ELECTRIC POWER SYSTEM PRODUCTION COSTING AND RELIABILITY ANALYSIS INCLUDING HYDRO-ELECTRIC, STORAGE, AND TIME DEPENDENT POWER PLANTS

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Acknowledgments

An earlier version of this paper appeared as an Energy Lab Working Paper (75-009WP). Since then, the storage algorithm has been changed substantially, time-dependent generators have been included, and frequency and duration techniques have been incorporated. The impetus for changing the storage algorithm came from a discussion with Jacob Zahari of Tel Aviv University. Others who have contributed in one way or another to this paper are: Jerry Bloom, Jonathan Goodman, Edward Moriarty and Richard Tabors who are now or who have recently been at MIT, Bill Fleck and Ken Hicks of Stone & Webster, and James Platts of Northeast Utilities.

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I. Introduction

Production costing and reliability models of electric power systems are used to estimate the cost of operating the electrical generators and to estimate the probability that there will not be enough power to meet the customer demand. With these models, the effects on system cost and reliability due to different assumptions about customer demand, fuel costs, or generator characteristics can be studied.

Electric power systems are operated to meet the fluctuating power demand at minimum cost. Electric utilities monitor the power flow throughout the system to decide what the power output from each generator should be. These decisions are based on economic criteria, but are constrained by electric stability requirements imposed by the transmission network. A complete model of the cost of operating a power system requires detailed models of, and data on, each generator and each transmission line. Such models are too complex to be used for planning studies, so many simplifying assumptions must be made. For example, most production costing models, including the one presented here, do not consider transmission or stability constraints.

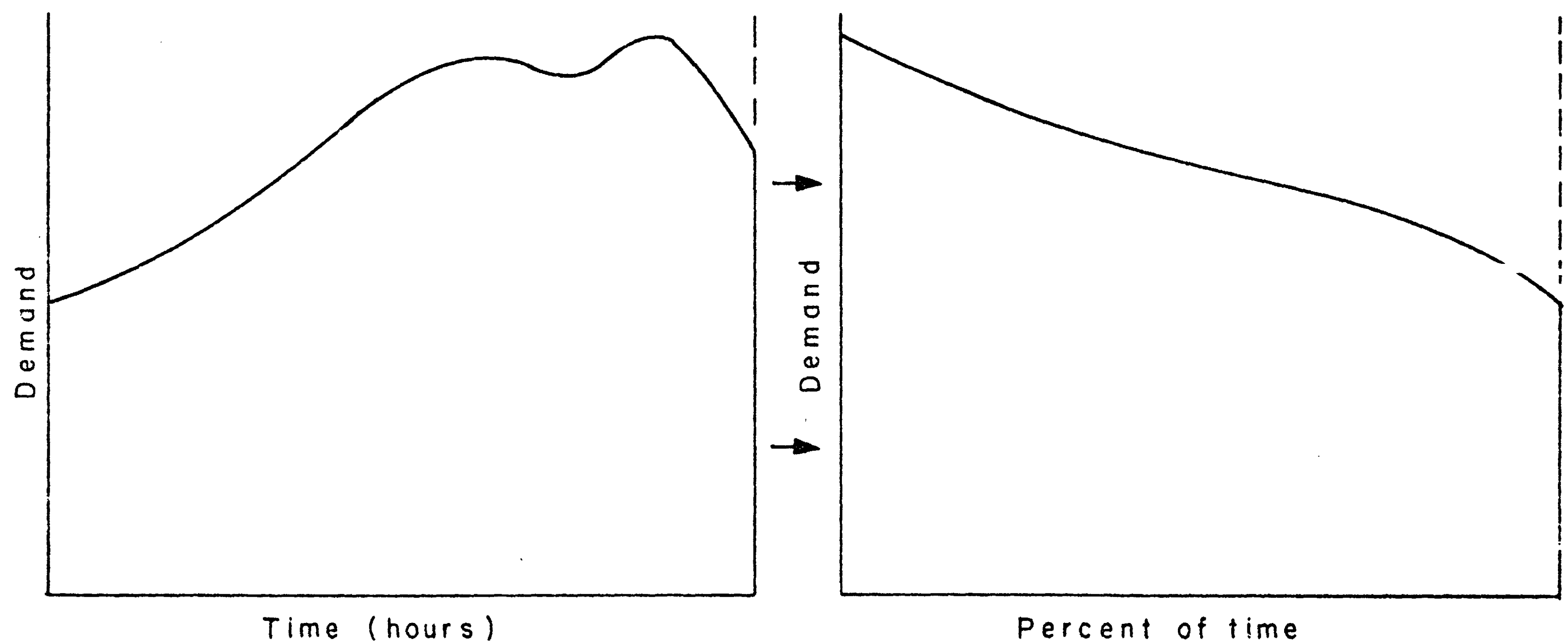
This paper discusses a standard production costing methodology that models the average generator output. The framework of the model is first presented as a deterministic model in which the customer demand is fixed and plants do not fail. Then, the model is expanded to a probabilistic model in which the customer demand and plant failures are random variables. Finally, the probabilistic model is extended to include hydro-electric, storage, and time dependent power plants and an alternative technique for computing the effective load carrying capability of a plant is developed.

The methodology presented in this paper has been implemented as three linked computer programs. ELECTRA models time dependent generators, SYSGEN performs the production costing analysis, and SCYLLA computes the load carrying capability of time dependent plants. (Conventional power plants can be evaluated within SYSGEN.) Documentation for these programs are available as Energy Lab Technical Reports. (See references 7,8 and 9.)

II. Deterministic Production Costing Model

Electric power systems are operated with the goal of meeting the electric demand at minimum cost. For a fixed set of generators, the dispatch strategy that results in the minimum operating cost is to use the generators in order of increasing marginal cost. In practice, this strategy may be modified to account for operating constraints such as spinning reserve requirements, high startup or shutdown costs and transmission constraints. The final ranking of generators is called the merit order or the economic loading order.

