May, 1994

LIDS- P 2247

Research Supported By:

NSWC N60921-92-C-0173

Tracking Maneuvering Targets Using H• Filters

Tsaknakis, H.
Athans, M.

May 1994

LIDS-P-2247

1994 ACC

TRACKING MANEUVERING TARGETS USING H_∞ FILTERS

By

Haralampos Tsaknakis ALPHATECH, Inc.

Michael Athans MIT

Abstract[1]

In this work we develop, test and evaluate H_∞ algorithms for tracking maneuvering targets as an alternative to traditional approaches based on Kalman filter extensions and modifications. A crucial property of H_∞ filters is an automatic increase of the filter bandwidth. We further explore this property versus appropriate artificial increases of the bandwidth of a Kalman filter and the H_∞ filter itself. H_∞ and Kalman filters are compared for RMSE performance as well as performance robustness to different maneuvering trajectories. Simulation results are presented showing the performance and robustness of the two filters for different maneuvering trajectories, data rates and bandwidth adjustments.

1. Introduction

In this paper we report the results of a study directed toward the development, testing and evaluation of H_∞-based state estimation algorithms for tracking maneuvering targets, under the assumption that the target maneuvers are unknown to the tracking algorithm. These H_∞ tracking algorithms offer a recent alternative to more traditional approaches that are almost exclusively based on Kalman Filter extensions and modifications. The main issue that has been investigated is that of the performance robustness of H_∞ estimation algorithms in tracking kinematically constrained targets subject to unknown and unmodeled acceleration disturbances. In addition, the performance trade-off between sampling rate and tracking performance is investigated, using RMS position and velocity errors. The H_∞ approach is compared in terms of tracking performance and robustness to traditional approaches that involve Kalman Filters with artificially increased (tuned) process noise covariance matrix (to increase filter bandwidth) to model the effect of unmodeled accelerations.

The problem of tracking maneuvering targets has received considerable attention due to its obvious importance in a wide variety of military applications, including air defense from missiles and aircraft, air-to-air warfare, naval surface warfare and strategic/theater defense. References [9], [11], [13], [15], [16], [17] provide a sample of the various approaches and studies reported in the literature. It should be pointed out that almost all the traditional approaches to target tracking are based on Kalman Filter ideas and the problem of tracking maneuvering targets that involves unknown acceleration disturbances has been traditionally dealt with by using various extensions and modifications of Kalman-like filters. Indeed tracking represents the vast majority of Kalman filter applications; and the Kalman filter (with its extensions) represents the most applied by-product of modern control theory. As such, Kalman-based target state estimation algorithms are one of the most studied and most effective practical algorithms.

H_∞-based control and estimation algorithms have been receiving an increased amount of attention in the past few years due to their potential for more robust performance to poor modeling of disturbances. Recent references in this vein are [1]-[5] and [14]. Traditional Kalman filtering and LQG approaches (or H_2 based algorithms as they are recently called) assume that the disturbances are stochastic processes modeled as the output of linear systems driven by white noise, and the performance yardstick is the minimization of RMS errors. In contrast, H_∞ approaches to control and estimation assume that the disturbances are bounded energy (L_2) signals, and they attempt to minimize the worst case error induced by this class of performance. Although H_2 and H_∞ algorithms have been compared for some linear time-invariant control systems, such a comparison did not exist in the context of tracking maneuvering targets, with significant unknown accelerations, using typical radar trajectories and sensor noise. The comparisons presented in this report are the first detailed comparisons between "dumb" and "tuned" H_∞ and Kalman filter algorithms, including the expected performance degradations as the radar sampling rate decreases.

The results of our study indicate that straightforward H_∞ tracking algorithms cannot deliver high tracking accuracies for maneuvering targets without further tuning. This broad conclusion seems to be valid over a variety of radar sampling rates. Furthermore, although at first glance the performance of H_∞ tracking filters is vastly superior to that of untuned ("dumb") Kalman filters, our studies demonstrate that one can "tune" both the Kalman-type and H_∞-based tracking algorithms to attain comparable RMS performance for highly maneuvering targets. Thus, the advertised robustness of H_∞ filtering algorithms is not enough to solve the problem of precise tracking of maneuvering targets. This suggests that one can, and should, develop a family of ***adaptive tracking algorithms*** that blend the best aspects of Kalman filters and H_∞ filters, and we suggest that such a development and evaluation be part of further research on the topic, including performance evaluations based on actual data.

In section 2 of this report, we present mathematical details for the derivation methodology and equation summary for different H_∞ filters. In section 3, we describe the dynamics used for simulating maneuvering target trajectories, radar-to-target geometries, radar measurement accuracies, and summarize the numerical results and performance comparisons between H_∞ filters and Kalman filters of different bandwidths, using "tuning" of the process noise covariance matrix (the Q matrix). Finally, in section 4 we discuss the numerical results and evaluate the performance of the algorithms.

[1] Work supported by the Naval Surface Warfare Center (NSWC) under Contract N60921-92-C-0173 under the direction of Gil Graff.

2. H_∞ Tracking Algorithms forContinuous-Time and Hybrid Target-Sensor Models

2.1 The Continuous-Time Equations of H_∞ Tracking Algorithms

The motion of maneuvering targets is generally described by nonlinear differential equations. However, for many practical situations these equations can be suitably linearized around the most likely state trajectory yielding an approximate linear time-varying system. In the sequel we shall be concerned with both linear and nonlinear target models and we shall discuss them separately in the context of continuous-time motion dynamics and continuous-time measurements.

Linear models for tracking maneuvering targets are typically of the form

$$\begin{aligned} \dot{X}(t) &= A(t)\,X(t) + B(t)\,w(t) \\ Y(t) &= C(t)\,X(t) + D(t)\,n(t) \\ Z(t) &= L(t)\,X(t) \end{aligned} \qquad (1)$$

where, X(t) is the state vector consisting of the position and velocity components of the target, Y(t) is the measurement vector, w(t) and n(t) denote process and measurement disturbances respectively, and A(t), B(t), C(t), D(t) are time-varying matrices of appropriate dimension. The vector Z(t) defines the variables to be estimated and is given by a linear combination, L(t)X(t), of the state variables. The fact that Y(t) and Z(t) involve different linear combinations of the state variables gives rise to the distinguishing "directional" properties exploited by H_∞ filters as we shall see later in this section.

