MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LABORATORY FOR INFORMATION AND DECISION SYSTEMS
CAMBRIDGE, MA 02139

STATUS REPORT #5

ON

NONLINEAR AND ADAPTIVE CONTROL

NASA GRANT NAG 2-297
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SUBMITTED TO:

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SUMMARY

This status report overviews the research on Nonlinear and Adaptive Control carried out at the MIT Laboratory for Information and Decision Systems under NASA grant NAG 2-297 for the time period January 1, 1987 to 30 June 1987. Participating faculty were Professors Gunter Stein, Lena Valavani, and Michael Athans (principal investigator). The grant monitors are Dr. George Meyer (NASA Ames Research Center) and Mr. Jarrell R. Elliott (NASA Langley Research Center).

The primary thrust of the research is to conduct fundamental research in the theories and methodologies for designing complex high-performance multivariable feedback control systems; and to conduct feasibility studies in application areas of interest to our NASA sponsors that point out advantages and shortcomings of available control system design methodologies.

The theoretical research overviewed in this status report is focused on adaptive and nonlinear systems. On-going feasibility studies during this reporting period relate to the multivariable control of twin-lift helicopter systems. Significant progress in all areas has been accomplished during this reporting period.

1. ADAPTIVE CONTROL THEORY

Following the completion of the Ph.D. thesis by D. Orlicki's Ph.D. thesis, publication [3], our thinking about adaptive control has changed in a significant manner. In this research we were able to develop new algorithms, of the MRAC type, which have guaranteed local stability properties in the presence of unmodeled dynamics and unmeasurable disturbances. To prevent the instability of the classical MRAC schemes, we have used the concept of *intermittent adaptation;* loosely speaking, this concept prevents the updating of uncertain plant parameters whenever the identification information is of dubious quality due to the simultaneous presence of unmodeled dynamics and disturbances which cannot be measured. Thus, we only adapt whenever we are sure that the real- time signals contain relevant information.

It is a highly nontrivial manner to decide, in real-time, when to adapt and when to (temporarily) stop the adaptation. The new algorithms involve the real-time monitoring of easily measurable signals, and require the capability of computing discrete Fast Fourier transforms (DFFT's) for those signals. Intermittent adaptation is implemented by blending the real-time spectral information generated by the DFFT's with variants of the model reference algorithms. The algorithms can be implemented through the use of a dead-zone nonlinearity whose width changes in real time based upon the DFFT calculations. To the best of our knowledge, this is the first time that an adaptive control algorithm has been developed that requires extensive spectral calculations so as to guarantee stability-robustness.

Although our intermittent adaptation algorithms represent an advance in the state of the art, and undoubtedly will become controversial because of their increased computational requirements, nonetheless the most important by-product of that research is a detailed appreciation of the immense complexity of the adaptive control problem. *In point of fact, we have become convinced that new and different approaches to the robust adaptive control problem must be developed.* There are simply too many hard questions, only tangentially related to adaptive control, that must be posed first, and of course answered, before we can proceed with confidence to using adaptive control to regulate physical systems, and especially multivariable ones. We briefly outline these questions that we have been investigating in the sequel.

Robust Adaptive Identification in the Time and Frequency Domains.

Research Goals. Classical adaptive control algorithms use a postulated dynamic system order, i.e. a transfer function with fixed numbers of poles and zeros, and then use (explicit or implicit) identification to improve the prior estimate of the model uncertain parameters. In robust adaptive control this is necessary, but by no means sufficient. What is required is the development of a new class of adaptive identification algorithms which, with a finite amount of data, produce not only a better nominal model, but in addition generate a bound in the frequency domain that captures the presence of possible high-frequency model errors. Such bounding of model errors in the frequency domain is required by all nonadaptive design methods so as to ensure stability-robustness by limiting the bandwidth of the closed-loop system. Such identification algorithms do not exist in the classical identification literature; such questions were not even posed. Thus, we believe that is essential to develop such algorithms and then to incorporate them in the adaptive control problem.

A major milestone along these lines has been completed with the publication of Richard LaMaire's doctoral thesis, under the supervision of Professors Valavani and Athans.

