimistic	***Pessimistic***	***Optimistic***	***Pessimistic***	***Optimistic***	***Pessimistic***
Demand uncertainty[33]	20%	5%	n/a	n/a	n/a	n/a
Scrap composition uncertainty[34]	n/a	n/a	n/a	n/a	90%	5%
Salvage ratio[35]	95%	55%	95%	55%	n/a	n/a
Secondary/ primary price gap[36]	55%	85%	55%	85%	55%	85%
Sorting Cost ($/t)	n/a	n/a	$10.0	$30.0	n/a	n/a

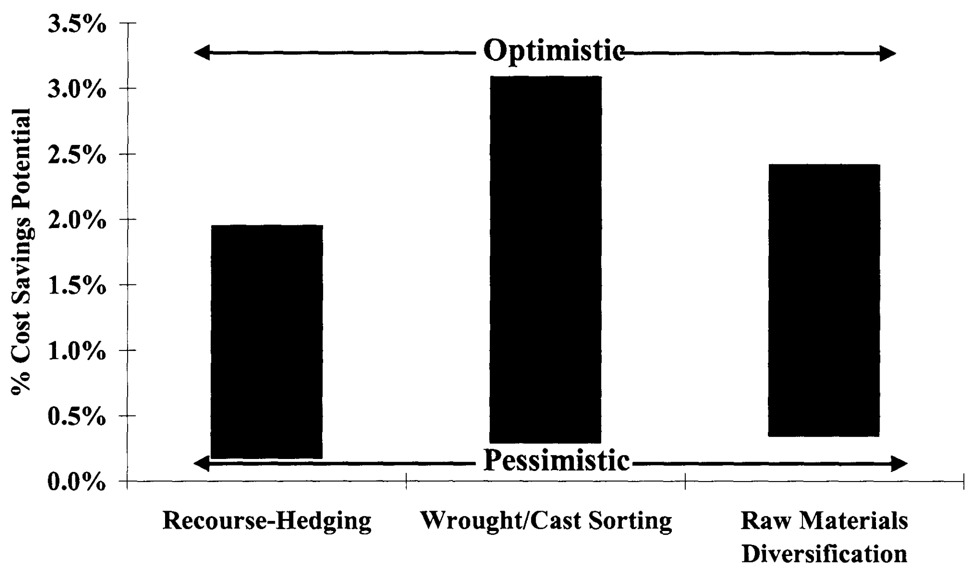

Figure 9.7 Estimated percentage cost savings from worst to best operating environment for various secondary aluminum consumption strategies

[33] This is measured by the coefficient of variation in demand defined as the standard deviation divided by the mean demand.

[34] This is measured by the coefficient of variation in composition defined as the standard deviation divided by the mean composition.

[35] It is defined as the salvage value of unused scrap, sorted or unsorted, divided by the original cost of acquiring that scrap material. A 5% discount has been added as an estimate for storage cost.

[36] Scrap price is a composite price weighted by the availability of scrap

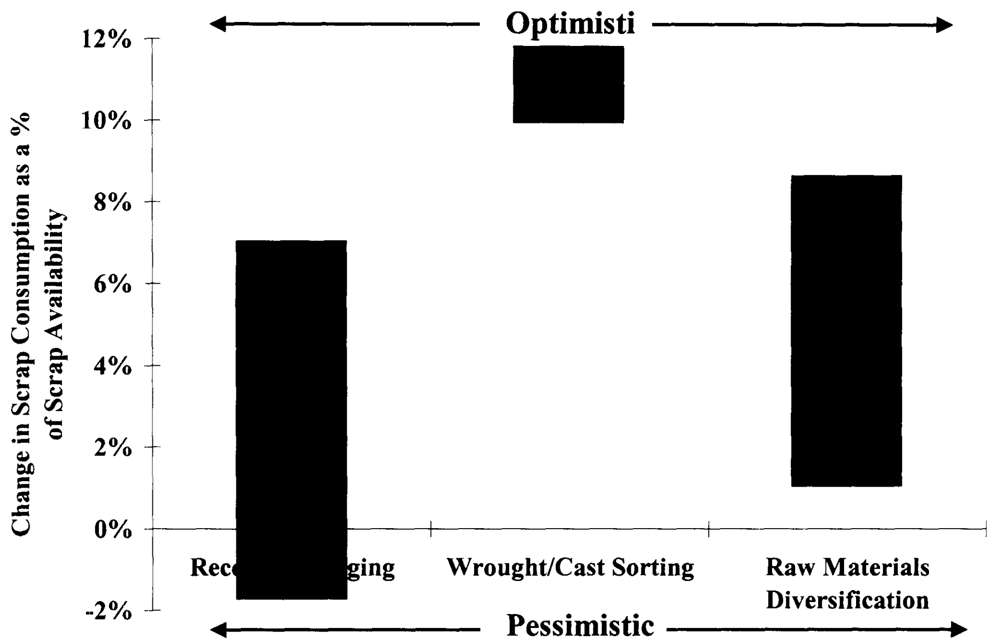

Figure 9.8 Change in scrap consumption as a percentage of scrap availability under worst to best operating environments

