

Anticipatory Behavior in Lane Changing Models

by

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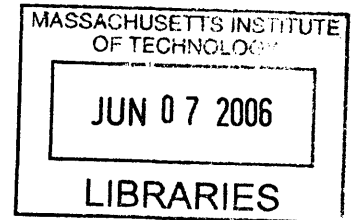
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BARKER

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Abstract

Actions performed by drivers in case of lane changing behavior are usually the result of some plan the driver has in mind. This involves anticipating future scenarios and persisting in order to execute the plan. The objective of this thesis is to develop a framework for modeling the lane-changing behavior that captures the anticipatory behavior of drivers. Two ways of capturing this behavior – a dynamic programming model and an explicit forced merging model – are developed in this thesis.

The fact that drivers constantly modify their plans in the light of new information, suggests the use of a dynamic programming approach, where the solution takes the form of an optimal decision rule that specifies drivers' optimal decisions as a function of their current information. A theoretical framework is developed and the advantages and disadvantages of the approach are discussed. The computational complexity of applying such a model suggests adopting an alternative approach to the problem.

The explicit forced merging model captures the planning and persistent behavior of drivers. The model is essentially a gap acceptance model that explicitly captures normal and forced merging behavior of vehicles merging from the on-ramp to the freeway. Aggressive drivers that tend to initiate forced merging persist in their plan to complete the merging process. The parameters of the model are estimated using detailed trajectory data. Estimation results show that the lane changing behavior is affected by relative speeds of the neighboring vehicles with respect to the merging vehicle, distance to the mandatory lane changing point and acceleration of the lag vehicle. They also show that the initiation of the forced merging process is dependent on unobserved driver characteristics like aggressiveness, driving experience etc. The model is statistically superior to another model estimated with the same dataset but which ignores the planning behavior of drivers.

Thesis Supervisor: **Moshe E. Ben-Akiva**

Title: Edmund K. Turner Professor of Civil and Environmental Engineering

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1. Introduction

1.1 Motivation

Traffic congestion is a growing problem in many parts of the world. Traffic demands vary significantly depending on the season of the year, the day of the week, and the time of day. Also, the capacity, often mistaken as constant, can change because of weather, work zones, traffic incidents, or other non-recurring events. A report on congestion mitigation by FHWA (2006) states that it is estimated that half of the congestion experienced is recurring congestion—caused by recurring demands that exist virtually every day, where road use exceeds existing capacity.

During the past years we have seen a phenomenal growth in motor vehicles, in comparison to which, the growth of roads is very slow. In the United States, construction of new highway capacity has not kept pace with increases in population and car use and the resulting increase in demand for highway travel. According to an FHWA report on traffic congestion and sprawl (Paniati, 2002), between 1980 and 1999 the route miles of highways increased by only 1.5 percent, while the total number of miles of vehicle travel increased by 76 percent. The solution of building more roads is not always feasible due to financial, geographical and environmental reasons. Moreover, building new roads induces greater demands for traffic flow.

According to the 2005 Urban Mobility Report (Schrank and Lomax, 2005), congestion in 2003 caused 3.7 billion vehicle-hours of delay and 2.3 billion gallons of wasted fuel, an increase of 79 million hours and 69 million gallons from 2002 to a total cost of more than \$63 billion. Traffic congestion is increasing in major cities, and delays are becoming more frequent in smaller cities and rural areas. Figure 1.1 shows the increase, from 1982 to 2002, in annual hours of delay experienced by drivers for the 85 urban areas in the United States.

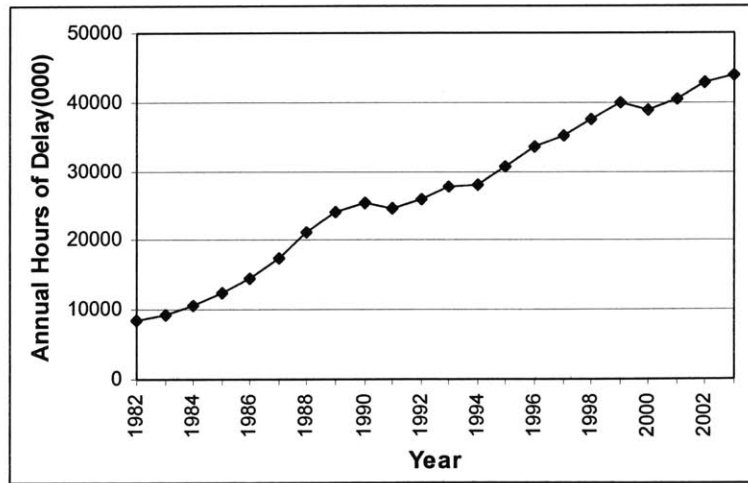


Figure 1.1-Increasing Level of Delay experienced by drivers

Source : 2005 Urban Mobility Study, TTI

The Travel Rate Index (TRI) measures the amount of additional time needed to make a trip in the peak period rather than at other times of the day. This measure is based solely on the regular traffic congestion on the roadways. This gives us an idea of how much of the change in traffic congestion is due solely to more cars using the roadways and/or not enough travelers choosing one of the other travel modes or travel options.

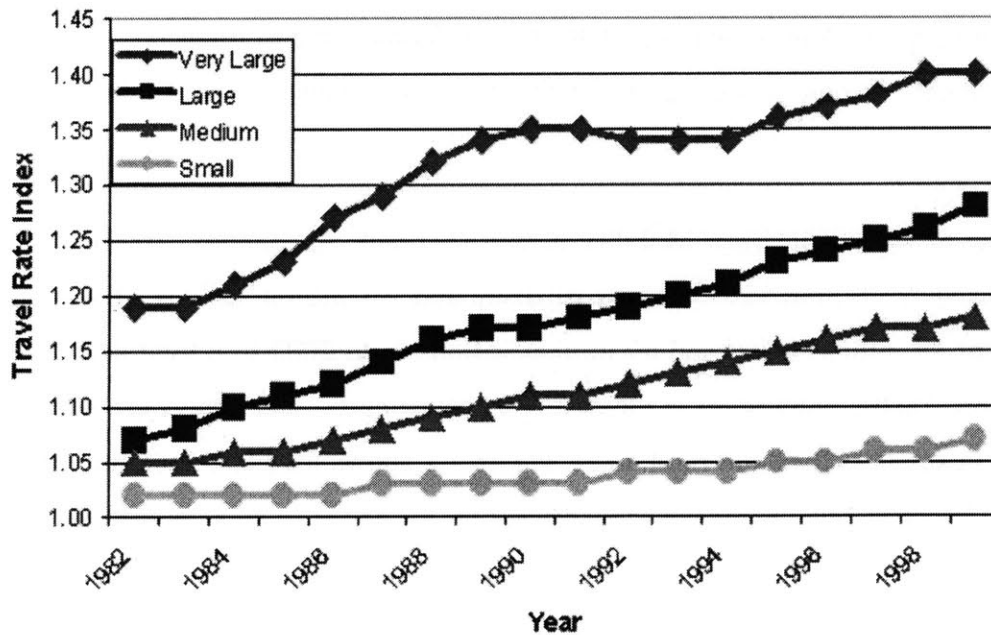


Figure 1.2-Congestion Trends in Urban Areas

Source : 2001 Urban Mobility Report

As shown in Figure 1.2 trends for the four urban area population categories in the 2001 Urban Mobility Report indicate increasing congestion since 1982.

Other than being a drain to the economy, congestion also causes environmental problems. California Air Resources Board estimates that emissions are 250% higher under congested conditions than during free-flow conditions (Schiller, 1998). A reduction in congestion therefore implies a reduction in pollution.

The various solutions suggested in the 2005 Urban Mobility Report are:

- Increased Capacity – New streets, urban freeways to serve new developments and public transportation improvements in congested corridors and to serve major activity centers.
- Greater Efficiency – More efficient operation of roads and public transportation by using information technology, educating travelers about their options and providing a diverse set of travel and development options.
- Demand Management – Using the telephone or internet and working from home, traveling in off-peak hours, using public transportation and carpools are some ways that the use of the transportation network can be modified.
- Development patterns – Techniques that can change the way that commercial, office and residential developments occur, while at the same time sustaining the urban quality of life and gaining an increment in urban development.

Intelligent Transportation Systems (ITS) are one way of alleviating the congestion problem. ITS apply emerging information systems technologies to address and alleviate transportation congestion problems. Solutions provided by Intelligent Transportation Systems applications are not only cost effective but a safe and efficient way to improve the mobility of passengers and freight. To be able to evaluate advanced traffic management systems and to incorporate the dynamic interaction between the traffic management system and drivers on the network, a microscopic simulation approach that captures movement of individual vehicles in the network is essential (e.g. MITSIMLab). This level of detail is necessary for evaluation at the operational level. Driving behavior theories and models at the microscopic level have the potential to help us understand the causes of congestion and to devise solutions for it.

Driving behavior models are an important part of microscopic traffic simulations and help predict driving maneuvers such as lane-changing, acceleration and route choice. They describe drivers' decisions with respect to their vehicle movement under different traffic conditions. These models include speed/acceleration models, which describe the movement of the vehicle in the longitudinal direction, and lane changing models, which describe drivers' lane selection and gap acceptance behaviors (e.g. Kazi 1999, Toledo 2003, Choudhury 2005).

To accurately model driving behavior it is essential to capture drivers' anticipatory and planning behavior. Implementation of models that do not capture such behavior in microscopic simulation tools may result in unrealistic representation of traffic flow characteristics and can lead to under-estimation of bottleneck capacities and over-estimation of congestion (e.g. DYMO 1999, Abdulhai et al 1999). Therefore, driving behavior models that can capture the planning behavior of drivers are required to realistically simulate traffic conditions.

1.2 The Problem

Individuals make decisions based on some plan that they have for the future. Actions performed by drivers in case of lane changing behavior are usually the result of some plan the driver has in mind. For instance, a driver may decide to move to the right lane now to eventually get to the exit ramp. Apart from anticipating the future scenarios, drivers are usually persistent in following their plan once they form one. For example, if a driver decides to force-in in front of a vehicle, he/she will continue that line of action and try to complete the merge.

Lunefeld (1989) characterizes the driving task as an information-decision-action activity, where information received in-transit is used with information and knowledge in-storage to make decisions and perform actions in a continuous feedback process. Drivers do not make single once and for all plans about their desired lanes. Rather, drivers constantly modify their plans in the light of new information. Toledo (2003) states that drivers may conceive an action plan and perform it over a length of time based on future anticipated conditions. He emphasizes the importance of this behavior in lane changing as drivers try to anticipate the

behavior of other vehicles and adjust their own behavior to facilitate completion of a lane change.

Existing models usually assume that drivers consider only current or past traffic conditions and make instantaneous decisions based on these conditions. However, in reality drivers tend to have expectations in the future and tend to anticipate future traffic conditions (Section 2.3). Based on these expectations, drivers form a plan and persist in their aim to execute the plan. By taking into account this anticipatory and persistent behavior of drivers, existing models can better represent driving behavior.

1.3 Objectives

In this thesis, we discuss various ways to enhance existing models by capturing the individual's expectations about the future, and his/her persistence to execute the plan. Two approaches that have the potential to capture the anticipatory and persistent behavior are discussed. The two models developed in this thesis for lane-changing decisions are:

- Dynamic Programming model
- Explicit Force Merging model

The objective of this thesis is to develop a framework for modeling the lane-changing behavior that captures the anticipatory and persistent behavior, estimate the model with the available data for freeway merging, implement the model in MITSIM – a microscopic traffic simulator and compare it with a model that does not capture the planning behavior. The model that captures this behavior is expected to be significantly better than the limited model.

1.4 Thesis Outline

The rest of this thesis is organized in seven chapters. The literature review in lane changing models and anticipatory behavior models is presented in Chapter 2. The literature review also covers state-of-the-art models that have incorporated anticipatory behavior in lane changing decisions. Chapters 3 and 4 cover two approaches toward including the anticipatory and persistent nature of individuals in lane changing models. Chapter 3 provides a theoretical

framework for applying dynamic programming to capture anticipatory and planning behavior in lane changing models. In Chapter 4 the estimation methodology and framework of capturing the planning behavior of drivers (the explicit force merging model) is presented. Chapter 5 gives an overview of the data used for estimation. The estimation results are presented in Chapter 6. The implementation results are discussed in Chapter 7. Finally conclusions and directions for further research are presented in Chapter 8.

2. Literature Review

This chapter reviews the research done in the area of lane changing models for freeway merging, used for micro simulation. This chapter also covers relevant research done to capture the anticipatory behavior of individuals focusing especially on the field of dynamic programming. Finally models that have in various ways captured the anticipatory behavior of individuals in lane changing behavior are described. The chapter is organized in three sections. Section 2.1 provides an overview of various lane-changing models developed. Section 2.2 covers models that capture anticipatory behavior and in Section 2.3 models that capture the anticipatory behavior of drivers are described.

2.1 Lane Changing Models

This section describes general lane changing models that have been developed but that do not capture the anticipatory behavior of individuals. This section also covers the relevant literature pertaining to gap acceptance models used in modeling lane changing behavior. As this section uses terms like lead and lag gaps, the following figure illustrates the relation between the subject and its lead and lag vehicles.

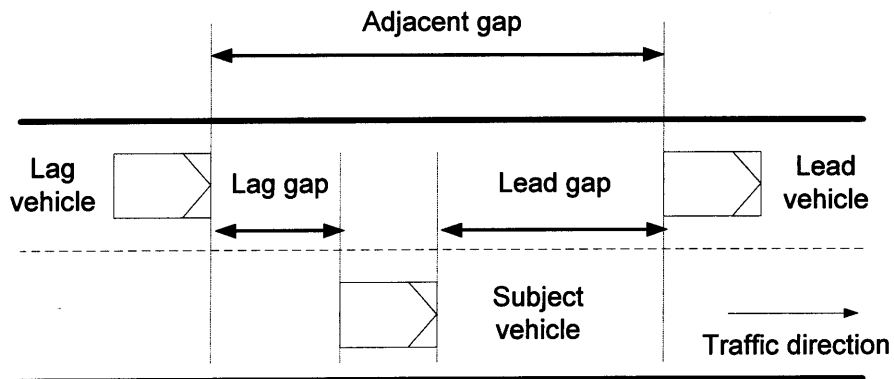


Figure 2.1-Relation between subject, lead and lag vehicles

Skabardonis (1985) developed a microscopic simulation model to investigate the interactions between traffic and geometric variables at grade-separated interchanges. The merging process is defined for a single vehicle and for queued vehicles. For a single driver, the critical lead and lag times depend on his speed relative to the main stream. If no gap is immediately

available the driver can either accelerate to create and merge in a gap in front or decelerate and merge in the following gap if acceptable. A queuing vehicle can evaluate mainstream gaps if the leading vehicle decides to merge; otherwise its acceleration is determined assuming it is a restrained vehicle. A large mainstream gap implies that several queuing vehicles can merge into the gap. Drivers are assumed to behave consistently. The critical lag time was assumed to vary between drivers according to a log-normal distribution. However, only the relative speed was assumed to affect the lag time. Moreover, the critical lead time was assumed to be a constant (1.0 sec) for all drivers. Most of the parameters are either assumed to be constants or take on a random value from an assumed range.

Gipps (1986) proposed a structure to connect the decisions a driver has to make before changing lanes. The model covers the urban driving situation, where traffic signals, obstructions and heavy vehicles all exert an influence. Gipps also considers the fact that the decision to change lanes may depend on conflicting objectives and that a driver must be able to reconcile between his short-term and long-term aims. Driver's behavior is governed by two considerations: attaining the desired speed and being in the correct lane to perform turning maneuvers. Zones based on the distance to the intended turn determine the relative importance of these two considerations. These zones are defined deterministically and variability between drivers and for the same driver over time is ignored. There is no proposed framework for rigorous estimation of the model parameters. Although the notion of anticipation is recognized, driver behavior is modeled by a set of rules in a decision-tree model; no rigorous model is developed that captures driver anticipation explicitly.

Hunt and Lyons (1994) used neural networks as an alternative method of modeling driver behavior within road traffic systems. Their main approach makes use of a learning vector quantization classification type of neural network. A driver is assumed to make a decision based on vehicle movements within a zone of influence, i.e., the activity within a certain distance behind the vehicle and a certain distance in front. The model is tuned to perform correctly by exposure to a large number of representative example inputs and desired decisions or answers. Their model uses visual pattern-based input to describe the driving environment around the vehicle about to make a lane change. However, their model does not consider possible cooperation between drivers during lane-changing.

In CORSIM (Halati et al 1997, FHWA 1998), a microscopic traffic simulation model developed by FHWA, lane changes are classified as either mandatory (MLC) or discretionary (DLC). An MLC is performed when the driver must leave the current lane (e.g. in order to exit to an off-ramp, avoid a lane blockage). A DLC is performed when the driver perceives that driving conditions in the target lane are better, but a lane change is not required. A risk factor is computed for each potential lane change and is defined in terms of the deceleration a driver would have to apply if its leader brakes to a stop. The risk is calculated for the subject with respect to its intended leader and for the intended follower with respect to the subject. The risk is compared to an acceptable risk factor, which depends on the type of lane change and its urgency. The model ignores variability in gap acceptance behavior.

Ahmed (1999) developed a lane changing model that captures both mandatory and discretionary lane changes. As shown in Figure 2.2, the framework is used to model three lane-changing steps: decision to consider a lane-change, choice of the target lane and gap acceptance.

The mandatory lane changing situation was found to be affected by the amount of time the driver has not been able to merge, also called the time delay. In case of discretionary lane change, the driver's satisfaction with the current lane is dependant on the difference between the current and desired speeds, presence of a tailgating vehicle and whether the vehicle is a heavy vehicle or not. If the driver is not satisfied with the driving conditions in the current lane, neighboring lanes are compared to the current one and a target lane selected. A gap acceptance model was used to represent the execution of lane changes. The parameters of the MLC and DLC components of the models were not estimated jointly.

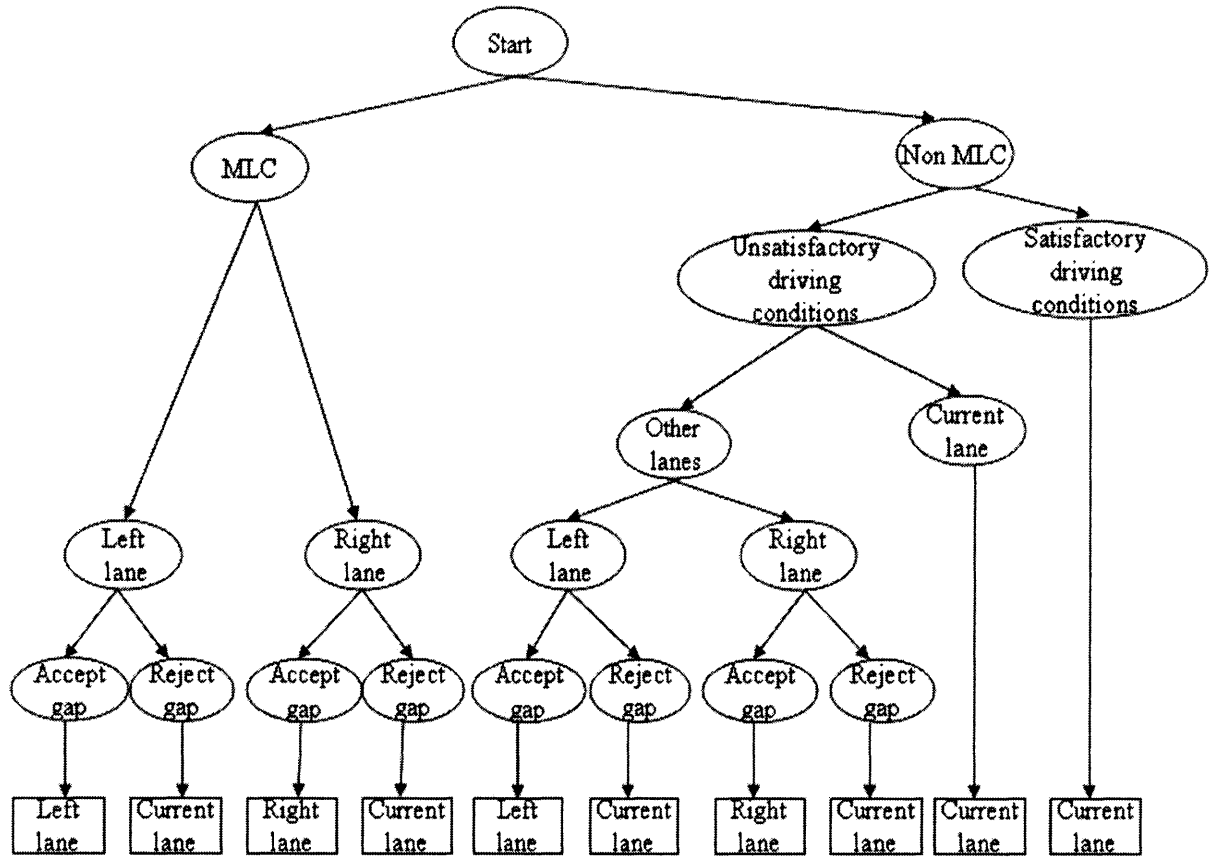


Figure 2.2-Structure of the lane changing model proposed by Ahmed (1999)

He also estimated a forced merging model that captures drivers' lane-changing behavior in heavily congested traffic. The model is described in Section 2.3.

2.1.1 Gap Acceptance Models

Gap acceptance models are an important component of most lane changing models. The driver decides whether a particular gap is acceptable or not based on his surrounding driving conditions. Every individual is assumed to have a critical gap based on which the available gap is either rejected or selected. For an individual n at time t , this can be modeled as:

$$Y_{nt} = \begin{cases} 1 & \text{if } G_{nt} \geq G_{nt}^{cr} \\ 0 & \text{if } G_{nt} < G_{nt}^{cr} \end{cases} \quad (2.1)$$

where, Y_{nt} is 1 if the gap is accepted, 0 otherwise. G_{nt} is the available gap and G_{nt}^{cr} is the critical gap.

In order to capture the probabilistic nature of gap acceptance decisions, critical gaps are modeled as random variables. Herman and Weiss (1961) assumed an exponential distribution, Drew et al (1967) assumed a lognormal distribution and Miller (1972) assumed a normal distribution for the critical gap lengths.

In CORSIM, the ramp merging process is based on gap acceptance. The considerations that 1) a vehicle will accept a smaller critical headway if it is going slower than the lead vehicle, 2) the deceleration required by the driver to adjust his position with respect to the new leader and 3) the new follower may cooperate with the lane changer by decelerating to increase the gap are combined into a measure called Risk. More aggressive drivers would accept higher risk values i.e., shorter gaps and higher acceleration/deceleration rates to complete the merge.

Kita(1993) used a logit model to estimate the gap acceptance model for the case of vehicles merging to a freeway from a ramp. The impact of different factors on driver's gap acceptance behavior was modeled using a random utility model. The gap length, relative speed of the subject with respect to the mainline vehicles and the remaining distance of the acceleration lane were found to have an impact on drivers' gap acceptance behavior.

The HCM (1997) uses gap acceptance models to describe the interaction between drivers at two-way stop controlled intersections. The critical gap is defined as the minimum time interval in the major-street traffic stream that allows intersection entry to one minor-stream vehicle. Thus, the driver's critical gap is defined as the minimum gap that the driver finds acceptable. Estimates of the critical gap are made on the basis of the largest rejected and smallest accepted gap for a given intersection. However this definition is not always true as driver behavior is dependent on many other factors and a driver may accept a gap that is smaller than previously rejected gaps.

Ahmed (1999) developed a gap acceptance model that requires drivers to accept both lead and lag gaps. The gaps are assumed to follow a log-normal distribution so that the gaps are

always non-negative. The mean of the distribution is the critical gap and an individual specific error term captures the heterogeneity among different drivers. The functional form of the critical gap is given by:

$$G_n^{cr,g}(t) = \exp\left(X_n^g(t)\beta^g + \alpha^g v_n + \varepsilon_n^g(t)\right) \quad g \in (\text{lead}, \text{lag}) \quad (2.2)$$

where $X_n^g(t)$ is the vector of explanatory variables, β^g is the vector of parameters associated with the variables. v_n is the individual specific random term assumed to be normally distributed, α^g is the parameter of v_n and $\varepsilon_n^g(t)$ is the normally distributed generic random term.

The subject vehicle, lead and lag vehicles and gaps as defined by Ahmed are shown schematically in Figure 2.3.

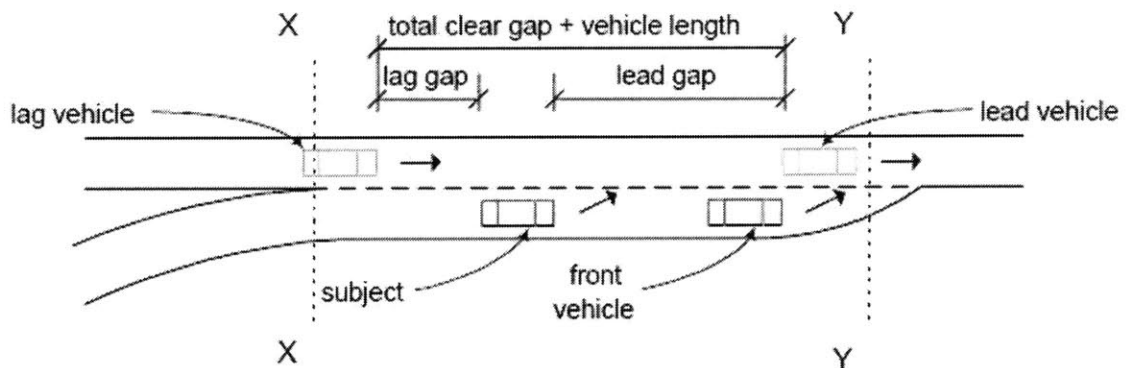


Figure 2.3-The subject, lead and lag vehicles, and the lead and lag gaps (Ahmed 1999)

Gap acceptance parameters were estimated jointly with other components of the model. Toledo (2003) used a similar critical gap approach in his lane changing model. The estimation results of the gap acceptance model are summarized in Table 2.1. Choudhury (2005) also used a similar gap acceptance model in her lane changing model.

Table 2.1-Estimation results for the gap acceptance model (Toledo 2003)

Variable	Parameter value	t-statistic
Lead Critical Gap		
Constant	1.127	2.78
$Max(\Delta V_n^{lead}(t), 0)$, m/sec.	-2.178	-0.63
$Min(\Delta V_n^{lead}(t), 0)$, m/sec.	-0.153	-1.86
Target gap expected maximum utility	0.0045	1.29
α^{lead}	0.789	2.46
σ^{lead}	1.217	2.55
Lag Critical Gap		
Constant	0.968	4.18
$Max(\Delta V_n^{lag}(t), 0)$, m/sec.	0.491	5.95
Target gap expected maximum utility	0.0152	1.65
α^{lag}	0.107	0.47
σ^{lag}	0.622	4.53

2.2 Anticipatory Behavior Models

This section covers models used to capture individual's anticipatory behavior. Dynamic Programming techniques have generally been used toward capturing individual's planning behavior. These models provide a broad framework for the approach developed in Chapter 3.

Dynamic Programming algorithms can be used to compute optimal policies given a perfect model of the environment. In the dynamic programming approach the solution takes the form of an optimal decision rule that specifies the optimal utility decisions as a function of the individual's current information.

Markov decision processes (MDP) provide a broad framework for modeling sequential decision making under uncertainty. MDP's have two kinds of variables: state variable s_t and control variables d_t . According to Rust (1994) a decision-maker can be represented by a set of primitives (u, p, β) where $u(s_t, d_t)$ is a utility function representing the agent's preferences at time t, $p(s_{t+1} | s_t, d_t)$ is a Markov transition probability representing the agent's subjective beliefs about uncertain future states, and $\beta \in (0, 1)$ is the rate at which the individual discounts utility in future periods.

