EXAMINATION OF WAVELET PACKET SIGNAL SETS FOR OVER-SATURATED MULTIPLE ACCESS COMMUNICATIONS

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ABSTRACT

This paper addresses the problem of signature set design for uncoded multiple access (MA) communication in which user signature waveforms are linearly dependent. In general, the linearly dependent or "over-saturated" scenario requires an exponentially complex detector.

The recent introduction of an optimal tree-structured joint detector of extremely low complexity ([1, 2]) has sparked our interest in signature set choice for over-saturated MA communications. The tree detector requires the set of linearly dependent user signatures to have tree-structured cross-correlations. This design guideline is a natural by-product of redundant wavelet packet signals. In this paper we explore, via simulations, the performance of wavelet packet signature sets and find them to do very well in general and to offer promising behavior with the introduction of arbitrary carrier phase.

1. Introduction

Due to natural limitations of any multiple access (MA) communication system, the user waveforms lie in a finite dimensional vector space. For some set of signature waveforms represented in signal space by the set of signature vectors, $\{\mathbf{s}_k\}_1^K$, the detection problem is to compute an estimate $\mathbf{b}$ from an observation $\mathbf{r} \in I\!R^N$

$$\mathbf{r} = \sum_{k=1}^{K} b_k \mathbf{s}_k + \sigma \mathbf{n} = \mathbf{S}\mathbf{b} + \sigma \mathbf{n}, \tag{1}$$

where K is the number of users, and $\mathbf{b} \in \{[b_1 \cdots b_K]^T \mid b_i \in P\}$, where P is some finite set of real amplitudes and the b_i's are iid uniform.[1] $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_K]$ is an $N \times K$ matrix whose columns are user signatures. The column vectors $\mathbf{s}_k$ of $\mathbf{S}$ span the space $I\!R^N$. The noise, $\mathbf{n}$, is a Gaussian vector of mean zero and identity covariance, and σ is the noise standard deviation.

Verdu has shown in [3] that for the general MA joint detection problem stated as above, i.e. for an arbitrary signature matrix, $\mathbf{S}$, the solution is N-P hard. In other words, the maximum likelihood estimate of $\mathbf{b}$ cannot be found by an algorithm having polynomial complexity in the number of users, K.

For the case in which the number of signal dimensions is equal to the number of users, $K = N$, the MA optimal joint detection problem can be reduced from the N-P hard problem to K single-user problems by imposing users to be orthogonal. Restricting the users in a MA communication system to be orthogonal results in suboptimal total information transmission rates ([4]). Linearly dependent user waveform sets ($K > N$ users in N dimensions) offer increased throughput relative to traditional orthogonal MA systems. A tree-structured joint detector and signal set design guidelines were developed by Learned et. al. in [1, 2].

The signal set guidelines allowed for optimal joint detection via a tree-structured iterative procedure of very low complexity. The signature set guidelines are detailed in Section 2.. The use of wavelet packet waveforms with the tree detector is motivated in Section 3. and experimental results are shown. The results are promising in that the wavelet packet signature sets offer excellent performance, even in the case for which carrier phases have been arbitrarily assigned to each user. We offer concluding remarks and future work in Section 4..

2. Signature Set Structure

The tree detector requires that the user signature signals exhibit tree-structured cross-correlations. This necessary cross-correlation structure is satisfied if the signature vectors can be assigned to the nodes of a tree like the one shown in Figure 1. The tree pictorially conveys the following required relationships among user signa-

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[1] Binary phase shift keying is a subset of this, namely, antipodal modulation $P = +1, -1$.

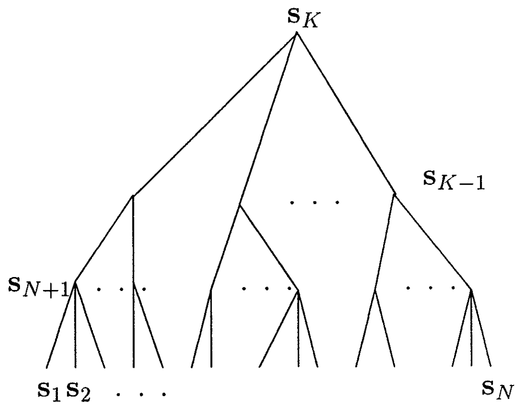

Figure 1: This example of a general tree shows the correlation structure needed among signature vectors within the signature set.

ture vectors.