Noticeably, the three strategies are not significantly differentiated on average with respect to cost savings potential under historically observed and estimated operating environment. Of course, as evident by the range of potential cost savings, unique operating variables such as the sorting cost can alter their actual relative economic impact. The lack of differentiation on cost impact, however, is not translated into the ranges on change in scrap consumption. Sorting stands out to carry the highest potential for scrap consumption improvement with the least variability as operating environment varies. This is perhaps, once again attributed to the disruptive nature of this technique. In fact, among all the concepts discussed throughout this thesis, sorting is the only one that requires technological change. This small range impact specific to sorting is due specifically to the assumption of a fixed 95% recovery rate, a factor on which scrap consumption via sorting is highly sensitive towards. While the sorting cost was varying between \$10/t to \$30/t, this range is smaller than the average price differential between scrap and primaries observed historically, leading to lessened sensitivity towards this parameter. Under most operating conditions, scrap consumption is improved with these strategies. The discussions previously on hedging made clear that whether the hedge is to be positive or negative depends on the option value of scrap. As the salvage value worsens and the price differential between scraps and primaries deteriorates, the option value can be negative. Nevertheless, this practice will lead to cost reduction on average for the remelter.

Overall this thesis presented a number of suggestions for deriving greater value from scrap consumption, while mindful of the restrictions on such methodologies due to real operating conditions. Each of them, not only enables scraps to be consumed in manners otherwise unavailable or unaware of, but also opens doors for continuing research in aluminum recycling strategies. With the exception of sorting strategies, the other strategies can be readily implemented with no additional capital investments. Over a broad range of historically justifiable operating conditions, sorting investments can be economically viable.

Chapter 10 : Future Work

This thesis has taken a progression from the point of scrap purchases to their usage in final production. Along the way a number of sources of inefficiencies were examined and proposed tactics and methodologies were introduced to address these issues. However, this thesis is by no means exhaustive in ways to promote scrap consumption, nor the methodologies restricted in application to those discussed above. For instance, the recourse model framework can be adopted to study scrap supplies and pricing volatility. Through an extension of that model, such supply-side uncertainties can be studied along side demand uncertainty.

As noted in Chapter 3, there is a multitude of sorting technologies being developed besides wrought/cast sorting. Each will have its merits and associated costs of implementation. For these technologies, the sorting and mixing model can be leveraged as a tool to answer many of the same questions that were posed in Chapter 6. All these sorting technologies have to be cost-effective. They have to demonstrate a certain level of recovery performance and utilization rate in order to be practical. Undoubtedly not all of these technologies will survive the market. The tools developed in this thesis can be used to gauge their relative competitiveness and to guide their development efforts. The sorting and mixing model itself will surely continue to evolve as well. Correlating the size of the scrap pieces to the ability of the sorter to differentiate and recover the alloys might be one refinement. In the case study that was carried out, the set of alloys produced and the scrap types available were varied but still not comprehensive. An extension of the case study that encompass a larger grouping of alloys, perhaps by composition or by application, can be useful as well.

Another possible enhancement to the sorting and mixing model, as well as the recourse model, is considerations for scrap composition uncertainties. The nonlinear formulations introduced as compositional chance constraints in Chapter 7 can be implemented as a standalone improvement on current practice at the production level. However, as partly demonstrated in Chapter 8, theoretically they can also be incorporated into the study of other sources of uncertainties in scrap management. The challenges for this implementation lie in scalability of the mathematical formulation, computational efficiencies and statistical data requirement. Despite these challenges, the questions surrounding how the various strategies discussed will interact are certainly interesting. For instance, will recourse-based hedging complement sorting technologies in promoting greater scrap consumption, or will they work against each other?

While the bulk of this thesis has been about the aluminum market, much of the work presented can also be directed towards the study of other light metal systems. While much smaller in market size than aluminum, magnesium is another rapidly growing light metal application in cars due to strict government requirements on fuel economy. As this market grows and matures, it will be interesting to track whether similar concerns regarding secondary consumption surface. As the types of alloys employed become more diverse in compositional specifications, a greater need for sophistication in the recycling efforts is likely.

Appendix A: Aluminum Sorting Technologies

Hand sorting of aluminum is time-consuming and inefficient. While low wages in developing countries can afford to hand-sort nonferrous scrap materials for $10 to $15 per ton (Spencer 2005), such a process is inaccurate and imprecise. As such, various parties have been developing automated techniques for sorting secondary aluminum. In the late 1990's, with the goal of retaining value in aluminum scrap, a number of aluminum producers, together with automotive manufacturers and the Department of Energy began a project to develop techniques to separate recycled wrought aluminum from cast and then into its alloy families. The idea is that the scrap will first be sorted into piles of wrought and cast products. The wrought products can conceivably be then separated into its alloy families or individual alloys for reprocessing into higher-value automotive wrought applications, such as hoods, door panels, and some chassis parts.

Although no automated sorting technologies have been widely adopted in commercial settings, the Auto Aluminum Alliance, which aims to boost automotive manufacturers' use of aluminum, is hoping to see a commercial sorting center able to filter 100 million pounds of aluminum per year (Aluminum Association Press 2004, Gesing 2002, Gesing 2001, Buchholz 2001). The initial capabilities will include wrought-to-cast sorting, but techniques are being tested that hold promise of separation of wrought scraps into compatible alloy families or maybe even individual wrought alloy in the future (Aluminum Association Press 1999, Vigeland 2001). The following sections give very brief introductions to various sorting techniques.