As individuals plan well ahead before making retirement decisions, dynamic programming models have been applied vastly in modeling retirement behavior (e.g. Karlstorm et al 2004, Lumsdaine et al 1992). In addition they have also been used to model the optimal replacement of bus engines (Rust, 1987).

The general form of dynamic programming models developed for choosing between two options, using the example of a retirement problem is presented in Appendix A.

Lumsdaine et al (1992) evaluate three different approaches to model retirement– option value, dynamic programming and probit – to determine which of the retirement rules most closely matches retirement behavior in a large firm. The dynamic programming model they developed is based on a recursive representation of the value function. At the beginning of the year, the individual has two choices: retire now and derive utility from future retirement benefits, or work for the year and derive utility from income while working during the year and retaining the option of retirement or work in the next year. The value function in the dynamic programming model is given by:

$$W_t = \max \left[U_w(Y_t) + \beta \pi(t+1|t) E_t W_{t+1} + \varepsilon_{1t}, \sum_{r=t}^S \beta^{r-t} \pi(r|t) (U_r(B_r(t)) + \varepsilon_{2t}) \right] \quad (2.3)$$

where β is the discount factor, $\pi(r|t)$ is the probability of survival at r given survival at t , S is the year beyond which the person will not live and E_t is the expectation operator. U_w and U_r are the utilities derived from choosing the option to continue working and from deciding to retire respectively. Y_t is the earnings at time t , r is the retirement age, $B_r(t)$ are the retirement benefits at time t , if he retires at r and $\varepsilon_{1t}, \varepsilon_{2t}$ are the random perturbations to the age-specific utilities.

They recognize that because the dynamic programming decision rules evaluate the maximum of future disturbance terms, its implementation depends importantly on the error structure that is assumed. They assume two error structures – i.i.d extreme value distribution and normal distribution. It was found that inclusion of random individual-specific effects

improves the model fit in the case where the error structure is assumed to follow an extreme value distribution.

Rust (1987) formulated a regenerative optimal stopping model of bus engine replacement to describe the behavior of an individual who is assumed to make decisions based on an optimal stopping rule: a strategy which specifies whether or not to replace the current bus engine each period as a function of observed and unobserved state variables. He specified the value function as:

$$V_{\theta}(x_t) = \max_{i_t \in C(x_t)} [u(x_t, i_t, \theta_1) + \beta EV_{\theta}(x_t, i_t)] \quad (2.4)$$

where x_t is the state variable, i_t is the control/decision variable, $C(x_t)$ is the choice set which consists of the decision to replace or not (0,1), β is the discount rate and E is the expectation operator. The expected value function given by $EV_{\theta}(x_t, i_t)$ is defined as:

$$EV_{\theta}(x_t, i_t) = \int_0^{\infty} V_{\theta}(y) p(dy | x_t, i_t, \theta_2) \quad (2.5)$$

where $p(dy | x_t, i_t, \theta_2)$ is the transition probability, θ_1 and θ_2 are the parameters to be estimated.

Rust (1989) formulated a model of retirement behavior based on the solution to a stochastic dynamic programming problem. The worker's objective is to maximize the expected discounted utility and at each time period the worker chooses how much to consume and whether to work full-time, part-time or exit the labor force. He considered accumulated financial and non-financial wealth, total income from earnings and assets, health status, age etc. as state variables that represent workers' current information that affects their expectations about future earnings, retirement benefits and health status. The control variables chosen at each time period are a) the employment decision and b) the level of planned consumption expenditures. Thus, the sequential decision problem is to choose values of the control variables at each time t that maximize the expected discount value of his utility over his remaining lifetime.

Karlstorm et al (2004) proposed a simple dynamic programming model that aims at explaining the retirement pattern of blue-collar male workers in Sweden. The single period utility functions are given by:

$$U_t = u_t(x_t, d_t, \theta_u) + \varepsilon(d_t) \quad (2.6)$$

The value function is given by:

$$V_t(x_t, \varepsilon_t, \theta_u) = \max_{d \in D(x_t)} [v_t(x_t, d_t, \theta_u) + \varepsilon_t(d_t)] \quad (2.7)$$

where, x_t is the vector of state variables, d_t are the control variables, θ_u is a set of parameters to be estimated and $D(x_t)$ denotes the choice set available to the individual in state x_t , and v_t is the expected value function. They assumed the unobserved components $\varepsilon_t(d_t)$ to be i.i.d. Gumbel distributed and hypothesized that the individual chooses that action that maximizes the lifetime utility.

The expected value function is given by:

$$v_t(x_t, d_t, \theta) = u_t(x_t, d_t, \theta_u) + z_{t+1} \beta \int \left\{ \log \sum \exp [v_{t+1}(x_{t+1}, d_{t+1}, \theta)] p_t(dx_{t+1} | x_t, d_t, \theta_p) \right\} \quad (2.8)$$

where, θ_p is another set of parameters to be estimated, β is the discount rate, z_{t+1} is the survival probability from t to t+1 and p_t is the individual's beliefs about the future.

The maximum likelihood estimation method was used to estimate the model. The state variables include the person's age, earnings, average pension points, retirement age and marital status. They followed a two-stage estimation procedure described in Rust (1987), estimating the belief parameters θ_p first and then using these parameters to estimate the remaining parameters θ_u .

As can be seen the dynamic programming framework has been used to model the retirement behavior of individuals. Individuals plan well ahead before retiring and various factors can affect the decision of retirement. Having identified this planning and anticipatory nature of people, the models described above have incorporated this behavior.

2.3 Anticipatory Behavior in Lane Changing models

This section describes state-of-the-art lane changing models that have incorporated the anticipatory nature of drivers. Each sub-section covers various ways in which a driver's planning behavior has been captured. A lane-change plan as developed by Hidas (2005), the idea of capturing driver's persistent behavior in the form of a forced merging model as developed by Ahmed (1996) are discussed below, the concept of short-term goal and short-term plan as postulated by Toledo (2003) and the concept of a target lane developed by Choudhury (2005).

2.3.1 Lane-Change Plan

Hidas (2002) developed a merging model with components essential for lane changing under congested traffic conditions. If a vehicle cannot merge by normal gap acceptance then it evaluates the flow conditions in the target lane and by predicting the position and speed of adjacent vehicles in the target lane up to a few seconds ahead, it attempts to set an acceleration which may lead to a more favorable situation for lane changing. If a lane changing is essential but the maneuver not feasible, the process continues with the simulation of a forced lane changing maneuver where the mainline vehicles attempt to provide courtesy to the merging vehicle by slowing down.

Hidas (2005) developed a lane changing model that incorporates explicit modeling of vehicle interactions using intelligent agent concepts. The drivers have individual goals and while doing so they interact and cooperate with other drivers to solve many conflicting goals. He classified different lane change maneuvers based on the relative gaps between the leader and follower. The three types are defined as:

- 1) Free Lane change – when there is no noticeable change in the relative gap between the leader and follower during the whole process, indicating that there was no interference between the subject and the follower vehicle
- 2) Forced Lane change – when the gap between the leader and follower was either constant or narrowing before the entry point, but starts to widen after the subject

vehicle enters, indicating that the subject vehicle has forced the follower to slow down

- 3) Cooperative Lane change – when the gap between the leader and follower is increasing before the entry point and starts to decrease afterwards, indicating that the follower slowed down to allow the subject vehicle to enter.

The subject vehicle can merge into the target lane if, at the end of the maneuver, the space gaps in front and behind the vehicle are not less than some given minimum acceptable space gaps. The follower vehicle is assumed to have a certain maximum speed decrease that it is either willing to give up in case of cooperative merging or forced to give up in case of forced merging. The value of the maximum speed decrease is a function of the driver's aggressiveness and urgency of the lane changing maneuver. The minimum acceptable gaps are smaller in the case of cooperative and forced merging than in normal merging. This is because the drivers are willing to tolerate much shorter spacing when they can clearly see the situation and are able to anticipate the actions of the other drivers.

It is assumed here that if the estimated time before reaching the End-of-Lane is less than 10 seconds, the vehicle will try to force its way into the target lane. If the vehicle is in such a situation, then the follower vehicle in the next lane is checked to see if it is possible to force it to give way. Forcing is considered feasible if the follower is behind the subject vehicle and the gap between the follower and the subject is less than the critical gap required for a lane change. In reality however forced merges are not necessarily governed by urgency alone. Long waiting times leading to impatience, presence of heavy vehicles etc. can be potential factors in a driver's decision to initiate a force merge.

Because of the modeling procedures used, Hidas postulates that the vehicles involved in a lane change maneuver must be able to see and communicate with each other in order to make decisions, to resolve conflicts and to collaborate with each other. A lane-change plan is created when a vehicle determines that a lane change is essential, but is not immediately feasible. The lane-change plan is continuously updated to reflect any changes in the traffic environment and is destroyed upon completion of the lane change.

The estimated parameters are the average maximum speed decrease, average minimum safe constant gap and acceptable gap parameter. Gaps are not assumed to follow any distribution in the population; instead the parameters are modified for individual vehicles according to their aggressiveness parameter.

Ahmed (1996) estimated a forced merging model that captures drivers' lane-changing behavior in heavily congested traffic. A driver is assumed to evaluate the traffic environment in the target lane to decide whether to merge in front of the lag vehicle in the target lane and communicate with the lag vehicle to understand whether the driver's right of way is established. If a driver intends to merge in front of the lag vehicle and right of way is established the decision process ends and the driver gradually moves into the target lane. Once the forced merging has started the driver is assumed to remain in this state, persisting till the merge in to the target lane is completed.

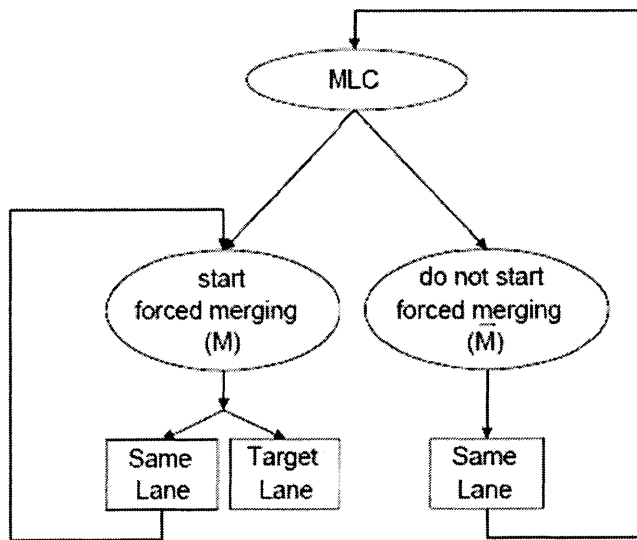


Figure 2.4- The forced merging model structure proposed by Ahmed (1999)

The forced merging model structure is shown in Figure 2.4. The estimation results of the forced merging model are shown in Table 2.2. The relative speed with respect to the lead vehicle, the remaining distance and the available gap were found to be important variables in predicting the decision to initiate a forced merge. Ahmed considers only the gap the driver is observed to change into- the last gap the driver is adjacent to before changing lanes-for

estimation purposes. Other gaps that the driver is adjacent to before the lane change is executed are not considered. Moreover, the model assumes that once a driver initiates a forced merge, he completes it. There is no gap acceptance level after the decision to initiate a forced merge is taken. In other words, the probability of completion of the merge is 1 if the driver has initiated a force merge.

Table 2.2-Estimation results for the forced merging model proposed by Ahmed (1999)

Variable	Parameter	t-statistic
Constant	-3.16	-10.59
min(0,lead vehicle speed -subject speed) (m/s)	0.313	2.66
Remaining distance impact *10	2.05	5.33
Total clear gap/10 (m)	0.285	2.85

2.3.2 Short-term goal and short-term plan

Toledo (2003) presented a framework that developed a driving behavior model based on the concepts of a short-term goal and short-term plan. Driving behavior consists of three main elements: the short-term goal, the short-term plan and the driver's actions. The short-term goal is defined by the driver's target lane. The driver constructs a short-term plan, which is defined by the target gap in the target lane that the driver wishes to use in order to accomplish his goal. The accelerations and lane changes are the driver's actions used to execute the short-term plan. The conceptual framework as described by Toledo is shown in Figure 2.5.

The concept of the target gap developed here is particularly unique. When the adjacent gap is rejected by the driver, the driver creates a short-term plan by choosing a target gap in the target lane traffic. The alternatives in the target gap choice set include available gaps in the vicinity of the subject vehicle. A gap which may not be acceptable at the time of the decision may still be chosen in anticipation that it will be acceptable in the future. The estimation results for the target gap model are summarized in Table 2.3.

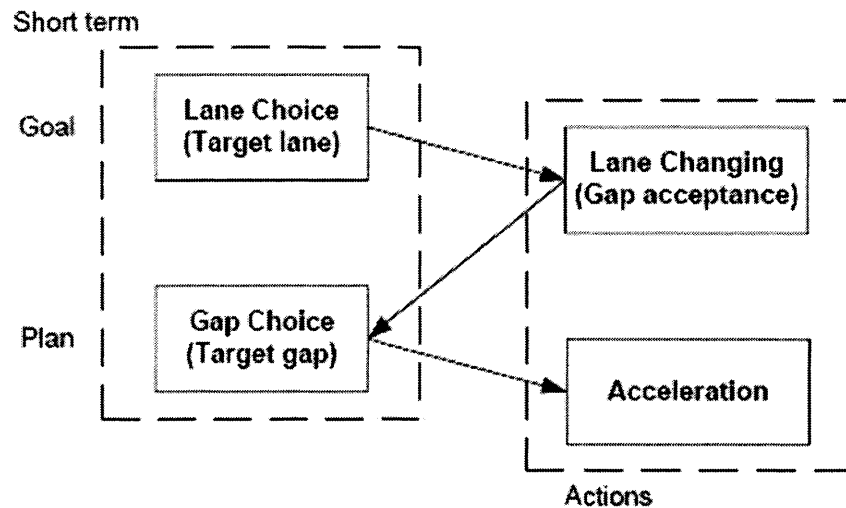


Figure 2.5- Conceptual framework for the driving behavior process (Toledo, 2003)

Toledo considers only the adjacent, forward and backward gaps in his model. Due to the computational difficulty of modeling all possible combinations of states of the short-term goal and short-term plan, which are unobserved, he assumed the concept of a partial short-term plan. It is assumed that the driver executes one step of the short-term plan, re-evaluates the situation and decides the next action to be taken. Thus, it is assumed that a driver formulates a plan at every instant which is not entirely realistic.

Table 2.3-Estimation results for the target gap model (Toledo 2003)

Variable	Parameter value	t-statistic
Forward gap constant	-0.837	-0.50
Backward gap constant	0.913	4.40
Distance to gap, m.	-2.393	-7.98
Effective gap length, m.	0.816	2.20
Front vehicle dummy	-1.662	-1.53
Relative gap speed, m/sec.	-1.218	-4.00
α^{bck}	0.239	0.81
α^{adj}	0.675	0.95

2.3.3 Target Lane Model

Most lane-changing models (e.g., Gipps 1986, Yang and Koutsopoulos 1996, Ahmed et al. 1996, Ahmed 1999, Hidas 2002, Toledo 2003) are based on the assumption that drivers evaluate the current and adjacent lanes and choose a direction of change based on the relative

utilities of these lanes only. Such models lack an explicit tactical choice of a target lane and can explain only one lane change at a time. Toledo et al (2005) developed a framework for modeling lane changing behavior in the presence of exclusive lanes. It is postulated that a driver may move to a 'worse' adjacent lane in order to get to a much better target lane further away, especially in the case where exclusive lanes are present. This captures an element of anticipatory or tactical planning behavior of drivers that previous models did not capture explicitly.

The target lane is the lane the driver perceives to have the highest utility taking a wide range of factors and goals into account. The factors include attributes of lanes, interactions between the subject vehicle and other vehicles around it, the driver's path plan and the driver's characteristics. An example of the proposed structure for a vehicle currently in the second lane to the right in a four-lane road is shown in Figure 2.6. The choice set consists of all four lanes in the road (the current lane, lane 3 and 4 on the left and lane 1 on the right). If the target lane is the same as the current lane, no lane change is required (NO CHANGE). If the target lane is Lane 1 then a change to the right is required (RIGHT) and if the target lane is Lane 3 or 4, a change to the left is required (LEFT). If a lane change is required, the driver evaluates the gaps in the adjacent lane corresponding to the direction of change based on a gap acceptance model. The driver either accepts the gap and moves to the adjacent lane (CHANGE RIGHT or CHANGE LEFT) or rejects the gap and stays in the current lane (NOCHANGE).

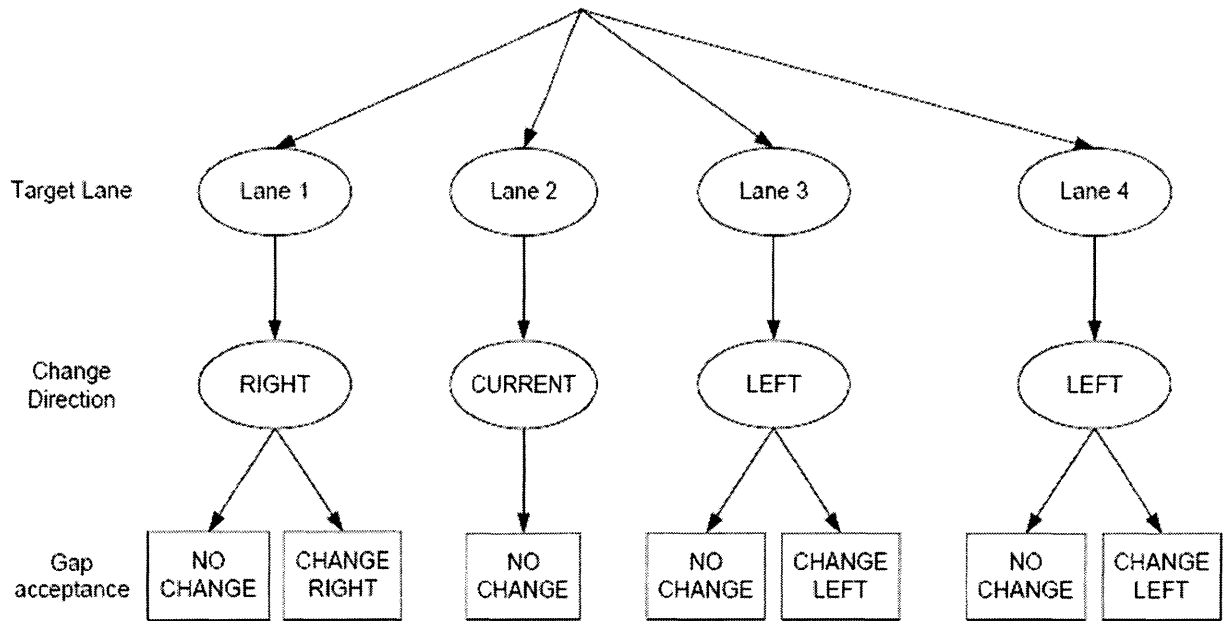


Figure 2.6- Example of structure of lane-changing model proposed by Toledo et al (2005)

The model is estimated using second by second trajectory data collected by the FHWA in a section of I-395 Southbound in Arlington, VA. The explanatory variables include neighborhood variables, path plan variables, network knowledge and experience. The heterogeneity variables capture the driver specific characteristics. The estimation results of the model are presented in Table 2.4.

An element of look ahead has been incorporated in this model. The model overcomes the myopic nature of previous models by capturing the fact that a driver's target lane need not be just the lane adjacent to him/her; the driver may make a lane change to the adjacent lane in order to get to a further lane. However, the target lane choice decisions are assumed to be instantaneous in this model. In reality drivers may have a short-term plan regarding their lane changing actions and this has not been captured in the model. For example, a driver deciding to overtake a slow moving vehicle front of it may move to the adjacent lane temporarily although the utility of the adjacent lane is lower than that of the current lane.

Table 2.4-Estimation results for the lane changing model (Choudhury 2005)

Variable	Parameter value	t-statistic
Target Lane Model		
Lane 1 constant	-1.570	-3.030
Lane 2 constant	-0.488	-1.552
Lane 3 constant	0.075	1.744
Lane density, vehicle/km	-0.011	-0.988
Average speed in lane, m/sec	0.119	1.560
Front vehicle spacing, m	0.022	2.879
Relative front vehicle speed, m/sec.	0.115	1.463
Tailgate dummy	-2.783	-0.176
CL dummy	1.000	1.485
Number of lane-changes from CL	-2.633	-0.270
Path plan impact, 1 lane change required	-2.559	-3.265
Path plan impact, 2 lane change required	-4.751	-3.584
Path plan impact, 3 lane change required	-6.996	-0.097
Next exit dummy, lane change(s) required	-0.980	-0.377
θ^{MLC}	-0.371	-2.608
π_1	0.001	-0.426
π_2	0.069	-8.101
α^{lane1}	-1.371	-2.582
α^{lane2}	-0.985	-0.510
α^{lane3}	-0.691	-3.441
Lead Critical Gap		
Constant	1.553	3.311
$Max(\Delta S_{nt}^{lead}, 0)$, m/sec.	-6.389	-3.793
$Min(\Delta S_{nt}^{lead}, 0)$, m/sec.	-0.140	-2.191
α^{lead}	-0.008	4.029
σ^{lead}	0.888	-1.229
Lag Critical Gap		
Constant	1.429	6.611
$Max(\Delta S_{nt}^{lag}, 0)$, m/sec.	0.471	4.907
α^{lag}	-0.234	0.469
σ^{lag}	0.742	4.802

2.4 Summary

The literature review shows that although driving behavioral models for lane changing have improved in the last decade, very few models are able to realistically capture the anticipatory or persistent behavior of drivers. Drivers are assumed to be myopic and most of the existing models assume that drivers react to existing or past traffic conditions and make instantaneous decisions.

A set of models that capture anticipatory behavior was investigated. It was seen that dynamic programming models have the potential to capture the anticipatory behavior of individuals. They involve future decisions that an individual can possibly make. Expectations about the future based on knowledge of current conditions are modeled. There are none or limited dynamic programming models that have been applied to the field of driver behavior or lane-changing decisions.

Various ways that have been used to capture anticipatory behavior in lane changing models were discussed. Toledo (2003) approached the problem by postulating that drivers have a short-term goal and a short-term plan. Hidas (2005) developed a lane changing model in which drivers have individual goals and while doing so they interact and cooperate with other drivers to solve many conflicting goals. Ahmed (1996) estimated a forced merging model that captures drivers' persistent lane-changing behavior in heavily congested traffic. Choudhury (2005) used the concept of target lane and captured the look-ahead nature of drivers.

The next chapter discusses an alternative way of capturing the expectations of drivers and their anticipatory behavior while making lane changing decisions using dynamic programming.

3. Dynamic Programming Framework applied to lane changing decisions

Individuals plan ahead in time and try to optimize their decisions with the information they currently have. Expectations about the future play an important part in many decisions taken by individuals. As was seen in Section 2.2, dynamic programming models have been used to capture the planning behavior of individuals in the case of retirement and other decisions. Section 2.3 reviewed state-of-the-art lane changing models that capture the planning behavior of individuals. In this chapter an alternative model framework is postulated which uses the concepts of dynamic programming for lane changing decisions and accommodates expectations about the future. The various advantages and disadvantages of applying such a model framework are also discussed.

3.1 Introduction

Individuals make decisions based on some plan that they have for the future. For example, in the case of lane changing behavior, one may decide to move to the right lane now to eventually get to the exit ramp. The time horizons for these plans can be as short as the next instant of time to as long as a period of minutes depending on whether the individual is myopic or a long-term planner.

Importantly, an individual can only plan for the future; he cannot with entire certainty say that he will perform a certain act in the future. Panel data has a wealth of information, including the future choices of individuals. One cannot use the future data to predict/estimate the current state as the future is uncertain and can only be anticipated. However, one can use current choices and states to evaluate a plan regarding the future. This involves anticipation of future possibilities given information about the current situation. The concept of expectations of future events is investigated further here.

The application of dynamic programming to model an individual's planning behavior in case of retirement decisions was seen in Section 2.2. This opens up the possibility of applying the dynamic programming model to capture a drivers' planning and anticipatory behavior. In this

chapter the application of dynamic programming to lane changing decisions is presented and its advantages and disadvantages discussed.

3.2 Model Framework

This section presents a theoretical model of driving/lane-changing behavior. Drivers do not make single once and for all plans about their desired lanes. Rather, drivers constantly modify their plans in the light of new information. This suggests the use of a dynamic programming approach, where the solution takes the form of an optimal decision rule that specifies drivers' optimal utility decisions as a function of their current information.

Markov decision processes (MDP) provide a broad framework for modeling sequential decision making under uncertainty. MDP's have two kinds of variables: state variable s_t and control variables d_t . According to Rust (1994) a decision-maker can be represented by a set of primitives (U, p, β) where $U(s_t, d_t)$ is a utility function representing the agent's preferences at time t , $p(s_{t+1} | s_t, d_t)$ is a Markov transition probability representing the agent's subjective beliefs about uncertain future states, and $\beta \in (0, 1)$ is the rate at which the individual discounts utility in future periods.

Individuals while driving consider a time-frame $[t, t+k]$ seconds while deciding their driving or lane-changing behavior. k can vary depending on the individual. For example, for a myopic driver k can be 0 or 1 as he/she may make decisions without planning ahead for them. On the other hand, a driver who plans well ahead in time will have a higher k value.

In this time-frame an individual may have a large choice set of options to choose from and he decides to choose that option that maximizes his utility. The concept of an individual's plan is illustrated with the following example. Assuming that there are 3 lanes on the highway and the driver is in the middle lane at time t and deciding whether to stay in the same lane (NC) or change left (L) or change right (R), the individual would be faced with a decision tree

fanning into the future is shown in Figure 3.1 for the option of staying in the same lane at time t:

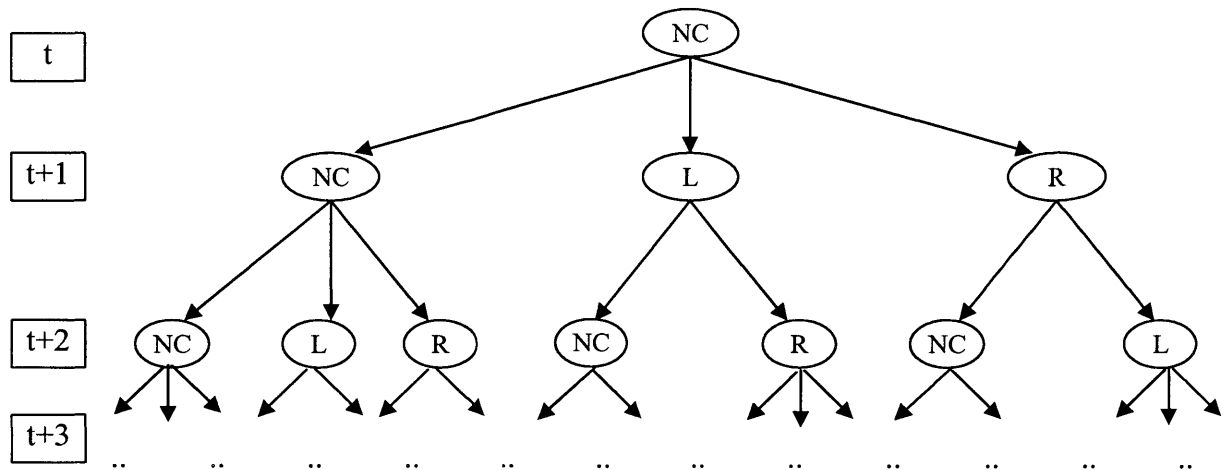


Figure 3.1-Possible paths if decision is to not change lane at t, time-frame of 3 seconds

The path diagram is not symmetrical because the driver's future choices are limited by decisions taken in the past. As can be seen, a driver in the middle lane who chooses to make a left lane change at time t+1 cannot have the option to make a left lane-change at time t+2. There will be similar path diagrams for the individual with the tree root as 1) L and 2) R. This shows that at instant t, the individual evaluates all possible paths and the benefits he associates with each path and makes the decision whether to stay in the same lane, move left or move right. We see that with just 3 possible states, and the individual considering just 3 future time intervals, the number of possible sequences is quite large (of the order of 3^4).

