DTQDB: A Fair and Fully Utilized Media Access Protocol for Dual Bus Networks

Angela L. Chiu
angela@lids.mit.edu
Tel: (617) 253-2147
Fax: (617) 258-8553

Robert G. Gallager *
gallager@lids.mit.edu
Tel: (617) 253-2533
Fax: (617) 258-8553

July 13, 1995

Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, MA 02139

Abstract

The distributed queue dual bus (DQDB) protocol with bandwidth balancing (the IEEE 802.6 standard for metropolitan area networks) achieves fairness by sacrificing a fraction of the bandwidth. Many existing protocols indicate that fairness and full utilization might be incompatible in high-speed-high-latency MANs or LANs. The main purpose of this research is to design a protocol called distributed total queue dual bus (DTQDB) that combines the two features together. In addition, the protocol provides bounded access delay that is linear in the round trip propagation delay. The basic concept is to compute, for each station, the latest estimate on the number of active downstream stations, according to the information available, and serve them in a round robin scheme.

Key words: dual bus network, propagation delay, fairness, full utilization, liveness

*corresponding author

1 Introduction

During the last fifteen years, we have witnessed a proliferation of proposals for high-speed bus networks. In many of these previous networks [3][4][5][6][7], fairness and full utilization are incompatible. The well known distributed queue dual bus (DQDB) protocol with bandwidth balancing (the IEEE 802.6 standard for metropolitan area networks) achieves a fair distribution of the bandwidth by requiring each station to use only a fraction of the available bandwidth for transmissions [2]. Thus, the protocol does not provide full utilization of the bandwidth. Moreover, the protocol converges slowly to a fair distribution of the bandwidth to the active stations.

In this study we introduce the new distributed total queue dual bus (DTQDB) protocol for a dual bus network. The protocol provides both fairness and full utilization. For a network with a steady set of active stations, a fair distribution of the bandwidth is obtained after one round trip propagation delay. Additionally, the protocol provides a finite access delay that is linear in the round trip propagation delay.

The remainder of this paper is organized as follows. In section II we describe the DTQDB protocol. We start with the basic dual bus topology, define what we mean by fairness and full utilization, and then give a full description of the DTQDB protocol. In Section III, we describe and prove some basic properties of the protocol. In Section IV we prove the correctness and liveness properties of DTQDB. Finally we conclude our results in Section V.

2 Distributed Total Queue Dual Bus (DTQDB) Protocol

2.1 Basic Dual Bus Metropolitan Network

The dual-bus topology we consider here is identical to that used in DQDB (see Figure 1). The two buses support unidirectional communications in opposite directions. Stations are connected to both buses and communicate by selecting the proper bus. A special unit at the head-end of each bus generates one slot at each unit of time. The stations are numbered from left to right as stations 1, 2, ..., K. Because of the symmetry of the dual bus topology, we can consider only transmission on one bus. The bus from station 1 to station K is used to transfer data and is referred as the data bus or downstream bus. The bus from station K to station 1 is used to make reservations and is referred as the reservation bus or upstream bus. Therefore station 1 is the most upstream station, and station K is the most downstream station. Assume all the stations are uniformly distributed over the dual-bus network. Suppose that the upstream and downstream propagation delays between any two adjacent stations, measured in slots, are given by D_u and D_d respectively, where D_u and D_d are integers. Each station has a local FIFO (first-in-first-out) queue to store data segments by local users while these segments wait for assignment to appropriate idle slots on the data bus.

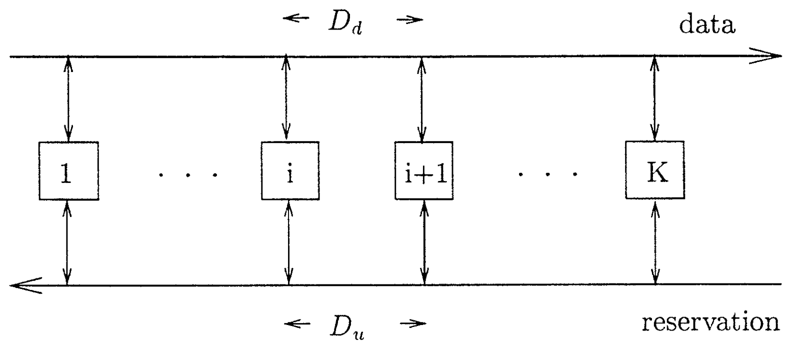

Figure 1: Dual Bus Topology

2.2 Definitions of Fairness and Full Utilization

Here, we define what we mean by fairness and full utilization.

Definition 1: Let $S_n = \{i_1 < i_2 < ... < i_n\}$ be the set of some n stations that have been "very active" since t_0, where being "very active" is defined later in Section 3.1.2. A protocol is *fair* if each station $i_k \in S_n$, $k = 1, ..., n$, starting from $t_0 + (i_k - i_1)D_d$, transmits one data segment in every n time slots for as long as S_n remains the set of "very active" stations and all the other stations remain idle (i.e., with empty queues).

Definition 2: If an idle slot arrives at a station $i, i \in [1,\ K]$, with a nonempty FIFO queue and is propagated to downstream stations, a protocol has *full utilization* if the idle slot is always used by one of the downstream stations.

That is, full utilization means that an idle slot is never wasted. It must be used if it could be used by one of the stations with nonempty queues.

2.3 Basic Concept and Implementation

2.3.1 Basic Concept

Since all the idle slots on the data bus are generated from the head-end, station 1, which is the most upstream station, has first access to idle slots. The basic concept of the protocol is to give equal access to all the stations, according to the most updated information available through the reservation bus. In particular, according to the information available, each station computes the latest estimate on the number of active downstream stations, and serves them in a round robin scheme. The novel feature of this protocol is that each station takes account of the idle slots propagated previously to interpret the information from downstream (i.e., estimated total number of packets in queue downstream and estimated number of active downstream stations).

2.3.2 Parameters

Next, we define the parameters used in the protocol. At time t, the information available at station $i \in [1,\ K]$ is,

- $Q_i(t)$: number of data segments in the FIFO queue of station i,
- $I_i(t)$: indicator function whether station i is busy or not, i.e.,

$$I_i(t) \triangleq 1 \text{ if } Q_i(t) > 0,\ 0 \text{ otherwise}, \tag{1}$$

- $n_i(t)$: number of idle slots propagated by station i during the past $D_u + D_d$ time slots. This is also written as $n_i(t - D_u - D_d,\ t)$.