- Each vector at a given level of the tree is orthogonal to all other vectors at that level.
- A signature vector is correlated only with its ancestor vectors (parent, grandparent, etc.) and its descendent vectors (children, grandchildren, etc.).

With this cross-correlation structure we reduce the exponential computational complexity to polynomial complexity where the solution is found by casting the newly structured problem into a shortest path problem.[2] The optimal tree structured algorithm developed by Learned et. al. is detailed in [1, 2].

For any tree-structured signature set, the tree joint detector will give the optimal estimate of $\mathbf{b}$. The optimal estimate of $\mathbf{b}$, however, is not always reliable. The probability of error of the optimal estimator is a function of the distances between possible received signals. For example, denote the received vector set as $R = \{\mathbf{Sb} | \mathbf{b} \in \Gamma\}$, where Γ is the known finite set of information weights. If the set, R, has small to no separation between points, the optimal bit error rate will be poor. Arbitrary choice of tree structured signature sets is, therefore, not an option for an actual system.

One example of "good" tree-structured signature vector set is the minimum Euclidean distance sets developed by Ross and Taylor for M-ary pulse amplitude modulation (PAM) [5]. Ross and Taylor begin with N orthogonal users in N dimensions that give a minimum distance d between possible received points in R. More users are added so that the minimum distance d between received points is preserved.[3]

A specific example for equal energy binary PAM users, also shown in [5], is repeated below where each column of the signature matrix $\mathbf{S}$ is a user signature vector

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 \end{bmatrix}. \tag{2}$$

For this specific example, the vectors that comprise the Ross/Taylor set correspond to 4 fine scale Haar functions and one course scale Haar function.

The correlation matrix, $\mathbf{S}^T\mathbf{S}$, is given below.

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 & 1 \end{bmatrix} \tag{3}$$

The structure of $\mathbf{S}^T\mathbf{S}$ clearly shows that this minimum distance set of signature vectors may be cast onto a 2-level quad tree for which 4 children emanate from a root node. Ross and Taylor designed these sets for their minimum distance property; the tree hierarchy is a by-product.

3. Exploration of Wavelet Packet Sets

Given that the minimum distance sets of Ross and Taylor are a special case of wavelet packets we are motivated to explore wavelet packets as a rich source of good tree-structured signature sets for over-saturated MA communications.[4]

An example of a wavelet packet signature set built with the the Haar discrete wavelet is given in Figure 2. The signature matrix, $\mathbf{S}$, is shown in Figure 2-a and the absolute value of the associated cross-correlation structure, $(\mathbf{S}^T\mathbf{S})$, is shown in Figure 2-b. The wavelet packet signature vector set can also be shown in terms of the focus bins of the wavelet packet tableau.[5] As can be seen from the tableau representation of the signature vectors in Figure 2-c, this set may be cast onto a tree with two levels. Another wavelet packet set is also shown in the figure and is used in the simulations discussed below.

[2] For example, the computational complexity is $O(K^p)$, p small.

[3] In the equal energy binary PAM case, for example, Ross and Taylor fit 21 users in 16 dimensions and 85 users in 64 dimensions.

[4] Wavelet packet tree-structured signature sets were first proposed in [6] for over-saturated MA communications with a sub-optimal detector.

[5] See [6] for a detailed explanation of the wavelet packet tableau.

Through implementation of the optimal detection algorithm with wavelet packet signature sets, we experimentally show that the wavelet packet-based signature sets achieve performance (i.e. bit error rate) that is comparable to that of the more strictly configured minimum distance set. Figure 3 shows the results of our Monte-Carlo runs with three different user signature sets: the Ross/Taylor set discussed in Section 2. (which is also a special case of a wavelet packet set) and the two wavelet packet sets shown in Figure 2. These three sets comprise 5 unit-energy signal vectors in 4 dimensions and have a minimum distance of approximately 2, 2, and 1.53, respectively. For comparison, we also ran our simulations for a single user in one dimension and a set of 4 orthogonal users in 4 dimensions, both having minimum distance of 2. The Ross/Taylor and Wavelet Packet I sets give identical performance curves. This is not surprising since these sets have the same minimum distance.