Research Methodology. We view the robust adaptive control problem as a combination of a robust identifier (estimator) and a robust control-law redesign algorithm. Current robust control design methodologies, such as the LQG/LTR methodology, require: 1) a nominal model, and 2) a frequency-domain bounding function on the modelling error associated with the nominal model. A new robust estimation technique, which we call a 'guaranteed' estimator, has been developed to provide these two pieces of information for a plant with unstructured uncertainty and an additive output disturbance. This guaranteed estimator uses parametric time-domain estimation techniques to identify a nominal model, and non-parametric frequency-domain estimation techniques to identify a frequency-domain bounding function on the modelling error. This bounding function is generated using discrete Fourier transforms (DFT's) of finite-length input/output data.

Several assumptions are required by the guaranteed estimator. In addition to a priori assumptions of the structure of the nominal model along with coarse, worst-case values of the parameters, we assume that the unmeasurable disturbance is bounded and that a magnitude bounding function on the Fourier transform of the disturbance is known. Further, we assume prior knowledge of a bounding function on the unstructured uncertainty of the plant relative to our choice of nominal model structure. These assumptions allow our time-domain estimator to be made robust to the effects of unstructured uncertainty and bounded disturbances. That is, our time-domain estimator updates the parameters of our nominal model only when there is good (uncorrupted) information. Similarly, the frequency-domain estimator, which has been developed, only updates the model and current bounding function on the modelling error when there is good information. In summary, the guaranteed estimator provides a nominal model plus a guaranteed bounding function, in the frequency-domain, as to how good the model is. Accuracy guarantees in the identifier part of the adaptive controller can be used by the control-law redesign part of the adaptive controller to ensure closed-loop stability, assuming the control-law is updated sufficiently slowly.

Recent Research Progress: All equations for both the time-domain and the frequency-domain plant identifierhave been developed. Also, in the frequency-domain, all equations that are used to compute the bound on the modeling error have been derived. The maximum possible effect, in the plant output, due to the unstructured uncertainty and the disturbance is computed using real-time DFTs of the input and the a priori assumed bound on the disturbance. The identification algorithm only updates the parameter estimates when the output error between the actual and predicted plant output is greater than the maximum possible error signal due to the unstructured uncertainty and the disturbance.

Additional issues concerning the guaranteed estimator relate to the fact that we are estimating a continuous-time plant with a discrete-time identifier. For example, the choice of sampling period for the estimator limits both the bandwidth of the adaptive control system as well as the accuracy of the estimator at high frequencies.

All the equations necessary to simulate the performance of these identification algorithms were coded and debugged. Because of the extensive real-time spectral calculations, we decided to use the CYBER supercomputer at Princeton which is available for use by the MIT community at no cost for CPU time. Numerical examples which are simple enough to demonstrate the ideas yet rich enough to capture the potential pitfalls have been designed and simulated.

The simulation results indicate that for the systems tested the time-domain identification algorithm did not work very well. On the other hand, the frequency-domain algorithms worked much better.

In closed-loop identification simulations the richness of the command signal was often not sufficient to excite the plant dynamics so that the identification algorithms could work properly. For this reason, we developed an "intelligent"scheme which would monitor the progress of the identification algorithm and inject probing signals at the appropriate frequencies at the plant input so as to enhance identification. Of course, this would deteriorate (temporarily) performance since a disturbance was injected intentionally in the feedback loop. Better identification, accompanied by higher loop-gains and bandwidths, would improve overall command-following and disturbance-rejection performance after the probing signals were terminated.

The algorithms require extensive real time computations. For sluggish plants the computational requirements are not severe. However, in order to identify and control plants with very lightly damped dynamics truly extensive CPU requirements exist. For example, in our simulation studies involving a second order plant with lightly damped poles the Cyber 205 supercomputer was too slow, for real time control, by a factor of two so as to achieve a closed-loop bandwidth of 5 rad/sec.

These findings cast a tone of pessimism, with respect to CPU requirements, in using real-time identification and high-performance adaptive control for typical aerospace plants that are characterized by lightly damped dynamics and dominant high-frequency modeling errors. On the other hand, parallel computer architectures can be exploited in this class of algorithms. Thus, more research along these lines is required.