The information sent by station i to the upstream station $i-1$ is,

- $M_i(t)$: estimated number of active downstream stations (including itself $I_i(t)$),
- $m_i(t)$: estimated number of data segments in the FIFO queues of all the downstream stations (including the ones at station i, $Q_i(t)$).

The other parameters used in the protocol are $m_{i+1}^{up}(t - D_u)$, $M_{i+1}^{up}(t - D_u)$, and $C_i(t)$ which will be explained in the next section.

2.3.3 Distributed Algorithm

$\forall t < 0$, all the parameters defined in the previous section are set to 0. At time $t \geq 0$, the information available at station i, $i \in [1,\ K]$, is $Q_i(t)$, $I_i(t)$, and $n_i(t)$. The initial setting of the algorithm is, for $t \in [0,\ D_u - 1]$, and each station $i \in [1,\ K]$, $M_i(t) = I_i(t)$, $m_i(t) = Q_i(t)$, the counter $C_i(t) = 0$, and station i occupies idle slots if $I_i(t) = 1$, and propagates them otherwise. This is also the algorithm for the most downstream station K at all $t \geq 0$, i.e.,

$$m_K(t) = Q_K(t), \tag{2}$$

$$M_K(t) = I_K(t), \tag{3}$$

$$C_K(t) = 0. \tag{4}$$

In general, for $t \geq D_u$, and station $i \in [1,\ K-1]$, the algorithm runs as follows:

1. receive $m_{i+1}(t - D_u)$ and $M_{i+1}(t - D_u)$ sent by station $i+1$ at $t - D_u$,

2. obtain the updated information $m_{i+1}^{up}(t-D_u)$ and $M_{i+1}^{up}(t-D_u)$ which correspond to $m_{i+1}(t-D_u)$ and $M_{i+1}(t-D_u)$ as follows,

$$m_{i+1}^{up}(t-D_u) = [m_{i+1}(t-D_u) - n_i(t)]^+, \tag{5}$$

$$M_{i+1}^{up}(t-D_u) = \min\{M_{i+1}(t-D_u),\ m_{i+1}^{up}(t-D_u)\}. \tag{6}$$

3. update the counter and make a decision as follows:

$$C_i(t) = \min\{C_i(t),\ M_{i+1}^{up}(t-D_u)\}, \tag{7}$$

if $I_i(t) = 0$,

then $C_i(t+1) = K - i$,

propagate the slot passing by (either busy or idle),

else if the slot passing by station i at t is busy,

then $C_i(t+1) = C_i(t)$,

else the decision whether to propagate or use the idle slot is made as follows,

if $C_i(t) = 0$,

then $C_i(t+1) = K - i$,

occupy the idle slot with the first data segment in queue,

else $C_i(t+1) = C_i(t) - 1$,

propagate the idle slot.

4. obtain $M_i(t)$ and $m_i(t)$ as below, and send them to station $i-1$,

$$m_i(t) = Q_i(t) + m_{i+1}^{up}(t-D_u), \tag{8}$$

$$M_i(t) = I_i(t) + M_{i+1}^{up}(t-D_u). \tag{9}$$

Notice that the core of the algorithm is the second step where station i uses the extra piece of information $n_i(t)$ to update $m_{i+1}(t-D_u)$ and $M_{i+1}(t-D_u)$. For example, at time t, station i receives the information that there are 10 data segments and 5 active stations downstream ($m_{i+1}(t-D_u) = 10$ and $M_{i+1}(t-D_u) = 5$). On the other hand, station i needs to take consideration of $n_i(t) = 3$, the 3 idle slots that have been propagated downstream since $t - D_u$ when the information was sent. In the absence of new arrivals, station i knows that there are 7 date segments left at the queues of at most 5 downstream stations (i.e., $m_{i+1}^{up}(t-D_u) = 7$ and $M_{i+1}^{up}(t-D_u) = 5$). Consider the same example except that $n_i(t) = 7$. Again, without new arrivals, station i knows that there are only 3 date segments left at the queues of at most 3 downstream stations (i.e., $m_{i+1}^{up}(t-D_u) = 3$ and $M_{i+1}^{up}(t-D_u) = 3$).

In order to guarantee the full utilization property, the decision made on idle slots should be based on only the information received, not the probabilistic estimates of future arrivals. As a consequence, downstream stations still suffer from propagation delays. In order to compensate for this disadvantage, the protocol is designed with a bias towards downstream stations in the updating equation (6), where $M_{i+1}^{up}(t - D_u)$ takes its maximum possible value. This can be seen in the second example above. The 3 date segments left can be distributed at one station, or at most 3 stations, and $M_{i+1}^{up}(t - D_u) = 3$. On the other hand, the estimate $m_{i+1}^{up}(t - D_u)$, the total number of data segments downstream, is the true value without the knowledge of new arrivals. This ensures the full utilization property which is proved in Section 3.1.1.

2.3.4 Properties of the Algorithm

In order to describe some basic properties of the proposed algorithm, we first define the following parameters, for $t \geq 0$, $i \in [1,\ K-1]$, $k \in [1,\ K]$,

- $A_i(t,\ t+s)$: number of arrivals at station i during $[t,\ t+s)$, which is the time interval starting at the t-th time slot and ending right before the $t+s$-th time slot,
- $n_i(t,\ t+s)$: number of idle slots that station i propagates during $[t,\ t+s)$; for a special shorthand notation with $s = D_u + D_d$,

$$n_i(t,\ t + D_u + D_d) = n_i(t + D_u + D_d), \tag{10}$$

- $N_i(t,\ t+s)$: number of idle slots that station i uses during $[t,\ t+s)$; thus,

$$n_{i+1}(t,\ t+s) = n_i(t - D_d,\ t + s - D_d) - N_{i+1}(t,\ t+s), \tag{11}$$

- $\tau_k^i(t)$: time when the information arriving at station i at t was sent from downstream station k, for $k > i$, or time when the information arrives at upstream station k, for $k \leq i$,

$$\tau_k^i(t) \triangleq t - (k-i)D_u, \tag{12}$$

- $T_k^i(t)$: time when the slot sent by station i at t arrives at downstream station k for $k > i$, or time when the slot was propagated from upstream station k, for $k \leq i$,

$$T_k^i(t) \triangleq t + (k-i)D_d. \tag{13}$$

See Figure 2 for the case when $k > i$.

