Design Methodologies for Controlling Vibrations in Buildings

by

Errol Laub
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ABSTRACT

Although vibration absorbers have been employed in mechanical equipment mounting for over 100 years, they have only recently been used for isolating large-scale structures such as bridges and buildings. Their applications range from acoustic to seismic isolation of structures. Virtually any vibration magnitude can be prevented from degrading the performance of a structure. This thesis is intended to provide a practical introduction to the design and suitability of vibration isolators for various environments. It contains a discussion of the past, present and future of vibration isolation associated with buildings. Analysis and design examples are provided to illustrate the design methodology.

Thesis Supervisor: Jerome J. Connor, Jr.
Title: Professor of Civil and Environmental Engineering

Acknowledgments

I would like to first thank my wonderful family for bringing me to this stage in my life, and for their unbelievably supportive attitude toward my education. This thesis is dedicated to my exceptional parents Richard and Roselyn Laub, and my siblings Ethan, Asher and Abigail.

My thesis would not have been possible without the guidance of Professor Jerome J. Connor, Jr. and Cheryl Sirna, both of whom helped me find the most important resources I needed in order to complete this paper. In particular, I would like to give special thanks to Professor Connor for his helpful input throughout the Masters of Engineering program, and for the caring, thoughtful way he pushed me to discover more about the field of structural engineering.

I am also very grateful to those friends at MIT who helped and encouraged me throughout this project. I know how fortunate I have been to work with them over the past year, and I wish them the best of luck in their future endeavors.

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Chapter One: Introduction to Vibration Isolation of Buildings

Definition of Vibration Isolation and Problem to be Solved

All structures are subject to vibration. The vibration isolator is a device that is designed to effectively isolate such structures from harmful vibrations. A device's ability to isolate is a result of its ability to temporarily store energy and release it at a later, less time-critical moment. The most basic isolation system consists of an excitation source attached to a receiver structure through an isolation element, such as a rubber bearing or metal spring.

Two important components of isolation are motion isolation and force isolation. In isolation of motion, the engineer attempts to reduce stresses and deflections in the system in order to create a comfortable working environment in a critical area (for either humans or machines). In isolation of forces, the engineer concentrates on reducing the forces transmitted by the source to a critical area in order to avoid the structure itself having to absorb the transmitted force. [Fabreeka]

The list of isolator applications in building vibration control is endless. Vibration isolators are used to decrease the annoyingly large vibrations of forge machinery that disturb neighbors. They are used to decrease vibration originating from subway traffic that enters through the basement of buildings and disturbs sensitive laboratory equipment. The devices are also used to protect historic buildings from earthquakes. In each case, engineers must custom-design a solution to the particular levels and distribution of unwanted vibration.

Isolator Manufacturers and Examples of Isolated Structures

The range of isolator manufacturers is almost as broad as the field itself. Companies like Fabreeka, Vibro/Dynamics, Lord Corporation, and Kinetics manufacture numerous kinds of isolators for industrial applications. Figure 1 shows a few of the many models available.

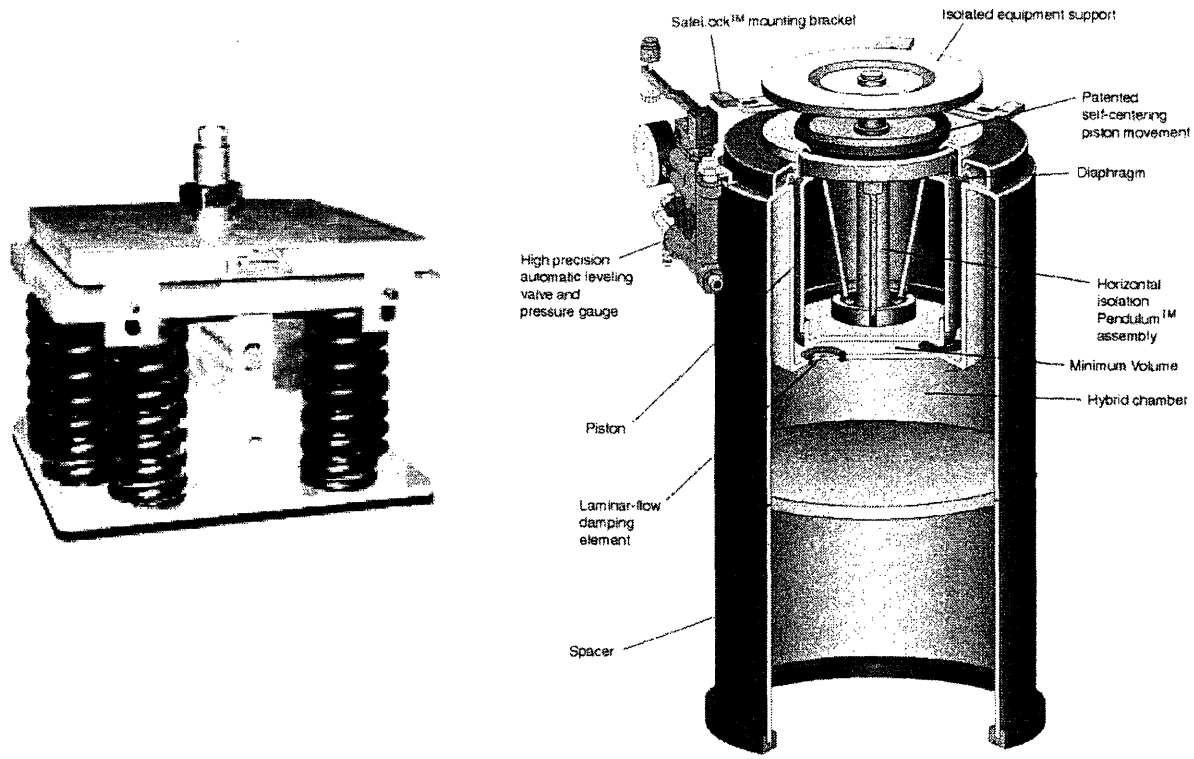

Figure 1: Typical vibration isolators [Vibro/Dynamics and Newport]

These devices have been used to isolate forge floors (from heavy machinery), coordinate leveling machines, semiconductor facilities, and even an atomic microscopy laboratory. For example, the Villa Farnesina in Rome, Italy was isolated in the 1950s from traffic on a nearby road that was causing deterioration of its external decoration and internal frescoes [Clemente]. Ordinarily, this building would have remained in good condition indefinitely. However, due to uncommonly high vibrations that the building experienced from several earthquakes, the Villa Farnesina's masonry was showing severe signs of deterioration by the year 1950. It was correctly believed at the time that vibrations of small amplitude but high frequency – such as traffic vibrations – could cause a significant reduction in masonry strength of a structure previously damaged by earthquakes.

From a traffic-induced vibration test, it was determined that the maximum acceleration due to traffic was about 0.0045 g, and the main frequency experienced by the Villa Farnesina was between 16 and 32 Hz. To decrease the vibrations originating from traffic, rubber bearings were placed in a grid pattern beneath the heavily traveled Lungotevere road.

In earthquake isolation applications, large DIS (Dynamic Isolation Systems) isolators have been used to isolate entire buildings and bridges. The DIS isolator is shown in Figure 2.

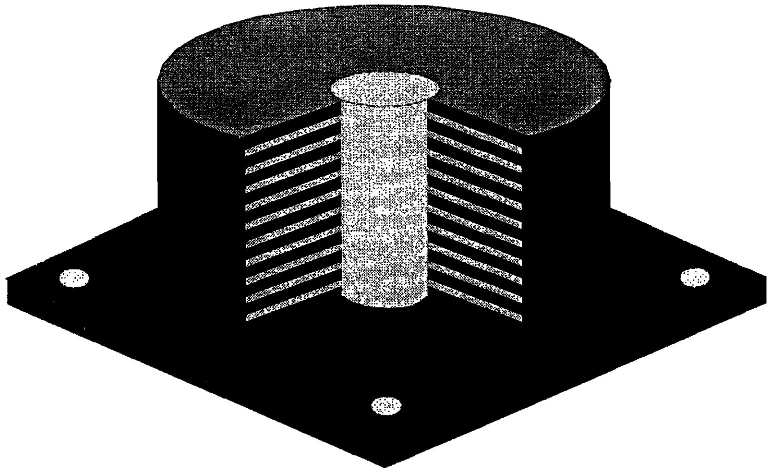

Figure 2: DIS Seismic Isolator [DIS]

This isolator has been used to protect many types of structures, including San Francisco and Oakland City Halls, and Stanford's Linear Accelerator Center. Although the field of seismic isolation is barely several decades old, there are several manufacturers of seismic isolator bearings in the U.S., and the industry is growing rapidly.

Chapter Two: Types of Vibration Isolators and their Uses

There are many different kinds of vibration isolators currently on the market, but the three most common are metal spring isolators, elastomeric isolators, and pneumatic isolators. Each of the three has advantages over the others for certain applications and frequency ranges. Other materials used for isolation are felt, cork, fabric and wool. These materials are used when the device must be very stiff.

Metal Spring Isolators

These devices are effective for vibration frequencies between 1 and 8 Hz [Bachmann], and have been used for industrial machinery for over 100 years [Fabreeka]. They act as isolators through their stiffness alone, since metal as a material does not provide damping. This means that metal spring isolators can only be designed for a single forcing frequency, and will not be effective if the forcing frequency changes. For this reason, isolator manufacturers often install separate damper parts into metal spring devices to artificially provide damping. Helical metal springs are effective even in spatial isolation, and can therefore be used in cases where unbalanced rotating masses produce low-frequency excitation. However, slender helical springs are sometimes lacking in global stability, and engineers must carefully consider the flexibility of the isolator in all directions before finalizing a design. This concern is particularly prevalent when low frequency isolation is required. Figure 3 shows a typical metal spring isolator.

Metal spring isolators have several disadvantages when compared with elastomeric isolators, however. Sound is transmitted well through a metal spring isolator, and therefore unwanted acoustic vibrations are still transmitted to the receiver. Metal spring isolators only provide stiffness, as noted above, and do not provide damping.

Metal spring isolators, however, have several advantages over elastomeric isolators. They are relatively ductile; they perform well even under large deflections. In many cases, they also perform well under wide temperature extremes. Metal isolators do not have a tendency to drift, or continuously deflect, under dead loads. For this reason, as well as several others, the characteristics of metal spring isolators can be predicted almost exactly.

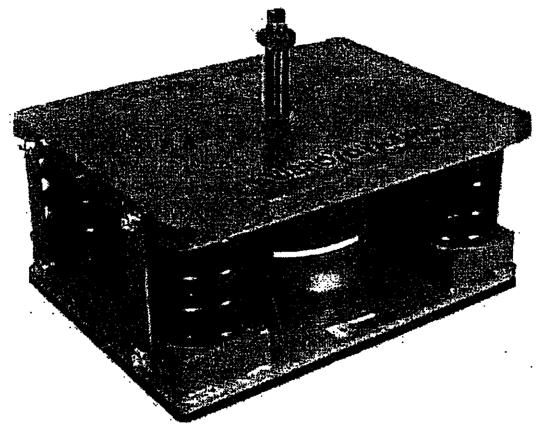

Figure 3: Typical Metal Spring Isolator [Vibro/Dynamics]

Elastomeric Isolators

The term "elastomeric" refers to rubber as an isolating material. These isolators are also over 100 years old, and have been used in high-frequency applications (from 5 to 20 Hz) [Bachmann]. Figure 4 shows a typical elastomeric isolator.

