1] F.W., 8.012 Cl. Mech. ①

web page web.mit.edu/8.012/www

- contact info.
- calendar
- pb. sets + answers
- sample exams; answers
- "lecture notes"

Maria Springer 4-352

Dan Kleppner
Scott Burles

recitations start next week

books: K.+K. - primary text, only required. problems
BCG - more leisurely exposition of ~~a subset~~ basic material

also useful: How Things Work www.wiley.com/college/howthingswork

my "office hours"

questions during lecture / break

problem sets. due W ~~at~~ 9:30 AM - boxes | 1st is posted due Sept. 18

~~we~~ ~~you should get them back promptly~~
answers will be posted right away
you should get them back promptly

special problems... same procedure (note typo) 17 → 18

~~policy on~~ grading .3, .15, .15, .25, .15
HW Q1 Q2 lab
Oct. 10 Nov. 14

~~objections~~ queries: in writing, ~~within 2 days~~ next ~~re~~ available recitation

policy on collaboration

lab ⟶ Rosenson

2]

scope and limits of classical mechanics

motion due to given forces.

some general rules about forces; some examples { gravity, springs, pulleys, ropes, " rigidity, constraints, friction }

[other courses for details of what the fundamental forces are]

~~fantastic~~ extraordinary richness of phenomena

orbital mechanics: recent results { sensitivity to initial conditions, Venus' spin }

Euler disc

rattleback

break ~~~

limits:

large velocities - special relativity
large distances } general relativity
very strong gravity }

fields, radiation
very small distances, structure of matter - quantum mechanics

BUT:
these builds on classical mechanics

p, E, L, symmetry. m

(x F!)

3) Elementary Concepts of CM

model of space:

triples of real numbers
(∞ in all directions: locality)

Euclidean distance. homogeneous and isotropic

to (x_1, x_2, x_3)
(y_1, y_2, y_3) $\longrightarrow \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + (x_3-y_3)^2}$

model of time

real #s.
"intervals" only (homogeneous)

models of matter

point masses

→ ~~fram~~ position, velocity, acceleration

vectors ($\vec{v}$ vs. speed)

→ vector, after fixing an origin

build up extended bodies from pts.!

→ intrinsic property. positive # × unit (gm., kg.)

we'll discuss its measurement + use next time

not weight : astronauts

(vs. supermarket)

0th law : Conservation of mass - for pt. mass, no change

build up + break apart bodies!

~~Central Thought:~~

~~4) The Primacy of Acceleration (Newton's 1st law)~~

~~(not velocity)~~

~~bicy~~

Sept. 5

4) Newton's 1st Law : "Natural Motion"

constant velocity { bicycling, canoeing, ice-skating, astronautics }

Galileo's ship ; Earth in space! (rotation + revolution)

vs. : { friction, bacteria }

5) Newton's 2^{nd} Law

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} \quad \left(\vec{F} = \frac{d}{dt}(m\vec{v}) \right)$$

empty? (defining force)

character of forces

must have a nearby source

simple dependence ~~on~~ on mass { gravity: force ∝ m (so just universal $\vec{a}$!)

all others*: force independent of m

walking chain demo.

6) Examples of forces

(better form for: subsystems, relativity, angular analogue)

near-Earth gravity

$$\vec{F} = m\vec{g} \quad (= m\vec{a})$$

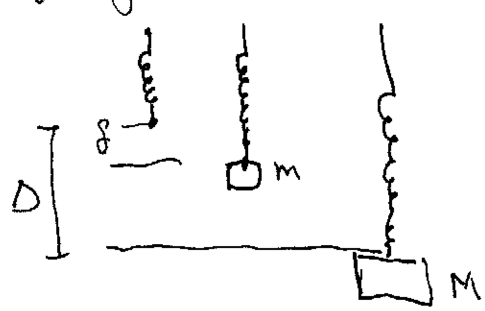
$$\vec{g} = 32\ \text{ft/sec}^2 \text{ or } 980\ \text{cm/sec}^2 \text{ down}$$

spring

$$\vec{F} = -k\vec{x} \qquad \vec{x} = \text{displacement from equilibrium}$$

(Hooke's law) $\quad k$ = spring constant

"weighing" mass



Force diagram: $\uparrow k\delta$, $\downarrow mg$

$$m = \frac{k\delta}{g}\ ; \quad M = \frac{k\Delta}{g},$$

$$m/M = \delta/\Delta$$

other ways ~~of~~ to weigh: balance, collisions

they must all turn out consistent!

