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Problem Set #3

Due October 10, 2001

1. Consider a model with consumers uniformly distributed on the interval $[0, 1]$. Two suppliers selling the same good are located at points a and $1 - b$ with $0 \leq a, b \leq \frac{1}{2}$. Their production costs per unit are c_1 and c_2, respectively. Consumers buy zero or one unit of the good. They receive zero utility if they don't buy the good and utility $v - p - tx^2$ if they buy the good from a firm at a distance of x from their location. Assume that the firms choose prices simultaneously, and that their objective is to maximize profits.

(a) Find the Nash equilibrium prices and profits in this model assuming that v is sufficiently large so that the equilibrium involves all consumers purchasing the good. How large can firm 1's cost disadvantage be if it does make positive profits in equilibrium?

(b) Suppose that before choosing prices the firms play a first period game where they simultaneously choose where to locate. Assume that the firms costs are equal, $c_1 = c_2 = c$. Show that in equilibrium the firms are maximally differentiated. What happens if firm 1 chooses its location before firm 2?

2. Consider a model of product differentiation along a line segment of length 1. Consumers are distributed uniformly along the line. They have unit demands with valuation $v - tx$ for the good at a distance x from their location. The cost of production is zero. Assume $v = t = 1$.

Assume that firm 1 has two products: one (call it L) located at the left endpoint of the line segment and the other (call it R) located at the right endpoint of the line segment. Firm 1 may charge distinct prices p_{1L} and p_{1R} for his two products. Firm 2 has only a single product which is also located at the right endpoint of the segment.

Assume that the firms compete by simultaneously choosing prices (p_{1L}, p_{1R}, p_2) in a one shot game. Find the equilibrium prices and show that the firms' profits are $\pi_1 = 1/8$ and $\pi_2 = 0$.

3. Consider the model of vertical differentiation discussed in class (and in section 7.5.1 of Tirole). Suppose that the firms' costs are higher than I assumed so that the equilibrium prices end up being such that some consumers do not buy the product. Write down the equations for demand when prices are such that the highest value consumers buy from firm H, some buy from firm L and some do not buy at all. Assuming that the best responses are always given by the first order conditions obtained by maximizing relative to these demands find the best response functions and solve for the Nash equilibrium. For what values of c do the equations you've written really give the Nash equilibrium of the game?

4. Firm 2 is known to have a constant marginal cost of c. Firm 1's cost is private information. Firm 2's prior is that c_1 is uniform on $[c - \sigma_c, c + \sigma_c]$.

Suppose also that when firms 1 and 2 set prices p_1 and p_2, consumers observe p_2 perfectly, but only get a noisy signal of p_1. Specifically, assume that when the price is p_1 consumers think the price is $\hat{p}_1 = p_1 + u$ where u is uniformly distributed on $[-\sigma_u, \sigma_u]$.

(a) Suppose σ_u is large. Find the Bayesian Nash equilibrium of the game where firms 1 and 2 simultaneously choose prices p_1 and p_2 and consumers buy from firm 1 if and only if $\hat{p}_1 < p_2$. How is the markup related to σ_u. Is the solution you've derived also correct if σ_u is smaller.

(b) Suppose that consumers had the option of paying s to observe p_1 perfectly. For what values of s would they choose not to conduct this extra price search.

(c) Suppose s is smaller than the value you found in part (b). What would you expect the equilibrium to look like?

5. Consider a market where two firms are located at the ends of a line segment of length one. Suppose there are two populations of consumers. Poor customers have utility $v - \alpha_p p_1 - \theta$ if they buy from firm 1 and $v - \alpha_p p_2 - (1-\theta)$ if they buy from firm 2. Rich customers have similar utilities, but with the parameter being α_r instead of α_p with $\alpha_r < \alpha_p$. Suppose that there are a unit mass of consumers in each population with θ being uniformly distributed on $[0, 1]$ within each population.

(a) Consider a standard game where firms 1 and 2 simultaneously choose p_1 and p_2. Assuming v is sufficiently large find the Nash equilibrium.

(b) Now suppose that firms engage in the practice of making some portion of the price of the good unobservable, e.g. the good is sold without a warranty or without a useful added feature than can can not be purchased elsewhere. Suppose that, for example, a poor consumer now receives utility $v - \alpha_p p_1 - \theta$ if he purchases the good with the add on and utility $v - \alpha_p p_1 - \theta - w$ if he purchases the good without the add on. Assume that there is no extra cost of providing the add on.

Consider a game where firms 1 and 2 simultaneously post prices p_1 and p_2 for the good without the add on. Consumers then choose which store to visit, and once at the store are able to observe the price with and without the add on. (Assume if you want that once they're at the store they don't have the opportunity to go to the other store. It doesn't matter.)

What prices will the firms charge for the good with the add on once consumers arrive at the store?

(c) Suppose α_r and α_p are sufficiently close together so that in equilibrium all customers buy the add on. What is the equilibrium price in this game? How does the presence of the add on affect equilibrium profits?

(d) Suppose α_r and α_p are farther apart and only rich customers buy the add on. What is the equilibrium price in this game? How does the presence of the add on affect equilibrium profits? How do the answers in parts (c) and (d) compare?

Describe informally why having the "hidden" add on price can affect prices even though the true prices are perfectly anticipated by consumers.