A.1 Thermo-Mechanical Sorting

This technique, also known as the "hot crush" was developed in the mid-1980's by the U.S. Bureau of Mines. It is 96% effective in segregating cast from wrought. As the name implies, it involves a series of heating, crushing and screening steps. Because the melting point of cast alloys are lower than that of the wrought alloys, during the heating step the wrought alloys stay solid while the cast alloys are weakened along their grain boundaries. Such weakened grain boundaries become the points of fracture during subsequent mechanical grinding. Finally a screening station segregates the larger wrought pieces from the smaller fractured cast items. An added bonus of this technique is that during the heating stage, the scraps are effectively decoated because the paints, lacquer, etc. will melt. This eliminates the need for an extra decoating step in the material recovery process (DeGaspari 1999).

A.2 Laser-Induced Aluminum Sorting

Huron Valley Steel Corp has been developing this technique since 1993 (Gesing 2002, DOE 2001). In their recent industrial prototype demonstration, successful segregation of 6111 from 5182 alloys was achieved. The major advantage of this technique is that it has the potential to identify specific alloys and not just group alloys by families. The technique is an effectively non-contact, non-destructive analytical method that performs sorting by explicitly checking the composition of each piece of scrap. As the pieces of scrap move through a conveyor belt, the first step is abrasion by laser to reveal the material beneath the surface. A subsequent pulse of the laser at the same spot vaporizes a small amount of the material and produces a plasma plume. This plume is then analyzed by Laser Induced Breakdown Spectroscopy (LIBS) for the material composition for individual alloy identification.

Another technique that relies on similar light emission plasma for sorting aluminum is called optical emission spectrograph (OES). Unlike LIBS, however, OES requires contact with the samples and at this time are only available as hand-held devices (Spencer 2005).

A.3 Color Identification Following Chemical Etch

Alcoa and Pacific Northwest National Laboratory have been developing this technique since the late 1990's (DeGaspari 1999). The material is first chemically etched to reveal a material-dependent color. This patented etching process involves a degreasing wash, hot water rinse, hot caustic-solution etch and a second water rinse followed by hot air drying. As the pieces of scrap move along the conveyor belt, their colors are then examined through video cameras used to identify and separate the different material streams. The colors are then mapped to the individual alloy families known to the system. This technique has potential for use to group wrought alloys into families: 2000 series (copper based), 3000 series (manganese based), 5000 series (manganese based), 6000 series (magnesium and silicone based) and 7000 series (zinc based). Recent demonstrations have shown recovery rates ranging from 70-100% depending on the particular alloy family (Gesing 2002).

A.4 Other Less Known Automated Sorting Techniques

Other lesser known sorting technologies beyond those just mentioned are also being studied. X-ray analysis is claimed as another technology under testing. However, it is unclear how it works. X-ray fluorescence (XRF) emissions from aluminum tend to be weak and quickly absorbed by ambient air, leading to difficulty in detection. A vacuum environment might be needed. Finally, wTe Corporation and National Recycling Technologies have teamed up to develop a Spectramet technology capable of high-speed sorting based on optoelectronics sensors and methods. Its exact details of operations have not been disclosed due to patent disclosure restrictions (Spencer 2005).

A.5 Summary on Sorting Technologies

It is likely that further research and development in sorting technologies beyond those mentioned above will continue. Their acceptance and competitiveness will depend on their ability to add value to the materials production, after accounting for the cost of implementation. These effects are captured in a mathematical programming framework described in detail in Chapter 5.

Appendix B: European Union Sorting Case Study Data

Average Compositions (wt%) of Scraps

Scrap	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Ni	Pb
Base Cast	6.75	0.80	2.95	0.16	0.62	0.00	0.03	0.02	0.06	0.15
Base Extrusion	0.44	0.22	0.02	0.04	0.33	0.00	0.06	0.00	0.00	0.00
Base Sheet	0.73	0.39	0.16	0.29	0.50	0.00	0.21	0.00	0.00	0.01
CoMingled	4.27	0.41	1.34	0.34	0.10	0.02	2.11	0.03	0.00	0.00

Chemical Compositions Specifications (wt. %) of Selected Alloys

Alloy	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Ni	Pb
230	12.5-13.5	<0.4	<0.03	<0.35	<0.05	<0.05	<0.1	<0.15	<0.05	<0.05
226	8-11	<1	2-3.5	0.1-0.4	0.1-0.5	<0.05	<1.2	<0.15	<0.3	<0.2
239	9-11	<0.8	<0.08	0.001-0.4	0.2-0.5	<0.05	<0.1	<0.15	<0.05	<0.05
6111	0.6-1.1	<0.4	0.5-0.9	0.1-0.45	0.5-1.0	<0.1	<0.15	<0.05	<0.05	<0.05
6082	0.7-1.3	<0.5	<0.1	0.4-1	0.6-1.2	<0.25	<0.2	<0.1	<0.05	<0.05
6060	0.3-0.6	0.1-0.3	<0.1	<0.1	0.35-0.6	<0.05	<0.15	<0.1	<0.05	<0.05
3104	<0.6	<0.8	0.05-0.25	0.8-1.4	0.8-1.3	<0.05	<0.25	<0.1	<0.05	<0.05
3105	<0.6	<0.7	<0.3	0.3-0.8	0.2-0.8	<0.2	<0.4	<0.1	<0.05	<0.05