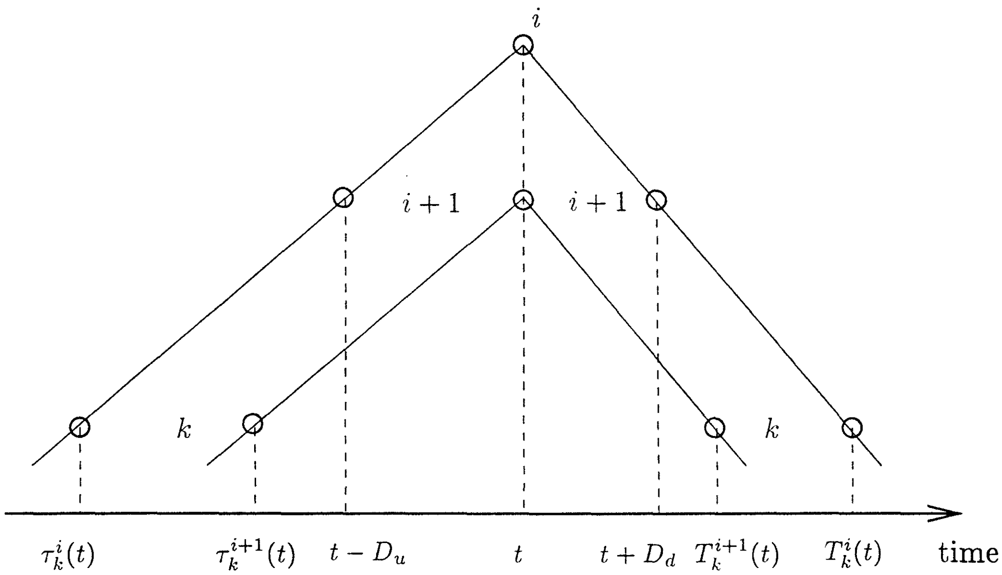

Figure 2: illustration of $\tau_k^i(t)$ and $T_k^i(t)$ for $k > i$

Propositions: The following propositions assume $t, s \geq 0$, and $i \in [1,\ K-1]$, . Noting that Proposition 1 to Proposition 7 can be easily verified for $t \in [0,\ D_u - 1]$, the proofs below are for $t \geq D_u$ only.

Proposition1: $Q_i(t) \geq I_i(t) \geq 0$.

Proof: By definition in (1), $I_i(t) = 1$ if $Q_i(t) > 0$, 0 otherwise. ■

Proposition2: $m_{i+1}^{up}(t - D_u) \geq 0$, $m_{i+1}(t - D_u) \geq 0$, $M_{i+1}^{up}(t - D_u) \in [0,\ K - i]$, $M_{i+1}(t - D_u) \in [0,\ K - i]$.

Proof: Use induction on i, from $i = K$ up to $i = 1$. ■

Proposition 3: $m_{i+1}(t - D_u) \geq m_{i+1}^{up}(t - D_u)$.

Proof: This is based on the updating equation (5) in the algorithm. ■

Proposition 4: $M_{i+1}(t - D_u) \geq M_{i+1}^{up}(t - D_u)$.

Proof: This is based on the updating equation (6) in the algorithm. ■

Proposition 5: $m_i^{up}(t - D_u) \geq M_i^{up}(t - D_u)$.

Proof: This is based on the updating equation (6) in the algorithm. ■

Proposition 6: $m_i(t) \geq M_i(t)$.

Proof: From Proposition 1 and Proposition 5,

$$Q_i(t) + m_i^{up}(t - D_u) \geq I_i(t) + M_i^{up}(t - D_u).$$

The result thus follows from (8) and (9). ■

Proposition 7: $M_{i+1}^{up}(t - D_u) \geq C_i(t) \geq 0$.

Proof: This comes from (7) in the counter updating step. ■

Proposition 8: $n_{i+1}(t,\ t+s) = n_i(t - D_d,\ t+s-D_d) - Q_{i+1}(t) - A_{i+1}(t,\ t+s) + Q_{i+1}(t+s)$.

Proof: From (11),

$$n_{i+1}(t,\ t+s) = n_i(t - D_d,\ t+s-D_d) - N_{i+1}(t,\ t+s).$$

The result follows since the change in queue size during an interval is the difference of arrivals and departures. ■

Proposition 9: $\sum_{k=i+2}^{K} N_k(T_k^{i+1}(t - D_u),\ T_k^i(t)) = n_{i+1}(t + D_d) - n_K(T_K^i(t))$.

Proof: Using (11) with $k-1$, $T_k^{i+1}(t - D_u)$, and $T_k^i(t)$ in places of i, t, and $t+s$,

$$\begin{aligned} N_k(T_k^{i+1}(t - D_u),\ T_k^i(t)) &= n_{k-1}(T_k^i(t) - D_d) - n_k(T_k^i(t)) \\ &= n_{k-1}(T_{k-1}^i(t)) - n_k(T_k^i(t)). \end{aligned} \tag{14}$$

Summing both side of (14) from $i+2$

$$\begin{aligned} \sum_{k=i+2}^{K} N_k(T_k^{i+1}(t - D_u),\ T_k^i(t)) &= \sum_{k=i+2}^{K} [n_{k-1}(T_{k-1}^i(t)) - n_k(T_k^i(t))] \\ &= n_{i+1}(T_{i+1}^i(t)) - n_K(T_K^i(t)) = n_{i+1}(t + D_d) - n_K(T_K^i(t)). \end{aligned} \tag{15}$$

Proposition 10: If station i has a nonempty queue and propagates all the idle slots arriving during $[t,\ t+s)$, then

$$C_i(t) - C_i(t+s) \geq n_i(t,\ t+s) \geq 0. \tag{16}$$

Proof: This follows from the counter updating step, where the counter is decremented at least by 1 every time a station with nonempty queue propagates an idle slot. ■

Proposition 11: $M_{i+1}^{up}(t - D_u) \geq [M_{i+1}(t - D_u) - n_i(t)]^+$.