The usual Kalman filter for estimating the current state of the target X(t) based on all past and present measurements $\{Y(t)\ ,\ 0 \le t \le t\}$ is described by the following differential equations:

$$\begin{aligned} \dot{\hat{X}}(t) &= A(t)\,\hat{X}(t) + P(t)\,C'(t)\,(D(t)D'(t))^{-1}\,[Y(t) - C(t)\,\hat{X}(t)] \\ \hat{Z}(t) &= L(t)\,\hat{X}(t) \end{aligned} \qquad (2)$$

where, P(t) is the symmetric nonnegative definite covariance matrix that solves the well-known matrix Riccati differential equation

$$\dot{P}(t) = A(t)\,P(t) + P(t)\,A'(t) - P(t)\,C'(t)\,(D(t)D'(t))^{-1}\,C(t)\,P(t) + B(t)\,B'(t) \qquad (3)$$

It should be pointed out here that the Kalman filter described by the above equations was designed to minimize the variance of the state estimation error under the modeling assumption that the disturbances w(t), and n(t) are continuous-time, white, zero-mean Gaussian processes with unit intensity covariance matrix; also, we assume that w(t) and n(t) are mutually independent. The value of P(t) in equation (3) is the covariance matrix of the optimal (in an RMS sense and also in the Maximum Likelihood sense) state estimation error at any time t:

$$P(t) = E\,\{\,(X(t) - \hat{X}(t))\,(X(t) - \hat{X}(t))'\,\} \qquad (4)$$

The initial conditions for differential equations (2) and (3) are the initial state estimate $\hat{X}(0)$, which is given as the expected value of the state at time 0, and initial covariance matrix P(0), which is given as the covariance of the initial state around its expected value. We note here that since the Kalman filter does not depend on L(t) at all (as can be clearly seen from equations (2) and (3)), it lacks what we refer to as the *directional performance property*.

The term B(t) B'(t) in eq. (3) is the process noise covariance matrix of the Kalman filter and we shall denote it by Q(t) = B(t)B'(t) (here we have an abuse of notation since Q(t) is the intensity of the equivalent process noise that defines its covariance matrix). The matrix Q(t) is often viewed as a design parameter reflecting uncertainty in the nominal state dynamic model. Larger Q's result in higher Kalman filter bandwidths and higher filter gains. When there is significant deviation from the assumed nominal state model due to target maneuvers, one way to deal with the problem in this context is to increase the Q matrix of the Kalman filter so as to provide protection against unknown dynamics not accounted for in the nominal model. However, the issue of how exactly to increase the Q matrix (bandwidth) of the Kalman filter is dealt with ad-hoc procedures requiring some knowledge of the anticipated maneuvers.

The H_∞ filter provides an alternative way to increase the bandwidth based on a logic designed to protect against the "worst" case disturbances. In fact, it seeks to minimize the maximum energy of the estimation error $Z(t) - \hat{Z}(t)$ over all possible disturbances w(t), n(t) with given finite energy. It is important to realize that in H_∞ theory the disturbances w(t) and n(t) are not modeled as white (indeed white noise is a process with infinite energy and is automatically excluded from H_∞ theory). The disturbances w(t) and n(t) that maximize the error energy are called the "worst" case disturbances. That performance criterion is called the H_∞ norm of a system driven by the disturbances w(t) and n(t) and having as output the estimation error. For a precise mathematical definition it is necessary to consider first the following norm of a time vector-valued signal s(t) from time 0 to time T, called $L_2(0,T)$ norm:

$$\|s\| = \left(\int_0^T s'(t)\; s(t)\, dt \right)^{1/2} \qquad (5)$$

Let $\|Z - \hat{Z}\|$ denote the $L_2(0,T)$ norm of the estimation error. Let $\|w\|$ and $\|n\|$ be the $L_2(0,T)$ norms of the process and measurement noises respectively. Also let P(0) be a weighting matrix representing the uncertainty due to the unknown initial state X(0). Then, the H_∞ norm of a filter F that produces estimates $\hat{Z}(t)$ is defined as follows:

$$J(T,F) = \sup \left\{ \left[\frac{\|Z - \hat{Z}\|^2}{\|w\|^2 + \|n\|^2 + (X(0) - \hat{X}(0))'\, P^{-1}(0)\, (X(0) - \hat{X}(0))} \right]^{1/2} \right\} \qquad (6)$$

where, the supremum is taken over all noise sequences w(t), n(t) with finite $L_2(0,T)$ norms and over all initial state vectors X(0).

From the above discussion it follows that the H_∞ formulation of the tracking problem provides a potentially useful and interesting alternative to a Kalman filter when unknown deterministic acceleration disturbances (jinking or other purposeful maneuvering) take place, and, also, when the measurement noises and the initial target state are unknown. We shall review some of the most recent results in H_∞ filtering theory for linear systems and present our extension of these results to nonlinear systems.

It has been recently shown ([1]) that for a linear time-varying system as described in equations (1), a filter F whose H_∞ norm (as in (6)) does not exceed a certain positive scalar γ can be designed by using a modified Riccati equation. Specifically, let us assume that a linear combination Z(t) = L(t) X(t) of the state variables is desired to be estimated.

Then, a filter satisfying the above requirements is given by the following equations:

$$\dot{\hat{X}}(t) = A(t)\,\hat{X}(t) + P(t)\,C'(t)\,(D(t)D'(t))^{-1}\,[Y(t) - C(t)\,\hat{X}(t)]$$

$$\hat{Z}(t) = L(t)\,\hat{X}(t) \tag{7}$$

where, the symmetric matrix function P(t) is determined from the following matrix Riccati -like differential equation:

$$\dot{P}(t) = A(t)\,P(t) + P(t)\,A'(t) - P(t)\,C'(t)\,(D(t)D'(t))^{-1}\,C(t)\,P(t)$$

$$+ \frac{1}{\gamma^2}\,P(t)\,L'(t)\,L(t)\,P(t) + B(t)\,B'(t) \tag{8}$$

We note here that the H_∞ filter has the important directional property in that it enables the designer to optimize performance along specified directions of particular interest in the state space. This property shows up explicitly in the modified Riccati equation (8) through its dependence on L(t). There is no analog to that in the conventional Kalman filter.