Normalized Price / Ton Assumptions for Primaries and Alloying Elements

Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Ni	Pb	P1020	P0508
1.88	0.32	2.66	2.02	2.27	9.95	0.98	10.67	15.11	0.76	1.36	1.36

Appendix C: Relevant Statistical Concepts

Part of this thesis relied heavily on statistical arguments and as such a brief review of statistical concepts particularly relevant to the work here will be summarized below for quick reference. A comprehensive review of probability and statistics can be found in a number of excellent texts (Bertsekas and Tsitsiklis 2002). The derivations here are meant to be succinct and at times serve as proofs as well. In the following let A, B and C be uncorrelated and independent stochastic variables with the possible values and associated probabilities of $\{A_1,A_2,A_3;\ p_1,\ p_2,\ p_3\}$, $\{B_1,B_2,B_3;\ q_1,\ q_2,\ q_3\}$ and $\{C_1,C_2,C_3;\ t_1,\ t_2,\ t_3\}$ respectively. K is just a constant.

$$E[A] = \sum_{i=1}^{3} p_i A_i = \mu_A$$

$$E[B] = \sum_{i=1}^{3} q_i B_i = \mu_B$$

$$E[C] = \sum_{i=1}^{3} t_i C_i = \mu_C$$

$$Var[A] = E[(A-\mu_A)^2] = E[A^2 - 2A\mu_A + \mu_A^2] = E[A^2] - \mu_A^2 = \sigma_A^2$$

$$Var[B] = E[(B-\mu_B)^2] = E[B^2 - 2B\mu_B + \mu_B^2] = E[B^2] - \mu_B^2 = \sigma_B^2$$

$$Var[C] = E[(C-\mu_C)^2] = E[C^2 - 2C\mu_C + \mu_C^2] = E[C^2] - \mu_C^2 = \sigma_C^2$$

Next, consider the statistical properties of product of two of these variables:

$$E[AB] = \sum_{i=1}^{3}\sum_{j=1}^{3} p_i q_j A_i B_j = (\sum_{i}^{3} p_i A_i)(\sum_{j}^{3} q_j B_j) = E[A]E[B] = \mu_A \mu_B$$

$$Var[AB] = E[(AB-\mu_A\mu_B)^2] = E[A^2B^2 - 2AB\mu_A\mu_B + \mu_A^2\mu_B^2] = E[A^2B^2] - \mu_A^2\mu_B^2$$

One can expand the expression $E[A^2B^2]$ to see if it is possible to simplify this further:

$$E[A^2B^2]=\sum_{i=1}^{3}\sum_{j=1}^{3}p_iq_jA_i^2B_j^2=(\sum_{i=1}^{3}p_iA_i^2)(\sum_{i=1}^{3}q_iB_j^2)=E[A^2]E[B^2]=(\mu_A^2+\sigma_A^2)(\mu_B^2+\sigma_B^2)$$

Putting these two expressions together:

$$Var[AB]=(\mu_A^2+\sigma_A^2)(\mu_B^2+\sigma_B^2)-\mu_A^2\mu_B^2=\sigma_A^2\mu_B^2+\sigma_B^2\mu_A^2+\sigma_A^2\sigma_B^2$$

It should also be clear from the above that the expressions for expectations and variances taken on products of independent variables are commutative. In other words:

$E[AB]=E[BA]$ and $Var[AB]=Var[BA]$

The following is a simple proof that the negation of a stochastic variable does not change its variance and that pure negation introduces a correlation of -1 with the original variable:

$$E[1-A]=1-\mu_A$$

$$Var[1-A]=E[(1-A-1+\mu_A)^2]=E[(\mu_A-A)^2]=E[A^2]-\mu_A^2=\sigma_A^2$$

What about the variance of (1-A)B? Note that this is not the same as the covariance between 1-A and B.

$$\begin{aligned}
Var[(1-A)B]&=E[((1-A)B-(1-\mu_A)\mu_B)^2]=E[(B-AB-\mu_B+\mu_A\mu_B)^2]\\
&=E[B^2+A^2B^2+\mu_B^2+\mu_A^2\mu_B^2-2AB^2-2\mu_BB+2B\mu_A\mu_B+2AB\mu_B-2AB\mu_A\mu_B-2\mu_A\mu_B^2]\\
&=E[B^2-2\mu_BB+\mu_B^2]+E[A^2B^2-2AB\mu_A\mu_B+\mu_A^2\mu_B^2]+2E[-AB^2+B\mu_A\mu_B+AB\mu_B-\mu_A\mu_B^2]\\
&=\sigma_B^2+Var[AB]+2[-\mu_AE[B^2]+\mu_A\mu_B^2]\\
&=\sigma_B^2+(\sigma_A^2\sigma_B^2+\sigma_B^2\mu_A^2+\sigma_A^2\mu_B^2)-2\mu_A(\mu_B^2+\sigma_B^2)+2\mu_A\mu_B^2
\end{aligned}$$

How about covariance, specifically the covariance between A and B assuming that the two are independent?