Documentation Status: Full documentation on this research is now available with the publication of the Ph.D. thesis of Mr. LaMaire [23], and an ACC paper [24]. Journal papers are in preparation.

Best Nonadaptive Compensator Design for Performance-Robustness.

Research Goals. Our research to date has pinpointed the need for a good initial guess for an adaptive compensator, whose parameters are then updated, in real-time, by the adaptive algorithm. We are developing techniques that design the best (from the viewpoint of good command-following and disturbance-rejection) nonadaptive compensator for the given prior plant uncertainty information. It is yet unknown how to design such nonadaptive compensators that exhibit this property of "best" performance-robustness.

Such a robust design technique will prove useful in a number of ways. First, it will yield a systematic procedure for designing feedback systems for uncertain plants with performance guarantees. Thus, the feedback loop will be guaranteed to be stable and, in addition, will meet minimum performance specifications for all possible plant perturbations. Second, the solution of this robust design problem will also enable us to quantitatively address one of the most fundamental questions in adaptive control: *what are the performance benefits of adaptive control?* While much attention has been paid to the development of many specific adaptive algorithms, very little consideration has been given to this issue at the heart of the adaptive control problem. Practical adaptive systems rely upon external persistently exciting signals (to ensure good identification), slow sampling (which helps stability-robustness to unmodeled high frequency dynamics) in addition to extensive real-time computation (to provide safety nets and turn-off the adaptive algorithm when it exhibits instability). All these "gimmicks" degrade command-following and disturbance-rejection performance and tend to neutralize the hoped-for benefits of an adaptive compensator. In light of these circumstances it is imperative that the decision to use adaptive control, for a real engineering application, must be based upon a quantitative assessment of costs and benefits. One of the main goals of this research project is to quantitatively evaluate the performance benefits of an adaptive control system vis-a-vis the best fixed-parameter nonadaptive compensator for a linear plant. Note that for a nonlinear system the parameters of such compensators can be fine-tuned using gain scheduling.

While a performance-robust design methodology will be useful in its own right, the positive implications for adaptive control should be clear. It is conjectured that such a fixed-parameter compensator design technique will form the basis of a practical robust adaptive control system. Compensator redesign will take place infrequently (when compared to the digital sampling rate) using information from a reliable system identification scheme.

Research Methodology. In his doctoral research Mr. David Milich, under the supervision of Professors Athans and Valavani, has examined design techniques which will yield the "best" fixed-parameter nonadaptive compensator for a plant characterized by significant structured, as well as unstructured, uncertainty. The "best" compensator is defined as the one that meets the posed performance (i.e. command-following, disturbance-rejection, insensitivity to sensor noise) specifications and stability-robustness over the entire range of possible plants.

Some of the key issues, and severe difficulties, in the design process have been identified. Conditions for stability-robustness and performance-robustness in the presence of significant structured and unstructured uncertainty have been developed. An a-priori magnitude bound, as a function of frequency, on the unstructured uncertainty is assumed known. In order to reduce the conservatism of the stability and performance conditions with respect to the structured uncertainty, directional information (in the complex plane) associated with the plant-parameter variations is exploited. Unfortunately, this directional information turns out to be closely associated with the so-called *Real-μ problem*, i.e. the problem of calculating structured singular values for real -- rather than complex-valued -- plant modeling errors; this problem has been studied by Doyle and is generically very difficult. Its solution appears to be beyond the state of the art, at least in the near future.

The only reasonable alternative appears to be to translate the prior knowledge of structured uncertainty into an equivalent unstructured uncertainty. It is still a very hard problem to design a compensator with guaranteed performance characteristics in the presence of these modeling errors. We have transformed the problem into what Doyle calls the *μ-synthesis problem,* which unfortunately is also very hard to solve. From a technical point of view, the *μ-synthesis problem* involves a blend a co-prime factorizations, structured singular value theory, and H^∞-optimization.

Recent Research Progress: A promising theoretical and algorithmic approach to the solution of the *μ-synthesis problem* is being developed. The theory utilizes the use of Hankel norms in approximating L^∞ functions using H^∞ functions. Certain procedures have been developed which would indicate whether or not the posed performance specifications are "too tight" for the level of modeling error present. In this case, the control system designer will have to relax the performance specifications, typically expressed as bounds on the sensitivity function maximum singular value, over some frequency ranges.