The matrix P(t) is not an error covariance matrix in the H_∞ context, not even when the noises w(t) and n(t) are generated as white noise processes, for simulation or other evaluation purposes. It simply defines the appropriate filter gains in equation (7) at any time instant. Recall that in H_∞ theory the disturbances are not modeled as stochastic processes, so the notion of covariance is not meaningful. It is clear from the above equations that smaller γ's result in larger P's which in turn result in higher H_∞ filter gains, higher filter bandwidth, and more weight on the actual measurements Y(t). However, there is a lower bound γ_{min} below which no value of γ can give a realizable filter. The filter corresponding to γ_{min} is typically called the optimal H_∞ filter for the system. Note that as γ goes to infinity, equations (7) and (8) reduce to the standard Kalman filter equations (2) and (3), provided we change our interpretation for w(t) and n(t) to represent white noise.

We consider now a more general *nonlinear* dynamic system described by

$$\dot{X}(t) = g(X(t)) + B(t)\,w(t)$$
$$Y(t) = h(X(t)) + D(t)\,n(t) \tag{9}$$
$$Z(t) = m(X(t))$$

where, it is assumed that a nonlinear function Z(t) = m(X(t)) of the state is desired to be estimated. The vector-valued nonlinear functions g(.), h(.), and m(.) are assumed to be differentiable with continuous partial derivatives. Let us define the following nonlinear (extended) filter:

$$\dot{\hat{X}}(t) = g(\hat{X}(t)) + P(t)\,C'(t)\,(D(t)D'(t))^{-1}\,[Y(t) - h(\hat{X}(t))]$$
$$\hat{Z}(t) = m(\hat{X}(t)) \tag{10}$$
$$\dot{P}(t) = A(t)\,P(t) + P(t)\,A'(t) - P(t)\,C'(t)\,(D(t)D'(t))^{-1}\,C(t)\,P(t)$$
$$+ \frac{1}{\gamma^2}\,P(t)\,L'(t)\,L(t)\,P(t) + B(t)\,B'(t)$$

where, in equation (10), the matrices A(t), C(t), and L(t) are computed from the Jacobian matrices of the nonlinear functions and their numerical values are defined in terms of the most current state estimates as follows

$$A(t) = \left[\frac{\partial g}{\partial X}\right]_{X = \hat{X}(t)}$$
$$C(t) = \left[\frac{\partial h}{\partial X}\right]_{X = \hat{X}(t)} \tag{11}$$
$$L(t) = \left[\frac{\partial m}{\partial X}\right]_{X = \hat{X}(t)}$$

Equations (10) and (11) define what we call an *extended H_∞ filter*. Their derivation is based on successive linearizations of the state dynamics and measurement equations around the current state estimates, in a way similar to the derivation of the extended Kalman filter for nonlinear systems with continuous-time measurements. In view of equation (11), the matrix Riccati differential equation (10) must be integrated together with the state estimate equations to generate in real time the estimate $\hat{Z}(t)$ of the vector Z(t).

2.2 The Discrete-Time and Hybrid Equations for H_∞ Tracking Algorithms

In this section we focus on sampled-data tracking models where noisy (uncertain) measurements are available at discrete time instants only. We will discuss the more general situation of discrete-time measurements in conjunction with continuous-time target dynamics (hybrid model). The purely discrete case (discrete measurements and discrete target dynamics) can be derived from the hybrid one by directly integrating the continuous-time equations of the hybrid model (wherever they appear) as we shall see in the sequel. It should be pointed out that the hybrid model is the most appropriate for studying filtering performance trade-offs versus data rates, since the sampling rates for the measurements can be changed without affecting in any way the target dynamics.

For the hybrid model the state and measurement equations are given as follows:

$$\dot{X}(t) = A(t)\,X(t) + B(t)\,w(t)$$
$$Y(kD) = C(kD)\,X(kD) + D(kD)\,n(kD), \quad k = 0,1,2\ldots \tag{12}$$
$$Z(t) = L(t)\,X(t)$$

where, D is the sampling interval (we assume uniform sampling). Notice that a continuous-time function Z(t) of the state X(t) is to be estimated based on the sampled measurements Y(kD). Following the definitions of section 2.1, the H_∞ norm of a filter F producing estimates $\hat{Z}(t)$ can be defined in a way completely analogous to that of equation (6), the only difference being in the definition of the $L_2(0,T)$ norm of the now discrete measurement noise sequence n(kD); in this case, the appropriate norm is the $l_2(0,T)$ norm, defined as

$$\|n\| = \left[\sum_{k=0}^{T/\Delta} n'(k\Delta)\,n(k\Delta)\right]^{1/2} \tag{13}$$

So, the H_∞ norm of a filter F for the hybrid model (12) is defined through equation (6) and the use in the denominator of the mixed L_2/l_2 norm on the unknown signals w(t) and n(kD) affecting the system. We stress that in this hybrid H_∞ filtering framework w(t) is *not* modeled as a continuous-time white noise process, and n(kD) is *not* modeled as a discrete-time white noise sequence.