Rubber vibration isolators were almost always made from natural rubber before World War II, and although during the War synthetic rubber isolators were developed, natural rubber is still the most widely used in elastomeric isolators [Crede]. Some of the synthetic rubbers that have been successfully used in vibration applications are Buna N, Nitrile, and Neoprene. Buna N and Neoprene are especially satisfactory in high-temperature operation, as they deteriorate more slowly than other rubbers at high temperature. Neoprene and natural rubber both withstand static loads at room temperature without much drift or creep, and Buna N is extremely good for resisting drift at high temperature.

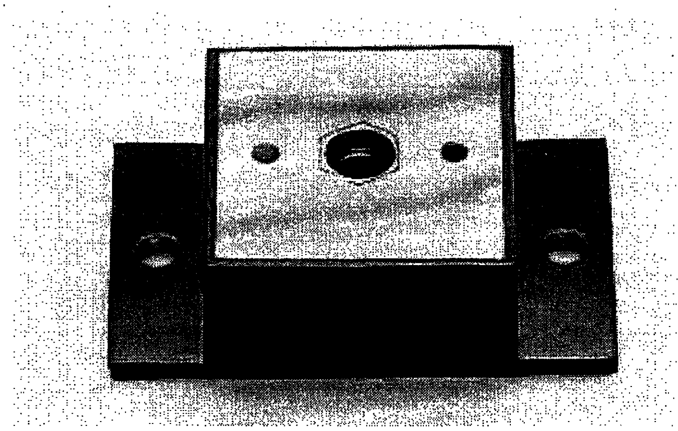

Figure 4: Typical Elastomeric Isolator [Fabreeka]

Elastomeric isolators have several disadvantages when compared with metal spring isolators. They are relatively brittle; they do not maintain their performance under large deflections. In most cases, they also perform badly when wide temperature extremes are encountered. Elastomeric isolators also have a tendency to drift, or continuously deflect, under dead loads. The characteristics of rubber isolators are difficult to predict well, as there is a fair spread in the properties of identically manufactured isolators.

Rubber isolators have several advantages over metal isolators, however. Sound is transmitted poorly through an elastomeric isolator, and thus acoustic isolation can be achieved to some degree by using a rubber isolator instead of a metal one. Elastomeric isolators provide damping as well as stiffness; in fact, elastomers provide damping ratios between 5% and 17% of critical.

Manufacturers of elastomeric isolators usually adhere many strips of premolded, ¼"-thick elastomers together to obtain the necessary thickness [Fabreeka]. This procedure is more cost-effective than cutting custom-sized bearings for each application, and the adhesion does not detract from an isolator's performance. To produce an isolator, a single base material – such as Nitrile or Neoprene – is selected and is varied in profile, hardness, and/or load capacity depending on the application. This material automatically sets the damping ratio of the device. To obtain a desired stiffness value, which depends on the thickness of the isolator, one increases the number of elastomer layers to decrease the stiffness k. Similarly, seismic isolators are composed of many layers of elastomer and metal plates to provide high vertical stiffness and low horizontal stiffness. Of course, the stiffness and damping characteristics of an elastomeric isolator are by no means constant [Halvorsen]. Both properties vary significantly with temperature and frequency of excitation. For instance, a Nitrile isolator's shear modulus (hence stiffness) changes by a factor of 20 over a temperature range from –7 to 49°C when subjected to 1000 Hz excitation. At 1 Hz, however, Nitrile's stiffness has no significant variation with temperature. Loss factor, an indicator of the damping value, is almost a factor of 10 greater for Nitrile at 1000 Hz than at 1 Hz. See the left side of Figure 5 for the variation of Nitrile stiffness and damping values for various temperatures.

Displacement transmissibility is simply the ratio of vibration amplitude at the receiver to the amplitude at the source. When transmissibility is low – certainly less than one – the isolator is performing well for the particular situation. For means of design, the displacement transmissibility of an isolator is plotted for various temperatures versus frequency, as shown in the right chart of Figure 5. This gives the engineer a feel for the behavior of a particular isolator in a given configuration. This graph also helps the engineer understand how the isolator will react when ambient temperature is altered.

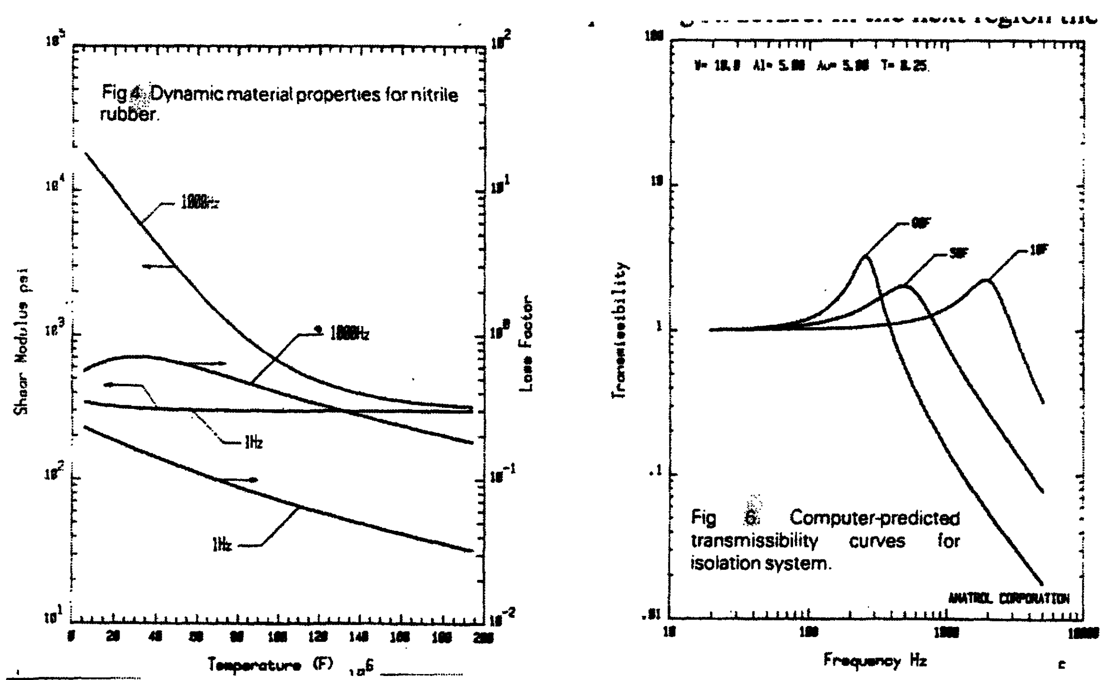

Figure 5: Isolation properties and transmissibility of Nitrile, for various frequencies [Halvorsen]

Pneumatic Isolators

"Pneumatic" denotes an air spring, and these isolators are used especially for low frequency forcing of 0.4 to 5 Hz (or large shock displacement) [Fabreeka]. Figure 6 shows a typical pneumatic isolator.

Pneumatic isolators are used for many different applications, including vehicle suspensions, coordinate measuring machines, electro/optical instruments and precision machine tools. They are usually at least semi-portable, and thus an engineer may decide to consider several uses for the isolator over its lifetime. Pneumatic isolators are precision equipment, and are increasingly being used in many different industries.

In some cases, an engineer may decide to combine two or even more types of isolators to create a composite that has advantages over each of its separate components. Figure 7 is an example of an isolator with pneumatic and elastomeric components.

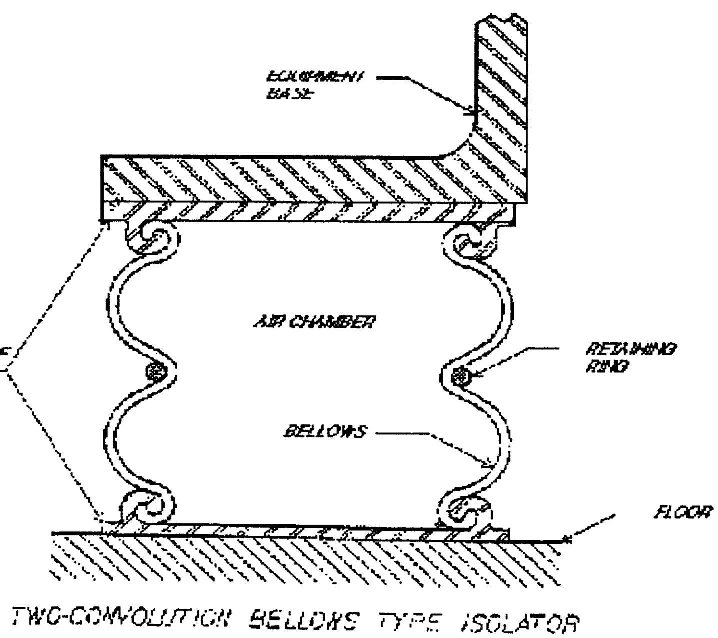

Figure 6: Typical Pneumatic Isolator [Fabreeka]

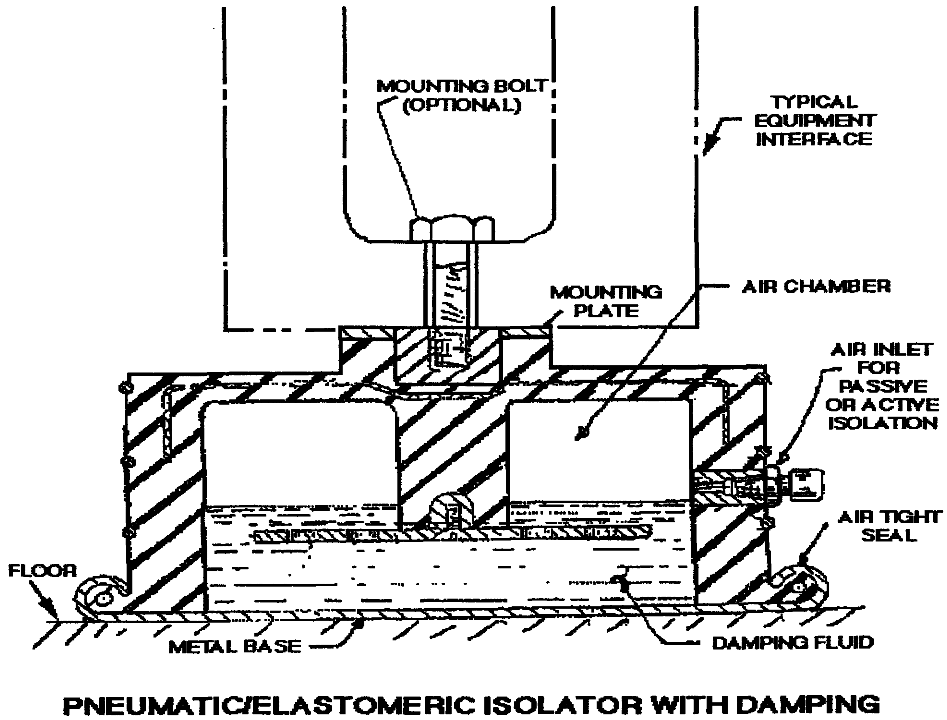

Figure 7: Schematic of typical Pneumatic / Elastomeric Isolator [Fabreeka]

Chapter Three: Vibration Measurement and Design Issues for the Engineer

Vibration Measurement

Table 1 lists some of the more common sources of vibration, and the frequencies and amplitudes – in inches or % of gravity – of the excitations [Fabreeka].