Proof: According to the updating equation (6) in the algorithm, we can break the proof into two cases.

- $M_{i+1}^{up}(t - D_u) = M_{i+1}(t - D_u)$. We have

$$M_{i+1}(t - D_u) \geq [M_{i+1}(t - D_u) - n_i(t)]^+.$$

- $M^{up}_{i+1}(t-D_u) = m^{up}_{i+1}(t-D_u)$. We have

$$\begin{aligned} m^{up}_{i+1}(t-D_u) &= [m_{i+1}(t-D_u) - n_i(t)]^+ \\ &\geq [M_{i+1}(t-D_u) - n_i(t)]^+, \end{aligned}$$

  which follows from the updating equation (5) in the algorithm and Proposition 6.

Thus, the proof is complete. ■

Next, we prove two lemmas here and two later in Section 3.1.2, which are useful in the proofs of correctness and liveness.

Lemma 1: $\forall t \geq 0$, $\forall i \in [1,\ K-1]$, $m_{i+1}(t+D_d) \geq m^{up}_{i+1}(t-D_u)$.

Remark: The main point of Lemma 1 is as follows. If a slot passing by station i at time t receives estimation information that there are $m^{up}_{i+1}(t-D_u)$ date segments waiting at downstream stations $i+1$, $i+2$, ..., K, then when the slot arrives at station $i+1$ at $t+D_d$, the estimate $m_{i+1}(t+D_d)$ should be at least as large as before. It might be larger due to the new arrivals at all the downstream stations $i+1$, $i+2$, ..., K while the idle slots considered in the estimation remains the same. Thus, the lemma is useful in the proof of full utilization.

Proof: Use induction on i from $i = K-1$ to 1.

1. Let $i = K-1$. Using Proposition 8 with K, $t-D_u$ and $t+D_d$ in places of $i+1$, t and $t+s$ respectively,

$$\begin{aligned} Q_K(t+D_d) &= Q_K(t-D_u) - n_{K-1}(t) + n_K(t+D_d) + A_K(t-D_u,\ t+D_d) \\ &\geq Q_K(t-D_u) - n_{K-1}(t), \end{aligned}$$

   where the inequality is due to the nonnegativity of $n_K(t+D_d)$ and $A_K(t-D_u,\ t+D_d)$. Combining with (2), i.e. $m_K(s) = Q_K(s) \geq 0$, we have

$$m_K(t+D_d) = Q_K(t+D_d) \geq [Q_K(t-D_u) - n_{K-1}(t)]^+ = m^{up}_K(t-D_u),$$

   where the last equality follows from (5) in the updating step. Hence, we have established the basis.

2. For arbitrary i, (5) and (8) yield

$$m_{i+1}(t+D_d) = Q_{i+1}(t+D_d) + [m_{i+2}(t-D_u+D_d) - n_{i+1}(t+D_d)]^+. \tag{17}$$

Using Proposition 8 with $t - D_u$ and $t + D_d$ in places of t and $t + s$ respectively, we have

$$\begin{aligned} n_{i+1}(t + D_d) &= n_i(t) - Q_{i+1}(t - D_u) - A_{i+1}(t - D_u,\ t + D_d) + Q_{i+1}(t + D_d) && (18) \\ &\leq n_i(t) - Q_{i+1}(t - D_u) + Q_{i+1}(t + D_d), && (19) \end{aligned}$$

where (19) is due to the nonnegativity of $A_K(t - D_u,\ t + D_d)$. Combining (17) with (19), we have

$$\begin{aligned} m_{i+1}(t + D_d) \geq\ & Q_{i+1}(t + D_d) + \\ & [m_{i+2}(t - D_u + D_d) - n_i(t) + Q_{i+1}(t - D_u) - Q_{i+1}(t + D_d)]^+. \end{aligned}$$

By induction,

$$\begin{aligned} m_{i+1}(t + D_d) &\geq Q_{i+1}(t + D_d) + \\ &\quad [m_{i+2}^{up}(t - 2D_u) - n_i(t) + Q_{i+1}(t - D_u) - Q_{i+1}(t + D_d)]^+ \\ &= Q_{i+1}(t + D_d) + [m_{i+1}(t - D_u) - n_i(t) - Q_{i+1}(t + D_d)]^+ && (20) \\ &= Q_{i+1}(t + D_d) + [m_{i+1}^{up}(t - D_u) - Q_{i+1}(t + D_d)]^+ && (21) \\ &\geq m_{i+1}^{up}(t - D_u), \end{aligned}$$

where (20) and (21) are based on (8) and (5) respectively. This completes the induction. ∎

Lemma 2: For all $t \geq 0$, and all $i \in [1, K - 1]$, the following two statements are true:

$$m_{i+1}^{up}(t - D_u) > 0 \text{ iff } M_{i+1}^{up}(t - D_u) > 0. \tag{22}$$

$$m_i(t) > 0 \text{ iff } M_i(t) > 0 \tag{23}$$

Remark. Lemma 2 formulizes the intuitive fact that, the estimated number of downstream data segments is positive if and only if the estimated number of active downstream stations is positive.

Proof: The backward statements can be seen from Proposition 5 and Proposition 6. We prove that the pair of forward statements are true by induction on $i = K - 1, ..., 1$.

1. Based on (2) and (3),

$$Q_K(t) = m_K(t) > 0 \Rightarrow M_K(t) = I_K(t) > 0,$$

Then,

$$
\begin{aligned}
m_K^{up}(t-D_u)>0 \;&\Rightarrow\; m_K(t-D_u)>0 \\
&\Rightarrow\; M_K(t-D_u)>0 \\
&\Rightarrow\; M_K^{up}(t-D_u)=\min\{M_K(t-D_u),\; m_K^{up}(t-D_u)\}>0 \qquad (24)
\end{aligned}
$$

according to Proposition 3, induction assumption (25), and (6) in the updating step respectively. Based on (8) in the algorithm, i.e.,

$$m_{K-1}(t)=Q_{K-1}(t)+m_K^{up}(t-D_u),$$

we have

$$
\begin{aligned}
m_{K-1}(t)>0 \;&\Rightarrow\; \text{at least one of } Q_{K-1}(t) \text{ and } m_K^{up}(t-D_u) \text{ is nonnegative} \\
&\Rightarrow\; \text{at least one of } I_{K-1}(t) \text{ and } M_K^{up}(t-D_u) \text{ is nonnegative} \\
&\Rightarrow\; M_{K-1}(t)=I_{K-1}(t)+M_K^{up}(t-D_u)>0.
\end{aligned}
$$

Thus we have established the basis for both statements.