The power demand on an electric utility varies with the season and the time of day. Figure 1a shows a typical daily variation in power demand. Although the overall pattern is predictable, there is a large random component that makes hourly predictions difficult. For this reason, most planning studies use load duration curves that give just the percent of time that each demand level occurs. Figure 1 shows how a time-dependent curve can be converted into a load duration curve. Although detail is lost in the conversion, the load duration curve is easier to work with for time periods longer than a day and for future time periods for which there is not enough information to create hourly curves.



Ia. Time-dependent load curve for a typical day. Ib. Load duration curve of Ia.

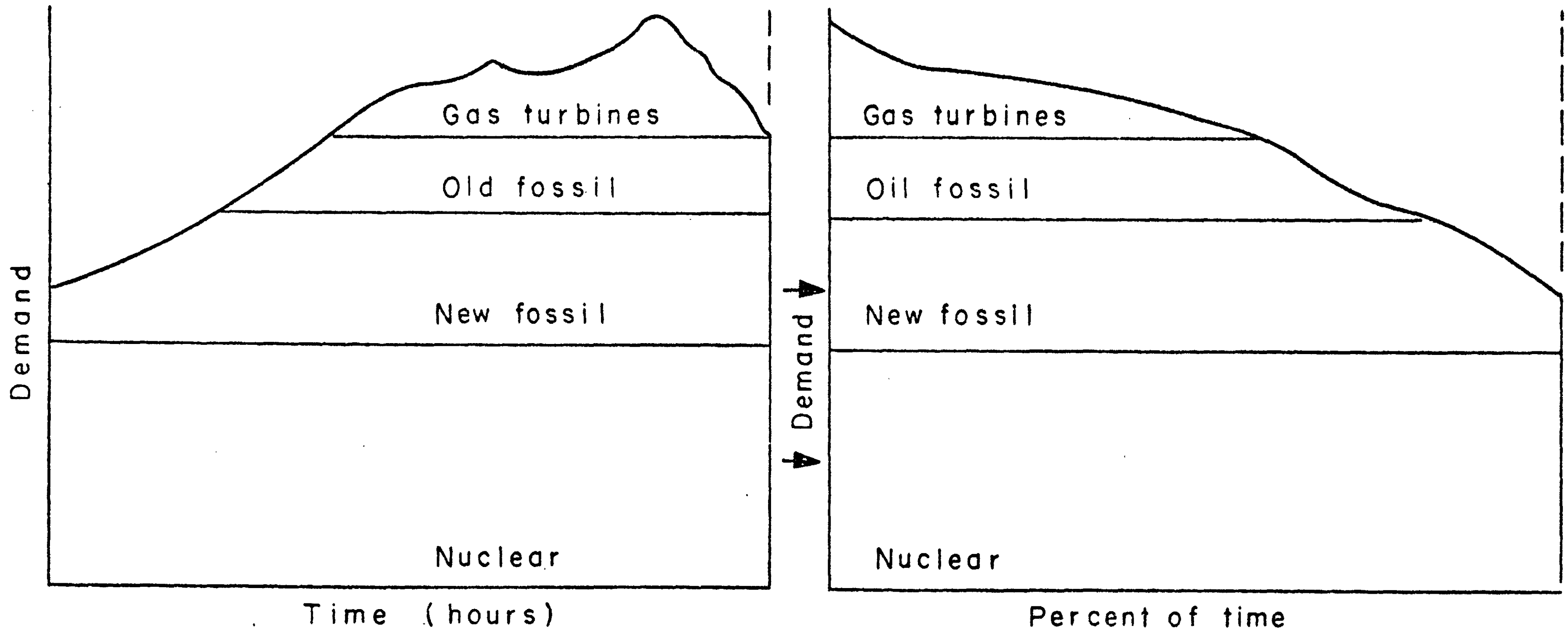
Figure I. Conversion of time dependent curve to load duration curve.

The operation of the power system can be modeled by plotting the capacity of the generators, in merit order, along the vertical axis of the customer demand curve as shown in Figure 2a. The demand level at which a unit starts to generate is called its loading point. The energy that a unit generates is the area under the customer demand curve between its loading point and the loading point of the next unit. Converting the time-dependent curve into a load duration curve, as shown in Figure 2b, leaves the loading point and the energy the same as in 2a.

II.A. Conventional Power Plants

Conventional central station power plants are plants that can generate power at full capacity at any time, except when they are on maintenance or forced outage. These plants are much easier to model than hydro, storage, or solar plants that have limited energy and time-dependent power output. Nonconventional power generation will be discussed in later sections.

In the deterministic model, the conventional power plant with the lowest marginal cost is loaded under the customer demand curve at a derated capacity that reflects the plant's availability. For example, a 1000 MW plant with an 80% availability factor would be brought up to 800 MW. This plant generates as much energy as it can to meet the customer demand. Since there is still unmet demand, the unit with the next lowest marginal cost is brought on line. This process continues until all the area under the load duration curve has been filled in. The total cost of



2a. Typical operating schedule.

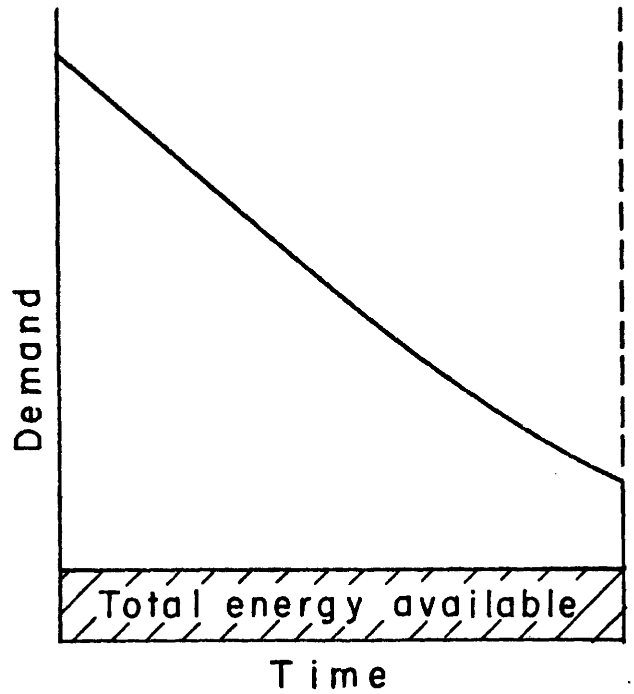
2b. Equivalent schedule on a load duration curve.