Sources	Frequency (Hz)	Amplitude (in. or % gravity)
Air Compressors	4-20	10^{-2}
Handling Equipment	5-40	10^{-3}
Pumps	5-25	10^{-3}
Building Services	7-40	10^{-4}
Foot Traffic	**0.5-6**	**10^{-5}**
Acoustics	100-10000	Various
Air Currents	Various	Various
Punch Presses	Up to 20	10^{-5} - 10^{-2}
Transformers	5-400	10^{-5} - 10^{-4}
Elevators	**Up to 40**	**10^{-5} - 10^{-3}**
Railroad	**5-20**	**+/- 0.15 g**
Highway Traffic	**35 MPH**	**+/- 10^{-3} g**
Earthquakes	**0.3-8**	**0.5 g**
Nuclear Detonations	0.3-8	0.5 g

Table 1: Common Vibration Sources [Fabreeka]

As any vibration consultant knows, there are numerous possible sources of vibration that should be considered when isolating any part of a building. Some of the most notable sources are written in the table above in bold lettering. Foot traffic, elevators and highway traffic near a building are especially important to notice where sensitive laboratory equipment is concerned. Recognizing the source(s) of vibration that excites a structure is not the only variable in isolation design, however. The distance between a structure and the source of the excitation(s) it is subjected to plays an extremely important role as well, since vibrations will usually attenuate (decrease in amplitude) over a short distance. Since no hard-and-fast rules exist for estimating the frequency or magnitude of vibrations a structure will experience by knowing the sources of vibration, vibration consultants always use portable measuring equipment to measure the level of vibration in key areas around the site in question. These surveys can determine considerable information about the excitation, including:

1. the direction of the vibration source,
2. the amplitude of vibration to see if it exceeds the sensitivity level required by the equipment or area to be isolated, and therefore the engineer can decide whether isolation need be considered at all,
3. the Power Spectrum Density of the source,
4. the effectiveness of the isolator, since the "floor" vibration can be compared with the isolated system.

Besides this extensive measurement survey of existing vibrations, the vibration consultant must also take measurements outside the everyday range of excitations, since the site is likely to experience unexpected vibrations – such as beat frequencies and shock pulses – frequently. To measure these vibration levels, an artificial excitation must be created using actuator devices to simulate these unexpected loads, and measuring equipment can then be used to determine the resulting vibration levels.

Suggested Overall Acceptance Levels of Vibration

A generalized guide to the acceptance levels of several environments is presented in Table 2 [Bachmann]. These vibration levels are not necessarily required according to law, but are global criteria derived from engineering experience and knowledge of the acceptable vibration levels stated in numerous building codes.

Environment	Acceptance Level
Pedestrian structures	a ≤ 5 – 10 % g
Office buildings	a ≤ 2 % g
Sports hall	a ≤ 5 – 10 % g
Dancing/Concert halls	a ≤ 5 – 10 % g
Factory floors	v ≤ ~ 10 mm/s

Table 2: Suggested Overall Acceptance Levels of Vibration [Bachmann]

Vibration tests on people have concluded that a vibration amplitude of 0.001 inch (0.1 mm.) is easily detected by office employees while working. This suggested acceptance level requires that floor deflections in office buildings also be limited due to vibration acceleration considerations.

Another consideration that enters the design of a building is that building damage will often occur when either large amplitudes or frequencies of excitation – or both – are experienced. Damage will probably occur when the vibration frequency is 8 Hz and the amplitude is 3900 microinches. Another damaging combination is 100 Hz and 400 microinches.

Stating vibration acceptance levels in terms of acceleration and velocity units may be convenient for the purposes of determining the need for isolating a structure, but once it has been decided to use vibration isolators in a design, the vibration's frequency becomes extremely important. This is because excitation frequency determines the type of isolator to employ in a design, as noted earlier.

To illustrate how frequency is used to determine acceptable vibration levels, let us consider an office building environment. The primary vibration of concern in an office building is foot traffic, unless the building is slender – in which case wind becomes a consideration – or unless vibration-sensitive equipment or other special cases are present. If a building structure is composed of materials that have natural frequencies at or near the walking pace of employees, the building's floors will probably vibrate in an uncomfortable way. Fortunately, this is almost never the case. The walking rate for most people is around 2 Hz, while concrete and steel floor systems usually have much higher natural frequencies. For the rare case that the walking rate reaches more than 2.4 Hz, the fundamental frequencies for floors should comply with the following guidelines:

- reinforced concrete: $f_1 > 7.5$ Hz
- prestressed concrete: $f_1 > 8.0$ Hz
- composite (in-situ concrete slab on steel): $f_1 > 8.5$ Hz
- steel (corrugated sheets with concrete infill: $f_1 > 9.0$ Hz

Massive concrete members often perform well even with fundamental frequencies below these suggested values, but they usually have frequencies above 7.5 Hz in any case. For special sensitive cases, the frequency bounds suggested above probably need to be raised.

Steps of Vibration Isolation Design

Vibration Control Engineering consists of five basic steps [Bramer, et al]. They include:

1. Analyze the environment of the vibration source and receiver, and form a simple mathematical model of the excitation properties.
2. Specify all of the design performance criteria, including vibration tolerance of the environment requiring isolation.
3. Decide which isolation method is most appropriate for the problem.
4. Perform a theoretical analysis of the new (isolated) system's vibration performance using the selected method.
5. Select an isolator product that meets the constraints imposed on the problem, including available space, loading, fastening possibilities, stability, and visual appearance.

The first step in design – analyzing the vibration environment around the site – can easily be taken to extremes if decisions are not made early in the analysis as to which sources are most likely to have the most effect on the receiver structure. The environment surrounding the receiver may actually provide substantial damping of its own, especially when vibrations are transmitted to the receiver by the ground [Fabreeka]. Table 3 shows typical damping factors of several soil types.

Soil Type	Damping Factor ξ
Dry sand and gravel	0.03 – 0.07
Dry and saturated sand	0.01 – 0.03
Dry and saturated sand and gravels	0.05 – 0.06
Clay	0.02 – 0.05
Silty sand	0.03 – 0.10

Table 3: Typical damping factors of several soils [Fabreeka]

There are several characteristics that an isolator must have to perform adequately. It must remain elastic over the life of the installation. Of course, it must be able to support both the static weight and the unbalanced dynamic force of the unit or system it is isolating. The isolator should also have a lower natural frequency than the excitation itself, otherwise the excitation may actually be amplified in force and amplitude. An isolator's natural frequency in Hertz is calculated according to the following relationship:

$$fn = 3.13 * \sqrt{\left(\frac{K_D}{W}\right)}$$

where K_D is the Dynamic spring rate in pounds/inch, and W is the static weight of the isolated unit in lbs.

An isolator is more effective when its natural frequency is low. At the very least, it should have a frequency such that the ratio of disturbing frequency to isolator natural frequency is 1.414 or $\sqrt{2}$.

The third step, deciding which isolation method is best for the particular application, may be avoided altogether with thoughtful consideration of the problem situation. For instance, the problem itself might be corrected by simply dynamically balancing the unbalanced machine or device that is producing the disturbance. The speed of the unbalanced rotation might be reduced without affecting the machine's overall performance, though this is also often unpractical. The most effective alternative to isolation is relocation of the vibration source to another site that is far enough from sensitive areas. This solution can only be effective if the unbalanced vibration is not harmful to the machine itself. Finally, one might modify the sensitive piece's response to excitation by adding mass or increasing stiffness.

Simplifications for Single-Degree-of-Freedom (SDOF) Systems

There are two basic cases of excitation that may be encountered in an SDOF vibration isolation problem [Connor]. The first involves external periodic forcing, while the second involves support motion.

In the first case, an isolator is used to isolate the support from the external excitation. This case can apply to an unbalanced machine that disturbs employees in a laboratory. Figure 8 is a schematic diagram of an unbalanced machine of mass m that produces a periodic load p and moves a varying distance u. An isolator is modeled as a spring of stiffness k, and a damper of damping c.

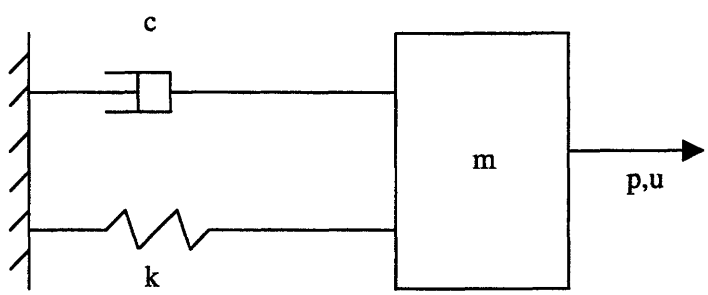

Figure 8: Single-Degree-of-Freedom system undergoing external loading [Connor]

In this diagram, $p = p_0\sin\Omega t$ and therefore $u = u_0\sin(\Omega t-\delta)$, where $u_0 = H1*p_0/k$, $H1 = ((1-\rho^2)^2+(2\xi\rho)^2)^{-0.5}$, $\tan\delta = 2\xi\rho/(1-\rho^2)$, and $\rho = \Omega/\omega$. The transmissibility or reaction force of the support divided by p is called H3. The function H3 is calculated as follows:

$$H3 = \sqrt{\frac{1+(2\xi\rho)^2}{(1-\rho^2)^2+(2\xi\rho)^2}}$$

One can see that for isolation to be effective (ie. H3 becomes less that one), the frequency ratio ρ should be such that $\omega < \Omega/\sqrt{2}$.

In the second case, an isolator is used to isolate the machine from the support excitation. This case can apply to sensitive coordinate measuring devices that must be isolated from floor vibrations. Figure 9 is a schematic diagram of a machine of mass m that is excited by a periodic load p originating at the support. For this case, the support moves a varying distance u_g, and the machine moves a corresponding distance u_t. An isolator is again modeled as a spring of stiffness k, and a damper of damping c.