Much more analytical and algorithmic research is needed to evaluate the advantages and shortcomings of this methodology.

Another recent result relates to the fact that the "best robust compensator" will have to be infinite-dimensional. Thus, from a pragmatic point of view, *we may have to use a very high-order dynamic compensator in order to "squeeze-out" the best possible performance from a highly uncertain plant.* This tentative conclusion raises some serious questions regarding the implementation of this compensator. Another critical issue is related with the use of this compensator within an adaptive control context. Presumably the posterior information generated by the real-time identification algorithm will be used to update the compensator parameters. If the "best" compensator is of very high order, much larger than the order of the plant, then new adaptive parameter-update algorithms will have to be developed. We speculate that the computational complexity of these, as yet undefined, adaptive algorithms will be far greater than those suggested in the past -- e.g. model reference adaptive control.

Documentation Status. Only partial documentation exists [20] for this research. The Ph.D. thesis of Mr. Milich is scheduled for completion in the winter of 1987-1988.

2. NONLINEAR CONTROL SYSTEMS.

A significant portion of the grant resources is devoted to the development of methodologies, theories, and design techniques that will advance the state of the art in multivariable control system design. During this reporting period we have made some significant progress in this area.

Systems with Multiple Saturation Nonlinearities.

Research Goals. The goal of this project is to develop new theory and methodologies for the analysis and synthesis of linear multivariable control systems that contain several saturation nonlinearities. We seek to develop modifications to the purely linear design methodologies, such as LQR, LQG, LQG/LTR, and H-∞ optimization, to explicitly take into account the problems associated with multiple saturation (magnitude and/or rate) nonlinearities in the control actuation channels.

There are several problems that can arise when a control system that has many saturation nonlinearities is designed by purely linear means. The most serious problem is that of stability; it is possible for a control system, which is stable when the actuators are not saturated, to become unstable when one or more controls become saturated. Such instability can happen if large command signals are applied or disturbances of large magnitude are present. The second class of problems are associated with performance. If the saturation limits are ignored in the purely linear design phase, it may happen that large crossover frequencies are specified by the designer. The actuators may not be able to provide the gain necessary to attain the required bandwidths; also, rate-limiting may not allow the physical controls to change as rapidly as a purely linear design demands. Hence, redesign must take place.

The difficulty is that there does not exist, as yet, a systematic methodology which will help the designer specify rational bandwidths consistent with the different magnitude and/or rate saturation limits. Also, transient performance suffers when saturation nonlinearities interact with integrators in the control loop; the so-called reset windup phenomenon. Reset windup keeps the nonlinearities saturated longer than necessary, and as a consequence transient responses are characterized by large overshoots.

Research Methodology. What we plan to do is to examine these stability and performance problems associated with multiple saturations in a unified manner. Most of the existing theory is either too complex or incomplete. It is possible to deal with saturation nonlinearities using optimal control theory, and derive necessary conditions using Pontryagin's maximum principle; unfortunately, this only provides us with open-loop solutions through the solution of complex two point boundary value problems for high-order plants. Most other approaches are based upon Lyapunov theory, which does not capture in a straightforward way the input-output behavior necessary for design.

This research is carried out by Mr. Petros Kapasouris as part of his Ph.D. thesis, under the supervision of Professor Athans. In our research to date, we have focused attention to the changes in the direction of the control signals that are induced by the saturating elements. The fact that we cannot deliver the "correct" magnitude should not produce any unpleasant effects except that the settling times should increase. What we want is to avoid is the highly oscillatory transients and unstable behavior. This appears to be more related to the changes in the directions of the control vectors.

Recent Research Progress. During this reporting period we were able to come up with simple, yet elegant, ways of attacking the problem. The problem is different depending on whether or not the controlled plant is stable or unstable.