2. Assume that the second statement is true for a given $i+1$, i.e.,

$$m_{i+1}(t-D_u)>0 \Rightarrow M_{i+1}(t-D_u)>0. \qquad (25)$$

Similar to the proof of (24), we have

$$
\begin{aligned}
m_{i+1}^{up}(t-D_u)>0 \;&\Rightarrow\; m_{i+1}(t-D_u)>0 \\
&\Rightarrow\; M_{i+1}(t-D_u)>0 \\
&\Rightarrow\; M_{i+1}^{up}(t-D_u)=\min\{M_{i+1}(t-D_u),\; m_{i+1}^{up}(t-D_u)\}>0
\end{aligned}
$$

Thus, we have shown that the first statement is true if the second one is true.

Based on (8) in the algorithm, i.e.,

$$m_i(t)=Q_i(t)+m_{i+1}^{up}(t-D_u),$$

we have

$$\begin{aligned} m_i(t) > 0 \quad &\Rightarrow \quad \text{at least one of } Q_i(t) \text{ and } m_{i+1}^{up}(t - D_u) \text{ is nonnegative} \\ &\Rightarrow \quad \text{at least one of } I_i(t) \text{ and } M_{i+1}^{up}(t - D_u) \text{ is nonnegative} \\ &\Rightarrow \quad M_i(t) = I_i(t) + M_{i+1}^{up}(t - D_u) > 0. \end{aligned}$$

Thus, the induction proof is complete. ■

3 Proof of Correctness and Liveness

3.1 Proof of Correctness

3.1.1 Proof of Full Utilization

Theorem 1: The protocol DTQDB has full utilization according to Definition 2 in section 2.2.

Proof: For the most downstream station K, an idle slot is allowed to pass by only if the FIFO queue is empty. Therefore, we only need to prove the full utilization statement for station $i \in [1,\ K-1]$. At time t if an idle slot gets propagated by station i which has a nonempty FIFO queue, then according to the algorithm, we must have

$$C_i(t) > 0.$$

Hence, by Proposition 7,

$$M_{i+1}^{up}(t - D_u) > 0. \tag{26}$$

Therefore, it is sufficient to show the following statement.

Statement 1: if an idle slot arrives at station $i \in [1, K-1]$ at t with $M_{i+1}^{up}(t - D_u) > 0$, and gets propagated, then the idle slot must be used by one of the downstream stations.

First, notice that,

$$m_{i+1}(t + D_d) \geq m_{i+1}^{up}(t - D_u) \geq M_{i+1}^{up}(t - D_u) > 0, \tag{27}$$

according to Lemma 1 and Proposition 5. Thus, based on Lemma 2,

$$M_{i+1}(t + D_d) > 0. \tag{28}$$

Now with this condition, we can prove Statement 1 by induction on i from $K-1$ to 1.

1. Let $i = K - 1$. We have

$$Q_K(t + D_d) = m_K(t + D_d) > 0,$$

according to (2) and (27). Then, station K uses the idle slot.

2. Assume that the statement is true for a given $i+1$. The idle slot propagated at time t by station i with $M_{i+1}^{up}(t-D_u) > 0$ arrives at station $i+1$ at $t+D_d$. There are three cases to consider at station $i+1$.

   Case 1: $Q_{i+1}(t+D_d) > 0$ and the idle slot is occupied by a data segment at station $i+1$.

   Case 2: $Q_{i+1}(t+D_d) > 0$ and the idle slot is propagated downstream. According to the algorithm, we have a similar argument as in (26) for station $i+1$, i.e., $M_{i+2}^{up}(t+D_d-D_u) > 0$. Based on the induction assumption for station $i+1$, the idle slot will be used by one of the downstream stations $k \in [i+2,\ K]$.

   Case 3: $Q_{i+1}(t+D_d) = I_{i+1}(t+D_d) = 0$, so that the idle slot is propagated downstream. Then,

$$M_{i+2}^{up}(t+D_d-D_u) = M_{i+2}^{up}(t+D_d-D_u) + I_{i+1}(t+D_d) = M_{i+1}(t+D_d) > 0$$

   based on (28). According to the induction assumption for station $i+1$, the idle slot will be used by one of the downstream stations $k \in [i+2,\ K]$.

   Hence the idle slot is used by one of the downstream stations from i in all three cases. ∎

3.1.2 Proof of Fairness

In order to prove the fairness property of the DTQDB protocol, we first need to prove Lemmas 3 and 4 below.

Lemma 3: For any given $i \in [1,\ K-1],\ t \geq 0$,

$$m_{i+1}^{up}(t-D_u) \geq \{ \sum_{k=i+1}^{K} [Q_k(T_k^i(t)) - A_k(\tau_k^i(t),\ T_k^i(t))]\ \}^+. \tag{29}$$

Proof: We use induction on i from $K-1$ to 1.

1. Let $i = K-1$. We have,

$$\begin{aligned} m_K^{up}(t-D_u) &= [m_K(t-D_u) - n_{K-1}(t)]^+ \\ &= [Q_K(t-D_u) - n_{K-1}(t)]^+ \\ &= [Q_K(t+D_d) - A_K(t-D_u,\ t+D_d) - n_K(t+D_d)]^+ \\ &= [Q_K(t+D_d) - A_K(t-D_u,\ t+D_d)]^+ \\ &= [Q_K(T_K^{K-1}(t)) - A_K(\tau_K^{K-1}(t),\ T_K^{K-1}(t)]^+. \end{aligned} \tag{30}$$

The first three equalities follow from (5), (2), and Proposition 8. The next to last follows from full utilization, which implies that $n_K(t+D_d) > 0$ only if $Q_K(t+D_d) - A_K(t-D_u,\ t+D_d) \leq 0$.