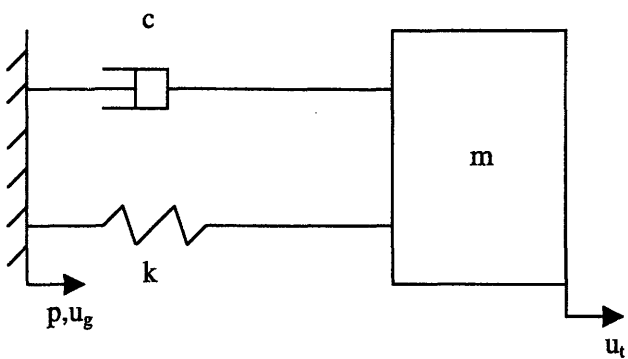

Figure 9: Single-Degree-of-Freedom System Undergoing Support Excitation [Connor]

In this diagram, H3 = u_{t0}/u_{g0}, where u_{t0} is the amplitude of support motion, and u_{g0} is the resulting amplitude of machine motion. One can see that in both cases, the period of the isolator should be less than the period of forcing.

In many instances, an isolated system is composed of a mass that itself has stiffness and damping, and this member is attached to an isolator that has mass m_f. Figure 10 shows a diagram of this model:

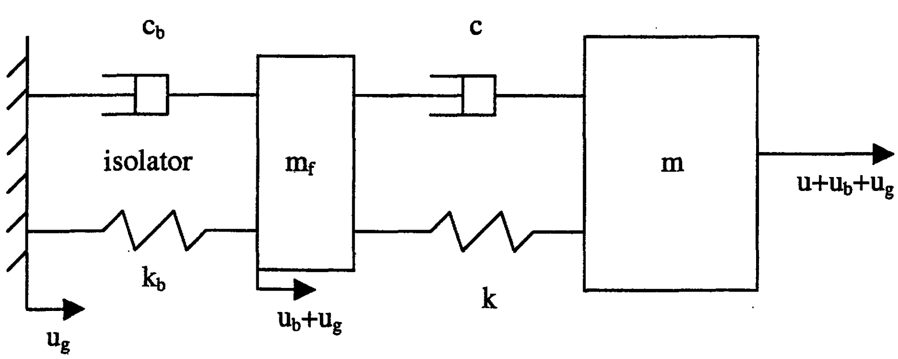

Figure 10: Model of isolated mass with internal stiffness and damping [Connor]

The equations of motion for an isolated machine that itself has a stiffness and damping quantity are:

$$md^2u/dt^2 + cdu/dt + ku = -m(d^2u_b/dt^2 + d^2u_g/dt^2)$$

and

$$m_f d^2u_b/dt^2 + k_b u_b + c_b du_b/dt = ku + cdu/dt - m_f d^2u_g/dt^2$$

These equations use u_g which is the displacement of the ground, u_b which is the displacement of the isolator, and u which is the displacement of the mass. For this isolator model, k_b is designed to be much less than k. If the isolator moves with a displacement amplitude u_{b0} approximately equal to u_{g0}, u_0 will be much less than the support displacement u_{g0}, and isolation will be almost perfect. In general, the frequency ratio is taken as approximately three, corresponding to an equivalent period of three times the period of excitation.

Rubber Bearing Models

Two main types of rubber isolators exist for vibration isolation: natural rubber and lead rubber isolators. Natural rubber isolators are usually modeled as simple shear members that consist of viscoelastic material. The term viscoelastic refers to a material's ability to provide elasticity as well as damping. Figure 11 is a diagram of a deformed natural rubber isolator.

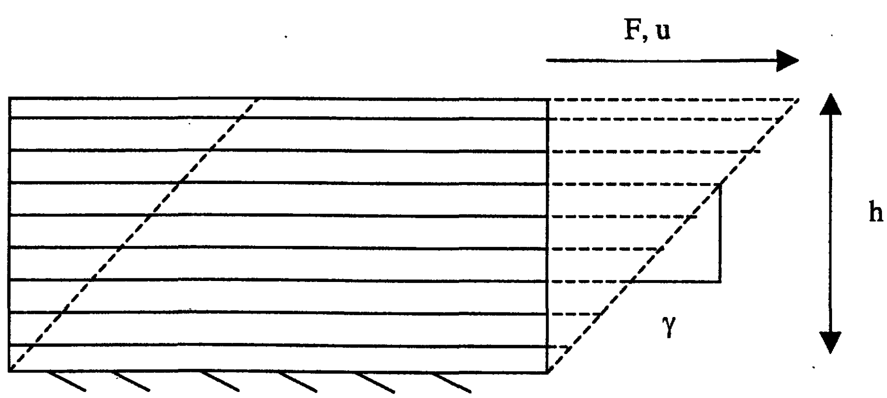

Figure 11: Model of Natural Rubber isolator under horizontal excitation [Connor]

From the above figure, $\gamma = u/h$, which means that for a periodic loading that deforms the bearing as $u = u_0 \sin\Omega t$, the strain deformation is $\gamma = \gamma_0 \sin\Omega t$. The forcing $F = k_{eq}u + c_{eq}du/dt$ – k_{eq} and c_{eq} are the equivalent stiffness and damping of the device, respectively – and the force required to deform the bearing u_0 is $F = f_d G_s u_0(\sin\Omega t + \eta\cos\Omega t)$, where f_d = bearing area divided by h, and G_s and η are material properties. u_0 is simply defined as $\gamma_0 h$, or the strain deformation amplitude times the isolator height.

The equivalent stiffness k_{eq} is defined as:

$$k_{eq} = f_d\left(\frac{1}{N}\sum_{i=1}^{N} G_s(\Omega_i)\right)$$

and the equivalent damping c_{eq} is simply a constant α times k_{eq}, where α depends on Gs and the excitation frequency Ω.

A lead rubber bearing (abbreviated LRB), on the other hand, is modeled as two distinct elements: the elastomeric element which is assumed linear elastic, and a lead plug which is assumed elastic-perfectly plastic. A diagram of this type of bearing can be found in Figure 2 (the DIS lead-steel-rubber isolator). The plug shear stiffness is found from the formulas $F/A = G\gamma$ and $T = ku$ where $\gamma = u/h$. Thus, $k_2 = AG/h$ in the shear (or transverse) direction. k_1 is typically one-tenth of k_2, where

$$k_1 = f_d\left(\frac{1}{N}\sum_{i=1}^{N} G_s(\Omega_i)\right)$$

N is the number of different material properties that cover the usual range of strain amplitude and frequency, f_d is A/h, and G_s is again a material property. The model for a lead rubber bearing is shown in Figure 12:

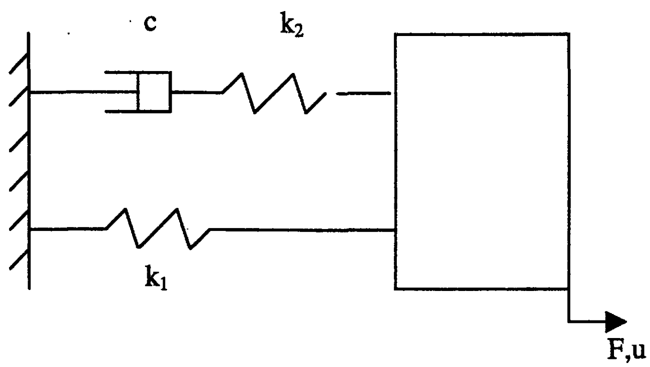

Figure 12: Model of Lead Rubber Bearing [Connor]

The equivalent stiffness k_{eq} is defined as:

$$k_{eq} = f_d\left(\frac{1}{N}\sum_{i=1}^{N} k_s(\mu_i)\right)$$

where k_s and μ_i are material properties, and N is defined as before for the natural rubber isolator. The equivalent damping c_{eq} is related to the excitation frequency and the material properties k_s and η as follows:

$$c_{eq} = \frac{\sum_{i=1}^{N} \Omega_i k_s(\mu_i)\eta(\mu_i)}{\sum_{i=1}^{N} \Omega_i^{\,2}}$$

General Applicability of Base Isolators

Base isolators are used to isolate a structure from its base vibrations, be they seismic, acoustic, or another type of excitation [Fabreeka]. There are several considerations one must take into account before installing a base isolator:

- If a base isolator isolates a structure from the ground itself, subsoil conditions must be investigated to ensure that the ground will not input long period vibration into the isolated structure above. If the ground has a low equivalent stiffness value, the isolators may actually *amplify* the ground motion and the structure then shakes uncontrollably.

- The structure together with the isolators beneath it must have a height-to-width ratio that prohibits overturning of the system. The problem of overturning is significant for designing isolators, since isolators are usually weak in vertical tension. If a structure pulls upward on an isolator, the isolator will almost surely fracture.
- Another important consideration is that the site allows the base to move an even greater distance than the isolator has been designed for. If the site has not been prepared for the large deformations the structure will undergo, the structure may disturb its surroundings.
- A final consideration is that there be little lateral loading on the structure – no more than 10% of the structure's weight.

A few comments about the problem of overturning are in order. There are three types of possible isolator configurations that can be used for base isolation [Encyclopedia]. They are called the Underneath Mounting System, where isolators are mounted beneath the structure; Center-of-gravity System, where the isolators are placed in the plane of the structure's centroid; and Radius-of-gyration System, where the isolators are mounted on both sides of the structure as shown in Figure 13.

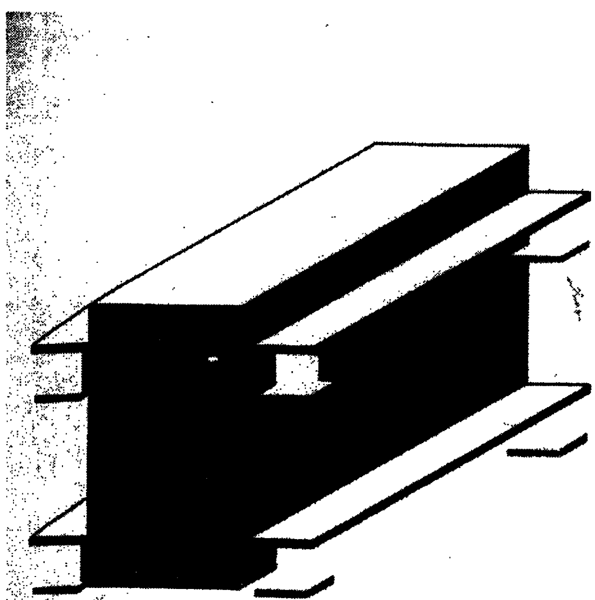

Fig. 4. Radius-of-gyration system.

Figure 13: Radius-of-gyration isolator configuration [Encyclopedia]

For the Underneath Mounting System, in general, the isolators that resist overturning should be spaced a minimum of twice the height of the center of gravity of the structure above the isolators. In the Center-of-gravity System, two conditions that must be met are: 1) the isolators that resist overturning should be spaced a minimum of twice the structure's radius of gyration, and 2) the horizontal stiffness of the isolators should be the same as the vertical stiffness. The ratio of height to width of the structure must be less than 2.8 for this system to perform well. For the Radius-of-gyration System, the height to width ratio can be up to 5. The main concern that must be addressed with this system is that a low rigidity may allow bending to occur between the upper and lower isolators.