The decision at time t (the control variable) is dependent on the state variables at that time, discussed in the following section. The state variables represent a subset of drivers' current information that affects their expectations about the future.

3.2.1 State and control variables

In order to represent the dynamics of driving behavior, the model should include state variables that describe the state the driver is in at that time. A few examples of state variables

which directly or indirectly affect drivers' realized utility levels can be v_{nt}^i -the mean speed in lane i at time t with respect to individual n , p_{nt}^i -the mean density in lane i at time t with respect to individual n and H_{nt}^i -the mean density of heavy vehicles in lane i at time t with respect to individual n .

Each variable is dependant on the location of the individual n at time t . For example, the mean speed of a lane would change with location of the driver from instant to instant. As individual specific variables are not usually available in such data-sets, driver-specific behavior is captured by means of ν_n , an individual-specific latent variable assumed to follow some distribution in the population. The attributes at the current time step, t , and the choice set available at t (L,R,NC) depend on the decision taken at time $t-1$ ($d_{n,t-1}$). For instance, if the individual was in the left-most lane at time $t-1$, the choice L is not available to him at time t .

The state variables represent a subset of drivers' current information that affects their expectations about the future. Given these state variables and the drivers' expectations, at each time t , the driver must choose values of the control variable d_{nt} -the lane-changing decision (stay in the current lane, change left if available, change right if available)

An individual's preferences can be given by $U_{nt}(x_{nt}, d_{n,t-1}, d_{nt}, \theta_u)$,

where x_{nt} is the vector of explanatory variables for individual n at t , θ_u are the parameters to be estimated, $d_{n,t-1}$ is the decision taken by the individual n at $t-1$ and d_{nt} is the decision/control variables for individual n at time t .

The utility function of an individual at time t can be given by:

$$\begin{aligned} U_{nt}(x_{nt}, d_{n,t-1}, d_{nt}, \theta_u) &= u_{nt}(x_{nt}, d_{n,t-1}, d_{nt}, \theta_u) + \varepsilon_{nt}^{d_{nt}} \\ &= \theta_u^{d_{nt}} x_{nt}^{d_{n,t-1} d_{nt}} + \varepsilon_{nt}^{d_{nt}} \end{aligned} \quad (3.1)$$

where $x_{nt}^{d_{n,t-1}d_{nt}}$ is the vector of explanatory variables for individual n at time t like relative speeds, acceleration, available gaps etc (for each possible value of d_{nt}), and $\varepsilon_{nt}^{d_{nt}}$ is a generic random term with i.i.d distribution across choices, time and individuals. In order to make the utilities individual specific, v_n an individual-specific variable assumed to follow some distribution in the population can be included along with its parameter $\alpha^{d_{nt}}$ which is to be estimated.

At any time t, an individual n is assumed to make an optimal decision with the current information available. Given a choice set of options to choose from (control variables), the driver chooses that option which provides the maximum utility. The Dynamic Programming model is based on a recursive representation of the value function which consists of the following terms:

- The utility at time t based on the individual's state variables
- A random error term
- The expectation of the value function at time t+1

It is through the value function that the anticipatory nature of drivers is captured. While making lane changing decisions, the driver considers not only the single period utility but also the expectation of utility of future choices. As the random error term is unobserved by us, we can only evaluate the expectation of the maximum utility that would be chosen by the driver. Thus the value function is given by the expectation of the maximum utility evaluated over the choice set available to the individual at time t.

Following the equation form specified in Lumsdaine et al (1992), the value function at time t for an individual n who has all 3 options (NC, L and R) available to him can be given by:

$$W_{nt}(x_{nt}, D(d_{n,t-1}), d_{n,t-1}, \theta_u, k) = E_{nt} \max \left[\begin{array}{l} u_{nt}(x_{nt}, d_{n,t-1}, NC, \theta_u) + \varepsilon_{nt}^{NC} + \beta E_{nt} W_{nt+1}(x_{n,t+1}, D(d_{nt} = NC), d_{nt} = NC, \theta_u, k), \\ u_{nt}(x_{nt}, d_{n,t-1}, L, \theta_u) + \varepsilon_{nt}^L + \beta E_{nt} W_{nt+1}(x_{n,t+1}, D(d_{nt} = L), d_{nt} = L, \theta_u, k), \\ u_{nt}(x_{nt}, d_{n,t-1}, R, \theta_u) + \varepsilon_{nt}^R + \beta E_{nt} W_{nt+1}(x_{n,t+1}, D(d_{nt} = R), d_{nt} = R, \theta_u, k) \end{array} \right] \quad (3.2)$$

where β is the discount rate, θ_u is a set of parameters to be estimated and $D(d_{n,t-1})$ is the choice set available to individual n at t on making a choice $d_{n,t-1}$ at t-1.

The first term within the maximization operator is the utility obtained if the individual choose to make no lane change at the current time-step, the second term is the utility obtained on making a left lane-change and the third term is the utility obtained on making a right lane-change. The value function is dependant on k, the time horizon the individual plans in. It essentially determines the number of recursive terms in the current instant's value function. For the sake of clarity, Equation 3.2 can also be written as:

$$W_{nt}(x_{nt}, D(d_{n,t-1}), d_{n,t-1}, \theta_u, k) = E_{nt} \max \left(\overline{W}_{nt}^{NC} + \varepsilon_{nt}^{NC}, \overline{W}_{nt}^L + \varepsilon_{nt}^L, \overline{W}_{nt}^R + \varepsilon_{nt}^R \right) \quad (3.3)$$

where,

$$\overline{W}_{nt}^{NC} = u_{nt}(x_{nt}, d_{n,t-1}, NC, \theta_u) + \beta E_{nt} \left(W_{nt+1}(x_{n,t+1}, D(d_{nt} = NC), d_{nt} = NC, \theta_u, k) \right) \quad (3.4)$$

$$\overline{W}_{nt}^L = u_{nt}(x_{nt}, d_{n,t-1}, L, \theta_u) + \beta E_{nt} \left(W_{nt+1}(x_{n,t+1}, D(d_{nt} = L), d_{nt} = L, \theta_u, k) \right) \quad (3.5)$$

$$\overline{W}_{nt}^R = u_{nt}(x_{nt}, d_{n,t-1}, R, \theta_u) + \beta E_{nt} \left(W_{nt+1}(x_{n,t+1}, D(d_{nt} = R), d_{nt} = R, \theta_u, k) \right) \quad (3.6)$$

The value function shows that at time t, the individual is trying to make an optimal choice between $d_{nt} = NC, d_{nt} = L$ and $d_{nt} = R$. $W_{nt}(x_{nt}, D(d_{n,t-1}), d_{n,t-1}, \theta_u, k)$ is the expectation of the maximum utility from the given choice set. If the $\varepsilon_{nt}^{d_m}$ are assumed to be i.i.d draws from an extreme value distribution, the value function can be written in the form of a logsum (Ben-Akiva and Lerman, 1985) as:

$$\begin{aligned}
& W_{nt+1} \left(x_{n,t+1}, D(d_{nt}), d_{nt}, \theta_u, k \right) \\
&= E_{n,t+1} \max_{d_{n,t+1} \in D(d_{nt})} \left(\overline{W}_{n,t+1}^{d_{n,t+1}} + \varepsilon_{n,t+1}^{d_{n,t+1}} \right) \\
&= \ln \sum_{d_{n,t+1} \in D(d_{nt})} \exp \left(\overline{W}_{n,t+1}^{d_{n,t+1}} \right) \\
&= \ln \sum_{d_{n,t+1} \in D(d_{nt})} \left[\exp \left(u_{n,t+1} \left(x_{n,t+1}, d_{nt}, d_{n,t+1}, \theta_u \right) \right) \cdot \right. \\
&\quad \left. \exp \left(\beta E_{n,t+1} W_{n,t+2} \left(x_{n,t+2}, D(d_{n,t+1}), d_{n,t+1}, \theta_u, k \right) \right) \right]
\end{aligned} \tag{3.7}$$

The expectation of the value function in the future - $W_{nt+1} \left(x_{n,t+1}, D(d_{nt}), d_{nt}, \theta_u, k \right)$ - is evaluated in the current time period t for values of the state variables in the future, $x_{n,t+1}$, and as the future is unknown, there exists a probability associated with the realization of various possible future states. The value function cannot be directly evaluated in the future states, and therefore we evaluate the expected value function $E_{nt} W_{nt+1} \left(x_{n,t+1}, D(d_{nt}), d_{nt}, \theta_u, k \right)$, given the information of the state variables at time t.

An individual evaluates the future states based on what he knows about the current state. There are various possible states and each of them has an expected value associated with it. In other words there is a probability associated with the realization of each of the possible future states. This is explained by the following equation where the integral integrates over all possibilities of future states.

$$\begin{aligned}
E_{nt} W_{n,t+1} \left(x_{nt}, D(d_{nt}), d_{nt}, \theta_u, k \right) &= \int W_{nt+1} \left(x_{n,t+1}, D(d_{nt}), d_{nt}, \theta_u, k \right) p_{nt} \left(dx_{n,t+1} \mid x_{nt}, d_{nt}, \theta_p \right) \\
&= \int \left\{ \ln \sum_{d_{n,t+1} \in D(d_{nt})} \exp \left[\overline{W}_{n,t+1}^{d_{n,t+1}} \right] \right\} p_{nt} \left(dx_{n,t+1} \mid x_{nt}, d_{nt}, \theta_p \right)
\end{aligned} \tag{3.8}$$

where $p_{nt} \left(dx_{n,t+1} \mid x_{nt}, d_{nt}, \theta_p \right)$ are the probabilities associated with realization of the various possible future states. The following sub-section provides illustrations of the formulation and explains the significance of the beliefs/probabilities associated with the possible future states.

3.2.2 State Variables and Beliefs

Based on the individual's current knowledge of the state variables, he/she forms beliefs about values for these variables in the future. These beliefs can be approximated by means of updating rules for each of the state variables. Equations which depend on past states can be used to predict the future.

The role and evaluation of the beliefs is illustrated by means of examples. Let the vector of explanatory variables be $x_{nt} = (v_t^i, \rho_t^i, H_t^i, \dots)$ where v_{nt}^i is the speed in lane i at time t , ρ_{nt}^i is the density of lane i at time t and H_{nt}^i is the heavy vehicle density in lane i at time t .

As the speed in lane i at time $t+1$ is a function of the speed at time t and other influencing factors:

$$v_{n,t+1}^i = f(v_{nt}^i, td, \dots, \eta_t) \quad (3.9)$$

where td is the time of day/peak hour dummy and η_t is an error term with assumed distribution.

One form of regression equation for speeds $(v_{n,t+1}^i | v_{nt}^i, d_{nt}, \gamma)$ following the form of regression equations for pension points in the retirement model of Karlstorm et al (2004) is:

$$\log(v_{n,t+1}^i) = \gamma_1 + \gamma_2 \log(v_{nt}^i) + \gamma_3 td + \eta_t \quad \eta_t \text{ is i.i.d } N(0, \sigma_\eta^2) \quad (3.10)$$

Similarly one can formulate regression equations for heavy vehicle density $(H_{n,t+1}^i | H_{nt}^i, d_{nt}, \gamma)$ where α, γ are parameters to be estimated. Various forms of regression equations need to be evaluated to find the equation that fits the data best.

To illustrate the above equations, consider an individual whose time frame is $[t, t+2]$ and who is in the middle lane of a 3-lane road. At time t the individual is trying to decide whether to change left, right or to continue on the same lane. This decision is based on factors like speeds and accelerations on all 3 lanes which serves as a LOS indicator – an individual

moves to the lane where traveling in higher speeds is possible if his/her VOT is positive. It is also based on the adjacent gaps available and his/her perception of the criticality of the gap.

The individual evaluates the available choices based on what he/she anticipates to happen in the future ($x_{n,t+1}$).

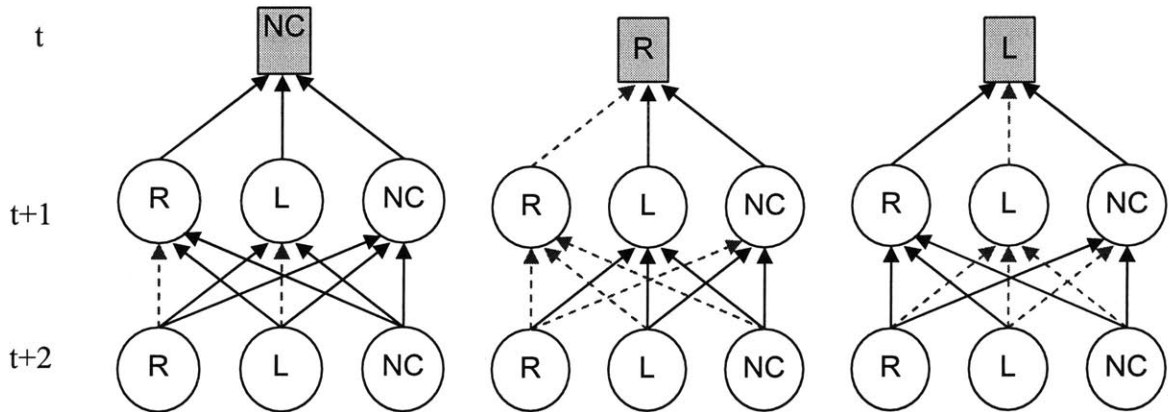


Figure 3.2-Variou possible decisions for a driver in the middle lane of a 3-lane road at t and their subsequent paths

As shown in Figure 3.2 a driver decides whether to make a left, right or no lane change at time t by evaluating the utility at time $t+2$, discounting it to $t+1$, then evaluating the utility at time $t+1$ and discounting it to the current time t . The driver looks ahead into the future and brings in the utility of the future possibilities into his current utility as can be seen by the direction of the arrows in the figure.

As can be seen in figure 3.2, only those paths are evaluated that are feasible to the driver given his current state (the dotted paths are infeasible in a 3-lane road if the driver is in the middle lane). The driver's decision to make a left, right or no-change is based on the evaluations of the feasible paths. For instance, the utility to move to the left lane in the current time step is based on the evaluation of all feasible paths present in column 2 of Table 3.1. Some of the paths may have low probability of occurrence like L-R-L.

Table 3.1-Possible future paths for a driver with look ahead of 3 seconds

NC	L	R
NC – NC – NC	L – NC – NC	R – NC – NC
NC – NC – L	L – NC – L	R – NC – L
NC – NC – R	L – NC – R	R – NC – R
NC – L – NC	L – L – NC	R – L – NC
NC – L – L	L – L – L	R – L – L
NC – L – R	L – L – R	R – L – R
NC – R – NC	L – R – NC	R – R – NC
NC – R – L	L – R – L	R – R – L
NC – R – R	L – R – R	R – R – R

Note: highlighted paths are not feasible in a 3-lane highway

Combining equation 3.5 and 3.8, if $d_{n,t} = L$:

$$\bar{W}_m^L = U_m(x_m, d_{n,t-1}, L, \theta_u) + \beta \left\{ \ln \sum_{d_{n,t+1} \in D(L)} \exp[\bar{W}_{n,t+1}^{d_{n,t+1}}] \right\} p_m(x_{n,t+1} | x_m, d_m, \theta_p) \quad (3.11)$$

The probability associated with the realization of the future states $p_{n,t}$ is dependent on the distribution assumed for the error terms in the regression equations for the beliefs. The term that contains expectations of the future consists of an evaluation by the driver of the following paths:

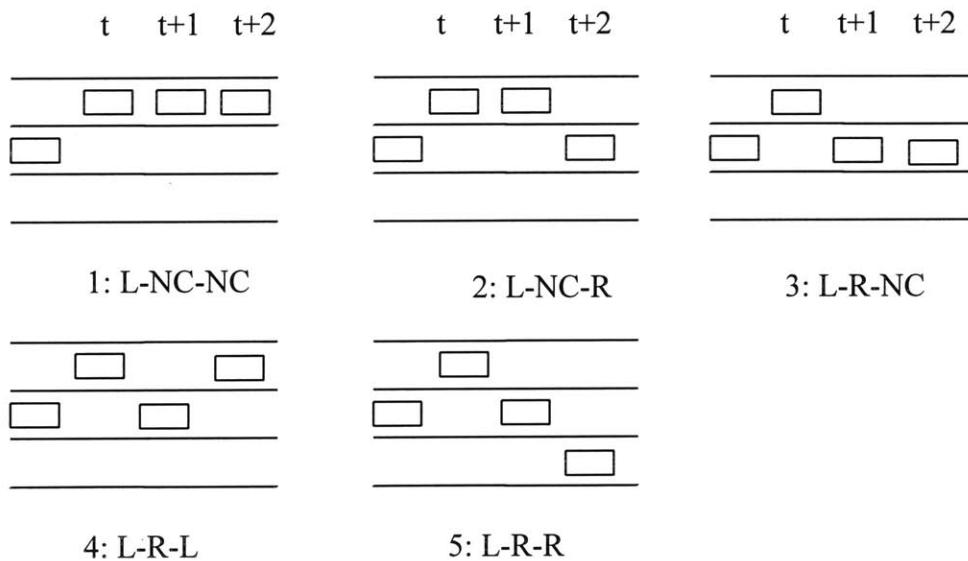


Figure 3.3-Trajectory of a single vehicle over timeframe [t,t+2] - Possible paths if $d_{n,t} = L$

The driver attaches a value with each of these possibilities in various anticipated future states. For example, if a driver anticipates that in the future lane 1 is going to be less congested, offer higher speeds compared to lanes 2 and 3, he will attach a higher probability to possibility 1 (L-NC-NC) than to other possibilities. Similarly, if he feels that lane 2 is better, but is being hindered by the driver in front of him, he would prefer a temporary lane change to 1 and back to his current lane (resulting in case 3 if he anticipates he can make the change back fast and case 2 if he thinks he needs more time to make the change back to the current lane).

The driver can anticipate different scenarios in the future and these scenarios are based on the knowledge the driver has in the current instant of time.

We assume that the driver is not trying to reach global optima here, but a local one – he tries to maximize his utility in the time-frame he is considering. Thus, we hypothesize that each driver evaluates k seconds into the future and makes the decision whether to perform a left, right or no-change action. Assuming the ε 's are i.i.d extreme value distributed, the probability of a left change is given by:

$$P_{nt}(L_{nt} | x_{nt}, \theta) = \frac{\exp\left(\overline{W}_{nt}^L\right)}{\exp\left(\overline{W}_{nt}^L\right) + \exp\left(\overline{W}_{nt}^R\right) + \exp\left(\overline{W}_{nt}^{NC}\right)} \quad (3.12)$$

θ – consists of all parameters required to be estimated, θ_u and θ_p

In general:

$$P_{nt}(d_{nt} | x_{nt}, \theta) = \frac{\exp\left(\overline{W}_{nt}^{d_{nt}}\right)}{\sum_{d'_{nt} \in D(d_{n,t-1})} \exp\left(\overline{W}_{nt}^{d'_{nt}}\right)} \quad (3.13)$$

This conditional probability can be used to estimate the model using the maximum likelihood estimation method specified in the next section.

Figure 3.4 shows the various possible trajectories for a vehicle with timeframe of 2 seconds and whose target lane is the left lane. It should be noted that an action such as L-NC-NC is

different from NC-L-NC, which is different from NC-NC-L. Although factors that govern the u_{nt} term may suggest that lane 1 is the better lane, it can so happen that lane 1 is not the better lane for time step $t+1$ because there is a merging ramp coming ahead and the driver may experience considerable weaving as a result. In that case, merging at $t+2$ may prove to be a better option. This behavior is accommodated because we consider future possibilities into account. A pattern like case 3 in Figure 3.4 may be observed due to inertia effects too, which is captured in the u_{nt} term.

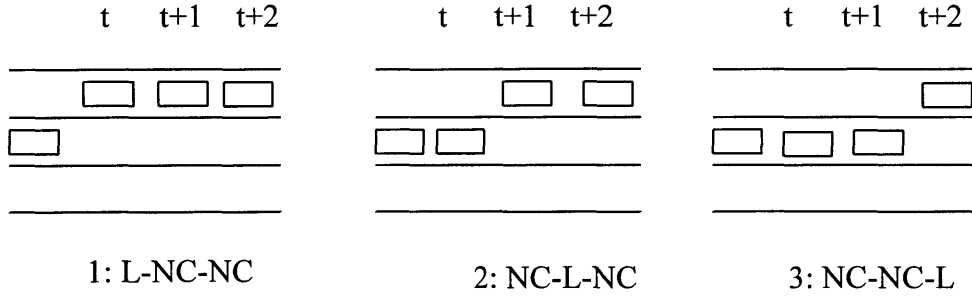


Figure 3.4- Trajectory of a single vehicle over timeframe $[t, t+2]$ - Decision of L can occur at t , $t+1$ or $t+2$ depending on expectation of the future

3.3 Likelihood Function

The maximum likelihood estimation function can be used to estimate the model. Given the panel decisions d_{nt} and the observed state variables x_{nt} , the likelihood function can be written as:

$$L(\beta, \theta_u, \theta_p) = \prod_{n=1}^N \prod_{t=1}^{k_n} P_{nt}(d_{nt} | x_{nt}, \theta) p_{nt}(x_{n,t} | x_{n,t-1}, d_{nt}, \theta_p) \quad (3.14)$$

where k_n is the value of k (the planning horizon) for individual n ,

$$P_{nt}(d_{nt} | x_{nt}, \theta) = \frac{\exp(\overline{W}_{nt}^{d_{nt}})}{\sum_{d'_{nt} \in D(d_{n,t-1})} \exp(\overline{W}_{nt}^{d'_{nt}})} \quad \text{and} \quad p_{nt}(dx_{n,t+1} | x_{nt}, d_{nt}, \theta_p)$$

associated with realization of the various possible future states.

The parameters can be jointly estimated or estimated by a two-stage estimation procedure followed by Karlstorm et al (2004) where θ_p is estimated first and then using these parameters, the remaining parameters in θ_u are estimated

Having discussed the basic dynamic programming framework as applied to lane changing decisions, the advantages and disadvantages of the approach are discussed in the following section.

3.4 Advantages and Disadvantages

The dynamic programming model assumes individuals to be planners and optimizers. People look ahead in time and make decisions that may not be reflected in most current models. The dynamic programming approach incorporates the fact that an individual plans ahead in time and bases his decisions of choice of target lane on his expectation/beliefs about the future. It includes the anticipatory behavior absent in many lane changing models and calculates the optimal choice an individual would choose. By incorporating expectations of the future, it has the potential to realistically model driver's planning capabilities and lane changing behavior. An example can be seen in the case of an overtaking maneuver. For example, the target lane may be the current lane but the driver decides to make a left lane change to overtake a vehicle and get back to the current lane. Such an action may not be captured in models that assume decisions to be taken instantaneously. Such a maneuver is illustrated in Figure 3.5 where the subject vehicle overtakes the slow moving vehicle in front of it by moving to a lane with low utility and changing back to its target lane.

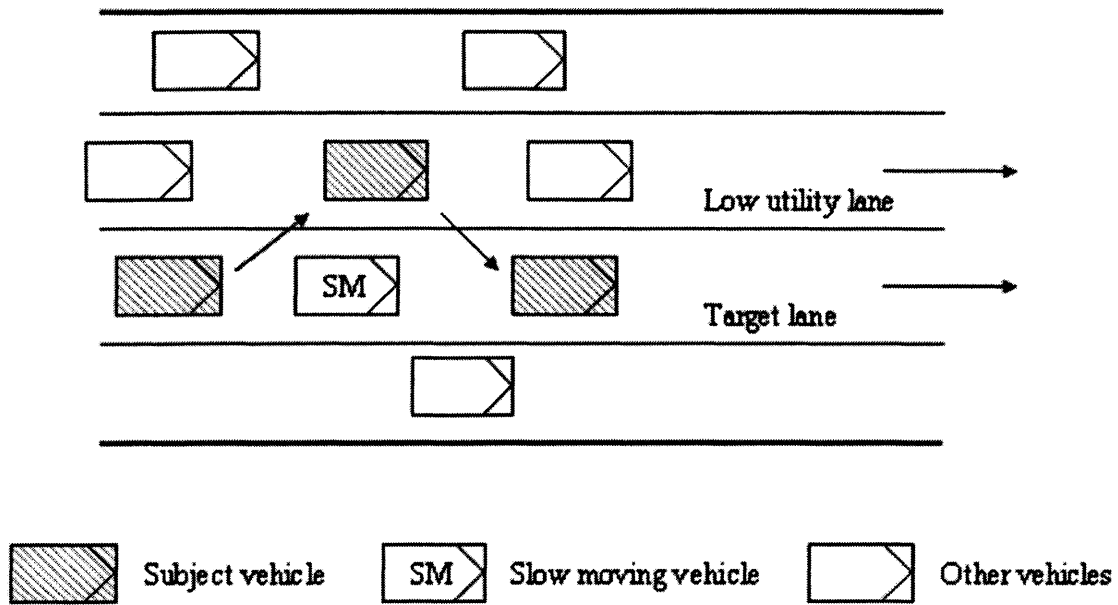


Figure 3.5-Capturing the overtaking maneuver through dynamic programming

Another example that highlights the importance of capturing the anticipatory behavior is illustrated. Consider an individual who has a high inertia to change lanes from his/her current lane and whose current lane is the 2nd lane of a 3-lane highway and changing to the left lane would give him/her a considerable gain in speed. In current models, which do not consider anticipation of the future, the inertia effect would overrule the utility due to higher speeds and the driver would continue on his/her current lane. However, if the anticipatory behavior is also accommodated, although the inertia term may be high and negative, the driver's expectations of the future in terms of speed gains would make the overall utility over time of a lane change to the left higher.

The dynamic programming model may prove to be computationally intractable. With just 3 possible states (no change, left or right) and the individual looking 3 seconds into the future the number of paths to be evaluated is of the order of 3^4 . Evaluating all future paths and the utility associated with each may prove to be cumbersome.

The main drawback of the DP model is the Bellman's 'curse of dimensionality' which leads to a high number of integrations over the state space. As the number of variables in the model

increases, the need to hypothesize beliefs about the values these variables would take in the future arises and with every new variable an added integration is required. Lane changing decisions may be governed by a large number of variables which implies a set of updating rules/beliefs for each of these variables. The form of the regression equation and the goodness-of-fit can be important factors in the predictive power of the dynamic programming model.

In light of the computational complexity, another model is developed in the following chapter that assumes drivers to be planners and persistent in their desire to pursue their plan.

3.5 Conclusions

In this chapter a theoretical framework, for capturing the anticipatory behavior of drivers during lane changing, using dynamic programming has been developed. Dynamic programming is a powerful tool to predict the outcome of driver's decision. Based on the current state and information, individuals have expectations of the future based on which they make lane changing decisions. However, due to the large number of variables that can potentially affect the individual's decision, the model becomes increasingly complex. Evaluating all possible future paths and computing the utilities and value functions with each of these paths can prove to be computationally cumbersome.