$$\sigma_{A,B}=E[(A-\mu_A)(B-\mu_B)]=E[AB-\mu_AB-A\mu_B+\mu_A\mu_B]=0$$

Note that this is only true when *A* and *B* are independent. What about the covariance of 1-*A* with *A* and the correlation? Is covariance also commutative?

$$\sigma_{A,1-A} = E[(1-A-1+\mu_A)(A-\mu_A)] = E[(\mu_A - A)(A-\mu_A)]$$
$$= E[-A^2 - \mu_A^2 + 2A\mu_A] = -E[A^2] + \mu_A^2 = -\sigma_A^2$$

$$\rho_{A,1-A} = \frac{\sigma_{A,1-A}}{\sigma_A \sigma_{1-A}} = \frac{-\sigma_A^2}{\sigma_A^2} = -1$$

$$\sigma_{1-A,A} = E[(A-\mu_A)(1-A-1+\mu_A)] = E[(A-\mu_A)(\mu_A - A)]$$
$$= E[-A^2 - \mu_A^2 + 2A\mu_A] = -E[A^2] + \mu_A^2 = -\sigma_A^2$$

Therefore covariance is commutative as well. Now consider the covariance between the products *AB* and *AC*. Recall that all three variables are independent and uncorrelated with each other.

$$\sigma_{AB,AC} = E[(AB - \mu_A\mu_B)(AC - \mu_A\mu_C)]$$
$$= E[A^2BC - AC\mu_A\mu_B - AB\mu_A\mu_C + \mu_A^2\mu_B\mu_C]$$
$$= E[A^2BC] - \mu_A^2\mu_B\mu_C - \mu_A^2\mu_B\mu_C + \mu_A^2\mu_B\mu_C$$
$$= E[A^2]\mu_B\mu_C - \mu_A^2\mu_B\mu_C - \mu_A^2\mu_B\mu_C + \mu_A^2\mu_B\mu_C$$
$$= (\mu_A^2 + \sigma_A^2)\mu_B\mu_C - \mu_A^2\mu_B\mu_C - \mu_A^2\mu_B\mu_C + \mu_A^2\mu_B\mu_C$$
$$= \sigma_A^2\mu_B\mu_C$$

Similarly the covariance between products *(1-A)B* and *AC* is:

$$\sigma_{(1-A)B,AC} = E[((1-A)B - (1-\mu_A)\mu_B)(AC - \mu_A\mu_C)]$$
$$= E[(B - AB - \mu_B + \mu_A\mu_B)(AC - \mu_A\mu_C)]$$
$$= E[ABC - A^2BC - A\mu_B C + AC\mu_A\mu_B - B\mu_A\mu_C - AB\mu_A\mu_C + \mu_B\mu_A\mu_C - \mu_A^2\mu_B\mu_C]$$
$$= -E[A^2BC] - \mu_A^2\mu_B\mu_C$$
$$= -E[A^2]\mu_B\mu_C - \mu_A^2\mu_B\mu_C$$
$$= -(\mu_A^2 + \sigma_A^2)\mu_B\mu_C - \mu_A^2\mu_B\mu_C$$
$$= -2\mu_A^2\mu_B\mu_C - \sigma_A^2\mu_B\mu_C$$

Oftentimes stochastic variables are summed:

$$E[A+B+C]=E[A]+E[B]+E[C]=\mu_A+\mu_B+\mu_C$$

$$Var[A+B+C]=E[(A+B+C-\mu_A-\mu_B-\mu_C)^2]$$
$$=E[(A^2+AB+AC-A\mu_A-A\mu_B-A\mu_C)+(AB+B^2+BC-B\mu_A-B\mu_B-B\mu_C)$$
$$+(AC+BC+C^2-C\mu_A-C\mu_B-C\mu_C)+(-\mu_A A-\mu_A B-\mu_A C+\mu_A^2+\mu_A\mu_B+\mu_A\mu_C)$$
$$+(-\mu_B A-\mu_B B-\mu_B C+\mu_B^2+\mu_A\mu_B+\mu_B\mu_C)+(-\mu_C A-\mu_C B-\mu_C C+\mu_C^2+\mu_A\mu_C+\mu_B\mu_C)$$
$$=E[A^2-A\mu_A-\mu_A A+\mu_A^2]+E[2AB-A\mu_B-B\mu_A-\mu_B A-\mu_A B+2\mu_A\mu_B]$$
$$+E[B^2-B\mu_B-\mu_B B+\mu_B^2]+E[2BC-C\mu_B-B\mu_C-\mu_C B-\mu_B C+2\mu_B\mu_C]$$
$$+E[C^2-C\mu_C-\mu_C C+\mu_C^2]+E[2AC-C\mu_A-A\mu_C-\mu_C A-\mu_A C+2\mu_A\mu_C]$$
$$=\sigma_A^2+\sigma_B^2+\sigma_C^2$$

This last simplification is contingent upon the variables *A, B* and *C* being independent and uncorrelated. In that case, the cross terms cancel out:

$$Var[A+B+C]=E[(A+B+C-\mu_A-\mu_B-\mu_C)^2]$$
$$=E[A^2-A\mu_A-\mu_A A+\mu_A^2]+E[B^2-B\mu_B-\mu_B B+\mu_B^2]+E[C^2-C\mu_C-\mu_C C+\mu_C^2]=\sigma_A^2+\sigma_B^2+\sigma_C^2$$