Chapter Four: Examples of Isolation Design Problems

Design of Metal Spring Isolator

The following example illustrates how a metal spring isolator is designed, assuming no damping is used [Crede]. An unbalanced industrial machine transmits large vibrations to the floor beneath, and it is decided to reduce the vibrations by placing isolators at both ends of the two beams supporting the machine. See Figure 14 for a diagram of the isolator configuration.

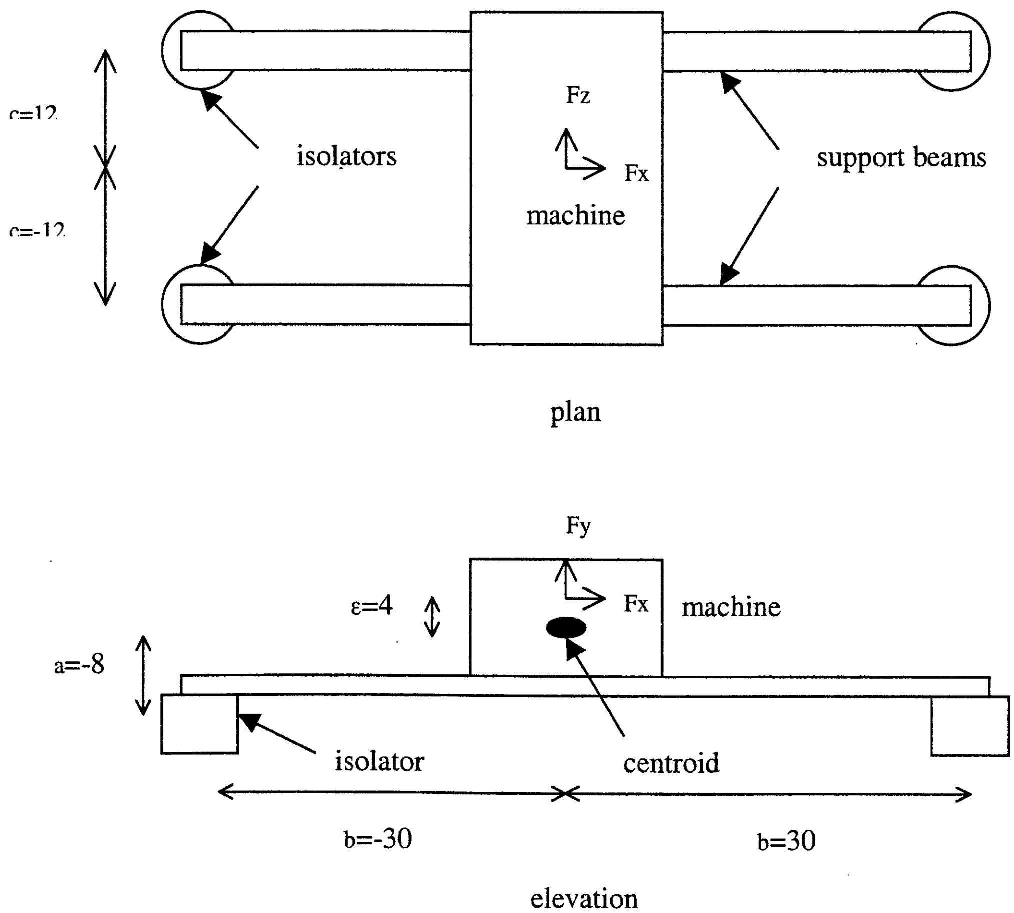

Figure 14: Metal Spring Isolator configuration supporting industrial machine [Crede]

Fx, Fy and Fz represent the unbalanced forces transmitted in the x, y and z directions to the support system by the machine. They are applied 4 inches above the centroid (denoted ε), and the isolators' mid-height is 8 inches below the centroid (denoted a). The machine and beam system weighs 1000 lbs. The radii of gyration of the system through the centroid is ρ_x=9, ρ_y=8.5 and ρ_z=6. The unbalanced force that must be absorbed completely by the metal spring isolators is periodic with a frequency f = 8 cycles per second. This force is taken as acting in the x, y and z-directions separately; thus, vibrations in the three directions are independent for design purposes.

To begin the design, one can form dimensionless ratios which facilitate the calculation of the isolators' required natural frequencies:

$\varepsilon/\rho_z = 0.667$	$\varepsilon/\rho_x = 0.444$
$a/\rho_z = -1.333$	$a/\rho_x = -0.889$
$b/\rho_z = +/-5$	$c/\rho_x = +/-1.333$
$a^2/\rho_z^2 = 1.78$	$a^2/\rho_x^2 = 0.79$
$b^2/\rho_z^2 = 25$	$c^2/\rho_x^2 = 1.78$
$n\rho_z^2/b^2 = 0.04$	$n\rho_x^2/c^2 = 0.561$

The next step is to find all natural frequencies of the machine-and-beams configuration. To determine the system's natural frequencies in terms of its vertical natural frequency Ω_y, one can use the chart on the left side of Figure 15 to find the two possible frequency ratios (the rightmost side of Figure 15 shows a design chart for a single-DOF system). The equation plotted in this chart is:

$$\frac{\Omega c}{\Omega y} x \frac{\rho_z}{b} = \frac{1}{\sqrt{2}}\sqrt{\frac{n\rho^2{}_z}{b^2}(1+\frac{a^2}{\rho^2{}_z})+1\pm\sqrt{\left[\frac{n\rho^2{}_z}{b^2}(1+\frac{a^2}{\rho^2{}_z})+1\right]^2-\frac{4n\rho^2{}_z}{b^2}}}$$

This equation uses n which is defined as k_x/k_y. Consequently, one calculates the quantity $\rho_z/b*\sqrt{k_x/k_y}$ which in this case is 0.2 for the z direction. Using $a/\rho_z = -1.333$, $(\Omega c/\Omega y)(\rho_z/b) = 0.19$ or 1.03. As can be seen from the above equation, the sign of a/ρ_z does not affect the natural frequencies on the left side. However, the sign of b/ρ_z should be the same as the sign of the radical on the right side of the equation. This procedure conveniently makes the frequency ratio $\Omega c/\Omega y$ positive. If $\rho_z/b = +0.2$, $\Omega c/\Omega y = 0.96$ and 5.15. Calculating $\rho_x/c*\sqrt{k_x/k_y}$, one gets 0.75

in the x direction. Since $a/\rho_x = -0.889$, $(\Omega c/\Omega y)(\rho_x/c) = 0.57$ and 1.29. This returns $\Omega c/\Omega y$ = 0.76 and 1.72.

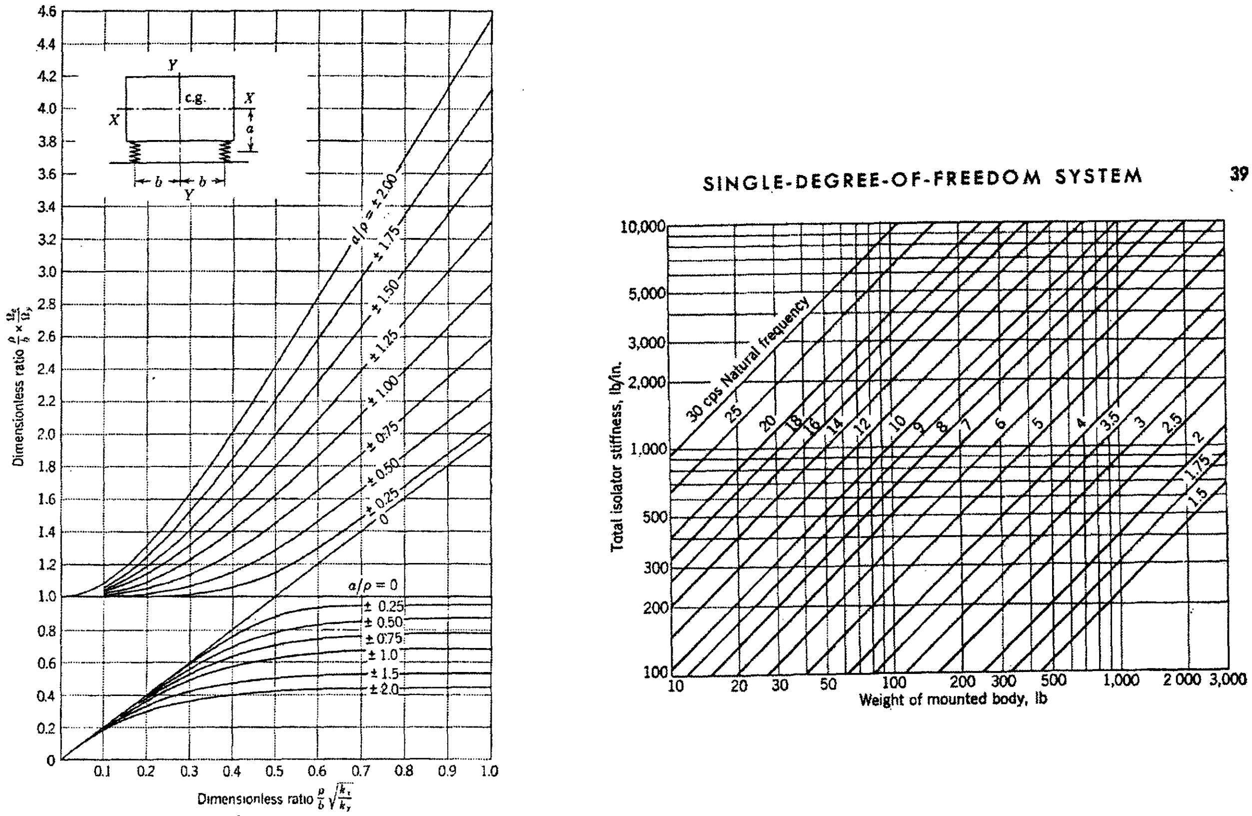

Figure 15: Design charts for Metal Spring Isolators [Crede]

The natural frequency in rotation about the y axis can be found from the following equation:

$$\frac{\Omega_B}{\Omega_y} = \sqrt{\frac{n(b^2 + c^2)}{\rho_y^{\ 2}}}$$

where $\Omega_y = \sqrt{(4k_y/m)}$ is the vertical, axial natural frequency of the machine system. This formula gives $\Omega_B/\Omega_y = 3.8$. We thus have six natural frequencies for the system: vertical (Ω_y), coupled in the x direction ($0.96\Omega_y$ and $5.15\Omega_y$), coupled in the z direction ($0.76\Omega_y$ and $1.72\Omega_y$), and rotational about the y axis ($3.8\ \Omega_y$).

The final step is to find a frequency ratio ω/Ω_y that avoids all six natural frequencies by examining the graphs of frequency ratio versus force transmissibility (T_F) and the natural frequency of rotation. See the Appendix for a plot of force transmissibility versus natural frequency. Figure 16 shows the graphs of frequency ratio versus T_F and one can easily see that a low transmissibility is achieved when $\omega/\Omega_y \sim 2.5$. This ratio also avoids the rotational frequency ratio of 3.8, and thus 2.5 is an effective frequency ratio for the isolator design.