Figure 2. Deterministic operating schedule.

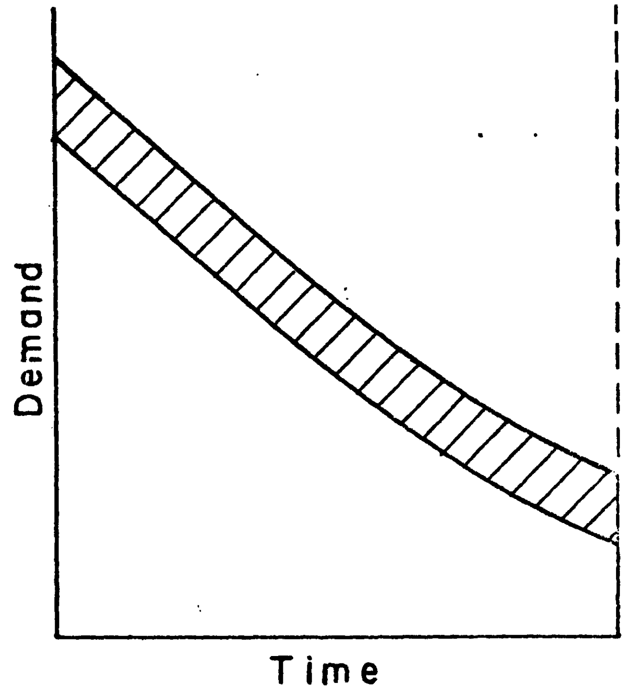
the system operation can be computed by multiplying each plant's total megawatt hours by the cost per megawatt-hour for that plant and then summing the costs over all plants.

II.B. Hydro-Electric Plants

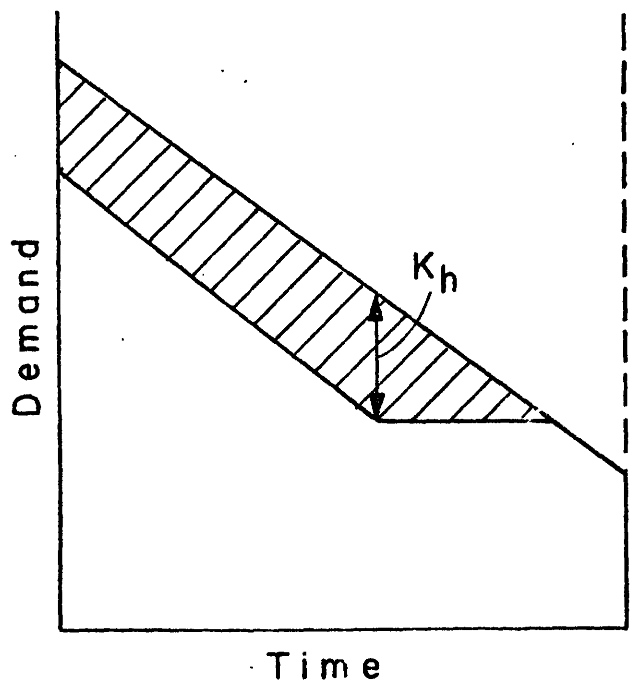
The inclusion of reservoir hydro-electric power complicates the problem of finding the minimum cost operating plan. The marginal cost of hydro-electric energy is essentially zero, since there are only operating costs and no fuel costs. This implies that hydro plants should be first in the economic loading order. However, the total amount of hydro energy available is limited by the river flows and the reservoir size. Usually, the total energy is not sufficient to run the hydro unit 100 percent of the time at full capacity. There are several possible strategies for discharging all of the hydro energy. One strategy is to load the hydro first, reducing the capacity until the area under the curve is equal to the total energy available. From Figures 3a and 3b, it is clear that this is equivalent to removing the same area from the top of the curve. But because the last units to be loaded are the most expensive to run, the operating cost would be reduced if as much area as possible were removed from the top of the curve. This can be achieved by removing the free hydro energy at full capacity as shown in Figure 3c. This is equivalent to finding the loading point for the hydro-electric unit such that, run at full capacity, the hydro energy is exactly equal to the area under the load duration curve. Figure 3d shows the result of these



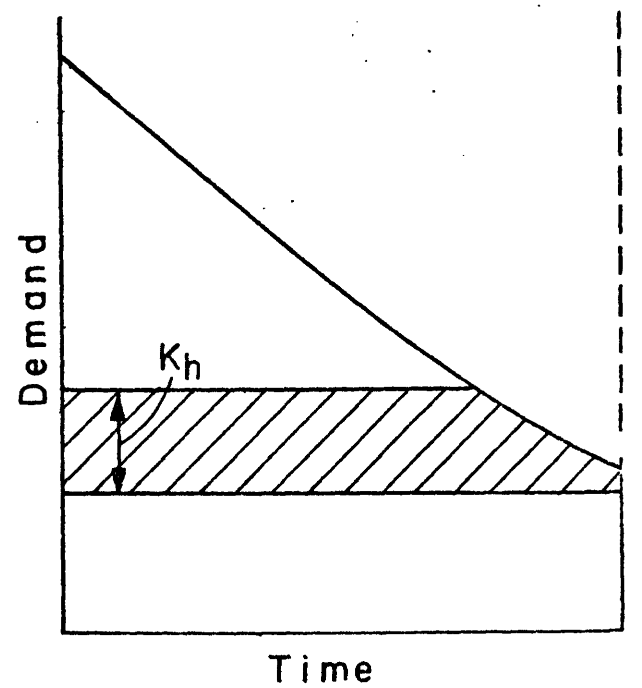
3a. Hydro capacity reduced to run unit as base loaded.