2. Assume the inequality (29) is true for a given $i+1$, i.e.,

$$m^{up}_{i+2}(t-2D_u) \geq \{ \sum_{k=i+2}^{K} [Q_k(T_k^{i+1}(t-D_u)) - A_k(\tau_k^{i+1}(t-D_u),\ T_k^{i+1}(t-D_u))]\}^+. \tag{31}$$

According to (5), (8), Proposition 8, and (31),

$$\begin{aligned}
m^{up}_{i+1}(t-D_u) &= [m_{i+1}(t-D_u) - n_i(t)]^+ \\
&= [Q_{i+1}(t-D_u) + m^{up}_{i+2}(t-2D_u) - n_i(t)]^+ \\
&= [Q_{i+1}(t+D_d) - A_{i+1}(t-D_u,\ t+D_d) - n_{i+1}(t+D_d) + m^{up}_{i+2}(t-2D_u)]^+ \\
&= \{Q_{i+1}(t+D_d) - A_{i+1}(t-D_u,\ t+D_d) - n_{i+1}(t+D_d) \\
&\quad +[\sum_{k=i+2}^{K} (Q_k(T_k^{i+1}(t-D_u)) - A_k(\tau_k^{i+1}(t-D_u),\ T_k^{i+1}(t-D_u)))]^+\}^+
\end{aligned}$$

Combining with the fact that,

$$\begin{aligned}
Q_k(T_k^{i+1}(t-D_u)) - A_k(\tau_k^{i+1}(t-D_u),\ T_k^{i+1}(t-D_u) = \\
Q_k(T_k^i(t)) - A_k(\tau_k^i(t),\ T_k^i(t)) + N_k(T_k^{i+1}(t-D_u),\ T_k^i(t)),
\end{aligned}$$

we have,

$$\begin{aligned}
m^{up}_{i+1}(t-D_u) &= \{Q_{i+1}(t+D_d) - A_{i+1}(t-D_u,\ t+D_d) - n_{i+1}(t+D_d) \\
&\quad +[\sum_{k=i+2}^{K} (Q_k(T_k^i(t)) - A_k(\tau_k^i(t),\ T_k^i(t)) + N_k(T_k^{i+1}(t-D_u),\ T_k^i(t)))]^+\}^+ \\
&\geq \{\sum_{k=i+1}^{K} [Q_k(T_k^i(t)) - A_k(\tau_k^i(t),\ T_k^i(t))] - n_{i+1}(t+D_d) \\
&\quad + \sum_{k=i+2}^{K} N_k(T_k^{i+1}(t-D_u),\ T_k^i(t))\ \}^+ \\
&= \{\sum_{k=i+1}^{K} [Q_k(T_k^i(t)) - A_k(\tau_k^i(t),\ T_k^i(t))] - n_K(T_K^i(t))\ \}^+ \qquad (32) \\
&= \{\sum_{k=i+1}^{K} [Q_k(T_k^i(t)) - A_k(\tau_k^i(t),\ T_k^i(t))]\ \}^+ \qquad (33)
\end{aligned}$$

where (32) is based on Proposition 9. (33) is based on the full utilization property, which implies that $n_K(T_K^i(t)) > 0$ only if $Q_k(T_k^i(t)) - A_K(T_k^{i+1}(t-D_u),\ T_k^i(t)) \leq 0\ \forall k \in [i+1,\ K]$.

Since $A_K(T_k^{i+1}(t-D_u),\ T_k^i(t)) \le A_K(\tau_k^i(t),\ T_k^i(t))$, this means that $n_K(T_K^i(t)) > 0$ only if $Q_k(T_k^i(t)) - A_K(\tau_k^i(t),\ T_k^i(t)) \le 0\ \forall k \in [i+1,\ K]$. ∎

Lemma 4: For any $i \in [1,\ K-1]$, and any $t \ge 0$,

$$\sum_{k=i+1}^{K} I_k(\tau_k^i(t)) \ge M_{i+1}(t-D_u). \tag{34}$$

Remark. Lemma 4 is intuitive based on the fact that taking extra idle slots into consideration in the information updating can only reduce the estimated number of active downstream stations among $i+1$, $i+2$, ..., K.

Proof: We use induction on i from $K-1$ to 1.

1. Let $i = K-1$. We have $I_K(\tau_K^{K-1}(t)) = I_K(t-D_u) = M_K(t-D_u)$, which establishes the basis.

2. Assume the statement is true for a given $i+1$. We have (34) with $i+1$ and $t-D_u$ in place of i and t as follows,

$$\sum_{k=i+2}^{K} I_k(\tau_k^{i+1}(t-D_u)) \ge M_{i+2}(t-2D_u) \tag{35}$$

Therefore,

$$\begin{aligned}\sum_{k=i+1}^{K} I_k(\tau_n^i(t)) &= I_{i+1}(t-D_u) + \sum_{k=i+2}^{K} I_k(\tau_k^{i+1}(t-D_u)) \\ &\ge I_{i+1}(t-D_u) + M_{i+2}(t-2D_u) \\ &\ge I_{i+1}(t-D_u) + M_{i+2}^{up}(t-2D_u) \\ &= M_{i+1}(t-D_u)\end{aligned}$$

according to (35), Proposition 4, and (9) respectively.

Thus, we have completed the induction. ∎

Definition 3: The set $S_n = \{i_1 < i_2 < ... < i_n\}$ is "very active" since t_0 if, for each station $i_k \in S_n$, and each $t \ge T_{i_k}^{i_1}(t_0) = t_0 + (i_k - i_1)D_d$, the queue length at time t exceeds the number of new arrivals in the interval from t back to the past round trip delay between i_k and i_1 , i.e.,

$$Q_{i_k}(T_{i_k}^{i_1}(t)) > A(\tau_{i_k}^{i_1}(t),\ T_{i_k}^{i_1}(t)). \tag{36}$$

See Figure 3 for an illustration.