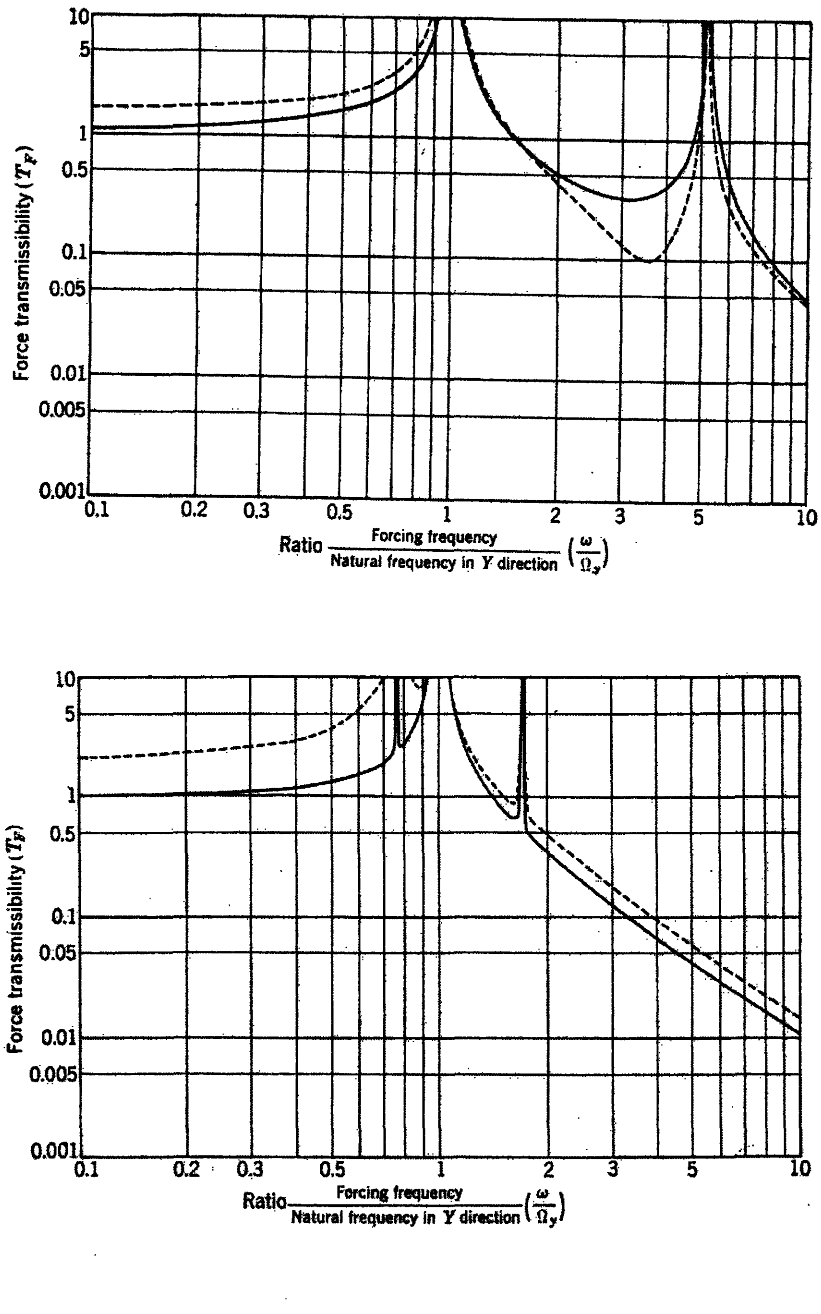

Figure 16: T_F versus Graphs of Frequency Ratio [Crede]

The desired vertical natural frequency of the isolators is 2*π*f/2.5 or 3.2 cycles per second, where f is the excitation frequency of the machine. The stiffness required of the isolators in all three directions is $k_y = fn^2*W/(3.13)^2$ or 262 lb./in., where fn is the desired natural frequency of the isolators and W is the weight of the machine-and-beams assembly.

One can examine the effect of frequency on the location of the axis of rotation by plotting lc/ρ versus the ω/Ω_y ratio. The graph of the x-direction motion is shown in Figure 17 and in the Appendix.

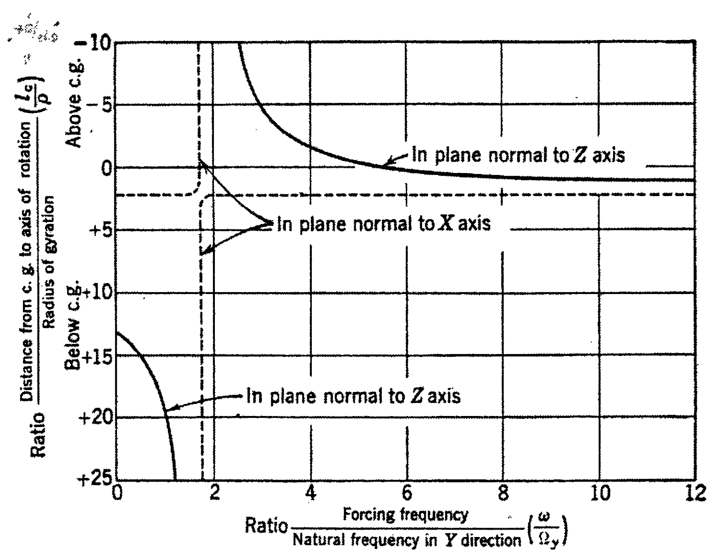

Figure 17: Graph of X-direction Motion versus Frequency Ratio [Crede]

Design of Fabreeka Isolator

The next example illustrates how a vibration isolator is designed, using an isolator manufactured by Fabreeka. This example considers both the stiffness and damping properties required for the isolator [Fabreeka].

The problem to be solved is: an industrial machine transmits annoying vibrations into its elevated floor support, and thus it has been decided to isolate the machine from the floor. The machine's weight is uniformly distributed and is 14,000 lbs., the speed is 45 Hz, and the machine's support is stiff, since it rests close to a column support. The centroid of the machine is in a stable location of the unit – it is not high on the unit and is near the geometric center of the machine. The machine is supported by two skids and is anchored with 1" diameter bolts to the floor as shown in Figure 18:

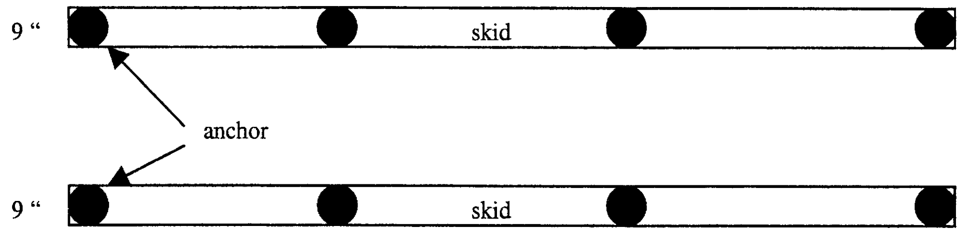

Figure 18: Skid supports for industrial machine [Fabreeka]

To begin the isolator design, it is decided that placing eight isolators beneath the machine, one at each anchor bolt, will stabilize the machine and make the problem conveniently symmetric. The static load that each isolator will need to support is then 14,000 lbs./8 or 1,750 lbs. Since the skid is 9" wide, it is best that the isolators should be 6" wide. The bearing area for each isolator will be the area that each isolator will cover of the skid, or 6" x 6", minus the area of the anchor bolt hole, or 1-1/8". The bearing area is then 35 sq. in. The isolator must be designed to carry the bearing stress of the machine, and since stress = load / bearing area, the stress is 50 psi.

The next decision that must be made is how much vibration the isolator should be designed to reduce. Since the disturbance is annoying but not severe, a 75% reduction is sufficient. Displacement transmissibility is 1 – 0.75 or 25%, and the isolator natural frequency can now be calculated using

$$T = \frac{1}{\left(\frac{fd}{fn}\right)^2 - 1}$$

where T is the displacement transmissibility, f_d is the disturbing frequency, and f_n is the isolator's natural frequency. f_n is 20.09 Hz using $f_d = 45$ Hz.

It is now possible to choose a suitable isolator. It must be tuned to have a natural frequency of 20 Hz, and must have a capacity of 50 psi. A Fabcel 50 pad is best for this application.

One more step: since the machine anchors are 1" bolts, the Fabcel washer must match the steel washer on the existing anchor bolts. The washer must have a 2.5" outside diameter, 1" inside diameter, and be 5/16" thick. This completes the Fabreeka isolator design.

It is interesting to note that Fabreeka's design procedure requires two input values: required isolator frequency and the bearing stress that will be carried by the isolator. In general, there are ten pieces of information that must be known in order to select an isolator:

1. Amount of isolation desired, or transmissibility
2. Fundamental frequency of the excitation
3. Maximum load for each isolator, the weight distribution, and the location of the centroid of the vibration source
4. Location and clearance around the isolator
5. Clearance around the receiver system
6. Height limitations
7. Support stiffness
8. Flexibility of plumbing, HVAC or other lines attached to the receiver structure
9. Allowable motion of the receiver (such as maximum acceleration)
10. Environmental conditions (such as temperature, radiation, etc.)

Design of DIS Isolator

This final example shows how a seismic isolator is designed, using a DIS (Dynamic Isolation Systems) isolator. As already shown in Figure 2, this isolator contains elastomeric pads separated by steel plates. DIS isolators usually contain lead cores which provide damping. The main difference between seismic isolators and Fabreeka isolators is that seismic isolators are used to isolate entire buildings instead of equipment. Seismic isolators are also designed to resist *horizontal*, not vertical, vibrations.

The building to be isolated is shown in Figure 19. It is located in San Francisco, which means that it must be designed to withstand the most severe earthquake possible. The building is founded on clay and silt, and the structure is a steel eccentric braced frame. Since this building is surrounded by taller buildings, wind speed and pressure are low.

