14.271 Handout 2 Cournot and Bertrand

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This handout introduces the simplest oligopoly models - Cournot and Bertrand.

1 Bertrand Equilibrium

Imagine a duopoly of two firms who play a one-shot game of setting prices (p_1, p_2) with no capacity constraints and constant, and equal marginal costs (c_1,c_2, $c_1 = c_2 = c$). Demand is given by:

$$
\begin{aligned}
D_i(p_i, p_j) &= D(p_i) \text{ if } p_i < p_j \\
&= 0 \qquad \text{if } p_i > p_j \\
&= \frac{D(p_i)}{2} \text{ if } p_i = p_j
\end{aligned}
$$

i.e. the last represents an arbitrary rule for splitting demand in the case of equal prices. Firms supply the demanded volume given their prices and take the price of the other firm as given in equilibrium.

It is straightforward to show that the only equilibrium in this game is $p_1 = p_2 = c$. The method of proof (which I'll go through in class) is to rule out all of the possible alternatives.

Three features of the "Bertrand Paradox":

1. Prices are at marginal cost of one of the firms
2. If marginal costs are different, one firm does not produce and the other makes profits

3. If marginal costs are equal, neither firm makes profits

4. If fixed costs, we would end up with monopoly

How do we overcome the Paradox?

1. Product differentiation

2. Capacity constraints (Edgeworth), Kreps-Scheinkman

3. Timing

4. Collusion

2 Cournot Equilibrium

Firms compete is quantities rather than prices. Total production determines the market price. Firms take the quantities of their rival as given. Let's assume constant marginal cost

$$\max_{q_i} q_i \left[P\left(q_i + \sum_{j \neq i} q_j \right) - c_i \right] - F_i$$

$$P'\left(q_i + \sum_{j \neq i} q_j \right) q_i - P\left(q_i + \sum_{j \neq i} q_j \right) - c_i = 0$$

Given an exact form of demand this can be re-arranged to give a reaction function for firm i

$$q_i^* = R(q_j)$$

Reaction functions slope down as long as marginal profit of firm i declines with q_j.

Rearrange the FOC to get

$$\frac{P(Q) - c_i}{P(Q)} = \frac{q_i/Q}{\epsilon}$$

which can be manipulated to give

$$\frac{P(Q) - \sum s_i.c_i}{P(Q)} = \frac{\sum s_i^2}{\epsilon}$$

where s_i is the market share of firm i. $\sum s_i^2$ is the HHI (Herfindahl Index) which forms the basis of the first stage of almost all merger analysis, for dubious reasons!

Implications:

1. similarity and difference to monopoly Lerner Index
2. for inelastic demand Cournot equilibrium does not exist
3. avoid Bertrand paradox
4. prices are above cost so *allocatively inefficient*
5. production can be inefficiently distributed so *productively inefficient* (X-Ref Farrell and Shapiro (AER, 1990))
6. symmetric case : as $n \to \infty, P \to c$

3 Strategic Substitutes and Complements

These are vital terms, which Glenn will discuss further.

$$\begin{aligned} \text{STRATEGIC SUBSTITUTES} &\quad \leftrightarrow \quad \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} < 0 \\ \text{STRATEGIC COMPLEMENTS} &\quad \leftrightarrow \quad \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} > 0 \end{aligned}$$

where a is the strategic variable chosen by the firms (prices in Bertrand, quantities in Cournot). The reaction curves for strategic complements slope upwards, for substitutes downwards.