Although the dynamic programming framework can result in more accurate predictions of lane changing decisions, the computational complexity is a serious drawback in model estimation. This leads us to develop another model that captures an element of planning behavior by means of the model structure. Such a model-the explicit force merging model-is developed in the next chapter.

4. Explicit Forced Merging Model

In the previous chapter it was seen that although dynamic programming has the capability to model the anticipatory behavior of individuals, the computational complexity involved is high to use the model framework for estimation. Thus, an alternative model framework is proposed here that captures the planning behavior by means of the model structure. In this chapter formulations of the various components of the explicit forced merging model are presented.

4.1 Modeling Framework

This model hypothesizes three levels of decision making: normal gap acceptance, decision whether to initiate a forced merge or not, and gap acceptance for forced merging. The framework of the model is summarized in Figure 4.1. The decision process is however latent, and only the end action of the driver (lane change to the target lane) is observed. Latent choices are shown as ovals and observed choices are represented as rectangles.

In the first level or the normal gap acceptance level, the driver evaluates the available gaps in the direction of the target lane for normal gap acceptance. In case of merging from the on ramp, the target lane is the right most lane of the mainline. If the available lead and lag gaps are acceptable, the driver makes a lane change under normal gap acceptance in the immediate time step. If the driver cannot merge through normal gap acceptance, he may decide to force his way to the mainline compelling the lag vehicle to slow down.

The planning behavior of drivers is captured in this level. It is assumed that if a driver does not accept a normal gap he looks ahead and decides whether to force merge. If the driver decides to initiate a forced merge process, the available lead and lag gaps in the immediate time step may not be acceptable in comparison to the critical gaps for the forced merge and in such cases, the actual merge is not completed in the immediate time step. However, once the driver has initiated a forced merge process, he remains in that state (unless the adjacent gap changes) and he can complete the merge at a later time step when the gaps are

acceptable. The driver is assumed to have entered the initiation state with a particular plan in mind to complete the merge by forcing and therefore does not decide to merge under normal gap acceptance to the same adjacent gap in a later time step. Unless the adjacent gap changes he remains in the initiated forced merging state till the forced merge is completed. Whenever the driver has a new adjacent gap, the driver is assumed to reformulate his plan of action i.e., the state of the driver is reset to the normal (not initiating forced merging) state as shown in Figure 4.1.

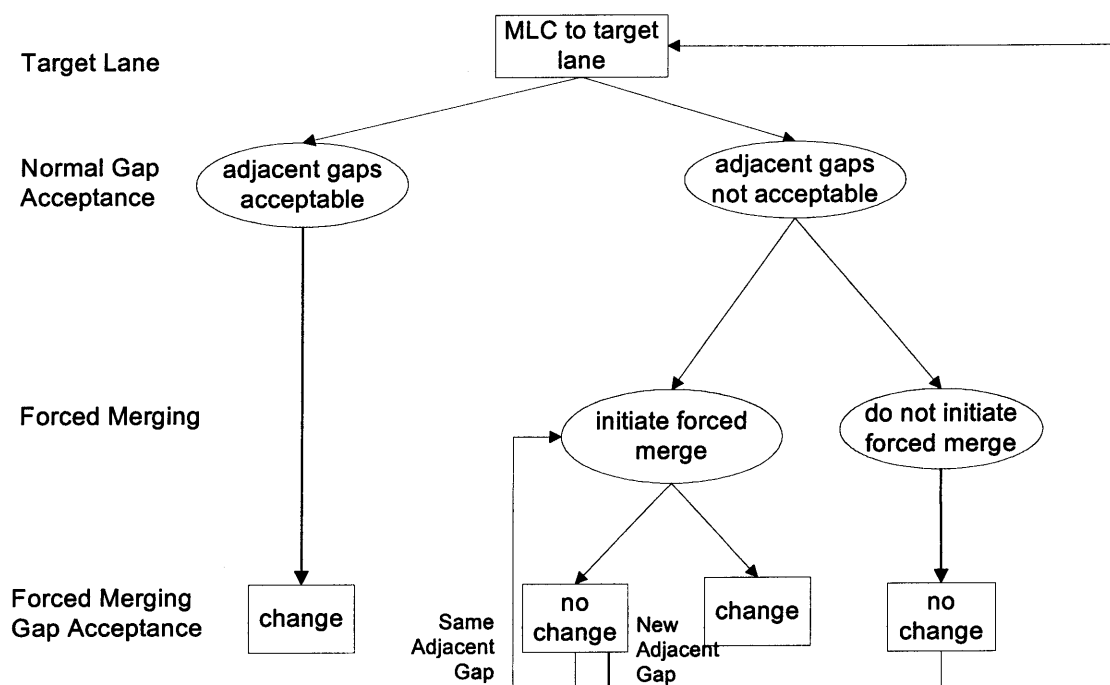


Figure 4.1-Framework of the explicit forced merging model

Each of these decision levels are discussed in detail below along with a list of candidate variables that are likely to affect the decisions. In each of these levels driver/vehicle specific latent variables are introduced to capture correlations between the decisions made by the same driver over time.

4.2 Normal Gap Acceptance

The normal gap acceptance model indicates whether a lane change is possible or not using the existing gaps. The driver first compares the available lead and lag gaps to the corresponding critical gaps for normal gap acceptance. An available gap is acceptable if it is greater than the critical gap. The terms are depicted in the following figure.

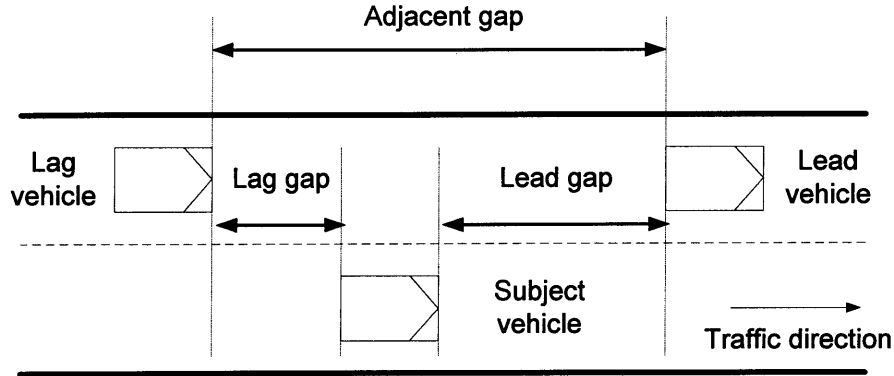


Figure 4.2-Relation between the subject, lead and lag vehicles

Critical gaps can be modeled as random variables. Their means are functions of explanatory variables. The individual specific random term captures correlations between the critical gaps of the same driver over time. Critical gaps are assumed to follow lognormal distributions to ensure that they are always non-negative:

$$\ln(G_{nt}^{Mg}) = \beta^{Mg^T} X_{nt} + \alpha^{Mg} \nu_n + \varepsilon_{nt}^{Mg} \quad g \in \{lead, lag\} \quad (4.1)$$

where, G_{nt}^{Mg} is the critical gap g of individual n at time t for normal gap acceptance (M), $g \in \{lead, lag\}$, X_{nt} is the vector of explanatory variables corresponding to the adjacent gap for individual n at time t , β^{Mg} is the corresponding vector of parameters for normal gap acceptance, ε_{nt}^{Mg} is the random term for normal gap acceptance of individual n at time t with $\varepsilon_{nt}^{Mg} \sim N(0, \sigma_{Mg}^2)$. ν_n is the driver specific random term and α^{Mg} is the coefficient of the driver specific random term for normal gap acceptance.

The driver/vehicle specific latent variables introduced in the model capture correlations between the decisions made by the same driver over time. The individual specific error term ν_n is included in the specification of the normal gap acceptance, initiate force stage and forced gap acceptance utility functions. The parameters associated with this variable in the various model components are estimated jointly, and so, capture correlations between these decisions, which may be attributed to unobserved individual specific driver/vehicle characteristics.

The gap acceptance model assumes that the driver must accept both the lead gap and the lag gap to change lanes. If a merging vehicle is in normal state, i.e., he has not initiated a forced merge ($s_{nt-1} = M$), the probability of a lane change through normal gap acceptance, conditional on the individual specific term ν_n is given by:

$$\begin{aligned}
 P(l_{nt} = 1 | s_{nt-1} = M, \nu_n) &= \\
 P(\text{accept lead gap} | s_{nt-1} = M, \nu_n) &\cdot P(\text{accept lag gap} | s_{nt-1} = M, \nu_n) \quad (4.2) \\
 = P(G_{nt}^{lead} > G_{nt}^{Mlead} | s_{nt-1} = M, \nu_n) &\cdot P(G_{nt}^{lag} > G_{nt}^{Mlag} | s_{nt-1} = M, \nu_n)
 \end{aligned}$$

where, l_{nt} is the lane-changing indicator of individual n at time t, 1 if a lane-change is performed by individual n at time t, 0 otherwise. s_{nt} is the state of the driver at time t (M or F), G_{nt}^{lead} is the available lead gap of individual n at time t and G_{nt}^{lag} is the available lag gap of individual n at time t.

If a driver had already initiated a forced merge in a previous time step, he cannot decide to merge to the same adjacent gap under normal gap acceptance. Therefore, if a merging vehicle is in initiated forced merging state at time t-1, unless there is a new adjacent gap, the probability of a lane change through normal gap acceptance at t is zero.

$$P(l_{nt} = 1 | s_{nt-1} = F, \nu_n) = 0 \quad (4.2a)$$

Assuming that critical gaps follow lognormal distributions, the conditional probabilities that gap $g \in \{lead, lag\}$ is acceptable is given by:

$$P\left(G_{nt}^g > G_{nt}^{Mg} \mid s_{nt-1} = M, v_n\right) = P\left(\ln\left(G_{nt}^g\right) > \ln\left(G_{nt}^{Mg}\right) \mid s_{nt-1} = M, v_n\right) = \Phi\left[\frac{\ln\left(G_{nt}^g\right) - \left(\beta^{Mg^T} X_{nt} + \alpha^{Mg} v_n\right)}{\sigma_{Mg}}\right] \quad (4.3)$$

$\Phi[\cdot]$ denotes the cumulative standard normal distribution.

Gap acceptance is affected by the interaction between the subject vehicle and the lead and lag vehicles in the adjacent lane. Candidate variables affecting normal gap acceptance include:

- relative speed of the subject vehicle with respect to the lead vehicle
- relative speed of the subject vehicle with respect to the lag vehicle
- acceleration of the lag vehicle
- remaining distance to the mandatory lane changing point

4.3 Decision to initiate a forced merge

If the normal gaps are not acceptable, the driver looks ahead and plans whether to initiate a forced merge in that gap ($s_{nt} = F$) or not ($s_{nt} = M$). In Figure 4.3 the normal gaps are not acceptable for the subject vehicle but it decides to initiate a forced merge with the plan that it can force the gaps to open up by imposing a deceleration on the lag vehicle and establishing a right of way. Note that the vehicle is assumed to have merged only if the center point of the vehicle crosses the lane boundary. If the adjacent gap changes while the driver is in this state, the driver abandons his current plan of action and reformulates his plan to merge into the freeway by checking if the new adjacent gap satisfies the normal gap acceptance criteria. Otherwise, the driver enters the decision to initiate a force merge level and the process repeats.

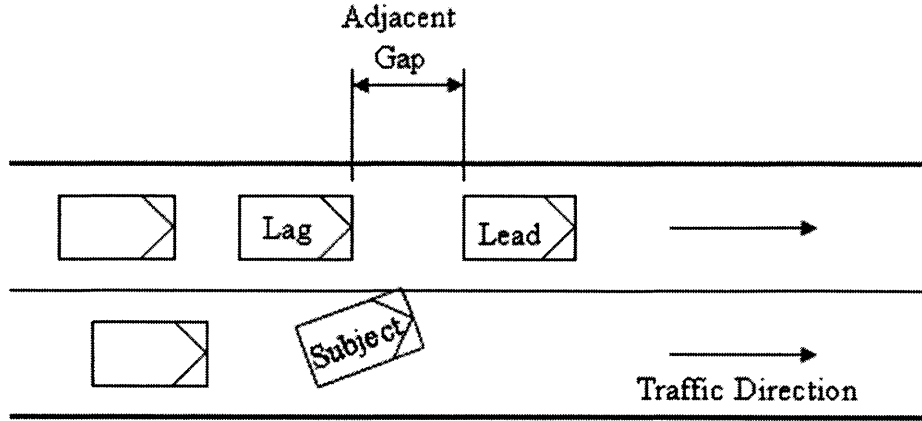


Figure 4.3-Example of a vehicle that has initiated force merging

By initiating a forced merge, the merging driver takes a risk and imposes a deceleration on the lag vehicle in the mainline. The utility of initiating a forced merge can be expressed as follows:

$$U_{nt}^F = \beta^{F^T} X_{nt} + \alpha^F v_n + \varepsilon_{nt}^F \quad (4.4)$$

where, U_{nt}^F is the utility of initiating a forced merge by individual n at time t , β^F is the corresponding vector of parameters for initiating a forced merge, ε_{nt}^F is the random term for initiating a forced merge. v_n is the driver specific random term and α^F is the coefficient of the driver specific random term for forced merging.

By assuming that the random error terms ε_{nt}^F are iid Gumbel distributed, the decision to initiate a force merge can be modeled as a logit model. Thus, the probability to initiate a forced merge is given by:

$$\left[\frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F v_n\right)} \right] \quad (4.5)$$

If the driver is adjacent to the same gap, the conditional probability of an individual to initiate a forced merge at time t , given he has not initiated a forced merge in the previous time step and the normal gaps are not acceptable is:

$$\begin{aligned}
& P(s_{nt} = F \mid s_{nt-1} = M, \nu_n) \\
&= P(s_{nt} = F \mid \nu_n) \cdot (1 - P(l_{nt} = 1 \mid s_{nt} = M, \nu_n)) \\
&= \left[\frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F \nu_n\right)} \right] \cdot (1 - P(l_{nt} = 1 \mid s_{nt} = M, \nu_n))
\end{aligned} \tag{4.6}$$

If the driver had already initiated a forced merge in a previous time step and the adjacent gap has not changed, the probability of being in initiated forced merge state is 1. If the driver had not initiated a forced merge in the previous time step, the probability of being in the initiated force merging state is the probability of rejecting the normal gaps in the current time step and the probability of deciding to initiate a forced merge as seen in Equation 4.7. However, if the driver cannot finish the initiated forced merging within the time he is adjacent to the same gap and is adjacent to a new gap, the state of the driver is reset to the normal (not initiated forced merging) state.

Thus, the probability of initiating a forced merge when the driver is adjacent to the same gap is given by:

$$\begin{aligned}
& P(s_{nt} = F \mid s_{nt-1} = F, \nu_n) = \delta_{nt} \\
& P(s_{nt} = F \mid s_{nt-1} = M, \nu_n) = \left[\frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F \nu_n\right)} \right] \cdot \\
& \left[1 - \Phi \left[\frac{\ln(G_{nt}^{lead}) - (\beta^{Mlead^T} X_{nt} + \alpha^{Mlead} \nu_n)}{\sigma_{Mlead}} \right] \right] \cdot \Phi \left[\frac{\ln(G_{nt}^{lag}) - (\beta^{Mlag^T} X_{nt} + \alpha^{Mlag} \nu_n)}{\sigma_{Mlag}} \right] \cdot \delta_{nt}
\end{aligned} \tag{4.7}$$

Where, $\delta_{nt} = 1$ if the driver is adjacent to the same gap in both time t and t-1, 0 otherwise.

Candidate variables affecting the decision to initiate a forced merge, apart from the individual specific characteristics, include

- status of the merging driver
 - distance to the MLC point

- delay (time elapsed since the driver is in MLC condition, as a proxy for impatience)
- speed
- lag vehicle status
 - type of the lag vehicle (heavy vehicle or not)
 - current speed and acceleration
- traffic conditions
 - level of congestion in the mainline
 - tailgating dummy (indicating one/more merging vehicles waiting behind the subject vehicle)

4.4 Decision to make a forced lane change

Once the driver initiates a force merge he persists in trying to complete the merging process by forcing in to the adjacent gap. The actual merging into the adjacent gap may therefore take some time. This decision level is not present in Ahmed's (1999) forced merging model where he assumed that if a driver intends to merge in front of a vehicle and right of way is established, the decision process ends and the driver gradually moves into the target lane.

The forced merge is executed only when the available gaps are acceptable in comparison with the critical gaps for the forced merge. From the moment a driver initiates a forced merge up to T_n (the last time step the vehicle is observed as a merging vehicle) he is considered to be in initiated forced merging state if the adjacent gap does not change. In this state the driver is trying to establish a right of way and is imposing a deceleration on the lag vehicle. At every instant the driver checks if the gaps are acceptable and accepts the gaps when he feels that they have opened up sufficiently for him to merge safely. After the driver has initiated a forced merge it is possible that he does not complete the merge if the gaps do not open up and are thus not acceptable. This can happen when the lag vehicle is not willing to yield to the merging vehicle or the lead vehicle decelerates causing the lead gap to narrow.

The critical gaps for forced merging are assumed to follow lognormal distributions to ensure that they are always non-negative:

$$\ln(G_{nt}^{Fg}) = \beta^{Fg^T} X_{nt} + \alpha^{Fg} \nu_n + \varepsilon_{nt}^{Fg}, g \in \{lead, lag\} \quad (4.8)$$

where, G_{nt}^{Fg} are the critical gaps of individual n at time t for forced merging gap acceptance, X_{nt} is the vector of explanatory variables corresponding to the adjacent gap for individual n at time t , β^{Fg} is the corresponding vector of parameters for forced gap acceptance, ε_{nt}^{Fg} is the random term for forced merge gap acceptance of individual n at time t with $\varepsilon_{nt}^{Fg} \sim N(0, \sigma_{Fg}^2)$. ν_n is the driver specific random term and α^{Fg} is the coefficient of the driver specific random term for forced merge gap acceptance.

The probability of changing lanes through forced merging given the adjacent gap is p conditional on the individual specific terms ν_n can be given by:

$$P(l_{nt} = 1 | s_{nt} = F, \nu_n) = P(G_{nt}^{lead} > G_{nt}^{Flead} | s_{nt} = F, \nu_n) \cdot P(G_{nt}^{lag} > G_{nt}^{Flag} | s_{nt} = F, \nu_n) \quad (4.9)$$

$$P(G_{nt}^g > G_{nt}^{Fg} | s_{nt} = F, \nu_n) = P(\ln(G_{nt}^g) > \ln(G_{nt}^{Fg}) | s_{nt} = F, \nu_n) = \Phi \left[\frac{\ln(G_{nt}^g) - (\beta^{Fg^T} X_{nt}^{Fg} + \alpha^{Fg} \nu_n)}{\sigma_{Fg}} \right] \quad (4.10)$$

where G_{nt}^{Fg} is the critical gap g of individual n at time t for forced merging gap acceptance and G_{nt}^g is the gap available to individual n at time t .

The variables influencing the critical gaps for forced merging may be the same as in merging under normal gap acceptance, but the parameters are likely to be different.

4.5 State Transitions

At time t given an adjacent gap, an individual n , can be in any one of the following states:

- Initiated forced merging ($s_{nt} = F$)
- Not initiated forced merging: normal ($s_{nt} = M$)

Once a driver has initiated forced merging to an adjacent gap, he persists in trying to complete the force merge. He does not consider normal gap acceptance in the subsequent time steps unless the gap changes. The decision in the subsequent time steps is only to evaluate the forced merging gap acceptance and decide whether or not to complete the forced merge in that time step. Thus once a transition is made from normal to forced merging state, the state cannot go back to normal unless the gap changes.

When the driver moves to a new adjacent gap, the state is reset to normal, that is, there is a transition from:

- normal to normal or
- initiated forced merge to normal

Therefore, the following 4 types of state transitions are possible:

1. Initiating forced merging from normal state:

$$s_{nt} = F \mid s_{nt-1} = M$$

If the driver was in the normal state, did not accept the available gaps and decides to initiate a force merging he moves from the normal to the force merging state.

2. Continuing the initiated forced merging state:

$$s_{nt} = F \mid s_{nt-1} = F$$

If a driver is in the force merging state, he persists in completing his plan and continues to be in the force merging state unless the adjacent gap changes. Thus, the probability associated with this state transition is 1 if the driver is adjacent to the same gap and 0 if the gap changes.

3. Continue being in the normal (has not initiated forced merging) state:

$$s_{nt} = M \mid s_{nt-1} = M$$

If the driver was in the normal state, did not accept the available gaps and decides not to initiate a force merging he continues to be in the normal state. Such a transition may occur for several reasons, for example a timid driver would not want to enter the force merging state, a myopic driver would not plan to initiate a force merge process and in these cases the drivers would continue being in the normal state until a suitable gap opens up.

4. Forced merging to normal state: (only if there is a new adjacent gap)

$$s_{nt} = M \mid s_{nt-1} = F$$

If a driver is in the force merging state, he can revert to the normal state only if the adjacent gap changes. Thus, the probability associated with this state transition is 1 if the gap changes and 0 if the driver is adjacent to the same gap.

The probability of each of these transitions is summarized below:

$$P(s_{nt} = F \mid s_{nt-1} = M, v_n) = \left[\frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F v_n\right)} \right] \cdot \left[1 - \Phi \left[\frac{\ln(G_{nt}^{lead}) - (\beta^{Mlead^T} X_{nt} + \alpha^{Mlead} v_n)}{\sigma_{Mlead}} \right] \cdot \Phi \left[\frac{\ln(G_{nt}^{lag}) - (\beta^{Mlag^T} X_{nt} + \alpha^{Mlag} v_n)}{\sigma_{Mlag}} \right] \right] \cdot \delta_{nt}$$

$$P(s_{nt} = F \mid s_{nt-1} = F, v_n) = \delta_{nt}$$

$$P(s_{nt} = M \mid s_{nt-1} = M, v_n) = \left[1 - \frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F v_n\right)} \right] \cdot \left[1 - \Phi \left[\frac{\ln(G_{nt}^{lead}) - (\beta^{Mlead^T} X_{nt} + \alpha^{Mlead} v_n)}{\sigma_{Mlead}} \right] \cdot \Phi \left[\frac{\ln(G_{nt}^{lag}) - (\beta^{Mlag^T} X_{nt} + \alpha^{Mlag} v_n)}{\sigma_{Mlag}} \right] \right]$$

$$P(s_{nt} = M \mid s_{nt-1} = F, v_n) = 1 - \delta_{nt} \quad (4.11)$$

where, $\delta_{nt} = 1$ if the driver is adjacent to the same gap in both time t and t-1, 0 otherwise.

The full merging trajectory is observable and the lane action is ‘no change’ in all but the last time period at which the individual makes a lane change. An individual can make a lane

change either through normal gap acceptance or through forced gap acceptance. Before the merge, the driver can be in any of the two states (M or F) and there are various combinations of these two states that can lead to the final outcome of a merge into the observed gap.

4.6 Conclusions

In this chapter, a framework for modeling driver's lane changing behavior using gap acceptance models was developed. A significant enhancement to existing models is the incorporation of anticipatory behavior into the model. Drivers are assumed to plan before forcing into an adjacent gap. If they anticipate that the gaps are likely to open up, they initiate a force merge and persist in their action and try to establish a right of way in the target lane. The fact that a driver has initiated a force merge does not necessarily mean that he will complete merge. There can be various factors that govern the completion of the merge and these have been taken into account in the explicit force merging model.

5. Data for Model Estimation

This chapter describes the data requirements for estimation of the proposed lane-changing model, characteristics of study area and the dataset used for estimating the driving behavior model.

5.1 Data Requirements

In the proposed model, most of the levels in the decision process involve latent choices and only the end action of the driver is observed. Estimation of the proposed lane-changing model requires detailed trajectory data in a merging section. The explanatory variables that can affect driving behavior are variables which:

- describe the relations in the traffic stream between the subject vehicle and the vehicles adjacent to it (lead and lag vehicles). Variables in this set may include subject speed, acceleration of the vehicles around the subject vehicle, relative speeds, the presence of heavy vehicles in the subject's lane or in the adjacent lane etc.
- capture traffic conditions not limited to the immediate surrounding vehicles. These variables also help capture the look-ahead or planning characteristics of a driver. Examples of these variables include average densities and average speeds upstream and downstream of the vehicle.
- capture the effect of the path plan on drivers' decisions. Variables in this group may include distance to the point where the driver must merge into the mainstream of vehicles.
- capture characteristics specific to each driver in the traffic stream. These variables capture the fact that individuals differ in their capabilities, their knowledge of the network, driving experience, reaction times and level of aggressiveness.

Detailed data in the form of trajectory data which consists of second-by-second (or even a finer resolution of time step) provides useful information about most of the variables described above. However, individual specific characteristics are not available from the trajectory data and therefore the heterogeneity term introduced in the models is used to

capture these characteristics. As the model aims at capturing behavior in congested situations, the time periods in the dataset should be representative of congested conditions.

5.2 Study Area Description

The data used in the estimation of the driving behavior model represents travel on the northbound direction of Interstate 80 in Emeryville, California. The data was collected and processed as part of the Federal Highway Administration’s (FHWA) Next Generation Simulation (NGSIM) project. Vehicles were tracked over a length of 1,650 feet. The data was collected using video cameras mounted on a 30-story building, Pacific Park Plaza, which is located in 6363 Christie Avenue and is adjacent to the interstate freeway I-80. The University of California at Berkeley maintains traffic surveillance capabilities at the building and the segment is known as the Berkeley Highway Laboratory (BHL) site.

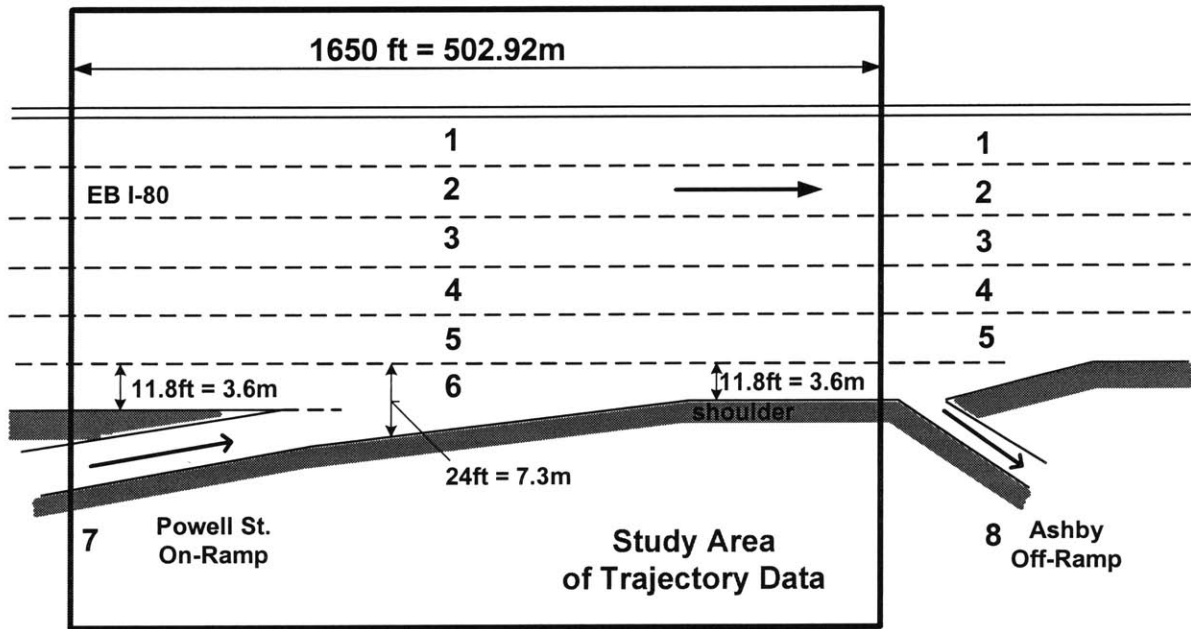


Figure 5.1-Data Collection Site (Source: FHWA 2005)

Figure 5.1 provides a schematic illustration of the location of the vehicle trajectory dataset. The site was approximately 502.9 meters long, with an on-ramp at Powell Street. The off-

ramp at Ashby Avenue is downstream of the study area. Lane numbering is incremental from the leftmost lane, which is the high occupancy vehicle (HOV) lane. Digital video images were collected using seven cameras, with camera 1 recording the southernmost and camera 7 recording the northernmost section of the study area as shown in Figure 5.2.