3b. Energy discharged at reduced capacity for peak shaving.



3c. Equivalent energy discharged to remove maximum energy from the peak.
K_h = Hydro capacity



3d. Equivalent loading point to remove maximum energy from the peak.

Figure 3. Equivalent loading point for conventional hydro units.

manipulations. The logic is explained here because it is easier to understand in the deterministic model, although it is only necessary in the probabilistic model where it is not possible to subtract demand from the top of the curve.

In the process of finding the optimal loading point for the hydro unit, it may be necessary to reduce the running capacity of the previously loaded plant. The remainder of the other unit's capacity can be loaded after the hydro energy has been discharged.

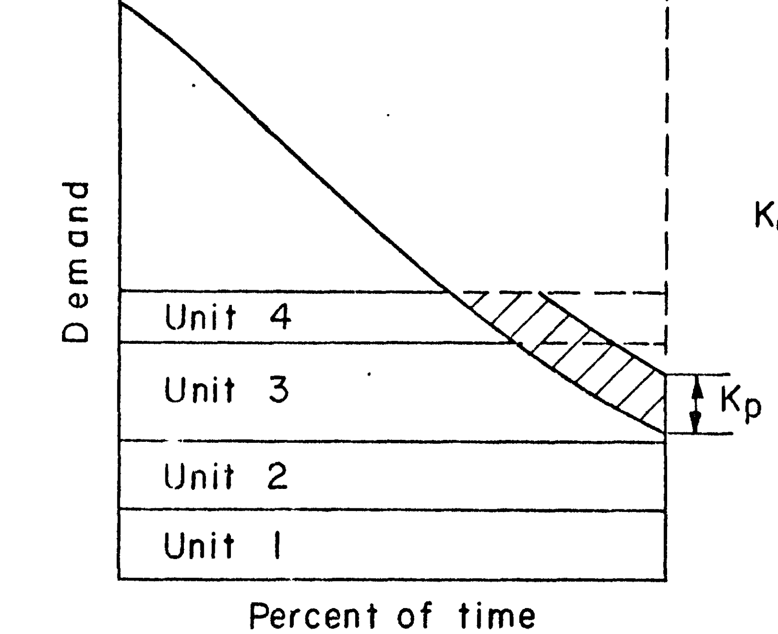
If there is more than one reservoir hydro plant, then they will be ranked in the loading order by the number of hours that they can generate at full capacity. That is, the reservoir with the largest ratio of energy to capacity will be the first to fit under the curve. This natural ordering can be used for reservoir hydro units, storage units, or any other limited energy units.

II.C. Storage Plants

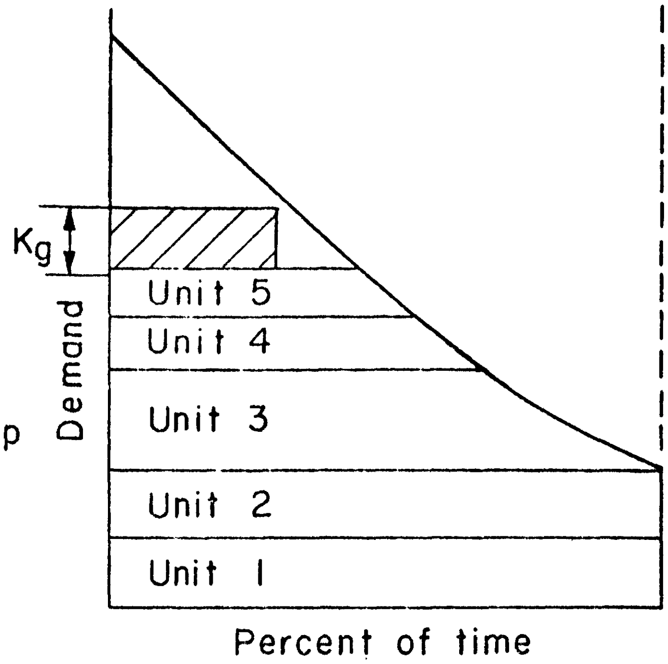
Electric utilities use storage plants to shift demand artificially from high marginal cost plants to low marginal cost plants. Currently, pumped hydro-electric storage is the only practical method available. Although the following section will refer to pumped hydro, the analysis is applicable to any central station storage unit that can be charged by all plants on the system.

Stored energy is generated by units which are low in the economic loading order, but that are not needed 100 percent of the time to meet the direct demand. Thus, an artificial demand is placed on these base loaded units by storage units . This stored energy can be released during periods of high demand when more costly units would normally be generating. Since the charging and discharging operations are not completely efficient, the energy available to meet demand using storage units is less than the energy generated by the base loaded units. Storage units are similar to conventional hydro units in that the amount of energy available is limited. However, modeling storage is complicated by the fact that the energy is not free and that the energy is generated on one part of the curve and discharged on another.