The filter whose H_∞ norm is less than a positive parameter γ was derived [14] as having finite jumps at the sampling instants. The filtering equations for that are as follows:

$$\dot{\hat{X}}(t) = A(t)\,\hat{X}(t), \quad t \neq k\Delta$$
$$\hat{X}(k\Delta) = \hat{X}(k\Delta^-) + P(k\Delta)\,C'(k\Delta)\,\big(D(k\Delta)D'(k\Delta)\big)^{-1}\,[Y(k\Delta) - C(k\Delta)\,\hat{X}(k\Delta^-)]$$
$$\hat{Z}(t) = L(t)\,\hat{X}(t) \tag{14}$$

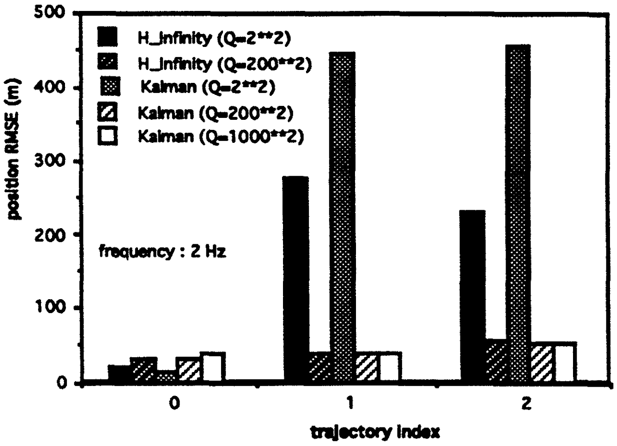

Figure 5. Performance Comparison Between H∞ and Kalman Filters for Different Bandwidths and Acceleration Disturbance Levels (2 Hz, position RMSE)

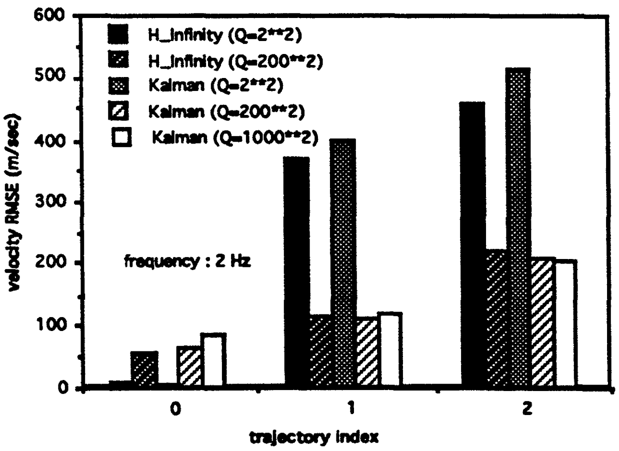

Figure 6. Performance Comparison Between H∞ and Kalman Filters for Different Bandwidths and Acceleration Disturbance Levels (2 Hz, velocity RMSE)

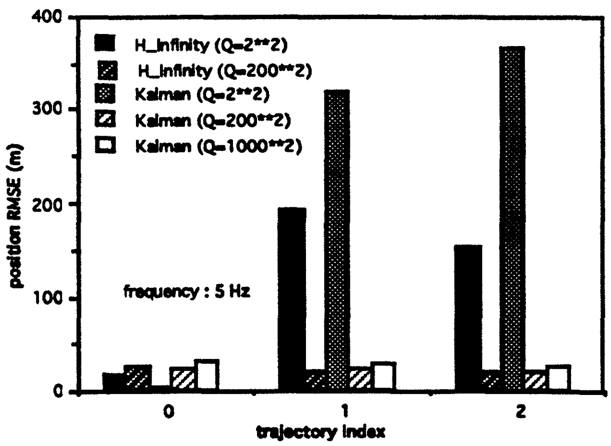

Figure 7. Performance Comparison Between H∞ and Kalman Filters for Different Bandwidths and Acceleration Disturbance Levels (5 Hz, position RMSE)

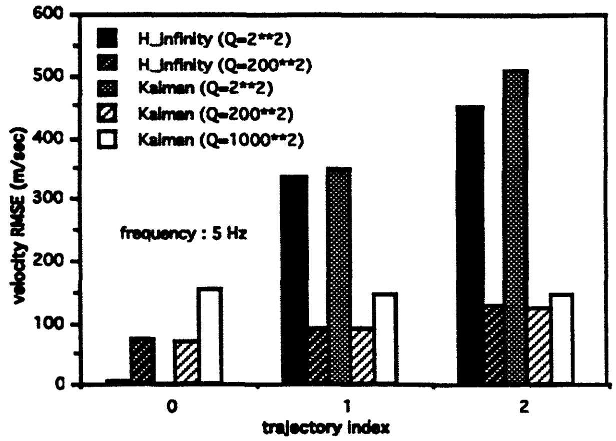

Figure 8. Performance Comparison Between H∞ and Kalman Filters for Different Bandwidths and Acceleration Disturbance Levels (5 Hz, velocity RMSE)

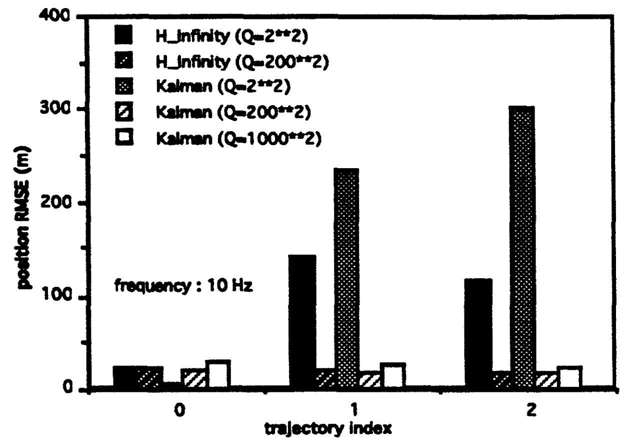

Figure 9. Performance Comparison Between H∞ and Kalman Filters for Different Bandwidths and Acceleration Disturbance Levels (10 Hz, position RMSE)

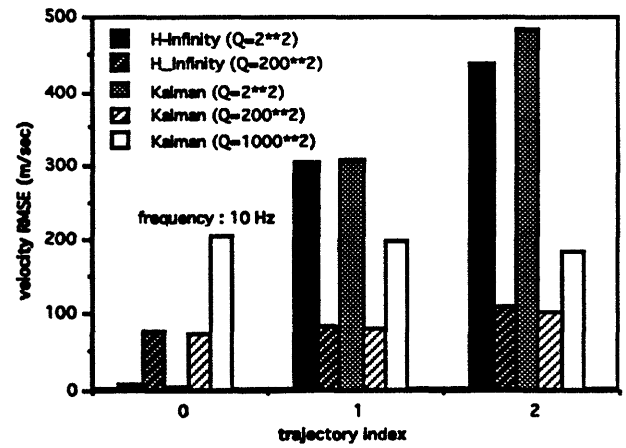

Figure 10. Performance Comparison Between H∞ and Kalman Filters for Different Bandwidths and Acceleration Disturbance Levels (10 Hz, velocity RMSE)