Otherwise the cross-terms will contribute to covariance terms as such:

$$Var[A+B+C]=E[(A+B+C-\mu_A-\mu_B-\mu_C)^2]$$
$$=E[A^2-A\mu_A-\mu_A A+\mu_A^2]+E[2AB-A\mu_B-B\mu_A-\mu_B A-\mu_A B+2\mu_A\mu_B]$$
$$+E[B^2-B\mu_B-\mu_B B+\mu_B^2]+E[2BC-C\mu_B-B\mu_C-\mu_C B-\mu_B C+2\mu_B\mu_C]$$
$$+E[C^2-C\mu_C-\mu_C C+\mu_C^2]+E[2AC-C\mu_A-A\mu_C-\mu_C A-\mu_A C+2\mu_A\mu_C]$$
$$=\sigma_A^2+\sigma_B^2+\sigma_C^2+2\sigma_{AB}+2\sigma_{BC}+\sigma_{AC}$$

Finally consider the effects of constants in probabilities and statistics.

$$E[KA]=KE[A]=K\mu_A$$

$$Var[KA]=E[(KA-K\mu_A)^2]=E[K^2(A-\mu_A)^2]=K^2\sigma_A^2$$

References

Aluminum Industry Roadmap For The Automotive Market: Enabling Technologies For Body Structures and Closures, The Aluminum Association, Inc., May 1999

Al-Futaisi, A. and Stedinger, J.R., *"Hydrologic and Economic Uncertainties and Flood-Risk Project Design,"* Journal of Water Resources Planning and Management, November 1999, p. 314

Aris, R.. *Discrete Dynamic Programming,* Blaisdell Publishing Company: New York, NY 1964

Bellman, R. E. and Dreyfus, S. E., *Applied Dynamic Programming*, Princeton University Press: Princeton, NJ 1962

Benedyk, J., Klimisch, R. and Skillingberg, M., *"Aluminum Automotive Body Sheet 2002",* Aluminum 2002 – Proceedings of the TMS 2002 Annual Meeting: Automotive Alloys and Aluminum Sheet and Plate Rolling and Finishing Technology Symposia

Bertsimas, D. and Tsitsiklis, J.N., *Introduction to Linear Optimization*, Athena Scientific: Nashua, NH, February 1997

Bertsekas, D., *Nonlinear Programming*, Athena Scientific: Nashua, NH, September 1999

Bertsekas, D. P and Tsitsiklis, J. N., *Introduction to Probability*, Quantum Books, Boston, MA, June 2002

Buchholz, K., *"Sorting Aluminum"*, Automotive Engineering International 2001 (109) p. 152

Buckingham, D.A. and Plunkert, P.A., *Aluminum Statistics*. August 26, 2002, USGS

Cattani, K. Cattani, Ferrer, G. and Gilland, W. *"Simultaneous Production of Market-Specific and Global Products: A Two-Stage Stochastic Program with Additional Demand After Recourse",* Naval Research Logistics 2003 (50) p.438 – 461

Charnes, A. and Cooper, W.W. *"Deterministic equivalents for optimizing and satisficing under chance constraints"*, Operations Research 1963 (11) p. 18-39

Chong, E. and Zak, S.H. *An Introduction to Optimization*. John Wiley & Sons, Inc: New York, NY 2001.

Choobineh, F. and Padmanabhan, V. *"Material Planning for Production Kits under Uncertainty,"* Production Planning & Control, Vol 15, No. 1, January 2004, 63-70.

Chriss, N.A., *Black-Scholes and Beyond.* Irwin Professional Publishing: Chicago, IL 1997

Cosquer, A. *"Optimizing the Reuse of Light Metals from End-of-Life Vehicles",* MIT Thesis (2003) supervised by Professor R. Kirchain.

Cosquer, A., Kirchain, R., *"Optimizing the Reuse of Light Metals From End-of-life Vehicles: Lessons from Closed Loop Simulations"*, The Minerals, Metals & Materials Society Meeting 2003

Dalmijn, W.L. et al, *"The Optimization of the Resource Cycle: Impact of the Combination of Technology, Legislation and Economy,"* Keynote Address, Int'l Minerals Processing Congress 2003, vol. 1 (2003), pp. 81-106

Datta, J., Dr. Ing, Aluminum-Verlag Marketing & Kommunication GmbH, Aluminum-Schlussel (Key to Aluminum Alloys) 6th Ed., Aluminum-Verlag Dusseldorf: 2002

De Weck, O.L., de Neufville R. and Chaize M., *"Staged Deployment of Communications Satellite Constellations in Low Earth Orbit"*, Journal of Aerospace Computing, Information, and Communication, 1, 119-136, March 2004.