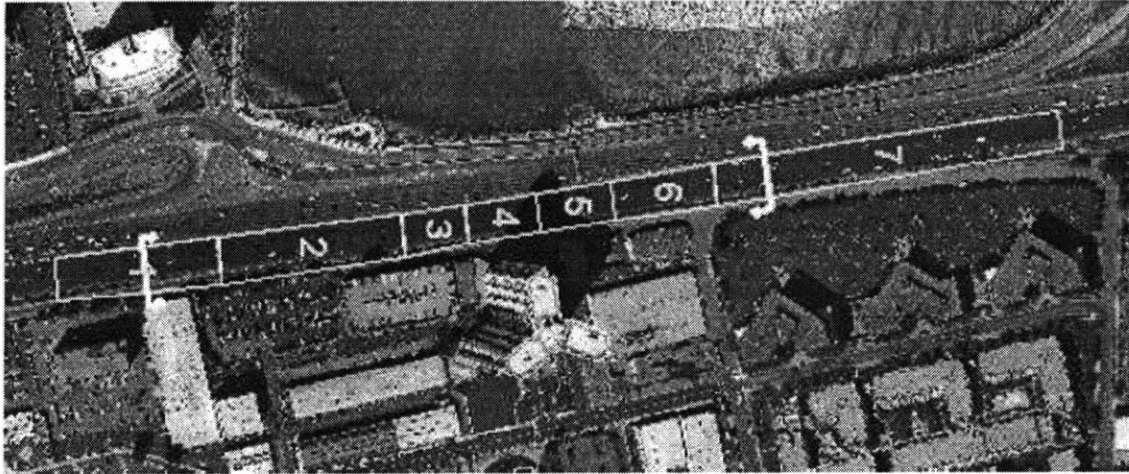


Figure 5.2-Camera coverage of the study area
(Source: NGSIM Data Analysis Report 2005)

Complete vehicle trajectories were recorded at a resolution of 10 frames per second. Thus, 45 minutes of data collected on April 13, 2005 at a resolution of $1/10^{\text{th}}$ of a second between the time intervals 4:00p.m-4:15p.m, 5:00p.m – 5:15p.m and 5:15 p.m-5:30p.m was available. The estimation is based on this dataset which consists of 540 vehicles and 17352 observations. The 4:00p.m – 4:15p.m period in the dataset is representative of a transitional traffic period in the build up to congested conditions and the 5:00p.m – 5:30p.m period is representative of primarily congested conditions.

5.3 Characteristics of the estimation dataset

As seen in the schematic representation of the study area in Figure 5.1, there are no physical lane marks separating the onramp vehicles from the mainline vehicles. The absence of a physical lane demarcation over a long stretch makes it difficult to specify when a lane change

has occurred and this necessitates the definition of an imaginary lane boundary.

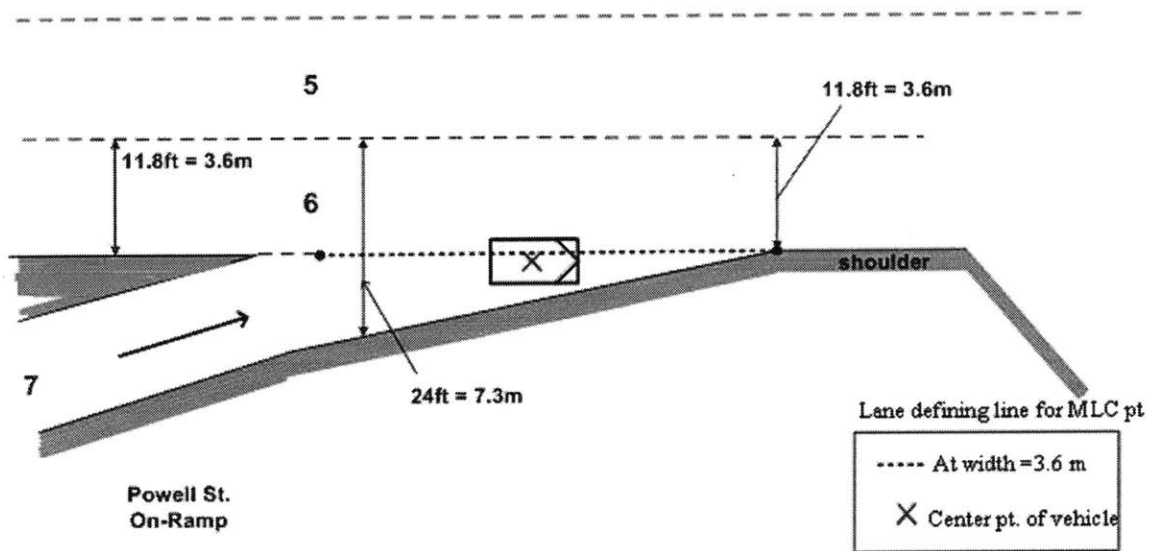


Figure 5.3-Imaginary lane boundary defined by the assumed MLC point

The mandatory lane changing (MLC) point, as shown in Figure 5.3 is defined as the point where the width of the rightmost lane assumes the single lane width (3.6 m). The definition of this point is important as it defines whether a merge has occurred or not.

A merge is assumed to be completed when the center point (B in Figure 5.4) of the vehicle has crossed the imaginary line/lane-mark defined above (Figure 5.3). Two other alternative definitions are the front left corner (A) and the right back corner (C) crossing this imaginary boundary line.

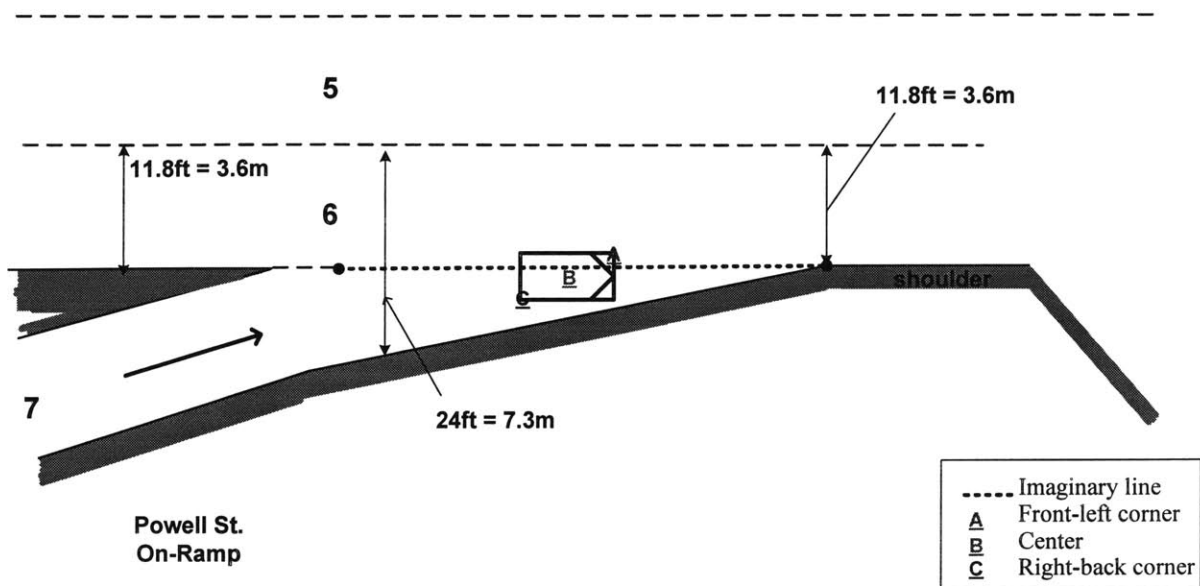


Figure 5.4-Definition of merge point with respect to the subject vehicle

In less congested situations, the choice between A, B and C does not result much difference since the execution of the lane change generally takes less than one time step, but in situations with high congestion level and low speeds, whether or not the merge is completed can depend on the choice of this point. For example, in Figure 5.4, the vehicle has already merged if point A is considered in the definition of the merge but not if point B or C is the point in consideration.

The vehicle trajectory data containing the coordinates of the various vehicles in the section were used to derive the required variables for estimation, like speeds, accelerations, average densities etc. For ease of data handling, the entire dataset was sampled at the rate of 1 in 10 observations. The resulting dataset had 540 vehicles with 17352 observations. Thus, on average a vehicle was observed for 32.1 seconds.

Speeds in the section vary from 0m/sec to a maximum of 20.7m/sec. with a mean of 4.2m/sec. There are many stop-and-go situations present in the dataset. Densities calculated 150m downstream of the merging vehicles in lane 6 range from 0.0 veh/km/lane to 126.7

veh/km/lane with an average of 61.9 veh/km/lane. 1.4% of the merging vehicles present in the dataset are heavy vehicles.

The distributions of speed, acceleration, density in Lane 6 and distance to the mandatory lane changing point in the entire dataset are shown in Figure 5.4.

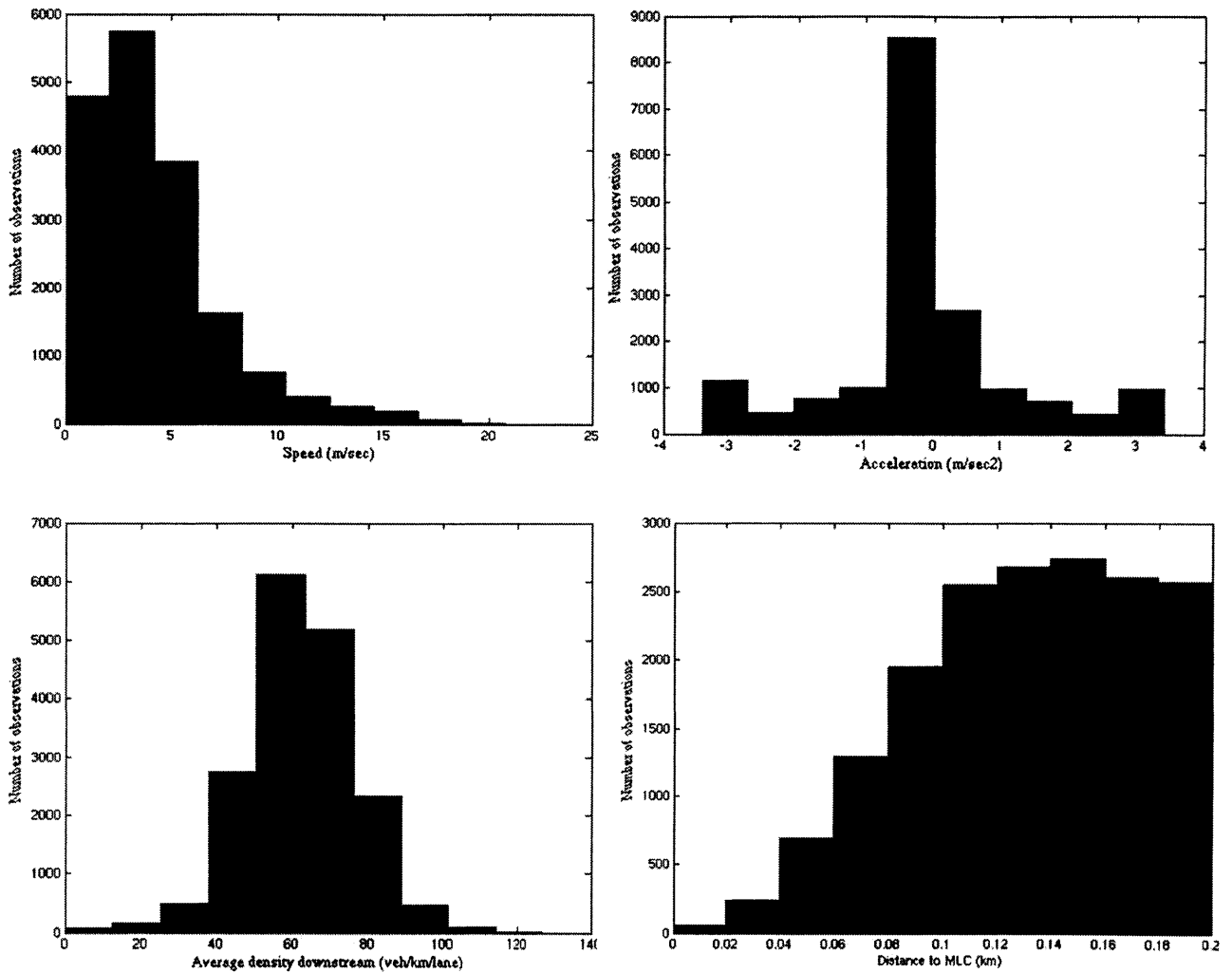


Figure 5.5-Distributions of speed, acceleration, density and distance to MLC in the data

The statistics relating to the subject vehicle are shown in Table 5.1. Table 5.2 gives the descriptive statistics for the lead and lag vehicle in relation to the subject vehicle. Relative speeds are defined as the speed of the lead vehicle or lag vehicles less the speed of the subject vehicle.

Table 5.1-Statistics of variables related to the subject vehicle

Variable	Mean	Std Dev	Median	Minimum	Maximum
Speed (m/sec)	4.2	3.11	3.34	0	20.7
Average Density d/s (veh/km/lane)	61.9	15.3	60.0	0	126.7
Distance to MLC (km)	0.13	0.04	0.13	0	0.20
Acceleration (m/sec ²)					
Positive	0.61	1.03	0	0	3.41
Negative	-0.65	1.07	-0.006	-3.41	0

Table 5.2-Statistics describing the lead and lag vehicles

Variable	Mean	Std Dev	Median	Minimum	Maximum
Lead Relative Speed (m/sec)	0.24 (-0.29)	1.26 (2.15)	0.24 (0.01)	-6.21 (-16.80)	5.60 (8.13)
Lead Gap (m)	9.92 (4.83)	9.01 (8.83)	7.57 (2.94)	0.13 (-19.43)	102.9 (160.6)
Lag Relative Speed (m/sec)	-0.55 (-0.41)	1.56 (2.15)	-0.51 (-0.15)	-10.98 (-14.25)	5.38 (18.09)
Lag Gap (m)	11.35 (5.25)	11.58 (8.85)	8.43 (3.39)	0.48 (-19.9)	172.9 (178.25)

- The values in parentheses are for the entire dataset

Table 5.2 summarizes statistics for the accepted lead and lag gaps. Thus, as can be seen, accepted lead gaps vary from 0.13m to 102.9m, with a mean of 9.92m. Accepted lag gaps vary from 0.48m to 172.9m. Figure 5.6 shows the relations between the subject, lead and lag vehicles. The definition of the lead and lag gap is clear from the figure. Negative gaps imply overlap between the subject and lead/lag vehicle.

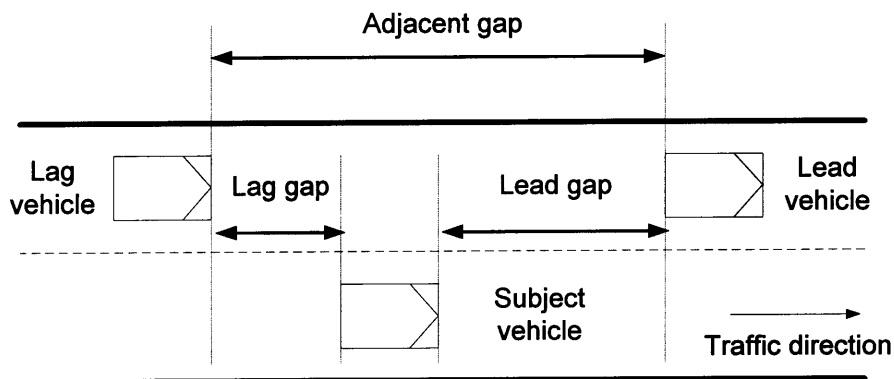


Figure 5.6-Relation between the subject, lead and lag vehicles

Statistics for the entire dataset are also shown in Table 5.2. As expected, the mean accepted gaps are larger than the mean gaps in the traffic stream for both the lead and lag gaps. Similarly, lead relative speeds in the accepted gaps are higher than in the mean of the entire dataset and lag relative speeds are lower in the entire dataset. This implies that when a gap is accepted, the subject vehicle is traveling slower compared to the lead vehicle and faster compared to the lag vehicle. The distributions of the speeds and spacing with respect to the lead and lag vehicles for the entire dataset are shown in Figures 5.7 and Figure 5.8 respectively.

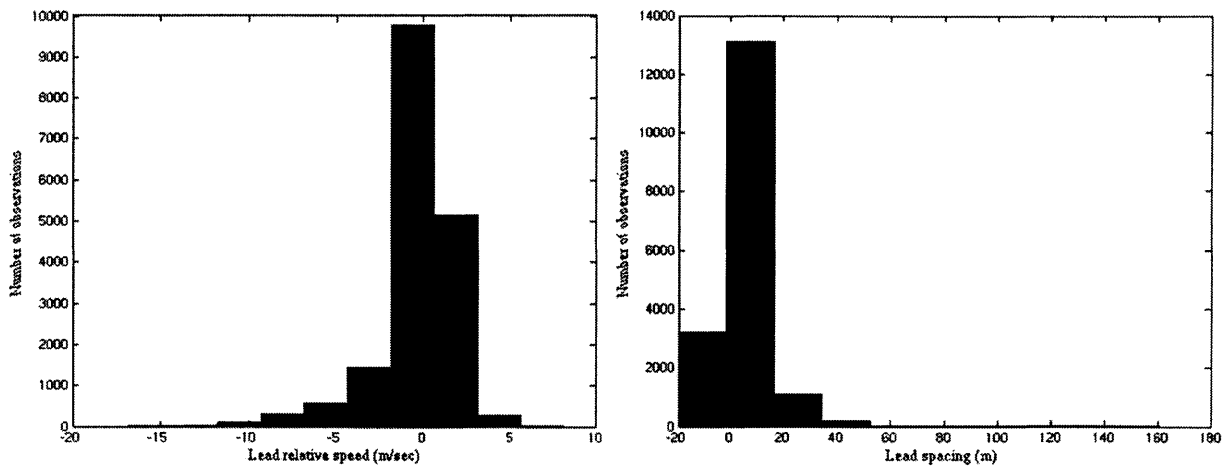


Figure 5.7-Distributions of lead relative speed and lead spacing in the entire dataset

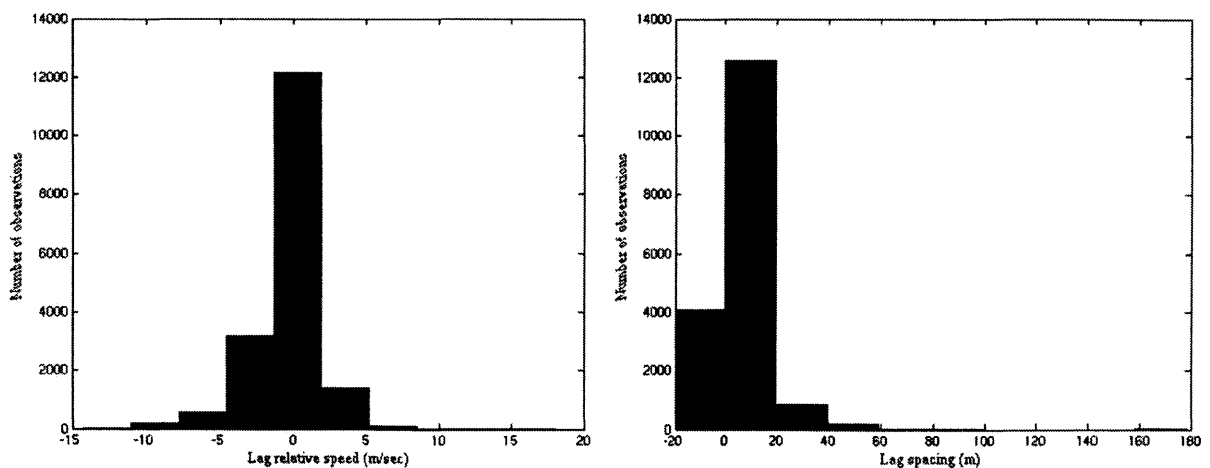


Figure 5.8-Distributions of lag relative speed and lag spacing in the entire dataset

The distributions of the speeds and spacing with respect to the lead and lag vehicles for the accepted gaps are shown in Figures 5.9 and Figure 5.10 respectively.

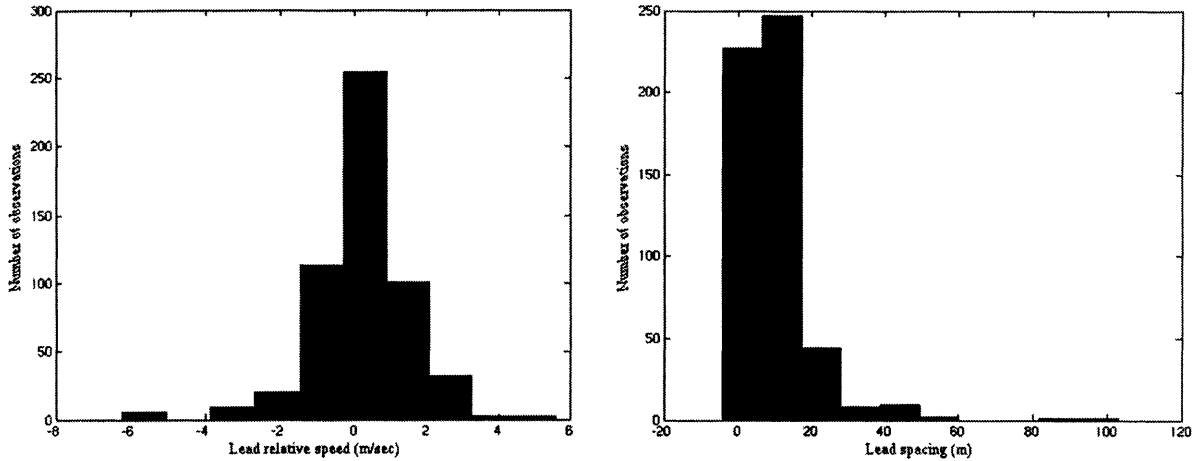


Figure 5.9-Distributions of lead relative speed and lead spacing for the accepted gaps

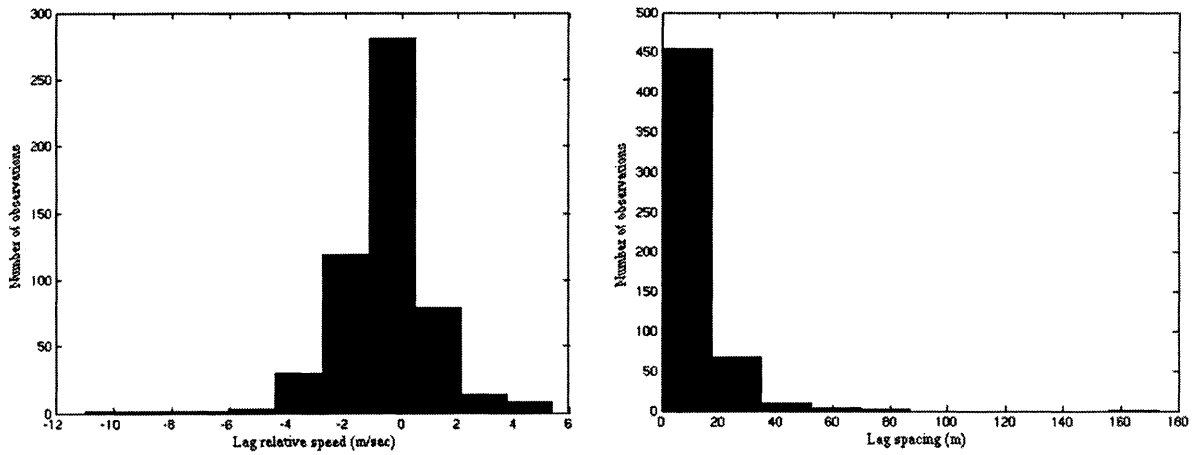


Figure 5.10-Distributions of lag relative speed and lag spacing for the accepted gaps

From the dataset it was observed that more than 80% of the merges occur when the distance to the mandatory lane changing point, as defined by the imaginary lane boundary, is less than

100m. Figure 5.11 shows the distribution of the number of merges with distance to the mandatory lane changing point in the section.

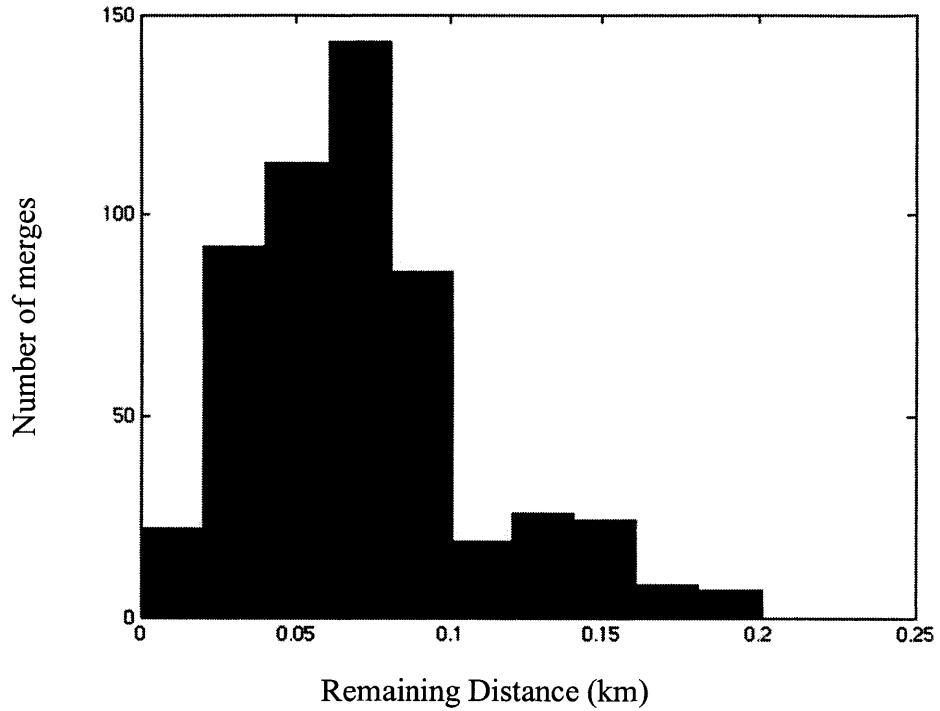


Figure 5.11-Distribution of number of merges with distance to MLC point

The number of vehicles in the trajectory dataset entering the lanes is summarized in the following table based on the time period. These values at a more detailed level (aggregated by minute) can be used as input to the micro-simulator at the implementation stage.

Table 5.3-Summary of vehicles entering by lane and time period

Lane	4:00 – 4:15 p.m.	5:00 – 5:15 p.m.	5:15 – 5:30 p.m
1	348	382	381
2	332	285	232
3	275	225	205
4	310	243	235
5	291	243	239
6	252	206	202
On-Ramp	190	205	200

The following graphs show the time mean speed and space mean speed on a lane by lane basis for the six lanes in the mainline. The time mean speed is defined as the average of the instantaneous speeds of all vehicles in a section during the specified time period. The space mean speed is calculated by dividing the sum of trajectory lengths traversed in a section by all vehicles by the sum of time take to traverse the section. The graphs have been taken from the Data Analysis Reports prepared for FHWA by Cambridge Systematics, Inc.