4. Discussion of the Results

The results shown in figures 5-10 provide performance summaries and comparisons of five different tracking algorithms on three different trajectories and for several sampling rates. These comparisons are quite representative;

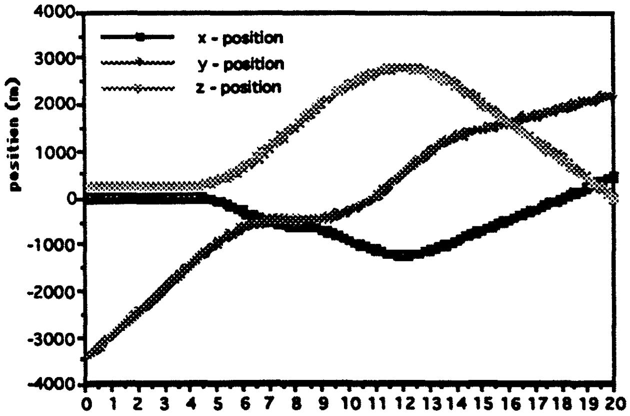

Figure 1. Target Position Profile Versus Time for Trajectory 1

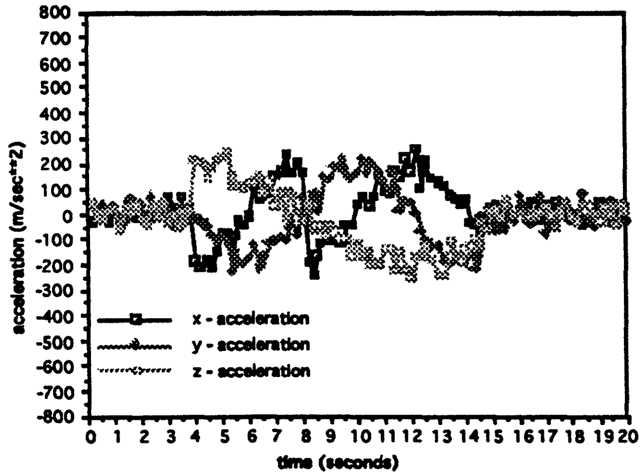

Figure 2. Target Acceleration Profile Versus Time for Trajectory 1

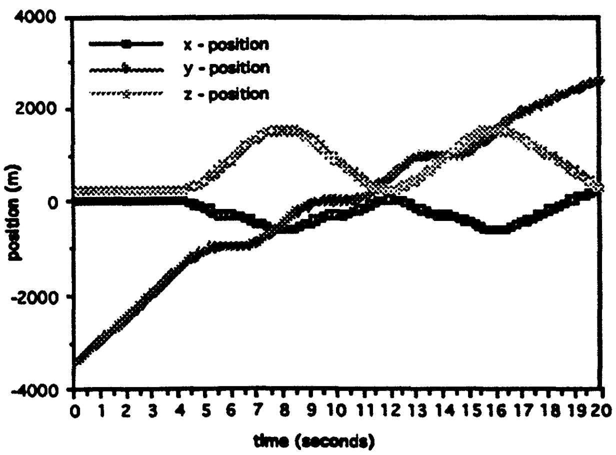

Figure 3. Target Position Profile Versus Time for Trajectory 2

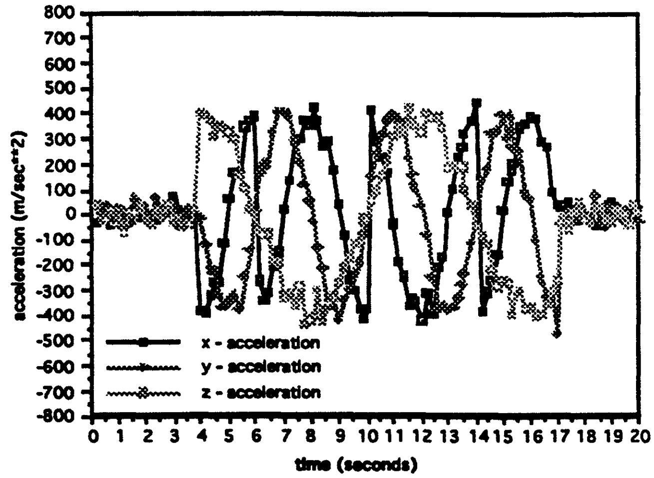

Figure 4. Target Acceleration Profile Versus Time for Trajectory 2

Regarding the radar measurement model, it is assumed that each measurement consists of a range-azimuth-elevation reading which is the true target position vector (r,θ,ϕ) in polar coordinates corrupted by additive white gaussian noises of standard deviations $(\sigma_r,\sigma_\theta,\sigma_\phi)$.

3.2 Algorithm Implementation. Performance trade-offs

The H_∞ filtering algorithm described in section 2.2 has been implemented and tested for the target maneuvering scenario described above. A single six-state (position and velocity) linear dynamic target model was used by the tracking algorithms together with a nonlinear measurement model (measurement accuracies are given in spherical coordinates). Thus, the filters assumed that the target was moving in a straight line without any severe maneuvers (with the exception of the small white acceleration noise discussed previously). Tracking performance was measured by the root mean squared error (RMSE) for position and velocity during the maneuvers, from 4 to 20 seconds (recall that the target is not maneuvering during the first 4 seconds of its flight).