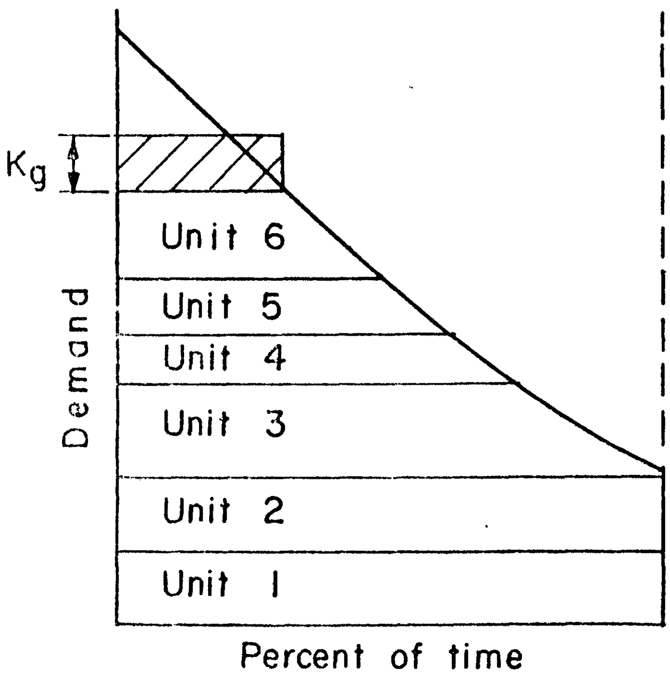
The total energy potentially available from a base loaded unit for storage can be found by computing the area above the load duration curve for the base loaded unit. Due to the limited capacity of the storage unit, some of this energy may be unavailable (see Figure 4a). Another limiting factor is the size of the reservoir. When the energy above the curve, subject to the limited capacity and the charging inefficiency, is equal to the storage capacity of the reservoir, then charging stops. Taking into account the inefficiencies of generating from storage, the total energy available to meet customer demand will be about two-thirds the energy generated for storage. This results in a marginal generating cost about one and a half times that of the base loaded unit used for storage.



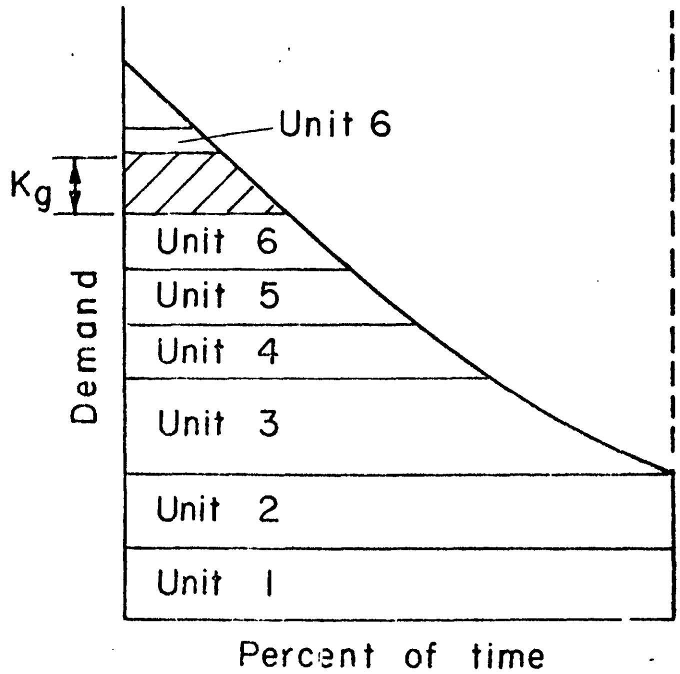
4a. Shaded area is energy available for pumped hydro.
K_p = pumping capacity



4b. Pumped hydro unit is next in the economic loading order. Energy is reduced by pumping/generating inefficiencies.
K_g = generating capacity



4c. Loading of the pumped hydro is postponed until the energy available exceeds the energy demand.



4d. Unit 6 is loaded in increments to allow maximum discharge of pumped hydro.

Figure 4. Loading sequence for pumped hydro.

Depending on the system and shape of the load curve, several base loaded units may fill a single storage unit, or one base loaded unit may fill several storage rervoirs. For the deterministic case, the marginal cost of the storage will be taken to be the average of the base loaded costs (with the inefficiencies factored in) weighted by the amount of energy each base loaded unit provides. If the storage units are ranked in order of decreasing number of hours at full capacity, then the first unit will be filled by the least expensive base loaded plant. Consequently it will be the first storage unit in the merit order after the storage units are sorted into the economic loading order based on the energy costs. When the first loading point is reached, the storage unit may have sufficient energy to discharge at full capacity, or it may not. In Section III.H it is shown that, in the deterministic case, the operating cost of the system is reduced if the pumped hydro is delayed in the loading order until the demand can be met by using the pumped hydro at full capacity. The argument is analogous to the one given for conventional hydro, even though the energy is no longer free. An illustration of the loading of pumped hydro is given in Figure 4.

III. Probablistic Production Costing Model

Two major factors affecting system operating costs are uncertainties in demand and random failures of plants. There are several models available that take these factors into account. The simplest is a deterministic model with heuristic calibration coefficients added to account for plant failures. Slightly more complicated is the method developed by Baleriaux and Jamoulle[1] which combines the probability distributions of customer demand and of plant failures to find the expected value of the energy produced by each plant and the probability that the customer demand will not be met. There is also a frequency and duration (FAD) method developed by Ringlee and Wood[2] that models both the load and plant failures as Markov chains. The FAD method gives information about the frequency and duration of system outages. Recently, Ayoub and Patton[3] have developed a method that includes frequency and duration in the Jamoulle-Baleriaux model and that requires fewer assumptions than the Ringlee-Wood model. The model described in this section is the combined method of Ayoub and Patton. In addition, several extensions are developed that allow the model to treat plants with limited energy and time-dependent power output.

The main difference between the deterministic model and the probabilistic model is that the electrical demand and electrical generation are treated as random variables in the probabilistic model. In the deterministic model, a plant's capacity is derated to reflect

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[1]Baleriaux, H., et al. "Simulation de l'exploitation d'un parc de machines thermiques de production d'électricité couplé a des stations de pompage," Revue E, Vol.V, No.7, 1967, pp.225-245.

[2]Ringlee and Wood "Frequency and Duration Methods for Power System Reliability Calculations - Part II - Demand Model and Capacity Reserve Model," IEEE Transactions, PAS-38, No. 4, April 1969.

[3]Ayoub, A.K., and Patton, A.D., "A Frequency and Duration Method for Generating System Reliability Evaluation." IEEE PAS Summer Power Meeting. F75 421-8.

random outages of the plant during its operating period. This assumes that the plant is always available at its derated capacity, or equivalently, that it has a forced outage rate of zero at its derated capacity, In fact, the plant is not always available. When a plant fails, more expensive generation must be brought on line to replace it. Since the deterministic model assumes that units never fail, the energy supplied by more expensive plants is underestimated. The deterministic model also assumes that the electrical demand is fixed. In the probabilistic model, uncertainty in the demand can be included in its probability distribution.