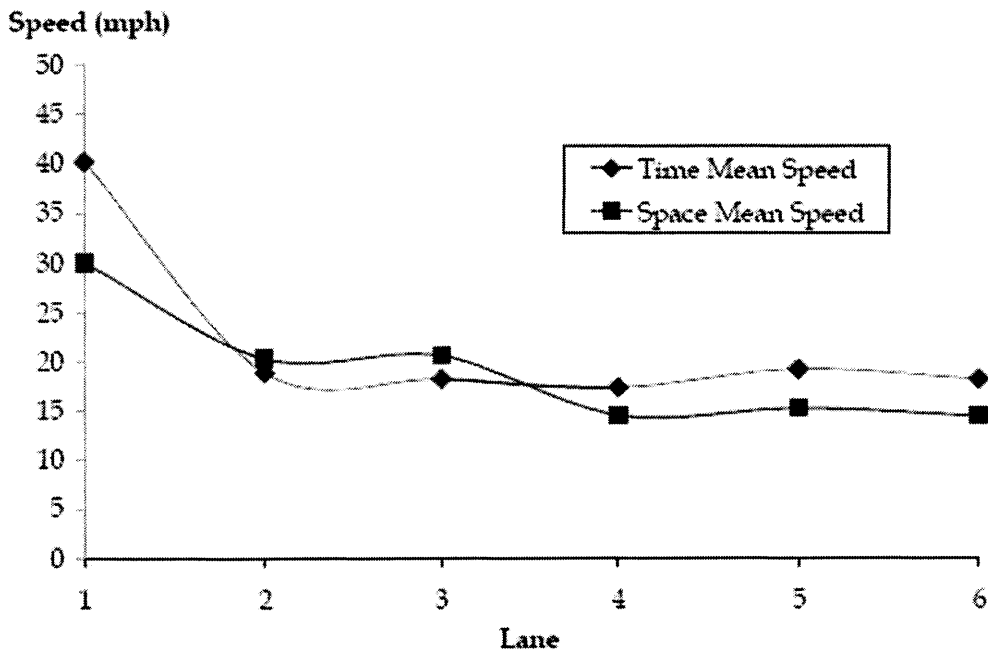


Figure 5.12-Time and Space Mean Speed for each lane during 4:00-4:15 p.m.

Source: NGSIM Data Analysis Report 2005

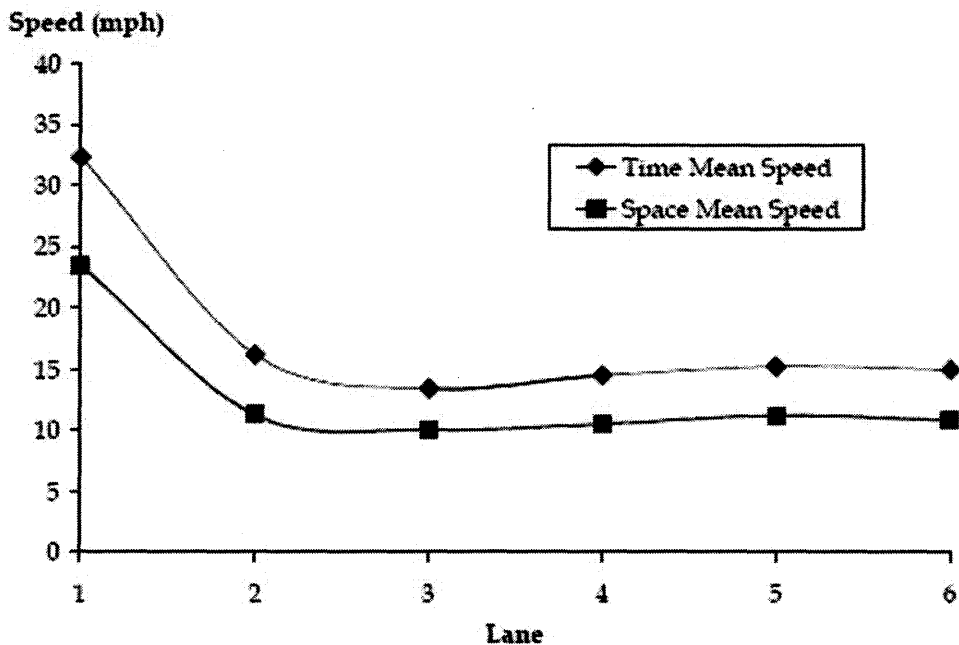


Figure 5.13-Time and Space Mean Speed for each lane during 5:00-5:15 p.m.

Source: NGSIM Data Analysis Report 2005

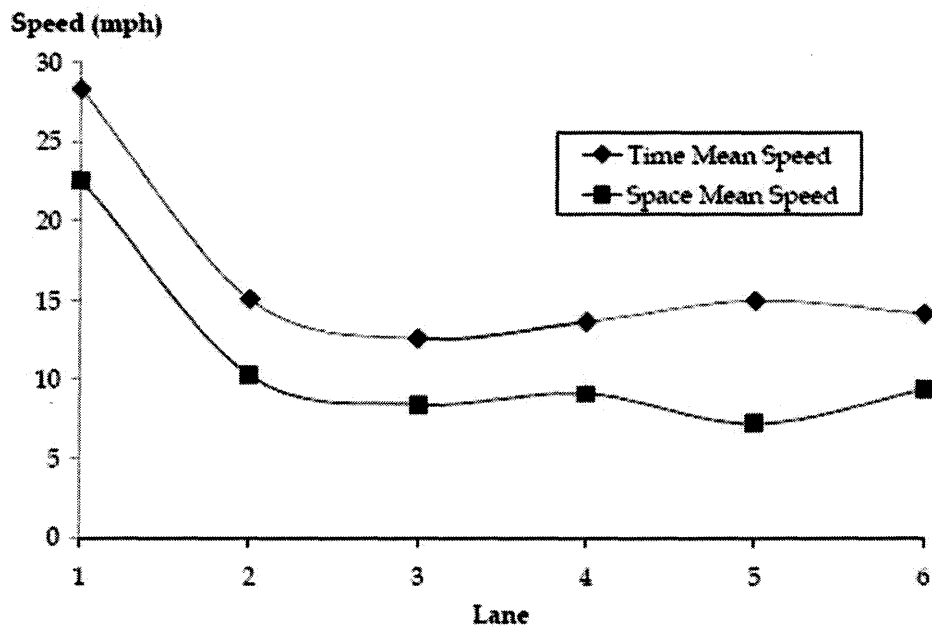


Figure 5.14-Time and Space Mean Speed for each lane during 5:15-5:30 p.m.

Source: NGSIM Data Analysis Report 2005

5.4 Conclusions

In this chapter the data requirements for estimation of the proposed lane-changing model were enlisted. The characteristics of study area and the dataset used for estimating the driving behavior model were described in detail. A particular characteristic of the I-80 dataset is the absence of physical lane marks over a long stretch making it difficult to specify when a lane change has occurred. The dataset is representative of congested conditions with mean speeds of 4.2m/sec. The traffic densities calculated 150m downstream of the merging vehicles in lane 6 ranged from 0.0 veh/km/lane to 126.7 veh/km/lane. The dataset provides detailed trajectory information like position of the vehicle, acceleration and speed of every vehicle at a resolution of 0.1 seconds. Distributions of relevant variables and descriptive statistics of the data used in estimation of the explicit forced merging model are also presented.

The aggregate origin/destination tables are tabulated and the distributions of time and space mean speeds for the observed section on a lane by lane basis are also presented.

6. Estimation Results

In this chapter estimation results of the explicit force merging model using the I-80 dataset are presented. All components of the model were jointly estimated using a maximum likelihood estimation procedure. The likelihood function used is described in the next section followed by a discussion and interpretation of the results. Finally, the model is statistically compared with another model that does not capture the persistent behavior of drivers.

6.1 Likelihood Function

An important limitation of the dataset is that there is no information about the driver/vehicle characteristics in the trajectory dataset (except for the length of the vehicle). To overcome this limitation driver/vehicle specific latent variables are introduced in the model. These variables capture correlations between the decisions made by the same driver over time. The individual specific error term ν_n is included in the specification of the normal gap acceptance, initiate force stage and forced gap acceptance utility functions. The parameters associated with this variable in the various model components are estimated jointly, and so, capture correlations between these decisions, which may be attributed to unobserved individual specific driver/vehicle characteristics.

In the dataset the full merging trajectory is observable. Only the final lane action of the driver l_{nt} for each gap is observed and the lane action is ‘no change’ in all but the last time period of the sequence. An individual can make a lane change either through normal gap acceptance or through forced gap acceptance. Before the merge, the driver can be in any of the two states (M or F) and there are various combinations of these two states that can lead to the final outcome of a merge into the observed gap. The latent state s_{nt} and the decision state sequence that led to that latent state are unobserved. The final lane actions of the driver are ‘no change’ for gaps $p=1$ to P_n-1 and ‘change’ for gap P_n (the last gap in the merging trajectory of the vehicle).

Since the decision state sequences are mutually exclusive, the joint marginal probability of observing a lane change for an adjacent gap p can be calculated by summation over all possible decision state sequences for that gap.

$$P_p(\mathbf{l}_n | \nu_n) = \sum_{\substack{\text{state} \\ \text{sequences} \\ \text{for gap } p}} P_p(\mathbf{l}_n, s_n | \nu_n) \quad (6.1)$$

If a driver is observed to have P_n adjacent gaps over his trajectory, the combined probability of observing a lane change by individual n can be expressed as:

$$P(\mathbf{l}_n | \nu_n) = \prod_{p_n} P_p(\mathbf{l}_n | \nu_n) \quad (6.2)$$

Another way of looking at the probability of individual n being in a particular state j at time t and performing lane action l_{nt} is given by the following equation:

$$\begin{aligned} P(l_{nt}, s_{nt} = j) &= P(l_{nt} | s_{nt} = j).P(s_{nt} = j) \\ &= P(l_{nt} | s_{nt} = j). \sum_i P(s_{nt} = j | s_{nt-1} = i).P(s_{nt-1} = i).P(l_{nt-1} = 0 | s_{nt-1} = i) \quad i, j \in M, F \end{aligned} \quad (6.3)$$

Since the initial state (at time $t=0$) is known to be normal (M),

For $t=1$, $P(l_{n1}, s_{n1} = j) = P(l_{n1} | s_{n1} = j).P(s_{n1} = j | s_{n0} = M), j \in M, F$

For $t=2$,

$$P(l_{n2}, s_{n2} = j) = P(l_{n2} | s_{n2} = j). \sum_i P(s_{n2} = j | s_{n1} = i).P(s_{n1} = i).P(l_{n1} = 0 | s_{n1} = i) \quad i, j \in M, F$$

For $t=3$,

$$P(l_{n3}, s_{n3} = j) = P(l_{n3} | s_{n3} = j). \sum_i P(s_{n3} = j | s_{n2} = i).P(s_{n2} = i).P(l_{n2} = 0 | s_{n2} = i) \quad i, j \in M, F$$

⋮

For $t=T_n$,

$$\begin{aligned} &P(l_{nT_n}, s_{nT_n} = j) \\ &= P(l_{nT_n} | s_{nT_n} = j). \sum_i P(s_{nT_n} = j | s_{nT_n-1} = i).P(s_{nT_n-1} = i).P(l_{nT_n-1} = 0 | s_{nT_n-1} = i) \quad i, j \in M, F \end{aligned} \quad (6.4)$$

where $P(s_{nT_n-1} = i)$ can be calculated recursively.

The state transition probabilities can be calculated using equations presented in Chapter 4 reiterated here:

$$\begin{aligned}
P(s_{nt} = F | s_{nt-1} = M, v_n) &= \left[\frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F v_n\right)} \right] \\
&\cdot \left[1 - \Phi \left[\frac{\ln(G_{nt}^{lead}) - (\beta^{Mlead^T} X_{nt} + \alpha^{Mlead} v_n)}{\sigma_{Mlead}} \right] \right] \cdot \left[\Phi \left[\frac{\ln(G_{nt}^{lag}) - (\beta^{Mlag^T} X_{nt} + \alpha^{Mlag} v_n)}{\sigma_{Mlag}} \right] \right] \cdot \delta_{nt} \\
P(s_{nt} = F | s_{nt-1} = F, v_n) &= \delta_{nt} \\
P(s_{nt} = M | s_{nt-1} = M, v_n) &= \left[1 - \frac{1}{1 + \exp\left(-\beta^{F^T} X_{nt} - \alpha^F v_n\right)} \right] \\
&\cdot \left[1 - \Phi \left[\frac{\ln(G_{nt}^{lead}) - (\beta^{Mlead^T} X_{nt} + \alpha^{Mlead} v_n)}{\sigma_{Mlead}} \right] \right] \cdot \left[\Phi \left[\frac{\ln(G_{nt}^{lag}) - (\beta^{Mlag^T} X_{nt} + \alpha^{Mlag} v_n)}{\sigma_{Mlag}} \right] \right] \\
P(s_{nt} = M | s_{nt-1} = F, v_n) &= 1 - \delta_{nt} \tag{6.5}
\end{aligned}$$

The probabilities of observing lane actions for each state can be calculated from equations:

$$\begin{aligned}
P(l_{nt} = 1 | s_{nt-1} = M, v_n) &= \\
P(\text{accept lead gap} | s_{nt-1} = M, v_n) &\cdot P(\text{accept lag gap} | s_{nt-1} = M, v_n) \tag{6.6} \\
= P(G_{nt}^{lead} > G_{nt}^{Mlead} | s_{nt-1} = M, v_n) &\cdot P(G_{nt}^{lag} > G_{nt}^{Mlag} | s_{nt-1} = M, v_n)
\end{aligned}$$

$$\begin{aligned}
P(l_{nt} = 1 | s_{nt} = F, v_n) &= \\
P(G_{nt}^{lead} > G_{nt}^{Flead} | s_{nt} = F, v_n) &\cdot P(G_{nt}^{lag} > G_{nt}^{Fflag} | s_{nt} = F, v_n) \tag{6.7}
\end{aligned}$$

The lane actions are ‘no change’ for time $t=1$ to $T_n - 1$ and ‘change’ at time T_n . Therefore, the probability of observing the entire set of lane actions of individual n can be given by:

$$\begin{aligned}
P(\mathbf{I}_n | \nu_n) &= \sum_j P(I_{nT_n} = 1, s_{nT_n} = j) \quad j \in M, F \\
&= \sum_j P(I_{nT_n} = 1 | s_{nT_n} = j) \cdot P(s_{nT_n} = j) \\
&= \sum_j P(I_{nT_n} = 1 | s_{nT_n} = j) \cdot \sum_i P(s_{nT_n} = j | s_{nT_{n-1}} = i) \cdot P(s_{nT_{n-1}} = i) \cdot P(I_{nT_{n-1}} = 0 | s_{nT_{n-1}} = i) \quad i, j \in M, F
\end{aligned} \tag{6.8}$$

where $P(s_{nT_{n-1}} = i)$ can be calculated recursively.

The unconditional individual likelihood

$$L_n = \int_{\nu} P(\mathbf{I}_n | \nu_n) f(\nu) d\nu \tag{6.9}$$

Where,

$f(\nu)$ is the standard normal probability density function.

The unconditional log likelihood summed over all individuals in the dataset is given by:

$$L = \sum_n \ln(L_n) \tag{6.10}$$

6.2 Estimation Results

The components of the model were jointly estimated using the maximum likelihood procedure described above. The estimation results of the explicit force merging model are presented here with a discussion and interpretation of the results. The model has three levels-normal gap acceptance, decision to initiate force merging and forced gap acceptance-which are estimated jointly.

6.2.1 Normal Gap Acceptance

The normal gap acceptance model indicates whether a lane change is possible or not using the existing gaps. The driver first compares the available lead and lag gaps, shown in Figure 6.1, to the corresponding critical gaps for normal gap acceptance. An available gap is acceptable if it is greater than the critical gap. Critical gaps can be modeled as random variables. Their means are functions of explanatory variables. The individual specific random

term captures correlations between the critical gaps of the same driver over time. The equations for the critical lead and lag gaps for normal gap acceptance can be given by:

$$\ln(G_{nt}^{Mg}) = \beta^{Mg^T} X_{nt} + \alpha^{Mg} v_n + \varepsilon_{nt}^{Mg} \quad g \in \{lead, lag\} \quad (6.11)$$

where, G_{nt}^{Mg} is the critical gap g of individual n at time t for normal gap acceptance (M), X_{nt} is the vector of explanatory variables corresponding to the adjacent gap for individual n at time t , β^{Mg} is the corresponding vector of parameters for normal gap acceptance, ε_{nt}^{Mg} is the random term for normal gap acceptance of individual n at time t with $\varepsilon_{nt}^{Mg} \sim N(0, \sigma_{Mg}^2)$. v_n is the driver specific random term and α^{Mg} is the coefficient of the driver specific random term for normal gap acceptance. The lead and lag critical gaps are assumed to follow lognormal distributions to ensure that they are always positive.

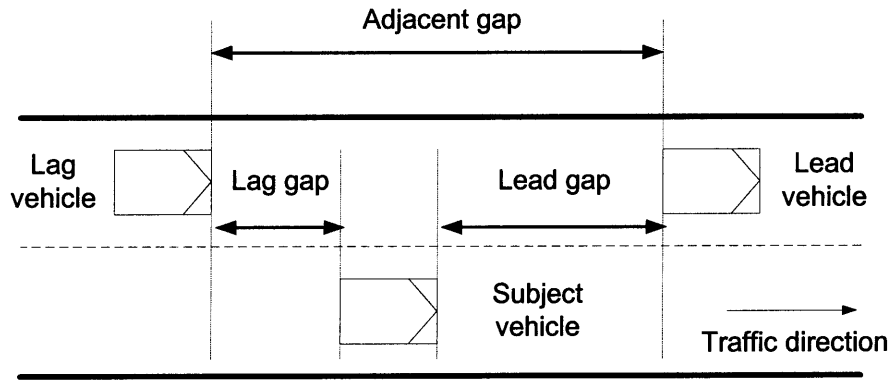


Figure 6.1-Relation between the subject, lead and lag vehicles

The estimation results for the lead and lag gaps for the normal gap acceptance are presented in Table 6.1.

Table 6.1-Estimation Results for Normal Gap Acceptance

Variable	Parameter Value	t-statistic
Normal Lead Gap		
Constant	1.18	1.32
Max(0,average speed - subject speed)(m/sec)	1.39	1.36
Min(0,lead speed - subject speed) (m/sec)	-0.595	-3.29
remaining distance to MLC point (10 meters)	1.59	3.28
remaining distance constant	0.561	1.13
$\alpha^{RemDisLead}$	0.570	6.19
σ^{MLead}	4.47	5.37
α^{MLead}	0.0169	0.58
Normal Lag Gap		
Constant	1.010	3.59
Max(0,lag speed - subject speed)(m/sec)	0.174	1.56
Min(0,lag speed - subject speed)(m/sec)	0.120	1.05
remaining distance to MLC point (10 meters)	0.0872	1.17
remaining distance constant	-3.066	-1.61
$\alpha^{RemDisLag}$	3.65	2.28
Max(0,acceleration of lag vehicle)(m/sec^2)	0.0941	1.21
σ^{MLag}	0.441	1.25
α^{MLag}	0.313	0.58

The estimated lead and lag critical gaps for the normal gap acceptance are given by:

$$G_{nt}^{MLead} = \exp \left(\begin{aligned} & 1.18 + \frac{1.39}{1 + \exp(-\max(0, \Delta V_{nt}^{avg}))} - 0.595 \text{Min}(0, \Delta V_{nt}^{lead}) \\ & + \frac{1.59 d_{nt}}{1 + \exp(0.561 - 0.570 v_n)} + 0.0169 v_n + \varepsilon_{nt}^{MLead} \end{aligned} \right) \quad (6.12)$$

$$G_{nt}^{MLag} = \exp \left(\frac{1.010 + 0.174 \text{Max}(0, \Delta V_{nt}^{lag}) + 0.120 \text{Min}(0, \Delta V_{nt}^{lag}) + \frac{0.0872 d_{nt}}{1 + \exp(-3.066 - 3.65 v_n)}}{+0.0941 \text{Max}(0, a_{nt}^{lag}) + 0.313 v_n + \varepsilon_{nt}^{Mlag}} \right) \quad (6.13)$$

$$\varepsilon_{nt}^{Mlead} \sim N(0, 4.47^2) \text{ and } \varepsilon_{nt}^{Mlag} \sim N(0, 0.441^2)$$

where G_{nt}^{Mlead} is the lead critical gap for the normal gap acceptance level (m), G_{nt}^{Mlag} is the lag critical gap for the normal gap acceptance level (m), ΔV_{nt}^{avg} is the relative speed of the average mainline speed with respect to the subject (m/sec), ΔV_{nt}^{lead} is the relative speed of the lead vehicle with respect to the subject (m/sec), d_{nt} is the remaining distance to the mandatory lane changing point (10m). ΔV_{nt}^{lag} is the relative speed of the lag vehicle with respect to the subject (m/sec), a_{nt}^{lag} is the acceleration of the lag vehicle (m/sec²). v_n are the unobserved driver characteristics. ε_{nt}^{Mlead} and ε_{nt}^{Mlag} are random error terms for the lead and lag critical gaps respectively.

The effect of the various variables is discussed in further detail here. The lead critical gap is a function of the average speed in the mainline relative to the subject vehicle's speed, the relative speed of the lead with respect to the subject and the remaining distance to the mandatory lane changing point.

The lead critical gap increases with increase in the average speed of the mainline. As the mainline average speed increases, the driver perceives increased risk to merging onto the mainstream of vehicles and requires larger critical gaps. However, it is reasonable to assume that the critical gap does not increase indefinitely with increasing average speeds in the mainline, but increases with a diminishing rate. Various functions were tried while estimating the model jointly and the following function was found to capture the behavior best:

$$\frac{\beta^{Mavg}}{1 + \exp(-\max(0, \Delta V_{nt}^{avg}))}$$

where β^{Mavg} was estimated to be 1.39. The more general function of the form

$$\frac{\beta^{Mavg}}{1 + \exp(\beta_1 - \beta_2 \max(0, \Delta V_{nt}^{avg}))}$$

was also tested but the parameters β_1 and β_2 were found to be insignificant upon estimation. Figure 6.2 shows the behavior of the critical lead gap with increasing mainline average speed.

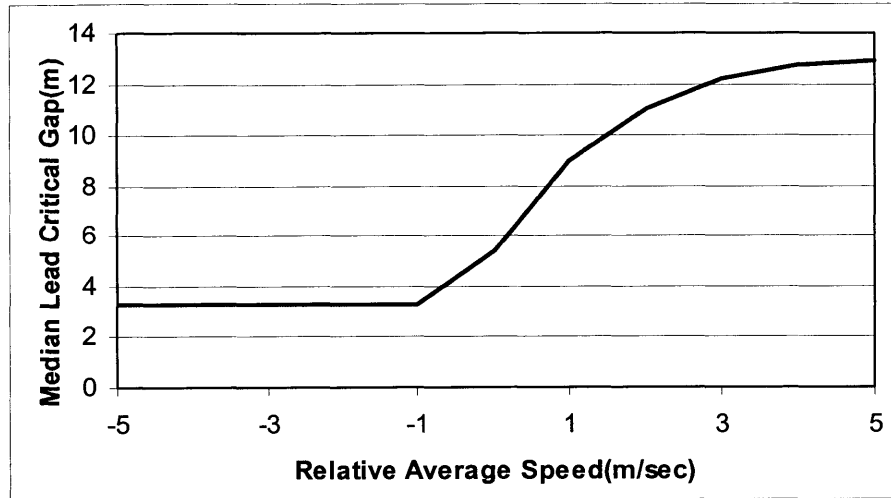


Figure 6.2-Median Lead Critical Gap as a function of relative average speed

The lead critical gap is larger when the lead vehicle is moving slower than the subject. This means that the lead is slowing down and the driver perceives increased risk as it gets closer to the lead vehicle, hence requiring a larger critical gap. Figure 6.3 illustrates this behavior.

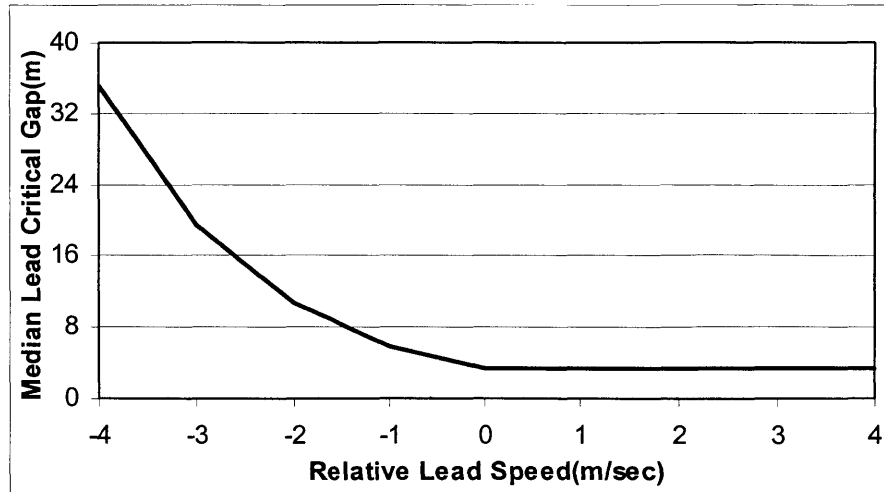


Figure 6.3-Median Lead Critical Gap as a function of relative lead speed

Both the lead and lag critical gaps decrease as the remaining distance to the mandatory lane changing point decreases. This is because as the driver nears the point where the ramp ends, his urgency to make the merge increases and he is willing to accept lower gaps to merge in to. Various functions were tried for capturing the effect of distance on the critical gaps. It was found that the individual specific factor can have an important impact in capturing this effect. Timid and myopic drivers have a very low probability of merging at the beginning, i.e, drivers who do not plan to make a merge and non-aggressive drivers tend to merge at the very end of the merge section. On the other hand, experienced and aggressive drivers require lower gaps to merge into the mainline.

The lag critical gap is a function of the subject relative speed with respect to the lag vehicle, the remaining distance to the mandatory lane changing point and the acceleration of the lag vehicle. As the model does not have an explicit courtesy merging level, variables that can signify courtesy like acceleration of the lag vehicle are used to capture this behavior.

The lag critical gap increases with the relative lag speed: the faster the lag vehicle is relative to the subject, the larger the critical gap is. The behavior of the lag critical gap with the relative lag speed is shown in Figure 6.4.