The results presented in figures 5-12 below show RMSE performance comparisons between H_∞ and Kalman filters of different bandwidths (obtained by changing the value of the Q matrix) when they all operate on three different trajectories; the first trajectory is a straight line trajectory with zero acceleration disturbance (trajectory index = 0), the second trajectory (index = 1) is the one depicted in figures 1-2 having peak acceleration 277 m/sec^2, and the third trajectory (index = 2) is depicted in figures 3-4 and has peak acceleration 554 m/sec^2. All these results were obtained at three different sampling frequencies: 2, 5, and 10 Hz. We shall discuss the implications of these numerical results in section 4.

where, the symmetric positive definite matrix P(t) is given by the solution of the following Riccati-like equation with jumps:

$$\dot{P}(t) = A(t)\,P(t) + P(t)\,A'(t) + B(t)B'(t) + \frac{1}{\gamma^2} P(t)\,L'(t)\,L(t)\,P(t), \qquad t \neq k\Delta \tag{15}$$

$$P(k\Delta) = P(k\Delta^-)\left[I + C'(k\Delta)\,\left|D(k\Delta)D'(k\Delta)\right|^{-1} C(k\Delta)P(k\Delta^-)\right]^{-1}$$

The initial conditions for the state, X(0), and the matrix P(0) depend on the information available to the filter before measurements start coming in. We assume that an estimate $\hat{X}(0)$ at time 0 is available with a weighting matrix P(0). The superscript ($^-$) in the above equations denotes function values just before the specified time argument. Notice that the state estimates evolve according to the nominal target dynamics between sampling instants (predict cycle), but at the sampling times a discontinuity jump is forced by the measurements as an instantaneous estimate update (update cycle). The differential equation for P(t) in equation (15) evolves as a Lyapunov equation between sampling instants with the additional quadratic term that depends on γ and the directionality matrix L(t) (predict cycle). At the measurement times, the Riccati-like equation undergoes a jump whose magnitude is directly related to the measurement matrix C (update cycle).

Extensions of the above results for linear dynamic systems and filters to nonlinear ones are readily obtained in a way similar to that for continuous systems described at the end of section 2.1, i.e. by linearizing the nonlinear equations around the most current state estimates. For such more general time-varying and nonlinear dynamic models a trial-and-error methodology based on direct numerical integration of the Riccati differential equation (15) was used for the approximate computation of the minimum value of γ that guarantees existence of solution of the differential equation. Furthermore, given the time-varying nature of the linearized system, an adaptive scheme was devised for the computation of γ between sampling instants; a different value of γ was computed at the beginning of each sampling instant taking thus into account the changing parameters of the dynamic and measurement model. Thus, the numerical value of γ will change as a function of time, reflecting that the filter that generates the estimates has a different H_∞ norm every time a new measurement is obtained. The time-varying nature of γ has not been discussed in either a theoretical or a practical context prior to this study, to the best of our knowledge.

3. Performance Analysis

3.1 Description of the experiments

The tracking algorithms presented above were simulated for a radar tracking system . The radar was placed at the origin of a rectangular coordinate system, and made noisy measurements of target range, with an RMS accuracy of 8 meters in range, and target azimuth and elevation with RMS accuracy of 5 mrad in azimuth and elevation. The radar measurements occurred at discrete times, and the sampling interval could be varied. The measurement accuracies did not depend on the radar sampling rate. The measurement errors was simulated as discrete-time white Gaussian noise (in polar coordinates) for all the experiments.

The class of target trajectories were selected so that interesting sensor/target geometries would take place while the target was maneuvering, including "overhead" and "broadside" configurations. The target initially moves at a constant speed of 500 m/sec toward the radar keeping a straight line horizontal motion from 0 to 4 seconds. After 4 seconds it starts maneuvering at a constant speed. The differential equations of the constant speed maneuvers are of the form

$$\begin{aligned} \dot{x}(t) &= V\cos(\theta(t))\cos(\phi(t)) \\ \dot{y}(t) &= V\cos(\theta(t))\sin(\phi(t)) \\ \dot{z}(t) &= V\sin(\theta(t)) \\ \dot{\theta}(t) &= \omega_1(t) \\ \dot{\phi}(t) &= \omega_2(t) \end{aligned} \tag{16}$$

where V is the constant speed of the target, (x(t), y(t), z(t)) is the target position vector in a Cartesian coordinate system centered at the radar, $\theta(t)$ and $\phi(t)$ are the instantaneous elevation and azimuth angles of the velocity vector respectively, and, finally, $\omega_1(t)$ and $\omega_2(t)$ are the corresponding vertical and horizontal turning rates of the target. The instantaneous acceleration a(t) for this maneuvering model has a magnitude

$$\| a(t) \| = V\sqrt{\omega_1^2(t) + \omega_2^2(t)\cos^2(\theta(t))} \tag{17}$$

so that the acceleration magnitude increases with the turning rates and with the speed of the target. By varying the turning rates appropriately, one can achieve a wide variety of constant speed maneuvering trajectories.

We considered numerous maneuvering trajectories during our study and sampling rates. For the purpose of exposition we shall present our results and numerical comparisons for two types of such maneuvering trajectories, each characterized by a different pattern and magnitude of the acceleration disturbance. In the first trajectory type, the target takes initially a turn of $\pi/2$ radians both vertically (upwards) and horizontally (clockwise) with the same rate of $\pi/8$ rad/sec ($w_1(t) = w_2(t) = \pi/8$ rad/sec) after which it turns to the other direction at the same rate ($w_1(t) = w_2(t) = -\pi/8$ rad/sec). The peak acceleration here is 277m/sec^2. Thus, the first trajectory type emulates a sea-skimming missile that climbs (and turns on the plane at the same time) before diving toward the target. Figure 1 shows in more detail the characteristics of this trajectory. In the second trajectory type, the target undergoes a jinking maneuver taking initially a sharper turn with $w_1(t) = w_2(t) = \pi/4$ rad/sec for the first $\pi/2$ radians and then switches turning directions every π radians at the same rate. The peak acceleration for the second type of maneuver is 554 m/sec^2. In the second trajectory we have two jinking maneuvers in both altitude and in the plane before the target dives toward its destination (see Figure 3). It is pointed out here that the acceleration levels were chosen deliberately high to amplify the performance and robustness differences (if any) between Kalman and H_∞ filters.

We note that all trajectories were chosen to have significant acceleration components in all directions. Initial positions and velocities were chosen as follows: initial position = (0, -3500m, 250m), initial velocity = (0, 500m/sec, 0). The time horizon for all experiments is 20 seconds and the discretization interval for integration of the continuous-time equations is 0.005 seconds. A white Gaussian acceleration process noise of intensity 4 m^2/sec^3 was added to the nominal state equations for both trajectories over and above the maneuver accelerations.

similar results were obtained for numerous other variants not included in this paper.