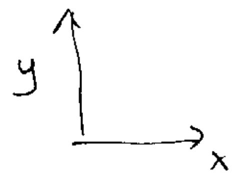
7] Examples of motions

a) constant acceleration (projectiles)

$$\ddot{x} = 0$$

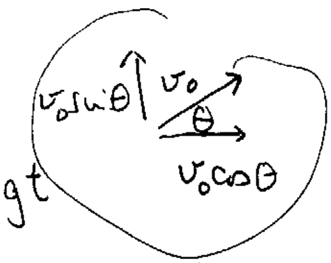
$$\ddot{y} = -g$$

(functions of t)



$$\dot{x} = c_1 \qquad \dot{y} = c_2 - gt$$

$$= v_{0x} \qquad\qquad = v_{0y} - gt$$



$$x = x_0 + v_{0x} t$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

Problem: given magnitude of initial velocity, determine angle for maximum range

Sol^n: Choose $x_0 = y_0 = 0$, $v_{0x} = v\cos\theta$, $v_{0y} = v\sin\theta$

$$x = v\cos\theta\, t$$

$$y = v\sin\theta\, t - \frac{1}{2} g t^2$$

$$\text{range} = v\cos\theta\, t_f$$

$$0 = v\sin\theta\, t_f - \frac{1}{2} g t_f^2 \Rightarrow t_f = \frac{2v\sin\theta}{g}$$

$$\text{range} = \frac{2v^2}{g}\sin\theta\cos\theta = \frac{v^2}{g}\sin 2\theta \Rightarrow \theta = \pi/4 \text{ for max.}$$

b) uniform circular motion

$$\theta = \omega t$$

$$x = R\cos\omega t$$
$$y = R\sin\omega t$$

$$\dot{x} = -R\omega\sin\omega t$$
$$\dot{y} = R\omega\cos\omega t$$
$$\ddot{x} = -R\omega^2\cos\omega t = -\omega^2 x$$
$$\ddot{y} = -R\omega^2\sin\omega t = -\omega^2 y$$

requires force pointing radially inward

~ break

8) Brief mathematical interlude: vectors

quantity with magnitude and direction

examples: position relative to a chosen origin

~~velocity~~
displacement
velocity
acceleration

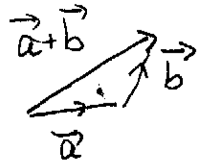
} different units!

After choosing axes, a vector can be written as a triple of numbers (with units) - the coordinates of its end-point

Notations: $\vec{v}$; (v_1, v_2, v_3) ; $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$; (v_x, v_y, v_z) ;

$\mathbf{v}$ (boldface), v

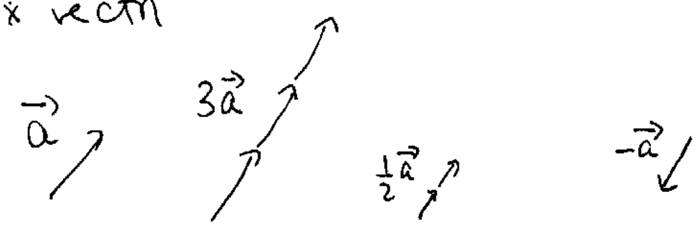
vector addition



~~Newton 2~~.

addendum to Newton 2: forces from different sources add as vectors!

scalar × vector



$$\vec{a} + (\overrightarrow{-a}) = 0$$

$$(\vec{a}+\vec{b}) + \vec{c} = \vec{a} + (\vec{b}+\vec{c})$$

$$\lambda(\vec{a}+\vec{b}) = \lambda\vec{a} + \lambda\vec{b}$$

If written as triples, just $\left\{\begin{matrix}\text{add}\\ \text{multiply}\end{matrix}\right.$ each piece independently

interpretation of components:

$$\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$$

$$(\text{or } v_1\hat{x} + v_2\hat{y} + v_3\hat{z})$$

unit vector in $\vec{x}$ direction

magnitude

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Sept. 10

you can take derivatives of vectors that depend on time, to get other vectors.