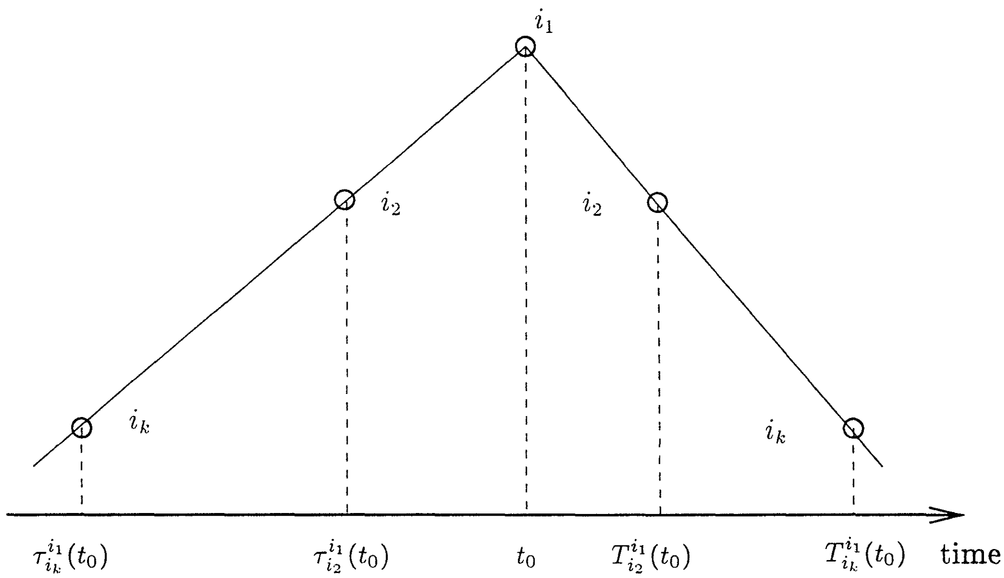

Figure 3: illustration of $\tau_{i_k}^{i_1}(t_0)$ and $T_{i_k}^{i_1}(t_0)$ for $i_k \in S_n$

Theorem 2: The protocol DTQDB is fair according to Definition 1 in Section 2.2.

Proof: For any station $i_k \in S_n$ that is "very active" at any $t \geq T_{i_k}^{i_1}(t_0) = t_0 + (i_k - i_1)D_d$, (36) implies that,

$$I_{i_k}(s) = 1 \ \forall s \in [\tau_{i_k}^{i_1}(t),\ T_{i_k}^{i_1}(t)], \tag{37}$$

$$Q_{i_k}(s_2) > A(s_1,\ s_2), \text{ for any } [s_1,\ s_2) \subseteq [\tau_{i_k}^{i_1}(t),\ T_{i_k}^{i_1}(t)). \tag{38}$$

Therefore,

$$M_{i_k+1}^{up}(t - D_u) \leq M_{i_k+1}(t - D_u) \leq \sum_{l=i_k+1}^{K} I_l(\tau_l^{i_k}(t)) = n - k, \tag{39}$$

according to Proposition 4, Lemma 4, and (37) with $\tau_l^{i_k}(t) \geq \tau_l^{i_1}(t)$, respectively.

Next, we show that for any $i \geq i_1$, and $t \geq T_i^{i_1}(t_0)$, the following inequality is true,

$$M_{i+1}^{up}(t - D_u) \geq \sum_{k=i+1}^{K} I_k(T_k^i(t)) \tag{40}$$

by induction on $i = K - 1,\ \ldots,\ i_1$.

1. Let $i = K - 1$. We need to prove that

$$M_K^{up}(t - D_u) \geq I_K(t + D_d). \tag{41}$$

This leads to two cases.

- Station K is one of the idle stations. Thus, $I_K(t+D_d)=0$, and (41) follows from Proposition 2.
- Station $K \in S_n$. Based on (3) and (37) with $t-D_u \geq \tau_K^{i_1}(t_0)$,

$$M_K(t-D_u) = I_K(t-D_u) = 1. \tag{42}$$

And, based on (30) and (38) with $[t-D_u,\ t+D_d) \subseteq [\tau_K^{i_1}(t),\ T_K^{i_1}(t))$,

$$m_K^{up}(t-D_u) = [Q_K(t+D_d) - A_K(t-D_u,\ t+D_d)]^+ \geq 1. \tag{43}$$

Therefore, according to the updating step (6) in the algorithm,

$$M_K^{up}(t-D_u) = \min\{M_K(t-D_u),\ m_K^{up}(t-D_u)\} \geq 1 = I_K(t+D_d).$$

2. Assume (40) is true for any given $i+1$. Therefore,

$$M_{i+2}^{up}(t-2D_u) \geq \sum_{k=i+2}^{K} I_k(T_k^{i+1}(t-D_u)). \tag{44}$$

Then,

$$\begin{aligned}
M_{i+1}(t-D_u) &= I_{i+1}(t-D_u) + M_{i+2}^{up}(t-2D_u) \\
&\geq I_{i+1}(t-D_u) + \sum_{k=i+2}^{K} I_k(T_k^{i+1}(t-D_u)) \\
&= I_{i+1}(t+D_d) + \sum_{k=i+2}^{K} I_k(T_k^{i}(t)) \\
&= \sum_{k=i+1}^{K} I_k(T_k^{i}(t))
\end{aligned}$$

according to (9), (44, (37) with the fact that $T_k^{i+1}(t-D_u) \in [\tau_k^i(t),\ T_k^i(t)] \subseteq [\tau_k^{i_1}(t),\ T_k^{i_1}(t)]$, and idle stations stay idle during $[\tau_k^{i_1}(t),\ T_k^{i_1}(t)]$. Moreover, based on Lemma 3,

$$\begin{aligned}
m_{i+1}^{up}(t-D_u) &\geq \{\sum_{k=i+1}^{K} [Q_k(T_k^i(t)) - A_k(\tau_k^i(t)),\ T_k^i(t))]\ \}^+ \\
&\geq \sum_{k=i+1}^{K} I_k(T_k^i(t))
\end{aligned}$$

where the second inequality is due to the fact that,

$$\text{for any } k \notin S_n,\ Q_k(T_k^i(t)) - A_k(\tau_k^i(t)),\ T_k^i(t)) = 0 = I_k(T_k^i(t)),$$
$$\text{for any } k \in S_n,\ Q_k(T_k^i(t)) - A_k(\tau_k^i(t)),\ T_k^i(t)) \geq 1 = I_k(T_k^i(t)) \quad (45)$$

where (45) is based on (38) with the fact that $[\tau_k^i(t)),\ T_k^i(t)) \subseteq [\tau_k^{i_1}(t)),\ T_k^{i_1}(t))$.