First of all, we wish to emphasize that the "untuned" Kalman and H_∞ filters use a covariance intensity matrix of Q = 2^2I This value of Q corresponds to the actual white acceleration noise injected in the target dynamics causing minor maneuvering. Thus, for all practical purposes the "untuned" Kalman and H_∞ filters assume that the target moves in a straight line (trajectory index = 0). Since we are measuring performance by RMSE in total position and velocity the Kalman filter is indeed optimal; the H_∞ filter has a higher bandwidth so that it has a worse performance when the target is indeed non-maneuvering. On the other hand, when we compare the performance of the "untuned" Kalman and H_∞ filters (with a covariance intensity matrix of Q = 2^2I) in either one of the maneuvering trajectories, it is evident from figures 13 to 20 that the "untuned" H_∞ filter yields better tracking accuracy than the "untuned" Kalman filter. The absolute RMS errors depend on the sampling rate (obviously the lower the radar sampling rate the worse the estimation accuracy), but the performance ratio is about the same. It is clear that with Q = 2^2I we have a very low bandwidth Kalman filter which makes it inappropriate for tracking highly maneuvering targets. The "untuned" H_∞ filter has a much higher bandwidth and does a better job in handling the maneuvering targets.

These above results can generate a sense of optimism about the benefit of using H_∞ filters in tracking maneuvering targets. However, an experienced engineer knows that there exists a vast array of tools for "tuning" Kalman filters for tracking maneuvering targets. The simplest way is to increase the intensity Q of the process white noise, so as to increase the Kalman filter bandwidth. In figures 5 to 10 we show the RMSE performance of two different "tuned" Kalman filters with covariance intensities of Q = 200^2I and Q = 1000^2I, generating higher bandwidth Kalman filters. From figures 13 to 20 we conclude that these "simply-tuned" Kalman filters outperform the "untuned" H_∞ filter.on both maneuvering trajectories (but not on the straight line trajectory). The Kalman filter with the intermediate bandwidth of Q = 200^2I performs better at all trajectories and is even less sensitive to different kinds of trajectories and acceleration intensity levels. This is particularly evident for the velocity RMS errors and at higher sampling frequencies, as demonstrated in figures 14 and 16. Part of the reason for this seems to be the fact that the bandwidth of Q = 200^2 was chosen to capture an "average" acceleration disturbance of all the examined trajectories, a choice that presupposes some knowledge of the acceleration intensities of the anticipated maneuvers. On the other hand, a very high bandwidth Kalman filter (the one with Q=1000^2) while it performs much better at high acceleration disturbances (trajectory index=1 or 2), when there is no maneuver (trajectory index=0) it has a relatively high RMS error. However, the overall sensitivity of the high bandwidth filter is significantly lower than that of the low bandwidth filter across trajectories. Thus, one may dismiss the need for H_∞ filters (with their increased computational complexity) as viable candidates for tracking maneuvering targets. This is also a premature conclusion.

Such considerations led us to increase Q for the H_∞ filter as well, and by setting Q = 200^2I we obtain what we refer to as a "tuned" H_∞ filter. Examination of figures 5 to 12 leads to the conclusion that for all three classes of targets all three tuned filters have more or less the same performance. Thus, it is worthwhile to tune H_∞ filters as well, in a manner similar to Kalman filters. Indeed, we do not have a solid argument for selecting one class of filters over another. Indeed, tuned H_∞ filters may lead to performance improvements if position prediction accuracy is the performance objective (this exploits the directional properties of H_∞ filters that were exercised in the simulations reported herein).

In summary, it looks like a bandwidth adjustment is necessary for the H_∞ filters just like it is necessary for Kalman filters when it comes to tracking maneuvering targets. Numerous additional experiments with many intermediate bandwidths for both the Kalman and the H_∞ filters (not shown here) supported even further these observations.

So, there are two basic issues here that call for an explanation: (a) the inability of an H_∞ filter to outperform a well tuned Kalman filter, and (b) the lack of an automatic bandwidth adjustment property of the H_∞ filter. Before we embark on answering these questions, it is necessary to take a look first into the characteristics of the experimental model.

First, we should note that the measurement noise model used in the experiments is the classical additive white gaussian noise model of known covariance and is exactly the one for which a Kalman filter is optimal when the RMS error is the performance measure. So, the questions of filter robustness, performance comparisons and associated trade-offs are relevant only with regard to the target dynamic model. The acceleration disturbances due to maneuvering represent unknown deviations from the assumed nominal dynamic model (straight line constant speed model), and one issue here is whether or not these deviations can be modeled as white gaussian noises of appropriate covariance, i.e. whether or not we are again within the realm of optimality of a Kalman filter with only one parameter to adjust, namely, the process noise covariance. To phrase it in a different way, the issue is whether or not the maneuvering trajectories shown in figures 1-4 are "probable" sample trajectories of the nominal target dynamic model driven by white process noise of appropriate covariance.

The acceleration magnitudes versus time, as shown in figures 2 and 4 for the two types of trajectories, exhibit a certain amount of randomness due to the small white process noise added to the nonlinear constant speed dynamic model that generated the maneuvers. Also, it is clear that for all acceleration trajectories there is a single zero-mean gaussian distribution of appropriate covariance so that any individual point of each acceleration trajectory can be a "probable" sample point drawn from that distribution. However, as the existence of patterns in the acceleration trajectories shows, it is not true that sample points of these trajectories are independent of one another, so the entire trajectory is not a probable sample path of a white noise stochastic process (such a choice of maneuvers would be extremely unrealistic, unusual and probably nonrealizable). But since there is no information whatsoever about the acceleration disturbance to any of the filters, a more conservative and robust filter design would be the one based on a "least favorable" or "least informative" process noise which is nothing but a white noise stochastic process. The results we have obtained here show that this is exactly the case, that a Kalman filter based on a "white noise" interpretation of the acceleration disturbance with an appropriate choice of the process noise covariance (to take care of the average magnitude of the acceleration) is indeed an optimal robust filter for the types of disturbances considered and in the absence of any adaptation. The H_∞ filter with a similar bandwidth adjustment performs almost identically to the Kalman filter and this suggests that the quadratic term involving the parameter γ in the H_∞ Riccati equation (see eq. (15)) becomes immaterial in these cases; indeed, the adjustment of the Q matrix has much more effect on the H_∞ filter performance.