dot product

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

$$= |\vec{v}|\,|\vec{w}| \cos \angle(\vec{v}, \vec{w})$$

~~see any text!~~

$$|\vec{v} + \vec{w}|^2 = (\vec{v} + \vec{w}) \cdot (\vec{v} + \vec{w}) = |\vec{v}|^2 + |\vec{w}|^2 + 2\,\vec{v} \cdot \vec{w}$$

"law of cosines"

$$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$$

$$\vec{v} \cdot (\vec{w} + \vec{x}) = \vec{v} \cdot \vec{w} + \vec{v} \cdot \vec{x}$$

$\vdots$

$$\vec{v} \cdot \vec{v} = |\vec{v}|^2$$

$$\vec{v} \cdot \vec{w} = 0 \iff \vec{v} \perp \vec{w}$$

example: circular motion

$$\vec{r}(t) = (R\cos\omega t,\ R\sin\omega t,\ 0)$$

$$\vec{v}(t) = (-R\omega\sin\omega t,\ R\omega\cos\omega t,\ 0) \qquad |\vec{v}| = R\omega$$

$$\vec{a}(t) = (-R\omega^2\cos\omega t,\ -R\omega^2\sin\omega t,\ 0) \qquad |\vec{a}| = R\omega^2$$

$$\vec{r} \cdot \vec{v} = 0\ ; \quad \vec{a} = -\omega^2 \vec{r}$$

why? $\vec{r}^{\,2} = R^2 \Rightarrow 2\,\vec{r} \cdot \frac{d\vec{r}}{dt} = 0$ $\qquad \vec{v} \cdot \vec{a} = 0$ 9/12

9) Inertial and non-inertial frames

Postulate: It is possible to set up observers (or, in practice, apparatus) with identical clocks, rulers, have them "stand still" and communicate instantaneously (to synchronize watches). Their measurements will conform to Newton's laws. Inertial frame

If you happen not to be working in an inertial frame, there are extra "fictitious" forces

$\uparrow kx$
$\downarrow mg$

$$m g_{grav.} - kx = m\omega^2 R$$

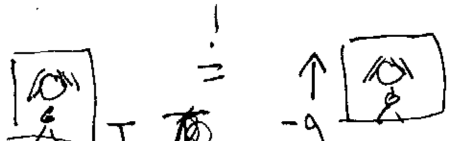
$$m(\underbrace{g_{grav.} - \omega^2 R}_{g_{eff.}}) = kx$$

Note "fictitious" forces are $\propto m$ (that was the *)

10) Einstein: ~~[illegible]~~ "happiest thought of my life"

effect of gravity equivalent to accelerated frame

mg $\downarrow$ (force)



I $\overset{!}{=}$ II

$-g$ $\uparrow$ (acceleration)

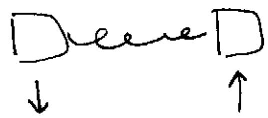
I: $m\vec{a}_{observed} = m\vec{g}$; $\vec{a}_{observed} = \vec{g}$

II: $m\vec{a}_{total} = 0$ $\quad \vec{a}_{total} = \vec{a}_{observed} + (-\vec{g})$; $\vec{a}_{observed} = \vec{g}$

Advantage of II: explains <u>why</u> $\vec{F} \propto m$!

10] Mach's Principle

Consider an isolated whirling pair

 vs. stationary 

From the length of the spring you can tell who's ~~they're~~ rotating (accelerating).

Mach: Accelerated relative to what?

fixed stars!?? - not in existing physics

~ 17 Sept.

11] Units and Dimensional Analysis

L, T, M

in any valid eq^n, "dimensions" ~~units~~ must match (choice of units can't matter)

e.g. $[a] = L/T^2$

$[s] = L$

uniform motion (from rest): $\cancel{[s]} = L$

$$s = f(a, t)$$

change unit of length $\to \times\lambda$, time $\to \times\varepsilon$

$$\lambda s = f\left(\frac{\lambda}{\varepsilon^2} a, \varepsilon t\right)$$

uniform motion with initial velocity

$$\left(s = vt - \frac{1}{2} a t^2\right)$$

$$\lambda s = f\left(\frac{\lambda}{\varepsilon^2} a, \frac{\lambda}{\varepsilon} v, \varepsilon t\right)$$

$$\varepsilon = 1/t, \quad \lambda = 1/at^2$$

$$\frac{s}{at^2} = f\left(1, \frac{v}{at}, 1\right)$$

in fact, $\frac{s}{at^2} = \frac{v}{at} - \frac{1}{2}$

(to 12)