In the probabilistic model, the electrical demand and power plant failures are modeled as random variables with memory. That is, a power plant has a probability of failure and an expected time that it remains in a failure state. The electrical demand has a probability of being at a given level and an expected time that it remains at that level.

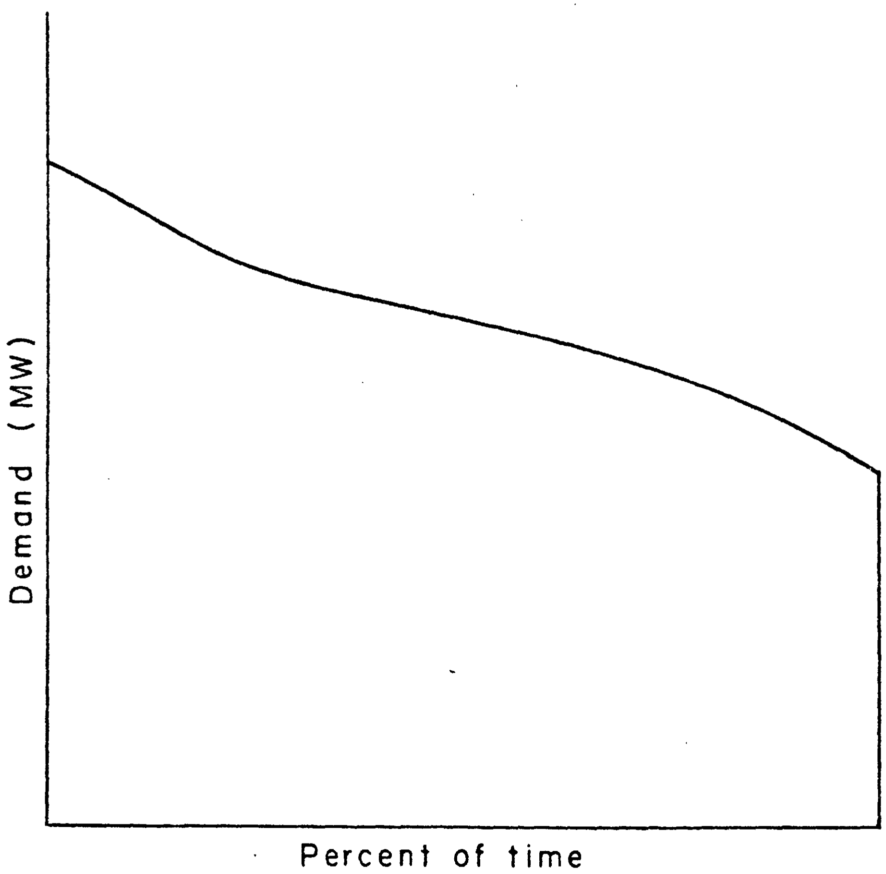
III.A Electrical Demand

III.A.1 Load Probability Distribution

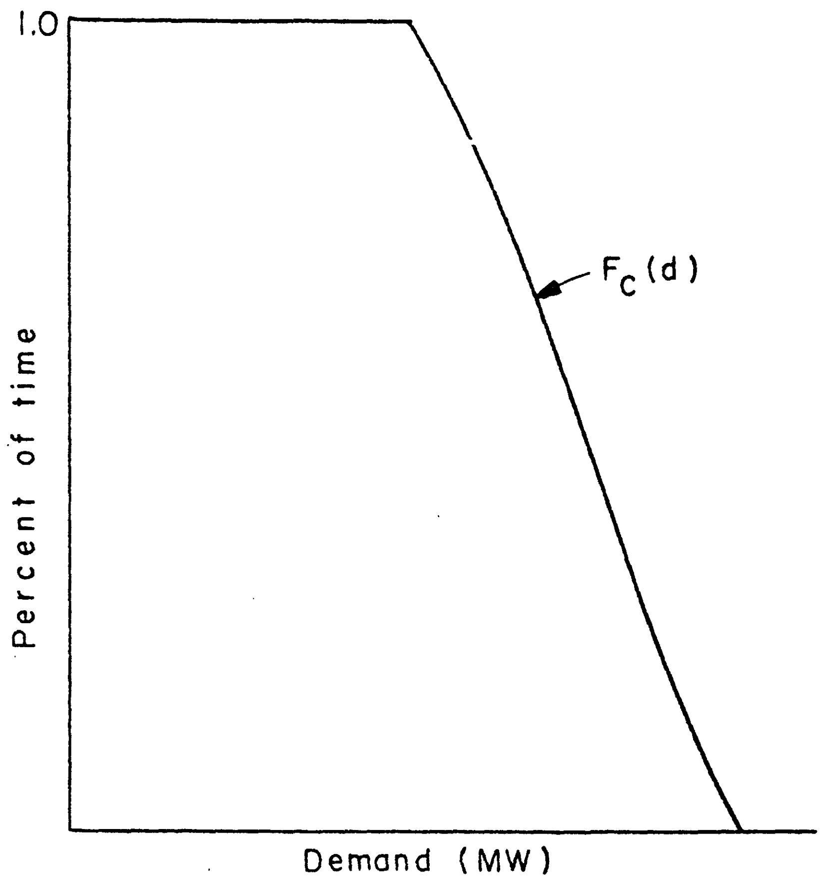
The probability curve for the electrical demand can be created either by using a demand model that estimates the distribution of demand or by equating the cumulative distribution function with the load duration curve. The latter procedure is done by rotating the axes of the load duration curve and normalizing the time period so that the vertical axis gives the percent of time that the demand level is exceeded. These operations are shown in figure 5. The percent of time that a given load level occurs can be interpreted as a probability. For example, there is a probability of 1.0 that the load will be greater than the minimum load at any given time, or equivalently the minimum load is exceeded 100 percent of the time. Further, there is a probability of zero that the load will be greater than the maximum load at any given time, or equivalently the maximum load is exceeded zero percent of the time.