For stable open-loop plants the concept is to have the command-following response of the MIMO system mimick, to the extent possible by the presence of the saturation nonlinearities, the transient response of the linear system. The idea is to monitor and adjust in real-time the tracking error vector, which acts as the input to the dynamic compensator so that the compensator never generates signals that will drive the system into saturation. In this manner, we are able to maintain the necessary "directional" properties of the design which are required to carry-out the *approximate plant inversion* and substitution of the "desired" dynamics in the forward loop. Note that, if we allow arbitrary saturation of the nonlinearities, the directional properties of the linear design become distorted; as a consequence, we destroy the approximate plant inversion property of our compensator. The method under study controls the signal levels so that the system always works in the linear region. *This key idea appears to solve all at once the undesirable stability, performance, and reset-windup issues.* Of course, as to be expected, the speed of response (rise time, settling time etc) to commands of large magnitude is reduced compared to the design without saturation nonlinearities.

In order to implement this scheme one has to execute some off-line and some on-line computations. The off-line computations require the computation of the boundary of a convex compact set, with several nondifferentiable points. This set is defined over a Euclidean space whose dimension is that of the dynamic compensator. The on-line computations calculate a (pseudo)gradient vector to the boundary of the set, and adjust a scalar which reduces the instanteneous size of the tracking error vector. This causes the dynamic compensator to generate a control signal that never saturates.

We have used some linearized dynamics of the F-8 aircraft, to which we added a fictitious flaperon, to test these ideas. In this setting we command changes in both the flight path and pitch angles; these are to be controlled using the elevator and the flaperon. In this set of transient simulations the results show excellent nonlinear responses.

Similar ideas can be used to handle rate saturation, and simultaneous magnitude and rate saturation.

For open-loop unstable plants, it is important to limit the set of initial states, disturbances and commands so that the system can be stabilized. Assuming that the system is at rest and that the disturbance environment is such that the system can be stabilized, then the problem is to limit in an intelligent manner the size of the command (reference) vector. This is accomplished by a method that modulates the size of the command vector so that the controls do not saturate; eventually, the full command vector is applied. The nature of the computations is similar as in the open-loop stable case. However, the dimension of the underlying sets is now much larger. We have used a model of the AFTI F-16 aircraft, which is open-loop unstable, to test the algorithm. As before, we are using the aircraft elevon and flaperon to control the pitch and flight path angles. Once more, the transient responses are excellent.

Documentation Status. Partial documentation of earlier research can be found in the paper by Kapasouris and Athans [5]. Full documentation will be found in Kapasouris' doctoral thesis, scheduled for completion in January 1988.

Gain Scheduled Control Systems

Gain scheduling is a common engineering method used to design controllers for systems with nonlinear and/or parameter varying dynamics. In the nonlinear case, the dynamics are linearized at several operating points, and a linear compensator is designed for each linearized plant. The parameters of the compensator are then interpolated, or scheduled, in between operating points, thus resulting in a global compensator. The procedure for linear parameter varying dynamics is identical to that above, except that the linearization is omitted.

Research Goals. Despite the lack of a sound theoretical foundation, gain scheduling has proven successfull in many engineering applications (e.g. jet engines, submarines, and aircraft). A goal of this project is to develop such a theoretical treatment of gain scheduled control systems. However, the ultimate goal is to use this analysis for the development of a complete and systematic gain scheduling design methodology. Given the success of current gain scheduled designs, such a development would prove very useful in better understanding and strengthening gain scheduled designs.

Research Methodology. Initial research has been directed at the linear parameter varying case in the doctoral research of Mr. Jeff Shamma, under the supervision of Professor Athans. An initial obstacle in the study of parameter varying, hence time varying, linear systems is that traditional linear time invariant analysis methods, in particular singular value loop shapes, are not immediately applicable. That is to say, if one designed a parameter varying compensator such that each "frozen parameter" design satisfied certain design specifications, there is no guarantee that the resulting time varying design will even be stable, let alone satisfy performance specifications. However, using input/output operator methods and conic sector stability results, some progress has been made in extending the notions of singular value loop shapes to time varying systems, thus significantly simplifying the analysis of closed loop feedback properties, e.g. sensitivity, robustness, etc.

Research was also conducted on how one should schedule the parameters of the compensator in between operating points. Initial results have shown that selection of the right parameterization of the compensator can yield an interpolation strategy which guarantees nominal stability in between operating points, *and* is apt to pick up any trends in design specifications which vary over the parameters. The key idea is that the design parameters, rather than the physical compensator parameters, are best suited for capturing the different specifications over the range of parameter variations. In particular, it has been shown that the LQG/LTR compensator is very well suited for such a parameterization since it provides enough a priori structure to the compensator to allow for a simple parameterization of compensator designs, while giving enough flexibility to satisfy design specifications.