choose $\varepsilon = 1/t$, $\lambda = \frac{\varepsilon^2}{a} = \frac{1}{at^2}$

$$\frac{s}{at^2} = f(1,1) = \cancel{\text{Force}}\ \text{Number}$$

(to $\parallel a$)

uniform circular motion

$[F] = ML/T^2$

$[m] = M$, $[R] = L$, $[\omega] = 1/T$

$$F = f(m, R, \omega)$$

$$F\,\frac{\lambda\eta}{\varepsilon^2} = f(\lambda m, \eta R, \omega/\varepsilon)$$

with $\lambda = \frac{1}{m}$, $\eta = \frac{1}{R}$, $\varepsilon = \omega$

$$\frac{F}{mR\omega^2} = f(1,1,1) = \text{number}$$

← wind tunnels, ...

~ 9/12

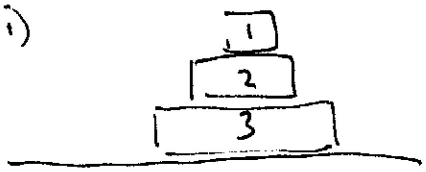
12) Newton's 3rd Law

$\overrightarrow{\text{action}} = (-)\,\overrightarrow{\text{reaction}}$

deep applications: cons. of momentum
extended bodies

example 1:

i)



Forces on 1: $\vec{F}_{12} + m_1\vec{g} = 0$

on 2: $\vec{F}_{21} + \vec{F}_{23} + m_2\vec{g} = 0$

on 3: $\vec{F}_{3f} + \vec{F}_{32} + m_3\vec{g} = 0$

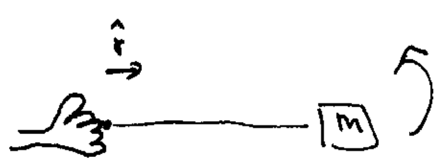
on floor: $\vec{F}_{f3}$ $(= M\vec{a})$

↑ very big!

$$
\begin{aligned}
\vec{F}_{f3} = -\vec{F}_{3f} &= \vec{F}_{32} + m_3\vec{g} \\
&= -\vec{F}_{23} + m_3\vec{g} \\
&= \vec{F}_{21} + m_2\vec{g} + m_3\vec{g} \\
&= -\vec{F}_{12} + m_2\vec{g} + m_3\vec{g} \\
&= m_1\vec{g} + m_2\vec{g} + m_3\vec{g} = (m_1+m_2+m_3)\vec{g}
\end{aligned}
$$

= W.I.S.B.
(What It Should Be!)

ii)



$$\vec{F}_{\text{block, rope}} = -mR\omega^2\,\hat{r}$$

$$\vec{F}_{\text{rope, hand}} + \vec{F}_{\text{rope, block}} \propto m_{\text{rope}} \approx 0 \quad \left(\begin{matrix}\text{simple}\\ \text{case}\end{matrix}\right)$$

$$\vec{F}_{\text{hand, rope}} = -\vec{F}_{\text{rope, hand}} \approx \vec{F}_{\text{rope, block}} = -\vec{F}_{\text{rope, block}} = mR\omega^2\hat{r}$$

"centrifugal force"

13) Constraint forces

Bodies supply them 'automatically' to keep matter from penetrating

(~Pauli exclusion principle)

14) Friction forces

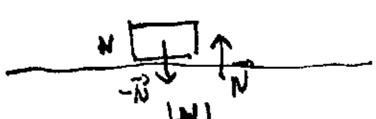
very difficult to explain from 1^{st} principles

(also a very important practical subject)

- lubrication, ball bearings, ⋮

+ salting, roughening of tracks, treads, ⋮

frequently useful phenomenological description:



static: $\vec F_{fr.}$ tangential

$$|\vec F| \le \mu_s |\vec N|$$

dynamics

$\vec F_{fr} = \mu_d |\vec N|$, directed against the motion (so $\vec F_{fr.} \cdot \vec v_{rel.} < 0$)

$$\vec F_{fr.} = -\hat v_{rel.} \mu_d |\vec N| \qquad \cancel{\vec F = -\hat v_{real} \mu_s |\vec N|}$$

$$= -\frac{\vec v_{rel.}}{|\vec v_{rel.}|} \mu_d |\vec N|$$

9/12

<u>example</u>: rolling without slipping (train)

It's static friction that moves trains!