Next, we introduce an element of an actual communication system, namely, arbitrary carrier phase. Specifically, each user modulates its waveform on a carrier frequency. The relative phases of the users cannot be controlled and are, therefore, modeled as being arbitrary. For example in the case of three users in two dimensions we may write a tree-correlated signature set as

$$\begin{aligned} s_1(t) &= a_1(t)cos(w_c t + \phi_1) \\ s_2(t) &= a_2(t)cos(w_c t + \phi_2) \\ s_2(t) &= \alpha(a_2(t) + a_2(t))cos(w_c t + \phi_3) \end{aligned} \quad (4)$$

with

$$\int_0^T a_1(t)a_2(t)dt = 0, \quad (5)$$

where $a_i(t), i = 1, 2, 3$ are the real-valued, time limited user signature envelop waveforms, α is a real scalar, w_c is the carrier frequency, and ϕ_i is user i's arbitrary but known carrier phase. Assuming binary PAM is employed, we have eight possible signals in the received set. The Euclidean distance between the two neighboring signals

$$\begin{aligned} r_a(t) = \; & a_1(t)cos(w_c t + \phi_1) + a_2(t)cos(w_c t + \phi_2) \\ & -\alpha(a_2(t) + a_2(t))cos(w_c t + \phi_3) \end{aligned} \quad (6)$$

and

$$\begin{aligned} r_b(t) = \; & -a_1(t)cos(w_c t + \phi_1) - a_2(t)cos(w_c t + \phi_2) \\ & +\alpha(a_2(t) + a_2(t))cos(w_c t + \phi_3) \end{aligned} \quad (7)$$

is

$$\begin{aligned} ||r_a(t) - r_b(t)||^2 = \; & 4(1 + \alpha^2)(E_1 + E_2) \\ & -8\alpha E_1 cos(\phi_1 - \phi_3) - 8\alpha E_2 cos(\phi_2 - \phi_3), \end{aligned} \quad (8)$$

where user i's bit energy is denoted by $E_i = \int_0^T a_i^2(t)cos^2(w_c t + \phi_i)dt$.

From Equation (8) we see that the Euclidean distance between neighboring points can either decrease or increase with the introduction of carrier phase. It is interesting to note that if $\alpha = 1/2$, the distance in (8) is $E_1(5 - 4cos(\phi_1 - \phi_3)) + E_2(5 - 4cos(\phi_2 - \phi_3))$. In this case, the distance between $r_a(t)$ and $r_b(t)$ can only *increase* with the introduction of carrier phase.

Analysis similar to the above 2-D example quickly becomes intractable for problems of larger dimension, hence, we re-run our simulations with arbitrary phase assignments for each user. The single and orthogonal users saw no degradation in performance, as expected.[6] The Ross/Taylor and wavelet packet sets all saw an improvement. The distance between points in these constellations was exhaustively calculated by computer. The histograms of these distances is shown in Figure 4. Notice that there is significant shifting of inter-point distances in the no phase and arbitrary phase cases for each user set.[7] This result is interesting, but not conclusive since a communications system would offer typical dimensions of 64, much larger than 4. In such a case, we would be interested in allowing the number of users to climb at least as high as 85. Further investigation of wavelet packet sets for higher dimensional over-saturated communications is soon to follow this work.

4. Conclusion

In this paper, we have shown some preliminary and promising results using linearly dependent signature sets based on wavelet packet waveforms in over-saturated multiple access communication. The recent introduction of an extremely computationally efficient optimal joint detection algorithm for over-saturated MA requires the use of tree-structured user signature signal sets. Wavelet packet waveform dictionaries offer a wide variety of such signature sets. This warrants further investigation for higher dimensional signal sets. In addition to studying the carrier phase problem in higher dimensions, future work will include studies of wavelet packet sets to find error concentrations for different users as a function of their position on the correlation tree.

[6] It is easy to show that the distances between points in these received constellation are not a function of the phases.