Recent Research Progress. A great deal of effort was devoted into understanding the stability-robustness properties of linear time-varying systems due to unmodeled time-invariant dynamics. As mentioned above, gain-scheduled designs, with slow parameter variations, can be modeled as linear time-varying systems. Thus, it is important to develop sufficient conditions for stability in the presence of unmodeled dynamics. Such sufficient conditions for stability-robustness have been derived. They take the form of frequency-domain inequalities. It is noteworthy to mention that the bound on the size of unmodeled dynamics must be calculated along a line parallel to the $j\omega$-axis in the s-plane. This implies that in order to guarantee the stability-robustness of gain scheduled designs, the frozen point designs must have a certain degree of relative stability.

At present, we are investigating the problems that arise when the gains are scheduled on the basis of "slow" state or output variables, rather than exogeneous parameters.

Documentation Status. No documentation of this research is available as yet. Mr. Shamma's doctoral thesis is scheduled for completion in 1988.

3. FEASIBILITY STUDIES

As mentioned before a small portion of the resources of this grant are devoted to the design of multivariable control systems for aerospace systems that are of direct interest to our NASA sponsors. These feasibility studies serve as a means for understanding the strengths and weaknesses of the theoretical results developed under the auspices of this grant. During this reporting period we only had a single active project in this area, namely the control of twin-lift helicopter systems.

This project has been described in the previous status report. All results are now fully documented in the SM thesis of A. Rodriguez [14].

During this reporting period we received from NASA Langley Research Center linearized dynamics of the F-18 aircraft which include the capability for thrust vectoring. This aircraft will be used by NASA for *supermaneuverability* studies. At present, we are studying the aircraft dynamics to evaluate their proper utilization within the guidelines of this grant.

PEOPLE

Professor Lena Valavani was appointed to the AIAA Guidance and Control committee.

Mr. Richard LaMaire received his Ph.D. degree in June 1987. He joined the technical staff of ALPHATECH Inc.

Mr. Armando Rodriguez received his MS degree in June 1987. He is continuing his doctoral studies at MIT.

Mr. Petros Kapasouris visited NASA/LaRC in May 1987 to brief them on his doctoral resaerch. Professor Athans also visited NASA/LaRC in May 1987 where he gave two seminars.

PUBLICATIONS

The following publications have been supported in full or in part by NASA grant NAG 2-297 since its inception 1 June 1984. Copies of these publications have been transmitted to the grant monitors, NASA headquarters, and publications office as required.

1. D. M. Orlicki, L. Valavani, M. Athans, and G. Stein, "Adaptive Control with Variable Dead-Zone Nonlinearities," *Proc. American Control Conference,* San Diego, CA, June 1984, pp. 1893-1898.

2. J. N. Tsitsiklis and M. Athans,"Guaranteed Robustness Properties of Multivariable Nonlinear Stochastic Optimal Regulators," *IEEE Trans. on Automatic Control,* Vol. AC-29, August 1984, pp. 690-696.

3. D. M. Orlicki, " Model Reference Adaptive Control Systems using a Dead-Zone Nonlinearity," LIDS-TH-1455, Ph.D. Thesis, MIT, Dept. of EE&CS, Cambridge, Mass., April 1985.

4. D. B. Grunberg and M. Athans, "A Methodology for Designing Robust Multivariable Nonlinear Feedback Systems," *Proc. American Control Conference,* Boston, MA, June 1985, pp. 1588-1595.

5. P. Kapasouris and M. Athans, " Multivariable Control Systems with Saturating Actuators and Anti-Reset Windup Strategies," *Proc. American Control Conference,* Boston, MA, June 1985, pp. 1579-1584.

6. M. Bodson and M. Athans, " Multivariable Control of VTOL Aircraft for Shipboard Landing," *Proc. AIAA Guidance, Navigation and Control Conference,* Snowmass, CO, August 1985, pp. 473-481.