Therefore, according to the updating step (6) in the algorithm,

$$M_{i+1}^{up}(t - D_u) = \min\{M_{i+1}(t - D_u),\ m_{i+1}^{up}(t - D_u)\} \geq \sum_{k=i+1}^{K} I_k(T_k^i(t)).$$

Thus, we have completed the induction proof for (40).

Based on (40), for any $i_k \in S_n$, and any $t \geq T_{i_k}^{i_1}(t_0) = t_0 + (i_k - i_1)D_d$

$$M_{i_k+1}^{up}(t - D_u) \geq \sum_{l=i_k+1}^{K} I_l(T_l^{i_k}(t)) = n - k.$$

Combining with (39), we have,

$$M_{i_k+1}^{up}(t - D_u) = n - k.$$

Hence, starting from $t_0 + (i_k - i_1)D_d$, each station $i_k \in S_n$ propagates an idle slot passing by when the counter $C_i(t)$ is nonzero, and decrements it by one until the counter gets to zero. When the counter gets to zero, station i_k occupies the next idle slot with its own data segment in queue, and resets the counter to $n - k$. This is a perfect round robin cycle, where during each n time units,, the most upstream station i_1 uses one of the idle slots and propagates the remaining $n - 1$ idle slots. The station i_2 uses one of these $n - 1$ idle slots and propagates the remaining $n - 2$, and so forth to station i_n. Then each station gets one of the n slots for its own transmission. ■

3.2 Proof of Liveness

Definition 3: An algorithm has the liveness property if the access delay of the first data segment in each queue is finite. Let P_i be some data segment at station i, $i \in [1,\ K]$, denote t_a^i as the time that P_i becomes the first segment in the queue, and t_d^i as the time that P_i departs from the queue. Then an algorithm has the liveness property if and only if $t_d^i - t_a^i < \infty$. Theorem 3 establishes something stronger - namely that $t_d^i - t_a^i \leq B_i$, where $B_i = (i-1)(D_u + D_d) + K - 1$.

Theorem 3: The protocol DTQDB has the liveness property and for each $i \in [1,\ K]$, $t_d^i - t_a^i \leq B_i$.

This is proven in LIDS Report LIDS-P-2308 [1].

Remark. Theorem 3 states that the access delay for the first data segment in queue at station i is upper bounded by $(i-1)(D_u + D_d) + K - 1$, the round trip propagation delay between station i

and the most upstream station 1, plus a constant $K-1$, where K is the total number of stations in the network.

For any general bus protocol with the full utilization property, any upper bound B_i to the access delay for the first data segment in queue at station i ($i \in [1, K]$) is at least as large as the round trip propagation delay, i.e., $B_i \geq (i-1)(D_u + D_d)$. This can be shown through a contradiction argument as below.

Assume the access delay of the first data segment in queue at station i is upper bounded by the round trip propagation delay minus one, i.e.,

$$t_d^i - t_a^i \leq (i-1)(D_u + D_d) - 1. \tag{46}$$

Consider the case where all the stations are idle except station 1 and station i. Station i is idle until t_a^i when date segment P_i arrives at the empty queue. Meanwhile, station 1 has a long queue and uses all the idle slots generated from the head-end except IS_{P_i}, the one used by data segment P_i. In order to satisfy condition (46), idle slot IS_{P_i} must be propagated by station 1 before the information on P_i reaches station 1. In other words, station 1 propagates an idle slot without knowing that any downstream station is active. Now, let's consider another case which is exactly the same as the previous one except that station i is always idle instead. Then the idle slot IS_{P_i} propagated by station 1 is wasted since there is no active downstream station. This is a contradiction to the full utilization property.

Thus in the DTQDB protocol, the access delay of the first data segment in queue is at most a constant $K-1$ away from its least possible value, the round trip propagation delay.

4 Conclusion

In this paper, we have designed and analyzed a distributed total queue dual bus (DTQDB) protocol. The basic concept of the protocol is to give equal access to all the stations according to the most updated information available through the reservation bus. In particular, according to the information available, each station computes the latest estimate on the number of active downstream stations, and serves them in a round robin scheme. It was shown that DTQDB achieves fairness with full utilization. Additionally, the protocol provides a finite access bound which is linear in the round trip propagation delay.

This research represents a new direction in the design and study of multiaccess protocols in high-speed-high-latency networks. The following issues warrant further research.

- Simulation results would be useful to analyze both steady state and transient state behaviors.

- The fairness defined here is for steady state behavior. It is desirable to analyze the protocol in transient states, where a protocol is defined to be "fair" if the number of idle slots used by any heavily loaded station during some interval T is at least as large as the number used by any other station, less some constant independent of T.

- It appears that the protocol can be generalized to the case of non-uniform geographic locations.

References

[1] Chiu, A. L., and Gallager, R. G., "DTQDB: A Fair and Fully Utilized Media Access Protocol for Dual Bus Networks," LIDS-P-2308, Laboratory for Information and Decision Systems, MIT, 1995.

[2] Hahne, E. L., Choudhury, A. K., and Maxemchuk, N. F., "Improving the Fairness of Distributed-Queue-Dual-Bus Networks," *Infocom*, San Francisco, Ca., pp. 175-184, 1990.

[3] Limb, J. O. and Flores, C., "Description of Fasnet – A Unidirectional Local-Area Communications Networks," *Bell Syst. Tech. J.*, 61:1413-1440, 1982.

[4] Maxemchuk, N. F., and Netravali, A. N., "Voice and Data on a CATV Network," *IEEE J. Select. Areas Comm.*, SAC-3:300-311, 1985.

[5] Newman, R. M., Budrikis, Z., and Hullett, J., "Standards for Metropolitan Area Networks," *IEEE Commun. Mag.*, 26:20-28, 1988.

[6] Tseng, C.-W. and Chen, B.-U., "D-Net, A New Scheme for High Data Rate Optical Local Area Networks," *IEEE J. Select. Areas Comm.*, SAC-1:493-499, 1983.

[7] Tobagi, F. A., Borgonovo, F., and Fratta, L., "Expressnet: A High-Performance Integrated-Service Local Area Network," *IEEE J. Select. Areas Comm.*, SAC-1:898-913, 1983.