DeGaspari, J., *"Making The Most of Aluminum Scrap"*, The American Society of Mechanical Engineers, 1999

Dempster, M.A.H.. *Stochastic Programming*. Academic Press: New York, NY 1980

Automotive Aluminum Scrap Sorting, Office of Industrial Technologies, Energy Efficiency, and Renewable Energy, DOE, July 2001

Ducker, *Passenger and Light Truck Aluminum Content Report,* 1999, Ducker Research Company

Ducker, *Global Automotive Aluminum Content Forecast Through 2010*, 2002, Ducker Research Company

Dupacova, J., *"Applications of stochastic programming: Achievements and questions,"* European Journal of Operational Research 140 (2002) pp. 281-290

Ellis, J.H., McBean E.A., Farquhar, G.J.. *"Chance-constrained/stochastic linear programming model for acid rain abatement – Complete collinearity and noncollinearity",* Atmospheric Environment 1985 (19) p. 925-937

Geldof, G.d., *"Coping with Uncertainties in Integrated Urban Water Management,"* Water Science Technology, Vol 36 (8-9), p. 265

Geoleian, K., Kar, K., Manion, M. and Bulkley, J., *Industrial Ecology of the Automobile*. Society of Automotive Engineers, Inc. 1997

Gesing, A., Berry, L., Dalton, R. and Wolanski, R., *"Assuring Continued Recyclability of Automotive Aluminum Alloys: Grouping of Wrought Alloys by Color, X-Ray Absorption and Chemical Composition-based Sorting"*, Aluminum 2002 – Proceedings of the TMS 2002 Annual Meeting: Automotive Alloys and Aluminum Sheet and Plate Rolling and Finishing Technology Symposia

Gesing, A., Steward, C., Hopson, G., Costello, S., Lambert, S., Good, T., Grieve, R., Dalton, R., Wolanski, R. and Berry, L., *"Separation of Wrought Fraction of Aluminum Recovered From Automobile Shredder Scrap",* Aluminum 2001 – Proceedings of the TMS 2001 Annual Meeting: Aluminum Automotive and Joining Sessions

Gorban, L.T., Ng, G.K. and Tessieri, M.B., *"An In-Depth Analysis of Automotive Aluminum Recycling in the Year 2010,"* SAE International Congress, Detroit 1994

Grassman, W.K., *Stochastic Systems for Management,* North Holland: New York, NY 1981

Growe, N., Romisch, W. and Schultz, R., *"A Simple Recourse Model For Power Dispatch Under Uncertain Demand,"* ANN OPER RES 1995 Vol. 59 pp. 135-164

Holland, P., *Copper's Cyclical Problems*, The Mining Journal, London, August 31, 2001

Howell, S., Stark, A., Newton, D., Paxson, D., Cavus, M., Pereira, J. and Patel, K., *Real Options: Evaluating Corporate Investment Opportunities in a Dynamic World.* Prentice Hall: London, UK 2001

Hull, J.C. *Options, Futures, and Other Derivatives.* Prentice Hall: Englewood Cliffs, NJ 2003

The Aluminum Industry's Sustainable Development Report, International Aluminum Institute, London, 2002

Kelly, T., et al., *Historical Statistics for Mineral and Material Commodities in the United States*. 2005, U.S. Geological Survey, US Dept of Interior, Reston, VA.

Kira, D., Kusy, M. and Rakita, I. *"A Stochastic Linear Programming Approach to Hierarchical Production Planning,"* J OPER RES SOC 1997 Vol. 48 (2)

Kulatilaka, A., *Real Options.* Harvard Business School Press: Boston, MA 1999

Lee, H. L., Padmanabhan, V., "The Bullwhip Effect in Supply Chains." Sloan Management Review, Spring 1997, Vol. 38 Issue 3, p93.

Léotard, J.P., *"Transmission Pricing and Incentives for Investments Under Uncertainty in the Deregulated Power Industry,"* MIT Laboratory for Energy and the Environment, Energy Laboratory Reports, February 1999

Li, S.X., Huang, Z.M., and Ashley, A., *"Manufacturer and Retailer Cooperation Through Franchising: A Chance Constrained Game Approach,"* Information Systems and Operational Research, 40 (2002), 131-148

London Metals Exchange (www.metalprices.com), Metal Bulletin Research.

Louveaux, B. *Introduction to Stochastic Programming.* Springer-Verlag: New York, NY 1997

Luenberger, D. G., *Linear and Nonlinear Programming.* Springer: New York, NY, September 2003

Lund, J., Tchobanoglous, G. and Lawyer, R., *"Linear Programming for Analysis of Material Recovery Facilities"*, American Society of Civil Engineers Journal of Environmental Engineering 1994 (5): p. 1082

Manno, I., *Introduction to the Monte Carlo Method.* Akademiai Kiado: Budapest, Hungary 1999

Martel, A. and Price, W., *"Stochastic Programming Applied to Human Resource Planning,"* J OPER RES SOC 1981 Vol. 32 (3)

Maurice, D., Hawk, J.A., and Riley, W.D., *"Thermomechnical Treatments for the Separation of Cast and Wrought Aluminum,"* TMS Fall Extractive Meeting, Pittsburgh, October 2000: Recycling of Metals & Engineering Materials

Mesina, M.B., de Jong, T.P.R., and Dalmijn, W.L., *"New Developments on Sensors for Quality Control and Automatic Sorting of Non-Ferrous Metals,"* 11th Symposium on Automation in Mining, Mineral and Metal Processing (MMM-IFAC 2004), September 8-10, 2004, Nancy, France

Mesina, M.B., de Jong, T.P.R., Dalmijn, W.L., and Reuter, M.A. *"Developments in Automatic Sorting and Quality Control of Scrap Metals,"* Light Metals 2004, TMS Annual Meeting, March 14-18, 2004, Charlotte, NC, USA