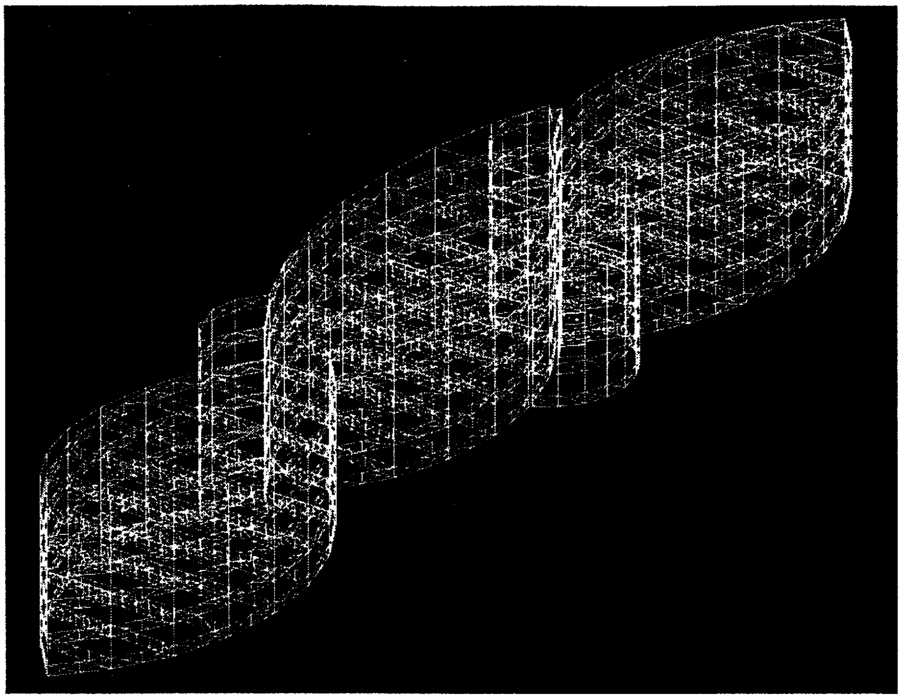

Figure 19: Example building requiring seismic isolation

The following assumptions are used for preliminary isolator design:

- Design Acceleration Level = 40% gravity (Zone 4 level earthquake, building is located >15 km from a fault)
- Soil Type = 2 (clay and silt)
- The structure is a steel eccentric braced frame: Rwi = 3 (Rwi is a reduction coefficient)
- Wind load is negligible, so lead cores do not yield due to wind
- Emce = 50% of Dead Load
- Isolators move together as a unit. This assumption requires that a rigid waffle slab be designed for the basement to connect the isolators.

The preliminary design is performed using a program called ISOLATE created by DIS. The preliminary design results for the isolators are as follows:

- Moat must be two feet wide to accommodate the isolator deformation.
- Each isolator is 39.5" in diameter and 14.5" in height
- For the lead core isolators, the cores are 6.5" in diameter.
- The vertical stiffness of the isolators is 18000 kip/inch.

- Lead-core isolators are placed on the perimeter of the building, and non-core isolators are placed in the interior. The vertical stiffness of the lead-core isolators is 11.26 kip/inch, and the vertical stiffness of the non-core isolators is 7.5 kip/inch.

Figure 20 shows the shape of the isolated building's first mode. It is apparent that the building translates laterally as a rigid body. This is indeed what would be expected for the first mode of an isolated, narrow building. The second mode is also a rigid body mode, but the building translates longitudinally instead. The third mode is a rotational rigid body mode about the building's center of mass. Thus, the base-isolated model behaves as expected.

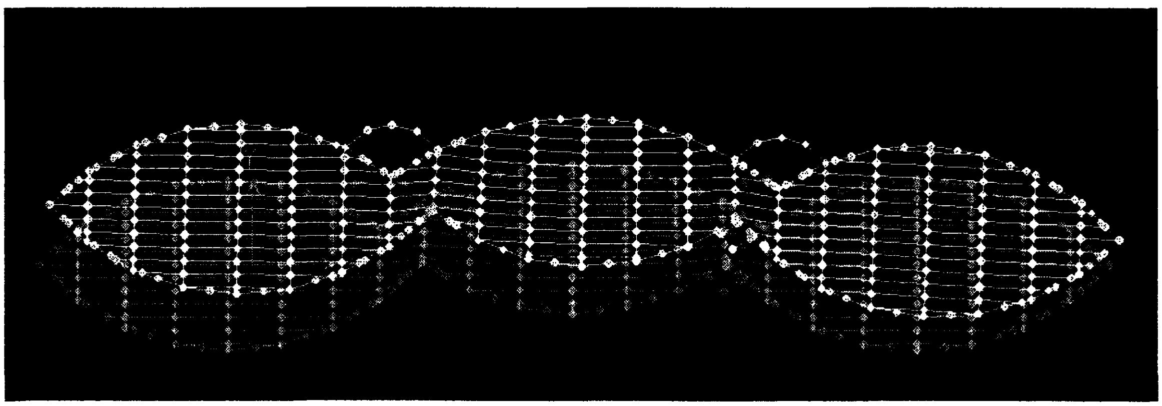

Figure 20: First mode shape of example building

Once the mode shapes have been checked, the displacement histories of the basement (Joint1298) and fourth floor (Joint2191) are compared, as shown in Figure 21. For the isolated model, the graph shows that the two floors move almost perfectly together, and there is thus little differential strain between floors.

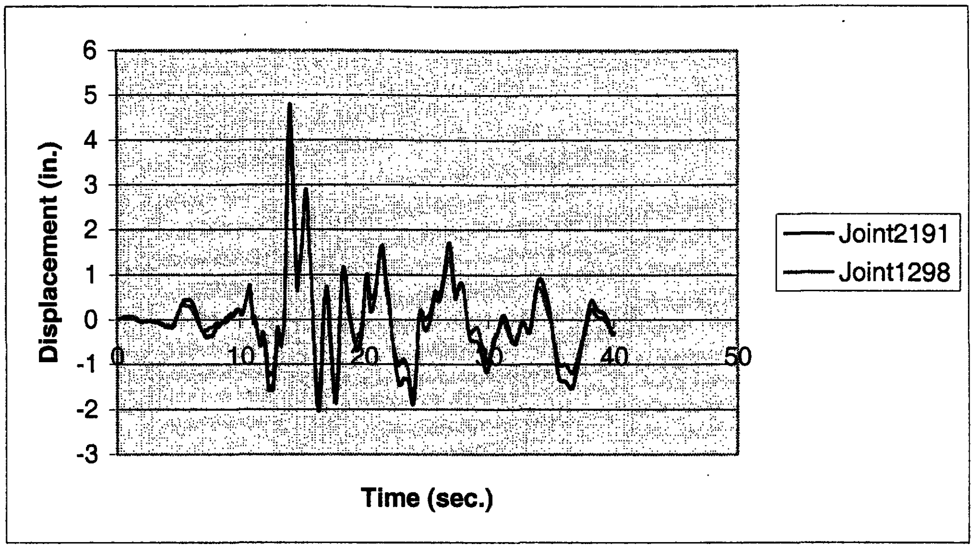

Figure 21: Time-history displacement comparison of basement and fourth floor

For the fixed-base building, however, the two floors have nearly the same *acceleration* history. This means that the fourth floor would experience the same high accelerations as the ground itself.

Table 4 presents a final comparison of the fixed-base and base-isolated models:

	Fixed-base	**Base-isolated**
Fundamental Period	0.6 seconds	2.5 seconds
Maximum Displacement	1.2 inches	9.6 inches
Maximum Acceleration	20% gravity	2% gravity

Table 4: Comparison of fixed-base and base-isolated building models

As the chart shows, the base-isolated building reduces the maximum acceleration to an acceptable level (2% of gravity is the threshold of human comfort in buildings under *wind* loads, whereas the acceptable threshold is higher for seismic loads). Although the fundamental isolated period is 2.5 seconds, which is usually sufficiently high for seismic isolation design, there remains the possibility that the soil beneath the building has a similar fundamental period. This issue must be investigated using a program such as SHAKE that calculates the soil's natural frequency. The properties of the isolators would then be designed such that the new fundamental period would be far away from both the earthquake period and the soil period.

Chapter Five: The History and Future of Vibration Isolation

Whatever the vibration problem may be, vibration control in buildings has emerged as an exciting, rapidly developing specialty. Since the late 1800s, when vibration absorbers were used exclusively in mechanical equipment mounting, isolator applications have been extended to include structures ranging from microscopy laboratories on the atomic level to entire buildings on the seismic level. The past decade, in particular, has featured the development and application of seismic isolators to bridges and buildings.

Structural control is an innovation that involves measuring the responses of a structure (through sensors) and sending an appropriate signal to a control device that acts to improve the structure's safety and serviceability [Spencer, et al.]. A diagram of the structural control algorithm is found in Figure 22.

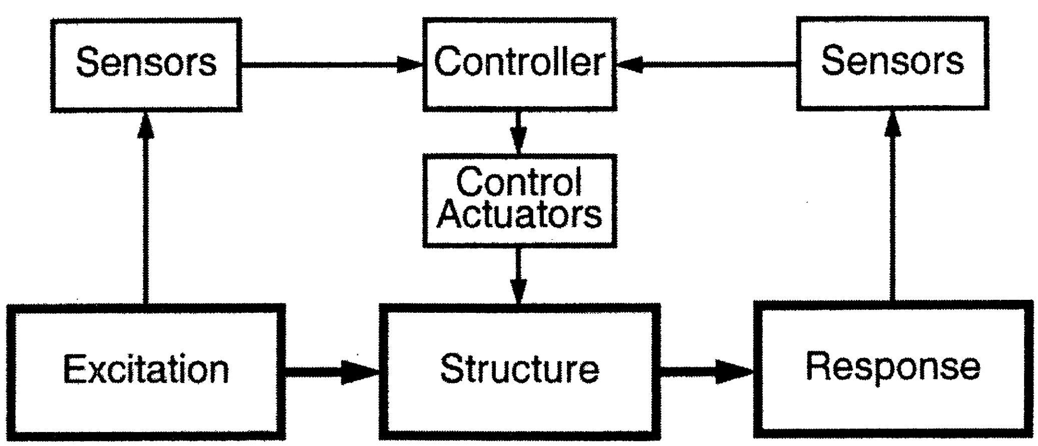

Figure 22: Structural control algorithm [Spencer, et al.]

The isolation industry has already taken large steps forward by developing active- and semi-active control isolation devices [Kinetics]. Active isolators are devices that actively move to oppose the motion of a structure. Active Mass Drivers, or AMD's, are a good example of an active isolator. Semi-active isolators, on the other hand, are passive isolators with actuator elements to change the properties of the isolators. Electrorheological – ER – and magnetorheological – MR – fluid dampers are good examples of semi-active isolators.

Active-control devices are already used to control the motion of buildings and bridges, such as the Nanjing Tower in China and the Rainbow Bridge in Japan [Spencer, et al.]. Both AMD's – passive tuned mass dampers with actuator attached to an auxiliary mass – and hybrid base isolators – passive base isolators with actuator supplements – are used worldwide. A diagram of an AMD used in building motion control is shown in Figure 23.

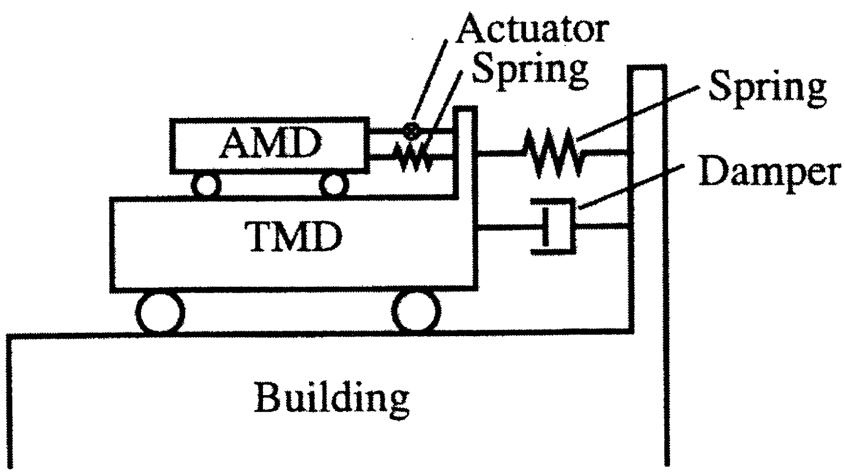

Figure 23: Active Mass Damper used for controlling building motion [Spencer, et al.]