III.A.2 Load Frequency Distribution

The expected time that the demand remains at a given level or load state can be derived from a demand model or from the original time-dependent demand curve. In the latter case, the expected duration can be found by measuring the lengths of time that the demand remains in a given state and then taking the expected value over all such time lengths. However, this procedure is rather awkward, and the same information is contained in the curve that gives the frequency with which the demand enters a given state. The load frequency curve is found by counting the number of times that the demand makes a transition from a level below to a level above the given demand. This is illustrated in figure 6. The relationship between the frequency curve and the duration curve is given below in equation 1.



5a. Original load duration curve.



5b. Inverted load duration curve.

Figure 5. Conversion of load duration curve to cumulative probability curve.

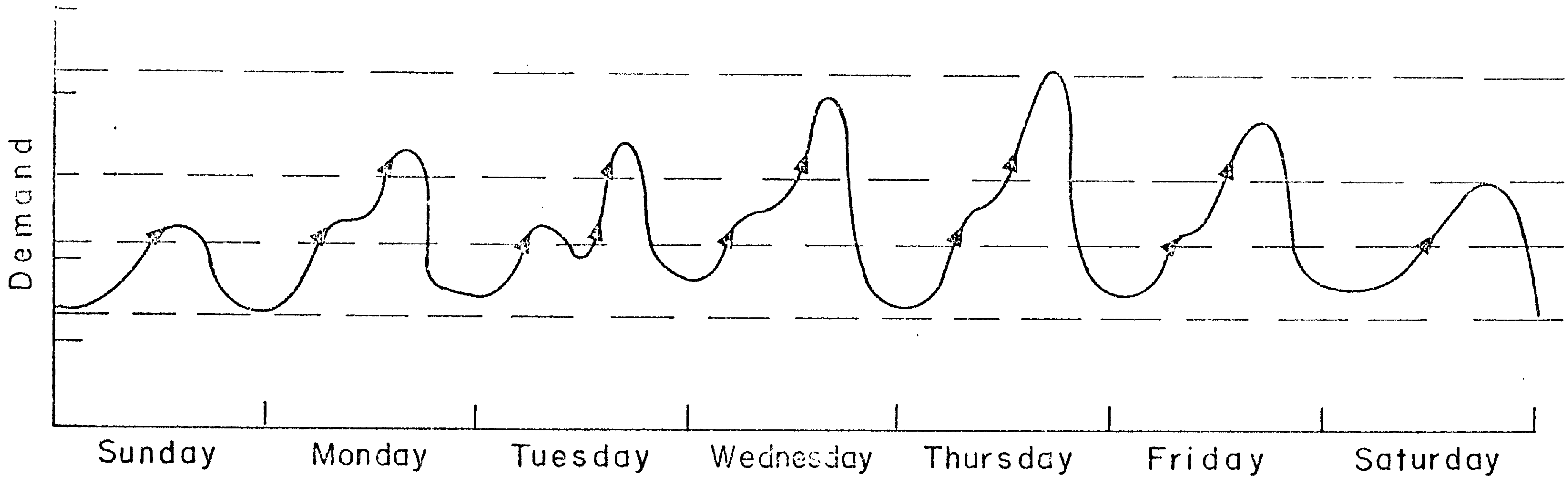


Figure 6a Weekly time dependent demand curve

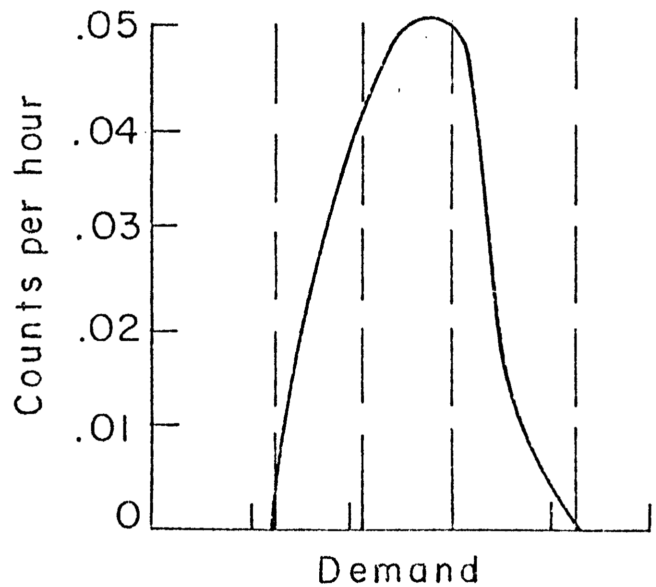


Figure 6b Frequency curve corresponding to curve 6a. The vertical axis plots the number of times the demand crosses a given level, normalized by the number of hours in a week

Figure 6 Demand frequency curve

The notation used in this section is explained in Section IV. In general, the letter 'f' is used for probability density functions, 'fq' for frequency curves, 'd' for duration curves, 'FQ' for reverse cumulative frequency curves, 'G' for cumulative probability functions, and 'F' for reverse cumulative probability functions . The subscript of the function tells which random variable the function describes. Thus $f_C(d)$ is the probability density function for the customer demand, D_C. By definition:

$$f_C(x) = \text{Probability } [x \leq D_C \leq x + dx]$$

$$G_C(x) = \text{Probability } [D_C \leq x] = \int_0^x f_C(y)\,dy \qquad (1)$$

$$F(x) = 1 - G_C(x) = \Pr [D_C \geq x] = \int_x^\infty f_C(y)\,dy.$$

$$fq_C(x) = \text{Frequency } [D_C = x]$$

$$FQ_C(x) = \text{Frequency } [D_C \geq x] = \int_x^\infty fq_C(y)dy$$

$$d_C(x) = \text{duration } [D = x]$$

$$= f_C(x)/fq_C(x),$$

or:

$$f_C(x) = fq_C(x)\ d_C(x).$$

The last equation states that the probability that the load is found in a particular state is the product of the frequency with which it enters that state and the length of time that it remains in that state.