This brings us to the second question, namely, why is it necessary to adjust the bandwidth Q of the H_∞ filter, a job that was expected to be done automatically by the very nature of that filter. The key to answering this question is a closer examination of the kinds of maneuvers (acceleration disturbances) we are dealing with versus the nature of the "worst case" disturbances the H_∞ filter is protecting against.

A thorough answer to the question would require an explicit characterization of the worst case disturbances w(t) and n(t) attaining the maximum in equation (6) that defines the H_∞ norm of the filter. The H_∞ filter can be thought as the equilibrium solution of a game played between the filter designer on the one side controlling the filter parameters, and "nature" on the other side controlling the disturbances. The payoff function of the game is the filter error energy per unit disturbance energy (the quotient inside the brackets in equation (6)) and depends both on the filter parameters and on the disturbances. "Nature" wants to maximize that function by creating the appropriate disturbances (worst case disturbances) while the filter designer wants to minimize it by employing the appropriate filter. So, first of all, the designer assumes that "nature" is an intelligent and rational player that has full information about the payoff function of the game. Furthermore, "nature" is assumed to have full information about the filter parameters based on which it employs its "strategy" of inflicting the maximum damage to the filter. It turns out that the worst case disturbance (that constitutes nature's strategy) is a function of the filter itself and its parameters, such as the filter gain, the matrices P, A, B, C, D etc. So, these disturbances will have to be sophisticated and knowledgeable of our target state estimation methodology and parameters. We have no knowledge whether or not disturbances with such characteristics are useful in real tracking applications. The H_∞ filter provides the highest degree of protection against the "worst case" disturbances by minimizing the error energy they induce but this alone does not guarantee the same degree of protection against all other kinds of disturbances. Indeed, one may have to introduce a frequency weighting function that indicates the range of frequencies that errors should be penalized, and these will be intimately related to the worst case maneuvers that one can expect. Indeed in recent feedback applications of both H_2 and H_∞ theory the usefulness of such frequency weights has been demonstrated. Such concepts have not been specialized to tracking applications. It is indeed interesting to note that the use of such weighting functions would increase the dimensionality of the Kalman or H_∞ filter, thus being analogous to introducing a reasonable dynamic model of the mechanism by which accelerations are generated!

In summary, it was found that an untuned H_∞ filter, as defined in sections 2.1 and 2.2, does not outperform an appropriately tuned Kalman filter in tracking a class of constant speed maneuvering targets, described in section 3.1. Furthermore, it was found that the H_∞ filter requires a tuning itself for robustness and performance, although the performance gap between a tuned and an untuned H_∞ filter is less dramatic than the gap between a tuned and an untuned Kalman filter.

5. References

1 Nagpal, K.M. and P.P. Khargonekar, "Filtering and Smoothing in an H_∞ Setting," *IEEE Trans. on Auto. Control*, Vol. 36, 1991, pp. 152-166.

2 Bernstein, D.S. and W.M. Haddad, "Steady-state Kalman Filtering with an H_∞ Error Bound," *System and Control Letters*, Vol. 12, 1989, pp. 9-16.

3 Krener, A.J., "Kalman-Bucy and Minimax Filtering," *IEEE Trans. on Auto. Control*, Vol. AC-25, 1980, pp. 291-292.

4 Doyle, J.C. *et al*, "State Space Solution to Standard H_2 and H_∞ Control Problems," *IEEE Trans. on Auto. Control*, Vol. 34, 1989, pp. 831-847.

5 Zames, G., "Feedback and Optimal Sensitivity: Model Reference Transformations, Multiplicative Seminorms, and Approximate Inverses," *IEEE Trans. on Auto. Control*, Vol. AC-26, 1981, pp. 301-320.

6 Francis, B.A., *A Course in H_∞ Theory*, Springer-Verlag, NY, 1987.

7 Gelb, A. (ed), *Applied Optimal Estimation*, MIT Press, Cambridge, MA, 1974.

8 Anderson, B.D.O. and J.B. Moore, *Optimal Filtering*, Prentice Hall, Englewood Hills, NJ, 1979.

9 Singer, R.A., "Estimating Optimal Tracking Filter Performance for Manned Maneuvering Targets," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-5, 1970, pp. 473-483.

10 Athans, M. and C.B. Chang, *Adaptive Estimation and Parameter Identification Using Multiple Model Estimation Algorithms*, ESD-TR-76-184, Technical Note 1976-28, MIT Lincoln Lab., Lexington, MA, June 1976.

11 Chang, C.B., R.H. Whiting, and M. Athans, "On the State and Parameter Estimation for Maneuvering Reentry Vehicles," *IEEE Trans. on Auto. Control*, Vol.AC-22, 1977, pp. 99-105.

12 Bloom, H.A.P. and Y. Bar-Shalom, "The Interacting Multiple Model Algorithm for Systems with Markovian Switching Coefficients," *IEEE Trans. on Auto. Control*, Vol. 33, 1988, pp. 780-783.

13 Bar-Shalom, Y. *et al*, "Tracking a Maneuvering Target Using Input Estimation Versus the Interacting Multiple Model Algorithm," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-25, 1989.

14 Sun Weiqian, Nagpal, K.M. and P.P. Khargonekar, "H_∞ Control and Filtering for Sampled-Data Systems", July 1991, to appear.

15 Blair W. D. and Watson G. A., "Interacting Multiple Bias Model Algorithm With Application to Tracking Maneuvering Targets", *31st IEEE Conf. on Decision and Control.*, Tuscon, AZ, Dec. 1992

16 Blair W. D. and Parker J. D., "Use of Target-Oriented Process Noise in Tracking Maneuvering Targets", *29th Annual Allerton Conference on Communications, Control and Computing*, Oct. 2-4, 1991.

17 Alouani A. T. and Blair W. D., "Use of Kinematic Constraints in Tracking Constant Speed, Maneuvering Targets", *30th IEEE Conf. on Decision and Control*, Brighton, UK, Dec. 11-13 1991.