The auxiliary mass used in Active Mass Dampers is usually less than 1% of the total structure mass, and is installed in one of the top floors of a building to resist the fundamental mode of the structure. Hybrid base isolators, on the other hand, are placed in the basement of a building. Since base isolators themselves decrease the interstory displacements as well as the absolute acceleration of a building, active control is only needed to limit absolute base displacement, and therefore hybrid base isolators cost approximately the same as passive base isolators.

Whereas passive isolators maintain a constant set of isolation properties for any excitation – such as stiffness and damping – semi-active isolators can "intelligently" alter their properties for various vibration levels [Kinetics]. Before semi-active devices, engineers could only design isolators for a certain predicted vibration environment. With semi-active isolators, however, devices can change their own properties in response to changes in temperature, frequency, or other important environmental parameters. Semi-active isolators are used primarily for low frequency levels, since these are the most sensitive vibration levels for structures. These isolators use sensors to determine how much vibration the isolator must absorb, and piezoelectric actuators – devices that send electrical pulses in response to pressure changes – to change the characteristics of the isolating material. Many semi-active isolators use three-dimensional sensors to accurately interpret the vibration environment surrounding the receiver. Semi-active isolators are already extensively used in Bose headsets for airplane pilots, certain types of sensitive machinery, ductwork and loudspeakers. In particular, acoustic isolation is increasingly becoming a semi-active control industry.

Variable Hydraulic Dampers, or VHD's, are semi-active damping devices that can change their own damping ratio through valve manipulation [Kurata, et. al.]. They consist of a cylinder and piston, a flow control valve, check valves, and an accumulator. The check valves are used to ensure that the hydraulic fluid (ie. oil) flows in only one direction through the flow control valve. The flow control valve is usually located between two hydraulic chambers. The accumulator's purpose is simply to supply fluid to the device to compensate for the loss of volume due to compression. A VHD performs as follows: the piston displaces under load, forcing oil through the flow control valve at a controlled rate, and the resulting fluid resistance determines the damping force of the isolator. The VHD isolator's damping properties are affected by fluid temperature and the frequency of excitation. Figure 24 shows a diagram of a VHD and how oil flow is controlled to counteract the motion of a structure, such as a building.

For seismic vibration of a building, a VHD should have an approximate damping coefficient range of $0 - 2x10^8$ Ns/m, a maximum damping force of $2x10^6$ N, and a maximum piston stroke of +/- 40 mm. The device usually consumes about 30W of power to perform correctly.

VHD's have been tested successfully on models of buildings subjected to seismic excitation, and it has been proven that they greatly reduce the displacement and acceleration response of structures [Kurata, et. al.]. The devices have even been installed on actual buildings to reduce their response due to stormy winds and earthquakes. However, the isolators are still costly to manufacture, require large amounts of power, and are limited in their capacity, and thus they remain to be widely used.

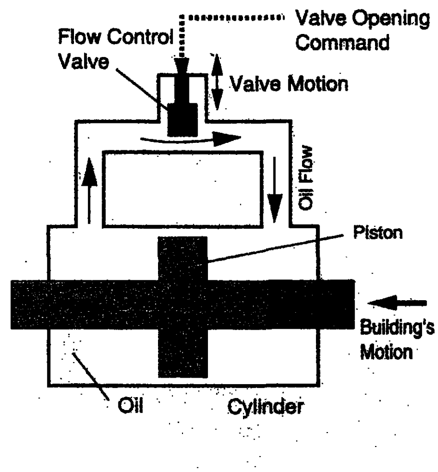

Figure 24: Schematic diagram of Variable Hydraulic Damper [Kurata, et.al.]

Electrorheological (ER) fluids are used in certain semi-active isolation devices [Lord]. Although the unique properties of electrorheological fluids have been known for many decades, it is only recently that ER isolators have been applied to industrial machines. These isolators can change both their stiffness and damping properties in milliseconds. When voltage is applied, control valves open or close, allowing the electrorheological fluid to escape or enter a cavity. This procedure changes the stiffness of the isolator. Applying voltage also changes the fluid's viscosity quickly, thereby altering the damping ratio of the device. Electrorheological isolators have already been used successfully as shock absorbers in the seats of trucks and on the tires of bicycles. See Figure 25 which shows an electrorheological fluid isolator used to cushion the response of a bicycle sprocket to vibration.

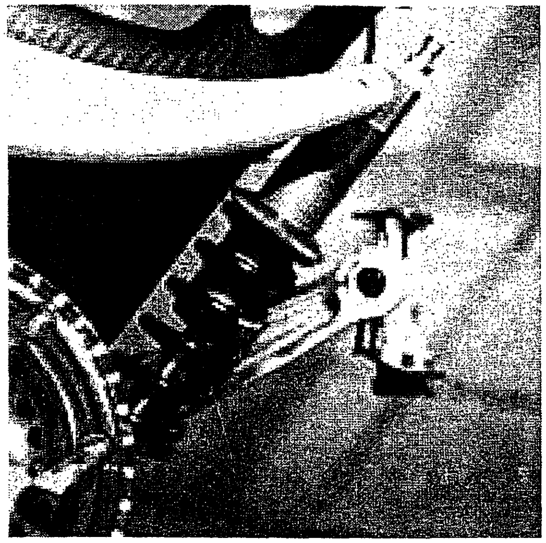

Figure 25: Electrorheological Fluid Isolator [AFS]

A more attractive alternative to ER isolators is magnetorheological (MR) isolators. These isolators act in the same way as ER devices, but instead the fluid is exposed to a *magnetic* field. The advantage of MR isolators over ER isolators for civil engineering applications is that MR devices have a maximum yield stress about ten times that of ER isolators. This fact makes MR isolators much more practical for use in building and bridge vibration control. In addition, MR fluids can draw power from low voltage sourcesand are relatively insensitive to contamination and temperature extremes. Figure 26 shows a schematic diagram of a 20-ton Magnetorheological Fluid Seismic Damper which has an inside diameter of 20.3 cm. and a stroke of ± 8 cm. It is 1 meter long and has a mass of 250 kg.

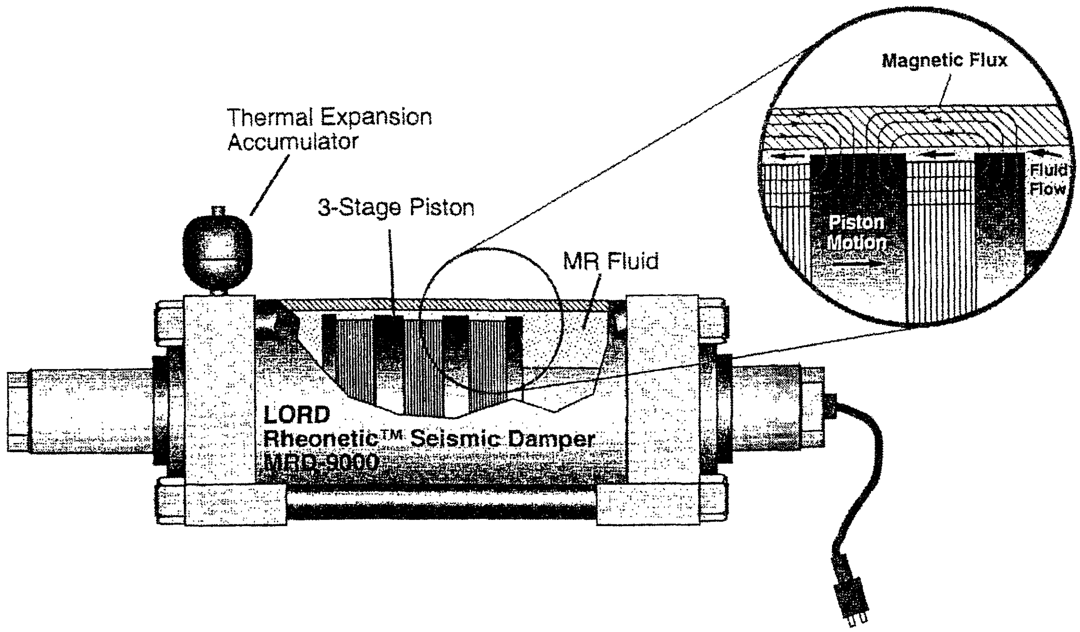

Figure 26: Diagram of Magnetorheological Fluid Damper [Lord and Spencer, et al.]

It is probable that MR isolators will be used soon in many civil engineering applications due to their simplicity, low power requirements and high force capacity.

Virtually every sensitive machine requires some device to reduce vibrations entering through its support. Thus vibration isolation will surely continue to play an important, exciting role in the future development of the semiconductor industry, research laboratories, and many more state-of-the-art facilities.

References

AFS (Advanced Fluid Systems, Ltd.). http://www.a-f-s.com/

Bachmann, Hugo and Walter Ammann. Vibrations in Structures Induced by Man and Machines. p. 63-85.

Bramer, T.P.C., G.J. Cole, J.R. Cowell, A.T. Fry, N.A. Grundy, T.J.B. Smith, J.D. Webb, and D.R. Winterbottom. Basic Vibration Control. Sound Research Laboratories, Ltd. 1977. p. 109.

Clemente, Paolo and Dario Rinaldis. "Protection of a Monumental Building Against Traffic-induced Vibrations." Soil Dynamics and Earthquake Engineering. Elsevier Science Ltd. Vol. 17, no. 5. July 1998. p. 289-296.

Connor, Professor Jerome J. and Boutros S.A. Klink. Introduction to Motion Based Design. Computational Mechanics Publications. Boston, MA.1996. p. 198-217.

Crede, Charles E. Vibration and Shock Isolation. John Wiley & Sons, Inc. New York, NY.1951. p. 2, 61.

DIS (Dynamic Isolation Systems): http://www.dis.com/

Fabreeka International, Inc.: http://www.fabreeka.com/

Halvorsen, William G. "Design of Rubber Isolation Systems." Automotive Engineer. Vol. 11, no. 6. Dec. 1986. p. 44-45.

Kinetics, Inc. http://www.kinetics.com/

Kurata, Narito, Takuji Kobori, Motoichi Takahashi, Naoki Niwa, and Haruhiko Kurino. "Shaking Table Experiment of Active Variable Damping System." Proceeding of First World Conference on Structural Control. 3-5 Aug. 1994. Vol. 2. Los Angeles, CA. p. TP2-108-117.

Lord Corporation. http://www.lordisolators.com/

Newport Corporation: http://www.newport.com/

Spencer, Jr., B.F. and Michael K. Sain. "Controlling Buildings: A New Frontier in Feedback." Special Issue of the IEEE Control Systems Magazine on Emerging Technology. Vol. 17, No. 6. Dec. 1997. p. 19-35.

Vibro/Dynamics Corporation: http://www.vibrodynamics.com/