7. G. Stein and M. Athans, "The LQG/LTR Procedure for Multivariable Feedback Control Design," *IEEE Trans. on Automatic Control,* Vol. AC-32, No. 2, February 1987, pp. 105-114.

8. G. Stein, "Beyond Singular Values and Loop-Shapes," LIDS-P-1504, MIT, Cambridge, Mass., January 1986.

9. M. Athans, P. Kapasouris, E. Kappos, and H. A. Spang III, "Linear-Quadratic-Gaussian with Loop-Transfer-Recovery Methodology for the F-100 Engine," *AIAA Journal of Guidance, Control, and Dynamics,* Vol. 9, No.1, January 1986, pp. 45-52.

10. M. Athans, "A Tutorial on the LQG/LTR Method," *Proc. American Control Conference,* Seattle, Wash., June 1986, pp. 1289-1296.

11. W. H. Pfeil, M. Athans, and H. A. Spang III, "Multivariable Control of the GE T700 Engine using the LQG/LTR Design Methodology," *Proc. American Control Conference,* Seattle, Wash., June 1986, pp. 1297-1312.

12. W. W. Quinn, "Multivariable Control of a Forward Swept-Wing Aircraft", MS Thesis, LIDS-TH-1530, Dept. of EE&CS, MIT, Cambridge, Mass., January 1986.

13. A. Rodriguez, "Multivariable Control of Twin Lift Helicopter Systems using the LQG/LTR Design Methodology," MS Thesis, Dept. of EE&CS, MIT, Cambridge, Mass., May 1987.

14. A. Rodriguez and M. Athans, "Multivariable Control of a Twin Lift Helicopter System Using the LQG/LTR Design Methodology," *Proc. American Control Conference,* Seattle, Wash., June 1986, pp. 1325-1332.

15. D. B. Grunberg and M. Athans, "A Methodology for Designing Robust Nonlinear Control Systems," LIDS-P-1558, MIT, Cambridge, Mass., May 1986, (accepted for *1987 IFAC World Congress,* Munich, West Germany, July 1987).

16. M. Athans, G. Stein, and L. Valavani, "Status Report #2 on Nonlinear and Adaptive Control; NASA Grant NAG 2-297," MIT, Cambridge, Mass., December 31, 1985.

17. M. Athans, G. Stein, and L. Valavani, "Status Report #3 on Nonlinear and Adaptive Control; NASA Grant NAG 2-297," LIDS-SR-1561, MIT, Cambridge, Mass., May 30, 1986.

18. G. C. Goodman, "The LQG/LTR Method and Discrete-Time Systems," LIDS-TH-1392, MS Thesis, Dept. of Mech. Engineering, MIT, Cambridge, Mass., August, 1984.

19. W. H. R. Lee, "On Robust Designs for Infinite Dimensional Systems", Ph.D. Thesis, Dept. of EE&CS, MIT, Cambridge, Mass., June 1986.

20. D. B. Grunberg, "A Methodology for Designing Robust Multivariable Nonlinear Control Systems," Ph.D. Thesis, LIDS-TH-1609, Dept. of EE&CS, MIT, Cambridge, Mass., August 1986.

21. D. Milich, L. Valavani, and M. Athans, "Feedback System Design with an

Uncertain Plant," *Proc. 25the IEEE Conference on Decision and Control,* Athens, Greece, December 1986, pp. 441-446.

22. M. Athans, G. Stein, and L. Valavani, "Status Report #4 on Nonlinear and Adaptive Control; NASA Grant NAG 2-297," LIDS-SR-1637, MIT, Cambridge, Mass., December 31, 1986.

23. R. O. LaMaire, "Robust Time and Frequency Domain Estimation Methods in Adaptive Control," Ph.D. Thesis, Dept. of EE&CS, MIT, Cambridge, MA, May 1987.

24. R. O. LaMaire, L. Valavani, M. Athans, and G. Stein, "A Frequency Domain Estimator for Use in Adaptive Control Systems," *Proc. American Control Conference,* Minneapolis, MN, June 1987, pp. 238-244.

25. M. Athans, G. Stein, and L. Valavani, "Status Report #5 on Nonlinear and Adaptive Control; NASA Grant NAG 2-297," LIDS-SR-1679, MIT, Cambridge, Mass., June 30, 1987.