[7] The minimum distance in the Ross/Taylor and and first wavelet packet sets remained at 2 with the introduction of phase. The number of points at this minimum distance, however, was cut in half. The most significant shifting in distances was seen by the second wavelet packet set which had an initial minimum distance of 1.53 and a shifted minimum distance of 1.7. This is reflected in the improvement between the BER curves of Figures 3 and 5.

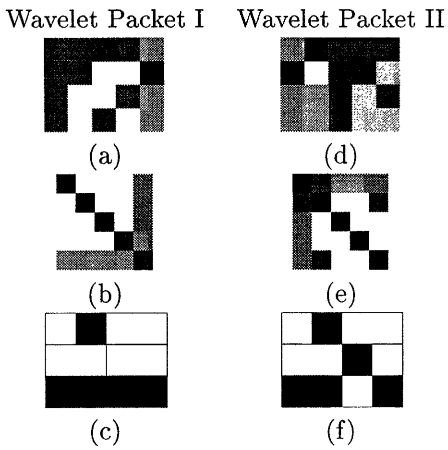

Figure 2: (a) wavelet packet signature matrix I, (b) cross-correlation matrix I, (c) wavelet packet tableau of set I, (d) signature matrix II, (e) cross-correlation matrix II, (c) tableau of set II

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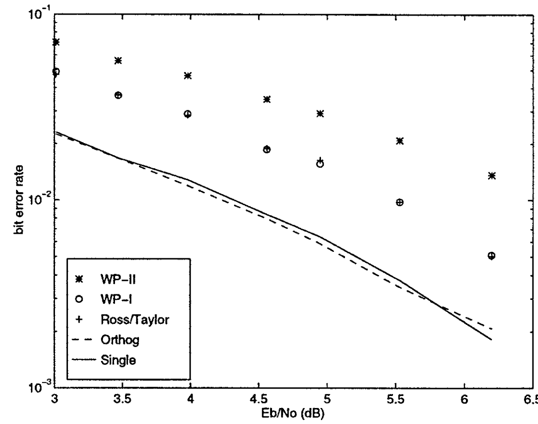

Figure 3: Bit error rate vs. signal energy to noise energy ratio in dB. The WP-I and WP-II sets are shown in Figures 2-a and d, respectively.

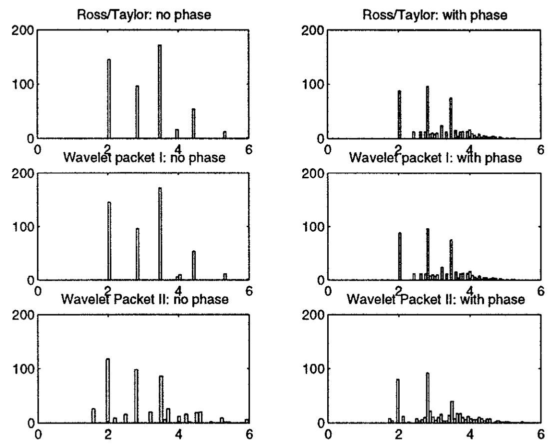

Figure 4: Histograms of inter-point distances

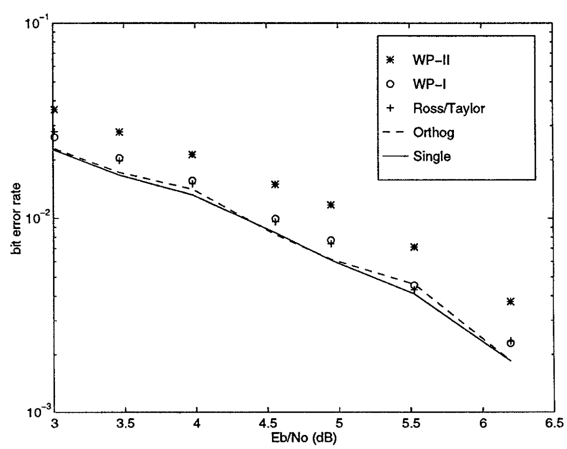

Figure 5: Bit error rate vs. signal energy to noise energy ratio in dB with arbitrary phase for each user.