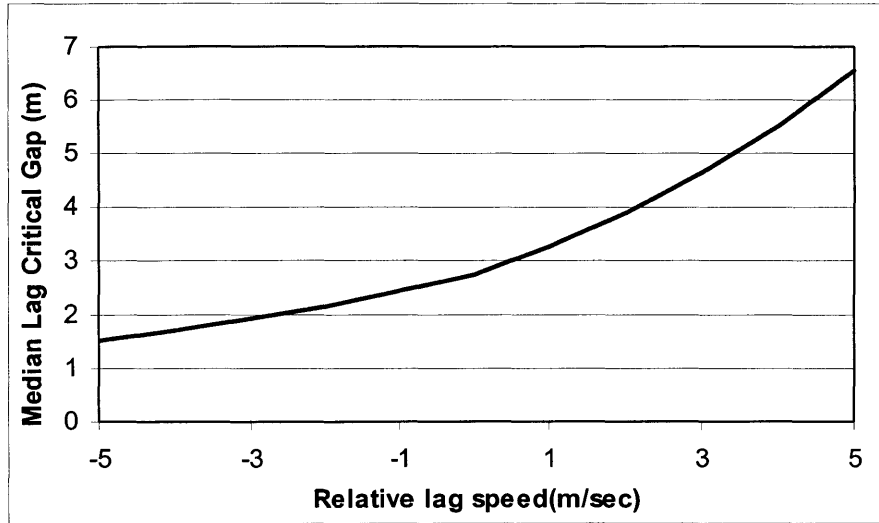


Figure 6.4-Median Lag Critical Gap as a function of relative lag speed

The lag critical gap increases as the acceleration of the lag vehicle increases, due to the higher perceived risk into merging onto the mainstream when the lag vehicle is accelerating. This variable is intended to capture the effect of courtesy behavior. Acceleration on the part of the lag vehicle can signify lack of courtesy shown by the lag vehicle. The behavior of the lag critical gap with the acceleration of the lag vehicle is shown in Figure 6.5.

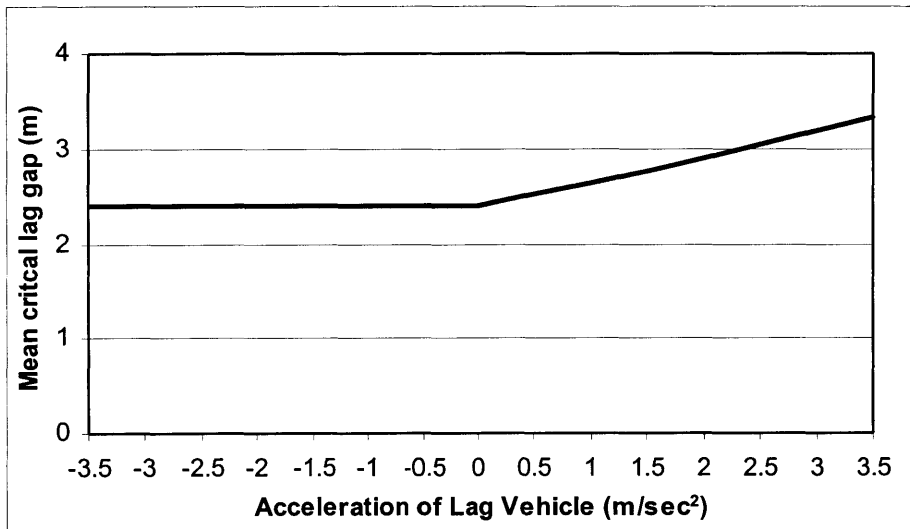


Figure 6.5-Median Lag Critical Gap as a function of acceleration of the lag vehicle

6.2.2 Decision to initiate a force merge

If a gap is not acceptable in the normal gap acceptance level, as shown in Figure 6.6, the driver plans as to whether to force merge into the adjacent gap or not. The decision to initiate a forced merge is modeled as a logit model with the utility of initiating a forced merge given by the following equation:

$$U_{nt}^F = \beta^F X_{nt} + \alpha^F v_n + \varepsilon_{nt}^F \quad (6.14)$$

where,

U_{nt}^F is the utility of initiating a forced merge by individual n at time t , β^F is the corresponding vector of parameters for initiating a forced merge, ε_{nt}^F is the random term for initiating a forced merge, v_n is the driver specific random term and α^F is the coefficient of the driver specific random term for forced merging.

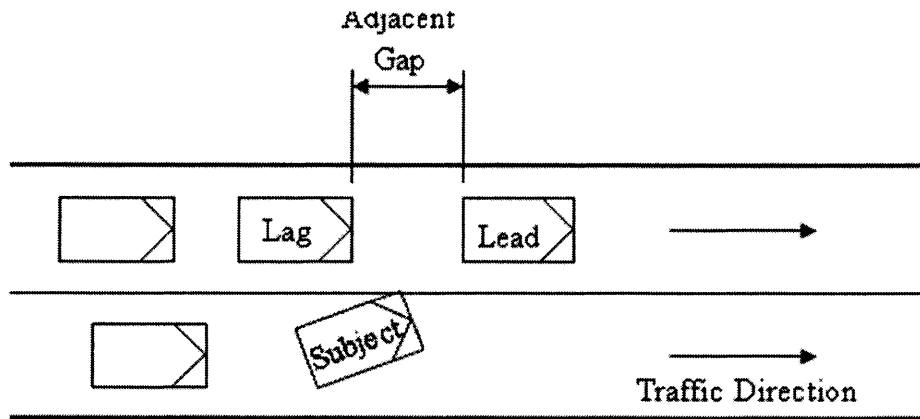


Figure 6.6-Driver initiating a forced merge

The estimated results for the decision to initiate force merge are presented in Table 6.2.

Table 6.2-Estimation results for the decision to initiate a forced merge

Variable	Parameter Value	t-statistic
Constant	-4.21	-1.70
Heavy Lag Vehicle Dummy	-2.041	-0.40
α^F	-30.6	-2.31

The probability of initiating a forced merge is given by the following equation:

$$P_{nt}^F = \frac{1}{1 + \exp(4.21 + 2.041\delta_{nt}^{hv} + 30.6v_n)} \quad (6.15)$$

where, P_{nt}^F is the probability of initiating a forced merge, δ_{nt}^{hv} is the heavy lag vehicle dummy and v_n is an individual specific error term which is assumed to be normally distributed in the population.

$$\delta_{nt}^{hv} = \begin{cases} 1 & \text{if lag is a heavy vehicle} \\ 0 & \text{otherwise} \end{cases}$$

The decision to initiate a forced merge is dependant on whether the lag vehicle is a heavy vehicle or not. For a driver to consider forcing a feasible means to change lanes, he/she should be confident that he/she can force the lag vehicle to decelerate in a short notice. Usually drivers perceive a risk in undertaking such a maneuver if the lag is a heavy vehicle as it is difficult to force a heavy lag vehicle to slow down in a short period of time. As a result if the lag is a heavy vehicle the probability to initiate a forced merge is low. The probability as a function of the heavy vehicle dummy is shown in Figure 6.7.

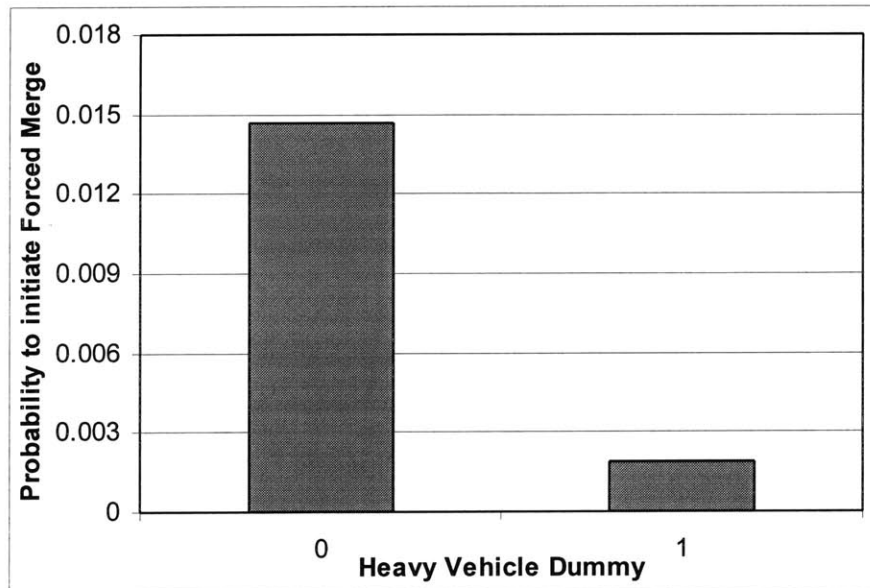


Figure 6.7-Probability to initiate a forced merge as a function of heavy lag vehicle dummy

The estimated coefficient of the unobserved driver characteristics is positive, indicating that an aggressive driver is more likely to initiate a forced merge whereas a timid driver is less likely to initiate a forced merge. During estimation many parameters that were thought to affect the decision to forced merge-the distance to the mandatory lane changing point, relative speed of the lag vehicle, level of congestion in the mainline-were tested but found to be insignificant. However, the coefficient for the driver/vehicle characteristics was found to be highly significant. This implies that the decision to initiate a forced merge is governed mainly by individual specific characteristics like aggressiveness, driving experience etc. and not by neighborhood variables or path plan variables. This finding is of interest as it states that a decision to initiate a forced merge is characteristic of an individual and not of the trajectory.

The following graph shows the variation of the probability to initiate a forced merge as a function of the individual specific term v_n .

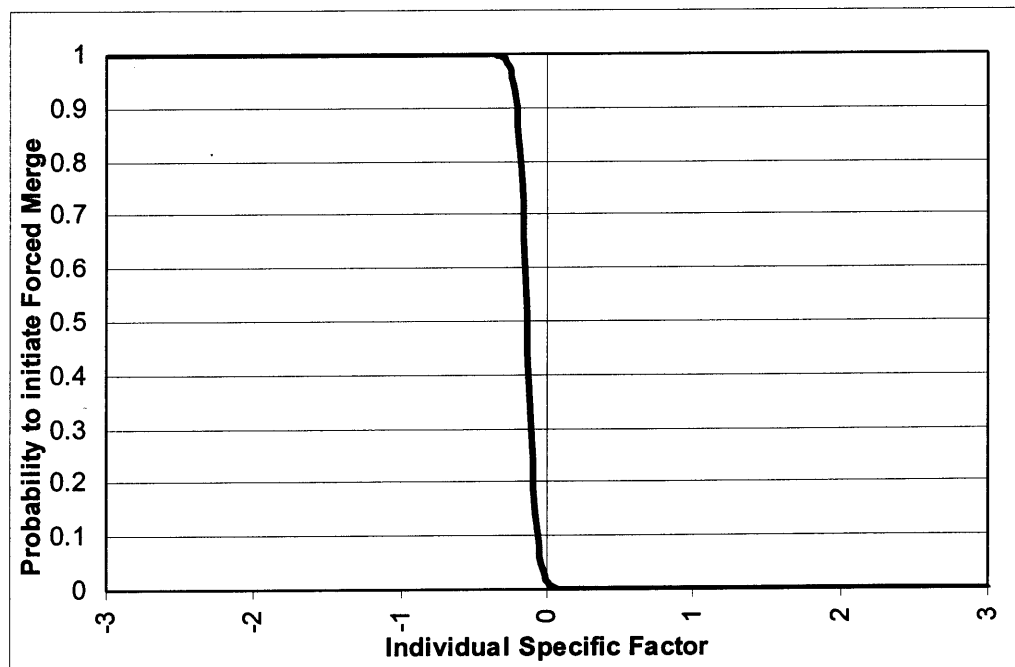


Figure 6.8-Probability to initiate a forced merge as a function of individual specific factor

The individual specific factor can reflect the aggressiveness, familiarity and planning capabilities of a driver. A negative value of this factor indicates an aggressive driver or a driver who plans ahead of time. A positive value indicates a timid or myopic driver. As can be seen from Figure 6.8, aggressive drivers and planners tend to initiate a forced merge with a very high probability while timid and myopic drivers do not initiate a forced merge.

6.2.3 Forced Gap Acceptance

Once the driver initiates a forced merge he persists in trying to complete the merging process by forcing in to the adjacent gap. The actual merging into the adjacent gap may therefore take some time. The forced merge is executed only when the available gaps are acceptable in comparison with the critical gaps for the forced merge. The critical gaps for forced merging are assumed to follow lognormal distributions to ensure that they are always non-negative:

$$\ln(G_{nt}^{Fg}) = \beta^{Fg^T} X_{nt} + \alpha^{Fg} \nu_n + \varepsilon_{nt}^{Fg}, g \in \{lead, lag\} \quad (6.16)$$

where G_{nt}^{Fg} are the critical gaps of individual n at time t for forced merging gap acceptance, X_{nt} is the vector of explanatory variables corresponding to the adjacent gap for individual n at time t , β^{Fg} is the corresponding vector of parameters for forced gap acceptance, ε_{nt}^{Fg} is the random term for forced merge gap acceptance of individual n at time t with $\varepsilon_{nt}^{Fg} \sim N(0, \sigma_{Fg}^2)$. ν_n is the driver specific random term and α^{Fg} is the coefficient of the driver specific random term for forced merge gap acceptance.

For estimation purposes, the coefficients for all variables except the constant, variance and coefficient of the driver specific random term were assumed to be the same as in the normal gap acceptance level.

The estimated results for the lead and lag gaps for the forced gap acceptance are presented in Table 6.3.

Table 6.3-Estimation Results for Forced Gap Acceptance

Variable	Parameter Value	t-statistic
Forced Lead Gap		
Constant	-0.734	-0.71
*Max(0,average speed - subject speed)(m/sec)	1.39	1.36
*Min(0,lead speed - subject speed) (m/sec)	-0.595	-3.29
*remaining distance to MLC point (10 meters)	1.59	3.28
*remaining distance constant	0.561	1.13
* $\alpha^{RemDisLead}$	0.570	6.19
σ^{FLead}	4.14	2.80
α^{FLead}	0.351	0.52
Forced Lag Gap		
Constant	-1.70	-1.83
*Max(0,lag speed - subject speed)(m/sec)	0.174	1.56
*Min(0,lag speed - subject speed)(m/sec)	0.120	1.05
*remaining distance to MLC point (10 meters)	0.0872	1.17
*remaining distance constant	-3.066	-1.61
* $\alpha^{RemDisLag}$	3.65	2.28
*Max(0,acceleration of lag vehicle)(m/sec^2)	0.0941	1.21
σ^{FLag}	0.152	0.60
α^{FLag}	0.0950	0.84

* same coefficient as in normal gap acceptance level

The estimated lead and lag critical gaps for the forced gap acceptance are given by:

$$G_{nt}^{FLoad} = \exp \left(\begin{aligned} & -0.734 + \frac{1.39}{1 + \exp(-\max(0, \Delta V_{nt}^{avg}))} - 0.595 \text{Min}(0, \Delta V_{nt}^{lead}) \\ & + \frac{1.59 d_{nt}}{1 + \exp(0.561 - 0.570 v_n)} + 0.351 v_n + \varepsilon_{nt}^{FLoad} \end{aligned} \right) \quad (6.17)$$

$$G_{nt}^{Flag} = \exp \left(\begin{array}{l} -1.70 + 0.174 \text{Max}(0, \Delta V_{nt}^{lag}) + 0.120 \text{Min}(0, \Delta V_{nt}^{lag}) + \frac{0.0872 d_{nt}}{1 + \exp(-3.066 - 3.65 v_n)} \\ + 0.0941 \text{Max}(0, a_{nt}^{lag}) + 0.152 v_n + \varepsilon_{nt}^{Flag} \end{array} \right) \quad (6.18)$$

$$\varepsilon_{nt}^{Flead} \sim N(0, 4.14^2) \text{ and } \varepsilon_{nt}^{Flag} \sim N(0, 0.152^2)$$

where,

G_{nt}^{Flead} lead critical gap for the forced gap acceptance level (m)

G_{nt}^{Flag} lag critical gap for the forced gap acceptance level (m)

ε_{nt}^{Flead} and ε_{nt}^{Flag} are random error terms

As can be seen from the estimates, all other things being the same, the critical gaps for forced merging are smaller than for normal merging. As the initiation process for a forced merge is usually begun by an aggressive driver, he/she is willing to take higher risks and accept a lower gap in order to complete the merge.

Both the lead and lag critical gaps decrease as the remaining distance to the mandatory lane changing point decreases. This is because as the driver nears the point where the ramp ends, his urgency to make the merge increases and he is willing to accept lower gaps to merge in to. Timid and myopic drivers have a very low probability of merging at the beginning, i.e, drivers who do not plan to make a merge and non-aggressive drivers tend to merge at the very end of the merge section. On the other hand, experienced and aggressive drivers require lower gaps to merge into the mainline.

The following graph shows the critical lead gap as a function of the remaining distance from the mandatory lane changing point for aggressive drivers. It was found that timid and myopic drivers do not consider the lead gap when they are far away from the merge point. The probability of such drivers merging at the beginning of the merge section is very low.

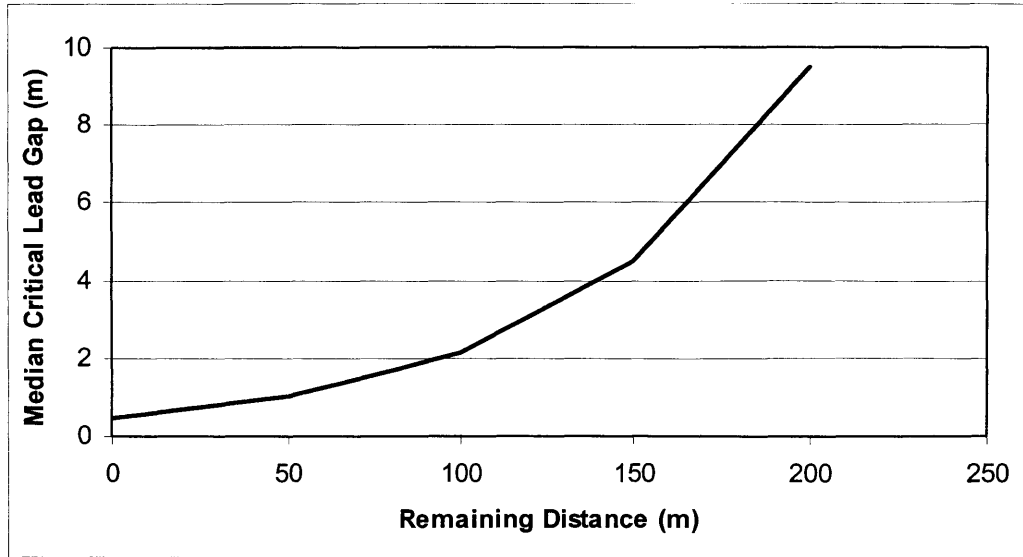


Figure 6.9-Median Critical Lead Gap as a function of remaining distance for aggressive drivers

Figure 6.10 shows the critical lag gap as a function of the remaining distance from the mandatory lane changing point for aggressive and timid drivers. As can be seen the critical lag gap does not vary much with distance for aggressive drivers, but increases with remaining distance for timid and myopic drivers.

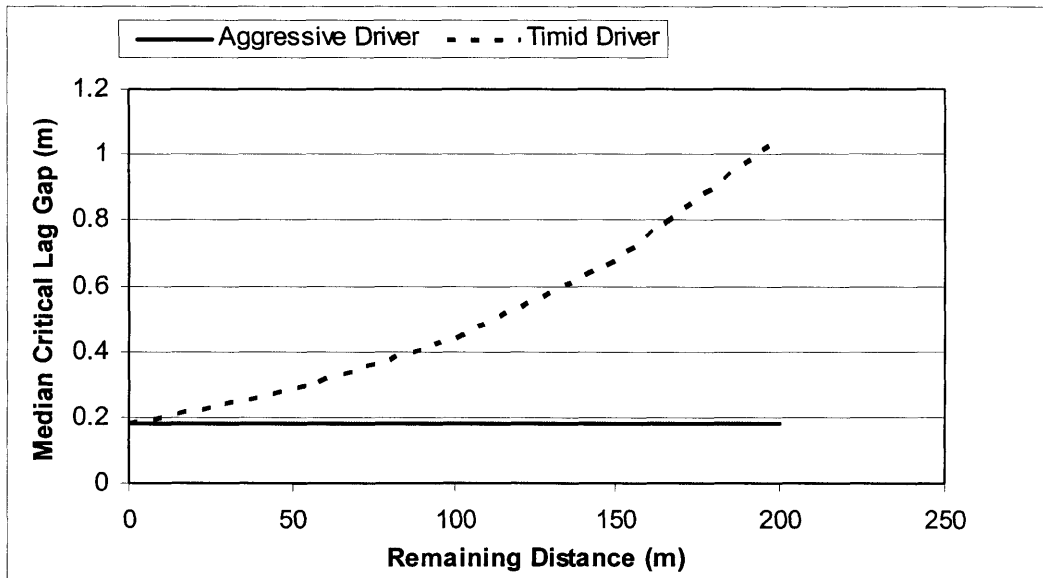


Figure 6.10-Median Critical Lag Gap as a function of remaining distance

6.3 Model Significance Test

The explicit forced merging model is compared against a simpler, single level model that does not capture the persistent behavior of drivers. The single level model aims at capturing the normal, forced and courtesy behavior of drivers through one gap acceptance level by means of relevant variables included. The model structure is shown in Figure 6.11.

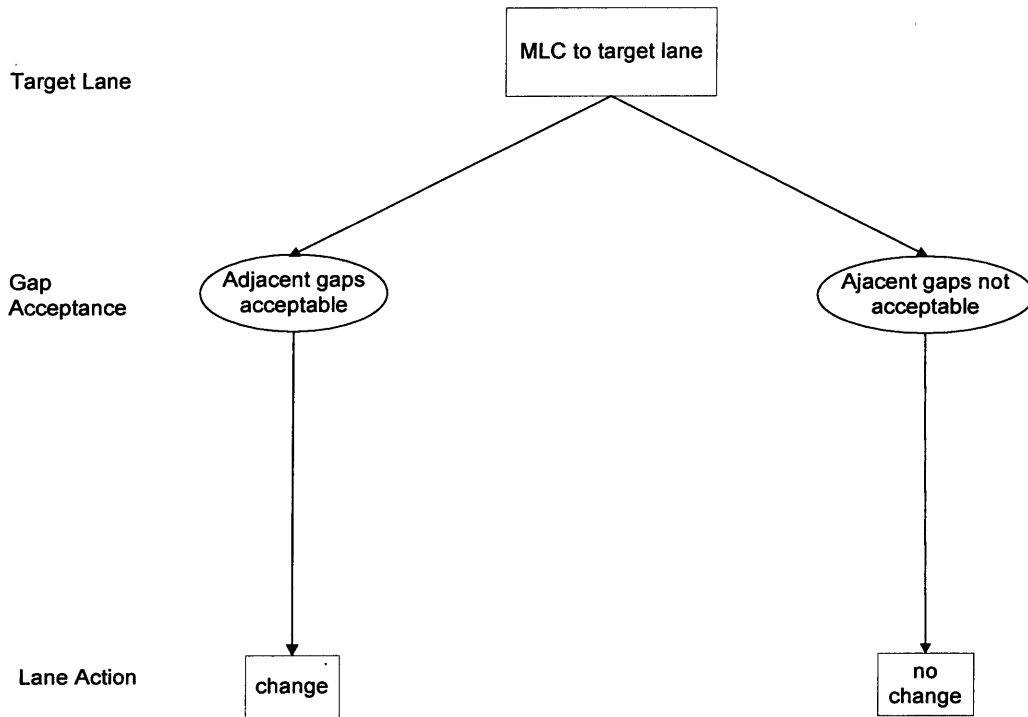


Figure 6.11-Framework of single level gap acceptance model (Lee 2006)

The estimation results of the single level gap acceptance model are presented in Table 6.4 and the estimation results of the explicit force merging model are summarized in Table 6.5.

Table 6.4-Estimation Results of the single level gap acceptance model

Single Level Model		
Final log-likelihood	-1639.69	
Number of cases	540	
Number of observations	17352	
Number of parameters	17	
Variable	Parameter Value	t-statistic
Normal Lead Gap		
Constant	0.181	0.203
Max(0,average speed - subject speed)(m/sec)	1.45	4.59
Min(0,lead speed - subject speed) (m/sec)	-0.571	-3.53
remaining distance to MLC point (10 meters)	1.029	4.29
Remaining distance constant	-0.492	-0.81
$\alpha^{RemDisLead}$	0.798	2.66
σ^{MLead}	4.27	5.86
α^{MLead}	-0.00016	-0.0033
Normal Lag Gap		
Constant	0.379	0.89
Max(0,lag speed - subject speed)(m/sec)	0.178	1.36
Min(0,lag speed - subject speed)(m/sec)	0.0909	0.707
remaining distance to MLC point (10 meters)	0.178	1.74
Remaining distance constant	-2.21	-0.55
$\alpha^{RemDisLag}$	2.88	0.73
Max(0,acceleration of lag vehicle)(m/sec ²)	0.0766	0.81
σ^{MLag}	0.914	5.63
α^{MLag}	-0.00012	-0.0025

Table 6.5-Estimation results of the explicit force merging model

Explicit Force Merging Model		
Final log-likelihood	-1628.64	
Number of cases	540	
Number of observations	17352	
Number of parameters	26	
Variable	Parameter Value	t-statistic
Normal Lead Gap		
Constant	1.18	1.32
Max(0,average speed - subject speed)(m/sec)	1.39	1.36
Min(0,lead speed - subject speed) (m/sec)	-0.595	-3.29
remaining distance to MLC point (10 meters)	1.59	3.28
remaining distance constant	0.561	1.13
$\alpha^{RemDisLead}$	0.570	6.19
σ^{MLead}	4.47	5.37
α^{MLead}	0.0169	0.58
Normal Lag Gap		
Constant	1.010	3.59
Max(0,lag speed - subject speed)(m/sec)	0.174	1.56
Min(0,lag speed - subject speed)(m/sec)	0.120	1.05
remaining distance to MLC point (10 meters)	0.0872	1.17
remaining distance constant	-3.066	-1.61
$\alpha^{RemDisLag}$	3.65	2.28
Max(0,acceleration of lag vehicle)(m/sec^2)	0.0941	1.21
σ^{MLag}	0.441	1.25
α^{MLag}	0.313	0.58
Initiate Force Merge		
Constant	-4.21	-1.70
Heavy Lag Vehicle Dummy	-2.041	-0.40

α^F	-30.6	-2.31
Forced Lead Gap		
Constant	-0.734	-0.71
*Max(0,average speed - subject speed)(m/sec)	1.39	1.36
*Min(0,lead speed - subject speed) (m/sec)	-0.595	-3.29
*remaining distance to MLC point (10 meters)	1.59	3.28
*remaining distance constant	0.561	1.13
* $\alpha^{RemDisLead}$	0.570	6.19
σ^{FLead}	4.14	2.80
α^{FLead}	0.351	0.52
Forced Lag Gap		
Constant	-1.70	-1.83
*Max(0,lag speed - subject speed)(m/sec)	0.174	1.56
*Min(0,lag speed - subject speed)(m/sec)	0.120	1.05
*remaining distance to MLC point (10 meters)	0.0872	1.17
*remaining distance constant	-3.066	-1.61
* $\alpha^{RemDisLag}$	3.65	2.28
*Max(0,acceleration of lag vehicle)(m/sec^2)	0.0941	1.21
σ^{FLag}	0.152	0.60
α^{FLag}	0.0950	0.84

* same coefficient as in normal gap acceptance level

The single level model can be obtained by imposing restrictions on the explicit forced merging model or the combined model. Thus, the Likelihood Ratio Test, which is used to compare log likelihood functions for unrestricted and restricted models of interest can be used here to compare the estimated models.

The test statistic for the null hypothesis that the restrictions are true is

$$-2(L^R - L^U) \quad (6.19)$$

which is asymptotically distributed as χ^2 with r degrees of freedom

where, L^R is the log-likelihood function value of the restricted model, L^U is the log-likelihood function value of the unrestricted model and r is the number of independent restrictions imposed.

In summary, the likelihood values of the estimated models are:

Model	Likelihood Function Value	Number of Parameters
Single Level Model	-1639.69	17
Explicit Forced Merging Model	-1628.64	26

The likelihood ratio is given by:

$$-2(-1639.69+1628.64) = 22.1$$

The number of degrees of freedom is 9 and the critical value of the chi-squared distribution with 9 degrees of freedom at a 0.95 confidence level is $\chi^2_{9,0.99} = 16.92$.