Neely III, J.E. and de Neufville, R., *"Hybrid Real Options Valuation of Risky Project Developments",* International Journal of Technology, Management and Policy 2001

Nemhauser, G.L., *Introduction to Dynamic Programming*, John Wiley & Sons, Inc.: New York, NY 1966

Ostroff, J., *Tighter Fuel Efficiency Standards Coming*, The Kiplinger Washington Editors, October 29, 2004

De Neufville, R., *"Uncertainty Management for Engineering Systems Planning and Design,"* MIT Engineering Systems Symposium, March 2004

De Neufville, R., *"Real Options: Dealing With Uncertainty in Systems Planning and Design",* 5th International Conference on Technology, Policy and Innovation 2001

Plunkert, P., *Minerals Yearbook*, in *Minerals Yearbook*, USGS, Editor. 2003, US Geological Survey, Department of Interior: Reston, VA.

Peterson, G.D., Cummings, G.S. and Carpenter, S.R., *"Scenario Planning: a Tool for Conservation in an Uncertain World,"* Conservation Biology, Vol 17 (2) April 2003, p. 358

Petruzzi, N.C. and Dada, M., *"Information and Inventory Recourse for a Two-Market, Price-Setting Retailer,"* Manufacturing & Service Operations Management 2001 (3) p. 242-263

Ralls, K. and Taylor, B.L., *"Better Policy and Management Decisions through Explicit Analysis of Uncertainty: New Approaches from Marine Conseration,"* The Journal of the Society for Conservation Biology, Vol 14 (5) October 2000

Reuter, M.A., Boin, U., Rem., P, Yang, Y., Fraunholcz, N., and van Schaik, A., *"The Optimization of Recycling: Integrating the Resource, Technological, and Life Cycles,"* Journal of Materials, 2004 August, pp. 33-37

Reuter, M.A., Boin, U., de Jong, T.P.R., Dalmijn, W.L., and Gesing, A., *"The Optimization of Aluminum Recovery from Recycled Material,"* 2nd Int'l Conference on Aluminum Recycling, Moscow 2004

ReVelle, C.S., Joeres E., Kirby W. *"The linear decision rule in reservoir management and design. Development of the stochastic model",* Water Resources Research 1969 (5) p. 767-777 van Schaik, A. and Reuter, M.A., *"Dynamic Modeling and Optimization of the Resource cycle of Passenger Vehicles,"* Minerals Engineering (11) 15 (2002), pp. 1001-1016

van Schaik, A. and Reuter, M.A., *"The time-varying factors influencing the recycling rate of products,"* Resources, Conservation and Recycling 2003 Vol. 40 pp. 301-328.

van Schaik, A. and Reuter, M.A., *"Minerals Engineering, The influence of particle size reduction and liberation on the recycling rate of end-of-life vehicles,"* Minerals Engineering, Vol. 17 (2), pp. 331-347.

van Schaik, A., Reuter, M.A., Dalmijn, W.L. and Boin, U., *"Dynamic modeling and optimization of the resource cycle of passenger vehicles,"* Minerals Engineering 2002, Vol. 15 (11), pp. 1001-1016.

van Schaik, A. and Reuter, M.A., *"The Influence of Particle Size Reduction and Liberation on the Recycling Rate of End-of-Life Vehicles,"* Minerals Engineering, (17) 2, 2004, pp. 331-347

Schultz, R. A, *"Aluminum for Light Vehicles: An Objective Look at the Next Ten to Twenty Years"*, Metal Bulletin 14th International Aluminum Conference, Montreal Canada 1999

Shih, J.S. and Frey H.C.. *"Coal Blending Optimization Under Uncertainty"*, Proceedings of the Tenth Annual International Pittsburgh Coal Conference 1993, p. 1110-1115

Spencer, D., *"The High-Speed Identification and Sorting of Nonferrous Scrap,"* JOM April 2005.

Stodolsky F., Vyas, A., Cuenca, R., Gaines, L., *"Life-Cycle Energy Savings Potential From Aluminum-Intensive Vehicles"*, 1995 Total Life Cycle Conference & Exposition, Vienna, Austria

Stuart, J.A. and Lu, Q., *"A Model for Discrete Processing Decisions for Bulk Recycling of Electronics Equipment"*, IEEE Transactions on Electronics Packaging Manufacturing 2000 (23) p. 314

Stuart, J.A. and Lu, Q., *"A Refine-or-Sell Decision Model for a Station with Continuous Reprocessing Options in an Electronics Recycling Center"*, IEEE Transactions on Electronics Packaging Manufacturing 2000 (23) p.321

Trigeorgis, L.. *Real Options: Managerial Flexibility and Strategy in Resource Allocation.* MIT Press: Cambridge, MA 1996

Xiao, Y. and Reuter, M.A., *"Recycling of Different Aluminum Scraps,"* Minerals Engineering 2002, pp. 763-970

Vigeland, P., *"Aluminum Recycling: The Commercial Benefits, the Technical Issues and the Sustainability Imperative,"* Metal Bulletin's 9th International Secondary Conference 7-9 November 2001, Prague

Waller, S.T. and A.K. Ziliaskopoulos *"A Dynamic and Stochastic Approach to Network Design,"* Journal of the Transportation Research Board No. 1771, pp. 106-114, 2001.