Thus, we can reject the null hypothesis at a 0.95 confidence level. Thus it can be concluded that the explicit forced merging model has a significantly better goodness of fit than the single level model.

Unobserved driver characteristics like aggressiveness play an important role in a driver's decision to initiate a forced merge. The unobserved driver characteristics were found to be insignificant in the single level model but are found to be significant in the explicit force merging model. The individual specific driver characteristics are found to play an important role in the decision to initiate a force merge. This implies that driver characteristics like aggressiveness, familiarity with the network, driving experience etc. which are unobserved in the dataset play an important role in the driver's decision to initiate a force merge.

The explicit force merging model which captures the persistent nature of drivers is able to capture the heterogeneity among drivers unlike the single level gap acceptance model. The model is also significantly better than the single level gap acceptance model.

6.4 Conclusions

In this chapter the joint likelihood functions for the lane changes observed in the I-80 trajectory dataset were derived. Estimation results of the explicit force merging model were presented.

Estimation results indicate that the decisions at the normal and forced gap acceptance levels are affected by the lead and lag relative speeds with respect to the merging vehicle and by the distance to the mandatory lane changing point. The relative value of the average speed of the mainline vehicles in the lane onto which the on-ramp vehicles merge with respect to the speed of the merging vehicle is also found to affect the lead critical gap. The critical lead gap increases at a diminishing rate with increase in positive values of this variable.

Merging through cooperative behavior is not explicitly modeled however variables that can capture the cooperative driver behavior are included. The acceleration of the lag vehicle is found to affect the lag critical gap and this is an indication of the courtesy provided by the mainline vehicle.

The probability to initiate a force merge is affected by the presence of heavy vehicles in the mainstream of vehicles and most importantly by driver characteristics like aggressiveness which are unobserved in the trajectory dataset. The decision to initiate a force merge is found to be individual specific. The constants for the forced gap acceptance are higher than for the normal gap acceptance indicating that once the driver initiates a force merge, he waits for the gaps to open up before completing the merge since the maneuver involves significantly higher risk compared to the normal gap acceptance.

The model is compared to a simpler single level gap acceptance model that does not capture the persistent behavior of drivers to follow through their plans once they have formulated it. The explicit force merging model captures driver specific characteristics better than the simpler model and is also found to be significantly better than the simpler model.

7. Implementation

The implementation results of the explicit force merging model are discussed in this chapter. The chapter begins with a section providing an overview of the MITSIMLab simulation tool in which the model was implemented. In the next section the implementation results are presented and a comparison is made with the single level gap acceptance model.

7.1 MITSIMLab

MITSIMLab is a simulation-based laboratory developed for evaluating the impacts of alternative traffic management system designs at the operational level and assisting in subsequent refinement. Examples of systems that can be evaluated with MITSIMLab include advanced traffic management systems (ATMS) and route guidance systems. MITSIMLab is a synthesis of a number of different models and represents a wide range of traffic management system designs. It has the ability to model the response of drivers to real-time traffic information and controls and can incorporate the dynamic interaction between the traffic management system and the drivers on the network.

The various components of MITSIMLab are organized in three modules:

1. Microscopic Traffic Simulator (MITSIM)
2. Traffic Management Simulator (TMS)
3. Graphical User Interface (GUI)

A microscopic simulation approach, in which movements of individual vehicles are represented, is adopted for modeling traffic flow in the traffic flow simulator MITSIM. The traffic and network elements are represented in detail in order to capture the sensitivity of traffic flows to the control and routing strategies. The road network is represented by nodes, links, segments (links are divided into segments with uniform geometric characteristics) and lanes. Traffic controls and surveillance devices are represented at the microscopic level.

The traffic simulator accepts time-dependent origin to destination trip tables as inputs. The OD tables represent either expected conditions or are defined as part of a scenario for evaluation. A probabilistic route choice model is used to capture drivers' route choice decisions. The origin/destination flows are translated into individual vehicles wishing to enter the network at a specific time. Behavior parameters (e.g., desired speed, aggressiveness) and vehicle characteristics are assigned to each vehicle/driver combination. MITSIM moves vehicles according to car-following and lane-changing models. The car-following model captures the response of a driver to conditions ahead as a function of relative speed, headway and other traffic measures. The lane changing model distinguishes between mandatory and discretionary lane changes. Merging, drivers' responses to traffic signals, speed limits, incidents, and toll booths are also captured.

The traffic management simulator (TMS) mimics the traffic control system under evaluation. A wide range of traffic control and route guidance systems can be evaluated. These include ramp control, freeway mainline control, lane control signs, variable speed limit signs, portal signals at tunnel entrances, intersection control, variable Message Signs and in-vehicle route guidance. TMS has a generic structure that can represent different designs of such systems with logic at varying levels of sophistication (pre-timed, actuated or adaptive). An extensive graphical user interface is used for both debugging purposes and demonstration of traffic impacts through vehicle animation. A detailed description of MITSIMLab appears in Yang and Koutsopoulos (1996) and Yang et al (2000).

7.2 Implementation Results

The simulation model was applied to the same road section (Interstate 80 in Emeryville, California), that was used to estimate the parameters of the explicit force merging model. The purpose of this exercise is two-fold: to verify the implementation of the model in the micro-simulator and to discuss the advantages, if any, of the explicit force merging model over the single level gap acceptance model.

The time dependent origin/destination tables extracted from the trajectory dataset were used as input to the simulation engine. The distributions of resulting travel times and remaining distance to merging point were compared with the real data.

The distribution of the travel time observed in the real data and simulated in the micro-simulator using the explicit force merging model are presented in Figure 7.1.

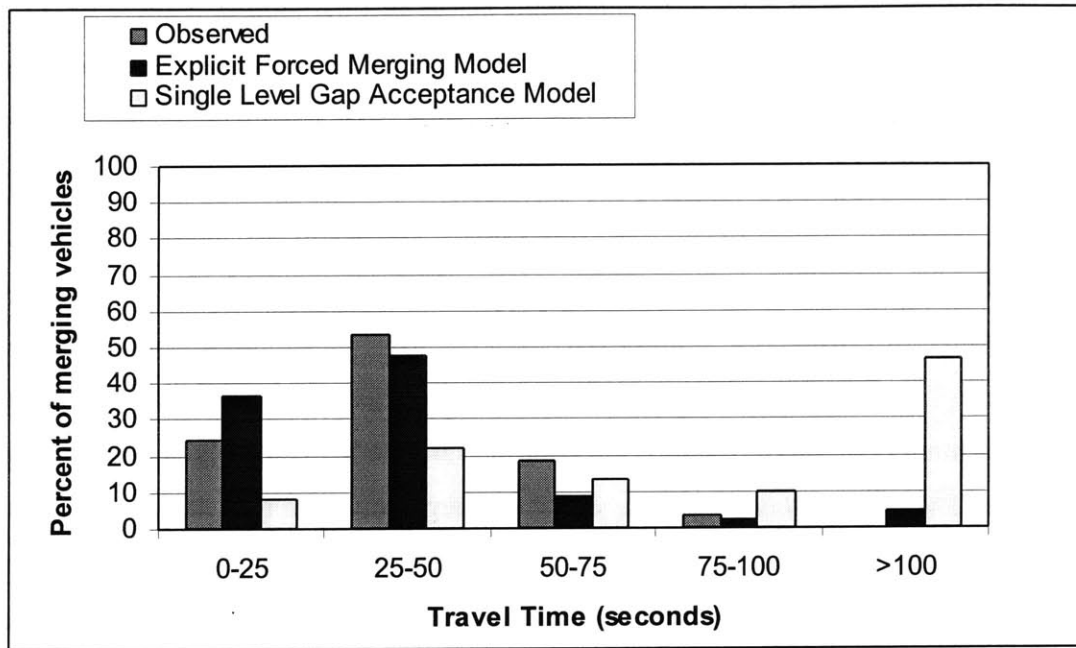


Figure 7.1-Observed and simulated travel times in the I-80 dataset

For the same inputs, it is seen that the single level gap acceptance model over predicts congestion in the merging section, resulting in queuing of vehicles and large travel times. This is because compared to the explicit force merging model the single level gap acceptance model does not capture the forcing and planning behavior of individuals. As a result, vehicles queue up waiting for a normal gap to open up. In the explicit force merging model however, if normal gaps are not available, individuals can formulate a plan and try to force merge into the available gaps. Thus, the explicit force merging model is able to better represent behavior in congested conditions than the single level gap acceptance model.

The distance between the point where the merging vehicle enters the mainstream of vehicles and the merging point (lane drop) is referred to as the remaining distance to the merging point. The distribution of the observed remaining distance to the merging point is compared to the distribution obtained by the explicit force merging and the single level gap acceptance model in Figure 7.2.

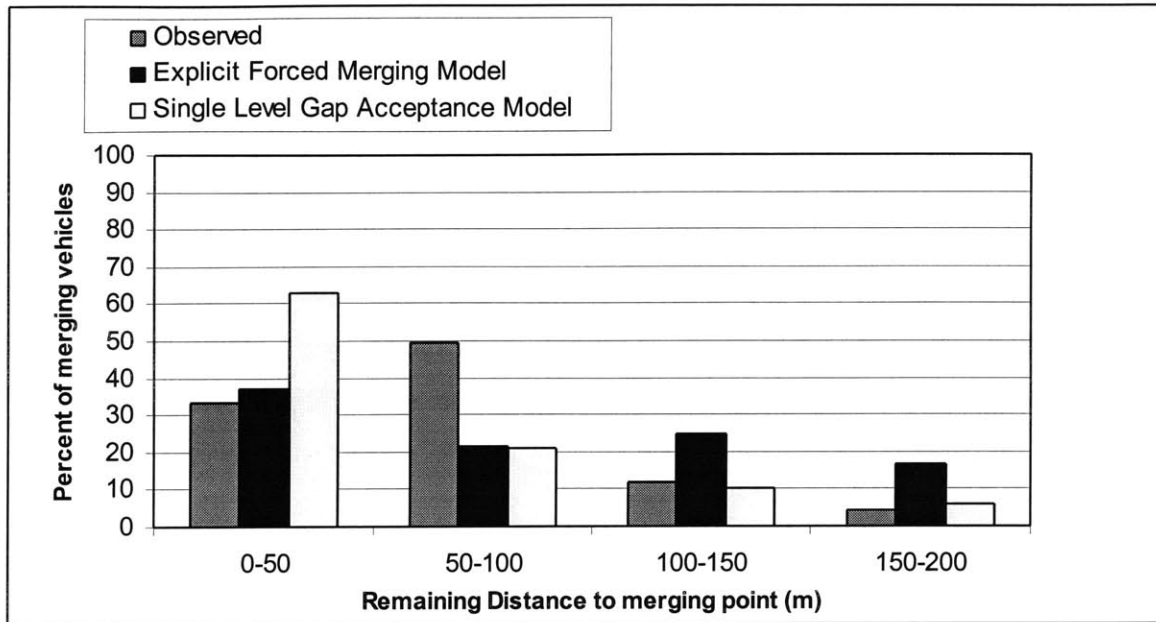


Figure 7.2- Distribution of observed and simulated remaining distance to merging point

In the single level gap acceptance model most of the merges happen near the merging point. This is because the normal gaps in the congested situations are not acceptable and the vehicle reaches the end of the merging lane where it waits for a gap to become acceptable. As a result, a queue forms behind this waiting vehicle. However, in the explicit force merging model if the normal gap acceptance criteria are not met, the driver has the option of formulating a plan and deciding whether to initiate a force merge or not. As the model captures the planning behavior of drivers, it is able to model behavior in congested scenarios better than the single level gap acceptance model. However the model seems to over predict the number of merges at the beginning of the merge section. As it was seen in Chapter 6, aggressive drivers and drivers who tend to plan well ahead in time have a higher probability of merging at the beginning. Thus, it is possible that the model over predicts the number of

force merges which may be due to the fact that the model does not capture courtesy merging explicitly and when a driver rejects a normal gap he has the option of initiating a forced merge. The option of anticipating an adjacent gap to open up in the future, thus leading to a merge further away from the beginning of the merging section, is not considered.

7.3 Conclusions

The explicit force merging model was implemented in the micro-simulator, MITSIMLab. The distributions of the travel time and remaining distance to the merging point indicate that the model is able to simulate behavior in congested situation well. The model was compared with the single level gap acceptance model and found to better model congested behavior. The model structure of the explicit force merging model captures the anticipatory behavior of drivers better than the single gap acceptance level model. As a result, if the normal gap acceptance criteria are not met, drivers can look ahead and decide if force merging is a better option.

8. Conclusions

This chapter summarizes the lane changing model frameworks developed in this thesis and the estimation results of the explicit force merging model. Directions for future research are suggested.

8.1 Summary

Two model frameworks that have the potential to capture the anticipatory behavior of drivers are developed in this thesis. The dynamic programming framework has been used mostly to model retirement decisions and it has been postulated in this thesis that such a framework can be used to model lane changing decisions more accurately. However due to the large number of state variables present the estimation of such a model can prove to be computationally intensive. The rest of the thesis focused on developing a framework for capturing the persistent behavior of drivers and the explicit force merging model was developed.

The explicit force merging model has three decision levels. The first level is the normal gap acceptance level. If the gaps are rejected in this stage, the driver decides whether to initiate a force merge or not. If he decides to initiate a force merge he enters the third level which is the forced gap acceptance level. Once a driver initiates a force merge, he remains in that state and persists to complete the merge by forcing unless the adjacent gap changes. The model structure accounts for correlations among the choices made by the same driver over different levels and time that are due to unobserved driver specific characteristics by introducing a driver specific random term. This driver-specific random term is included in all model components.

Parameters of the model are jointly estimated by maximizing the likelihood function. Detailed trajectory data is used for model estimation. Estimation results indicate that the decisions at the normal and forced gap acceptance levels are affected by the lead and lag relative speeds with respect to the merging vehicle and by the distance to the mandatory lane changing point. The average speed in the mainline relative to the merging vehicle affects the

lead critical gap. As cooperative behavior is not explicitly modeled, variables that can capture courtesy yielding like acceleration of the lag vehicle are included to capture this behavior. The decision to initiate a forced merge is found to be highly individual specific.

Statistical tests show that the explicit force merging model is significantly better than the single level gap acceptance model. The model also captures driver specific characteristics better than the simpler model. Implementation results also indicate that the model capture driver behavior in congested situations better. Thus it can be concluded that by incorporating the force merging behavior explicitly in the model and by capturing the planning behavior of drivers by means of the model structure the model captures the driver behavior more accurately.

8.2 Research Contributions

The objective of this research is to make driving behavior models more realistic. Most current models are reactive and myopic as drivers are assumed to make instantaneous decisions and are not characterized by any planning capabilities. The thesis contributes to driving behavior models in the following ways:

- A dynamic programming framework to capture the anticipatory behavior of individuals is developed. Such a framework has not been applied to lane changing decisions. Modeling lane changing behavior through dynamic programming can lead to realistic representation of driving behavior.
- A new lane changing model that explicitly captures the force merging behavior of drivers is proposed. This model is based on the idea that in congested situations drivers tend to force their way in to adjacent gaps and that drivers are persistent in order to follow a formulated plan.
- Based on this concept, an explicit force merging model structure is developed. The model has been applied to freeway merges from ramps, but the model structure has the potential to be applied to other lane changing scenarios in congested situations.
- The parameters of the explicit force merging model are estimated using detailed trajectory data. The parameters are jointly estimated using the maximum likelihood

estimator. The structure of the model accounts for correlations among the choices made by the same driver over different choice levels and time. Estimation results show that lane specific and driver specific variables are significant in gap acceptance. The model shows a significantly better fit than the simpler, single gap acceptance model.

8.3 Future Research

Absence of physical lane boundaries over a long stretch of the study area made it difficult to accurately specify a lane change. Estimation of the model using well specified datasets can increase the reliability of the model.

The explicit force merging model has been estimated only for the case of freeway merges from the on ramp. Other situations where the model can be applied need to be tested. Only the normal and forced merging behavior of drivers was explicitly captured in the model, the cooperative behavior was not explicitly modeled. Incorporation of the courtesy yielding behavior into the model can have the potential to improve the model fit.

The dynamic programming framework developed was not estimated because of the computational complexity involved. Further development of the model framework and estimation can result in more realistic driver behavior models.

The interaction between the gap acceptance and acceleration behavior of the driver is ignored in the current model whereas, in the real world, drivers are likely to accelerate and decelerate based on their tactical short-term plan to merge into the adjacent gap. Hence, there is the need to develop more detailed driving behavior models based on the concept developed here that is also capable of capturing interdependencies between gap acceptance and acceleration behaviors.

Appendix A: Illustration of the Dynamic Programming Framework

The general form of dynamic programming models developed for choosing between two options, using the example of a retirement problem specified in Lunsdaine, Stock and Wise (1992) is presented here:

If we assume that an individual has to choose between two decisions ‘w’-continue working and ‘r’-retire, at time instant t, an individual can be assumed to derive utility

$$U_w(Y_t) + \varepsilon_{1t} \text{ from option w and} \quad (\text{A.1})$$

$$U_r(B_t(s)) + \varepsilon_{2t} \text{ from option r.} \quad (\text{A.2})$$

where, Y_t is the earnings at time t, s is the retirement age, $B_t(s)$ are the retirement benefits at time t, if he retires at s and $\varepsilon_{1t}, \varepsilon_{2t}$ are the random perturbations to the age-specific utilities

The dynamic programming model is based on a recursive representation of the value function. At the beginning of the year, the individual has two choices: retire now and derive utility from future retirement benefits, or work for the year and derive utility from income while working during the year and retaining the option of retirement or work in the next year. The stochastic dynamic programming rule considers the expected value of the maximum of current versus future options. The value function can be written as follows:

$$W_t = \max \left[\bar{W}_{1t} + \varepsilon_{1t}, \bar{W}_{2t} + \varepsilon_{2t} \right] \quad (\text{A.3})$$

where,

$$\begin{aligned} \bar{W}_{1t} &= U_w(Y_t) + \beta \pi(t+1+t) E_t W_{t+1} \\ \bar{W}_{2t} &= \sum_{r=t}^S \beta^{r-t} \pi(r|t) U_r(B_r(t)) \end{aligned} \quad (\text{A.4})$$

where β is the discount factor, $\pi(r|t)$ is the probability of survival at r given survival at t, S is the year beyond which the person will not live and E_t is the expectation operator. Y_t is the

earnings at time t , r is the retirement age, $B_r(t)$ are the retirement benefits at time t , if he retires at r and $\varepsilon_{1t}, \varepsilon_{2t}$ are the random perturbations to the age-specific utilities.

\bar{W}_{1t} is the utility obtained when the individual chooses to continue working and \bar{W}_{2t} is the utility obtained when the individual decides to retire. E_t is the expectation that the individual has at time t about the value function in the future (W_{t+1}) obtained by making a decision at t . As can be seen, if the individual decides to retire the path is simplified. If he decides to continue to work, there are further paths that need to be evaluated.

If the error terms are assumed to be i.i.d draws from an extreme value distribution, the expectation of the maximum can be written as a log-sum.

The probability that an individual chooses to retire over continuing to work is :

$$\Pr\left(U_w(Y_t) + \beta\pi(t+1|t)E_t W_{t+1} + \varepsilon_{1t} < \sum_{d=t}^S \beta^{d-t} \pi(d|t) U_r(B_d(t)) + \varepsilon_{2d}\right) \quad (\text{A.5})$$

Assuming that the error terms are iid, they formulated the expectation of the value in the future as:

$$\begin{aligned} \frac{E_t W_{t+1}}{\sigma} = \mu_{t+1} = \gamma_e + \ln & \left[\exp\left(\frac{(U_w(Y_{t+1}) + \beta\pi(t+2|t+1)E_t W_{t+2})}{\sigma}\right) + \right. \\ & \left. \exp\left(\frac{\left(\sum_{d=t+1}^S \beta^{d-t-1} \pi(d|t+1) U_r(B_d(t+1))\right)}{\sigma}\right) \right] \\ = \gamma_e + \ln & \left[\exp\left(\frac{U_w(Y_{t+1})}{\sigma}\right) \exp(\beta\pi(t+2|t+1)\mu_{t+2}) + \exp\left(\frac{\left(\sum_{d=t+1}^S \beta^{d-t-1} \pi(d|t+1) U_r(B_d(t+1))\right)}{\sigma}\right) \right] \quad (\text{A.6}) \end{aligned}$$

where, γ_e is the Euler's constant and σ is the scale parameter.

The expectation of the value function at $t+1$ is the maximum of the utility obtained on following the path that results on choosing to continue working and the path that results on choosing to retire (evaluated as a log sum). The path on choosing to continue working requires an evaluation of the utility at $t+1$ and the value function at $t+2$ (which bring in recursion). The path on choosing to retire requires an evaluation of the utility that arises out of the benefits at each consecutive time-step after the individual decides to choose option r , until S .

$$\Pr(\text{option } r \text{ in } t) = \frac{\exp\left(\frac{\sum_{d=t}^S \beta^{d-t} \pi(d|t) U_r(B_d(t))}{\sigma}\right)}{\exp\left(\frac{U_w(Y_t) + \beta \pi(t+1|t) E_t W_{t+1}}{\sigma}\right) + \exp\left(\frac{\sum_{d=t}^S \beta^{d-t} \pi(d|t) U_r(B_d(t))}{\sigma}\right)} \quad (\text{A.7})$$

Thus, as can be seen the dynamic programming algorithm works on the principle that optimal solutions of sub-problems can be used to find the optimal solution of the overall problem.

Appendix B: Terms used in the model equations

Normal Gap Acceptance Level

G_{nt}^{Mlead} lead critical gap for the normal gap acceptance level (m)

G_{nt}^{Mlag} lag critical gap for the normal gap acceptance level (m)

ΔV_{nt}^{avg} relative speed of the average mainline speed with respect to the subject (m/sec)

ΔV_{nt}^{lead} relative speed of the lead vehicle with respect to the subject (m/sec)

d_{nt} remaining distance to the mandatory lane changing point (10m)

ΔV_{nt}^{lag} relative speed of the lag vehicle with respect to the subject (m/sec)

a_{nt}^{lag} acceleration of the lag vehicle

v_n unobserved driver characteristics

$\varepsilon_{nt}^{Mlead} \sim N(0, \sigma_{Mlead}^2)$ and $\varepsilon_{nt}^{Mlag} \sim N(0, \sigma_{Mlag}^2)$

ε_{nt}^{Mlead} and ε_{nt}^{Mlag} random error terms for the critical lead and lag gaps

σ_{Mlead} and σ_{Mlag} standard deviation for the critical lead and lag gaps

Initiate Forced Merging Level

P_{nt}^F probability of initiating a forced merge

δ_{nt}^{hv} heavy lag vehicle dummy

v_n unobserved driver characteristics

Force Merging Gap Acceptance Level

G_{nt}^{Flead} lead critical gap for the forced gap acceptance level (m)

G_{nt}^{Flag} lag critical gap for the forced gap acceptance level (m)

ΔV_{nt}^{avg} relative speed of the average mainline speed with respect to the subject (m/sec)

ΔV_{nt}^{lead} relative speed of the lead vehicle with respect to the subject (m/sec)

d_{nt} remaining distance to the mandatory lane changing point (10m)

ΔV_{nt}^{lag} relative speed of the lag vehicle with respect to the subject (m/sec)

a_{nt}^{lag} acceleration of the lag vehicle

v_n unobserved driver characteristics

$\varepsilon_{nt}^{Flead} \sim N(0, \sigma_{Flead}^2)$ and $\varepsilon_{nt}^{Flag} \sim N(0, \sigma_{Flag}^2)$

ε_{nt}^{Flead} and ε_{nt}^{Flag} random error terms for the critical lead and lag gaps

σ_{Flead} and σ_{Flag} standard deviation for the critical lead and lag gaps

Appendix C:
Terms used in tabulation of estimation results

	Parameters	Variable	Explanation
Normal lead gap	Constant	1	constant in normal lead critical gap
	Max(0,average speed - subject speed)(m/sec)	$\text{Max}(0, \Delta V_{nt}^{avg})$	Max(0,relative speed of the average mainline speed-measured 150m downstream of the subject- with respect to the subject)
	Min(0,lead speed - subject speed) (m/sec)	$\text{Min}(0, \Delta V_{nt}^{lead})$	Min(0,relative speed of the lead vehicle with respect to the subject)
	remaining distance to MLC point (10 meters)	d_{nt}	remaining distance to the mandatory lane changing point
	remaining distance constant	1	constant in the function capturing effect of remaining distance
	$\alpha^{RemDisLead}$	v_n	unobserved driver characteristics in the remaining distance function
	σ^{MLead}	σ_{MLead}	standard deviation for the critical lead gap
	α^{MLead}	v_n	unobserved driver characteristics
Normal lag gap	Constant	1	constant in normal lag critical gap
	Max(0,lag speed - subject speed)(m/sec)	$\text{Max}(0, \Delta V_{nt}^{lag})$	Max(0,relative speed of the lag vehicle with respect to the subject)
	Min(0,lag speed - subject speed)(m/sec)	$\text{Min}(0, \Delta V_{nt}^{lag})$	Min(0,relative speed of the lag vehicle with respect to the subject)
	remaining distance to MLC point (10 meters)	d_{nt}	remaining distance to the mandatory lane changing point
	remaining distance constant	1	constant in the function capturing effect of remaining distance
	$\alpha^{RemDisLag}$	v_n	unobserved driver characteristics in the remaining distance function
	Max(0,acceleration of lag vehicle)(m/sec^2)	a_{nt}^{lag}	acceleration of the lag vehicle
	σ^{MLag}	σ_{MLag}	standard deviation for the critical lag gap
	α^{MLag}	v_n	unobserved driver characteristics

Forced lead gap	Constant *Max(0,average speed - subject speed)(m/sec)	1 Max(0, ΔV_{nt}^{avg})	constant in forced lead critical gap Max(0,relative speed of the average mainline speed- measured 150m downstream of the subject- with respect to the subject)
	*Min(0,lead speed - subject speed) (m/sec) *remaining distance to MLC point (10 meters) *remaining distance constant * $\alpha^{RemDisLead}$ σ^{FLoad} α^{FLoad}	Min(0, ΔV_{nt}^{lead}) d_{nt} 1 v_n σ_{FLoad} v_n	Min(0,relative speed of the lead vehicle with respect to the subject) remaining distance to the mandatory lane changing point constant in the function capturing effect of remaining distance unobserved driver characteristics in the remaining distance function standard deviation for the critical lead gap unobserved driver characteristics
Forced lag gap	Constant *Max(0,lag speed - subject speed)(m/sec)	1 Max(0, ΔV_{nt}^{lag})	constant in forced lag critical gap Max(0,relative speed of the lag vehicle with respect to the subject)
	*Min(0,lag speed - subject speed)(m/sec) *remaining distance to MLC point (10 meters) *remaining distance constant * $\alpha^{RemDisLag}$ *Max(0,acceleration of lag vehicle)(m/sec^2) σ^{FLag} α^{FLag}	Min(0, ΔV_{nt}^{lag}) d_{nt} 1 v_n a_{nt}^{lag} σ_{FLag} v_n	Min(0,relative speed of the lag vehicle with respect to the subject) remaining distance to the mandatory lane changing point constant in the function capturing effect of remaining distance unobserved driver characteristics in the remaining distance function acceleration of lag vehicle standard deviation for the critical lag gap unobserved driver characteristics

Initiate Force	Constant	1	constant in the logit model for initiate forced merging
	Heavy Lag Vehicle Dummy	δ_{nt}^{hv}	1: if the lag is a heavy vehicle 0: otherwise
	α^F	ν_n	Heavy vehicle classification as in trajectory data. unobserved driver characteristics

* same coefficient as in normal gap acceptance level

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