III.B Conventional Power Plants

In the probabilistic model, the equivalent demand on a unit is defined to be the sum of the demand due to customers plus the demand due to failures of plants lower in the merit order. The equivalent demand D_E is the sum of two random variables:

$$D_E = D_C + D_F \qquad (2)$$

Where D_C is the direct customer demand and D_F is the demand due to forced outages of units already dispatched. From probability theory, the cumulative distribution for the sum of two random variables is given by:

$$G_E\ (d) = \int_0^d \int_0^{d-D_F} f_{C,F}\ (D_C, D_F)\ dD_C\ dD_F \qquad (3)$$

The function $f_{C,F}(D_C,D_F)$ is the joint probability density function of the customer demand, D_C, and the forced outage demand, D_F. Assuming these two random variables are independent implies that:

$$f_{C,F}\ (D_C, D_F) = f_C\ (D_C)\ f_F\ (D_F) \qquad (4)$$

Using equation (4), equation (3) can be simplified to:

$$G_E (d) = \int_0^d f_F (D_F) \int_0^{d-D_F} f_C (D_C) \, dD_C \, dD_F. \qquad (5)$$

From the definition of the cumulative distribution function for the customer demand given in equation (1), equation (5) becomes:

$$G_E (d) = \int_0^d f_F (D_F) \quad G_C (d - D_F) \, dD_F \qquad (6)$$

$$= \text{Probability [load + outages} \leq d].$$

The distribution of the equivalent demand is central to the probabilistic model. As will be shown below, the expected energy generated by each unit can be computed from it, as can the loss of load probability.

The equivalent load also has a frequency distribution that can be computed from the probability and frequency functions of the load and the outages:

$$FQ_e(d) = \int_0^d [fq_F(x) F_C(d - x) + f_F(x) FQ_C(d - x)] dx \qquad (7)$$

$$= \text{Frequency [load + outage} \geq d]$$

where

$fq_F(x)$ = Frequency [demand due to failures = x]

$FQ_C(x)$ = Frequency [customer demand $\geq$ x]

and F_C and f_F are the probability distributions for load and outages as defined in (1).

III.B.1 Single Increment Algorithm

III.B.1.a Outage Probability Distribution

For the case in which the forced outage rate of each plant is a discrete random variable, the integral over the probability density function $f_F(D_F)$, can be replaced by the sum over the probability mass function. For a plant with forced outage rate, q, and capacity, K, this probability mass function is given by:

$$P_F\ (D_F) = \begin{cases} p & \text{if } D_F = 0 \\ q & \text{if } D_F = K \end{cases} \qquad (8)$$

where p + q = 1. That is, there is a probability, q, that the plant will not perform and the demand on plants higher in the loading order due to its failure will be the capacity of the plant. There is a probability, p, that the plant will perform and the demand due to forced outage will be zero.

Replacing the integral with the sum, equation (6) becomes:

$$G_E(d) = pG_C(d) + qG_C(d - K)$$

or since p + q = 1 and $G_E = 1 - F_E$:

$$F_E(d) = pF_C(d) + qF_C(d - K). \qquad (9)$$

Equation (9) gives the new equivalent load duration curve. Figure 7 illustrates graphically how this curve is found using convolution. This

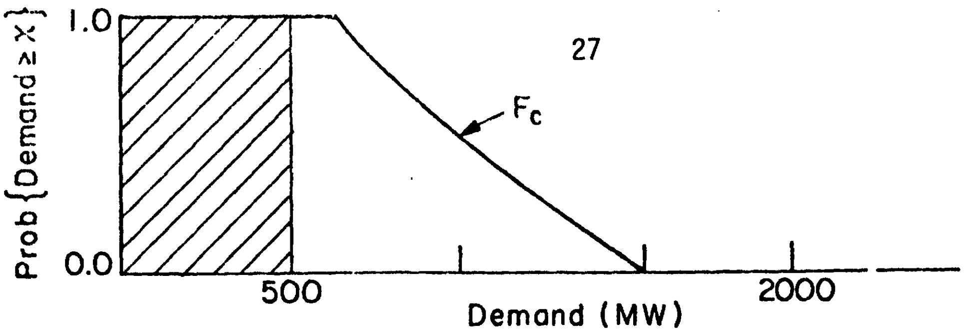


Figure 7a. Loading of a 500 MW plant with 90% availability. Expected capacity (shaded area) = 500 MW. Expected energy = expected capacity x availability x hours = 75600 MWH/week

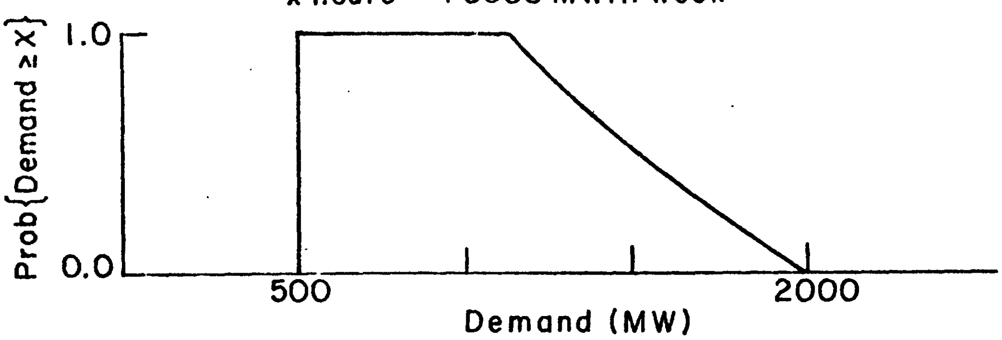


Figure 7b. If the first plant fails, the second plant sees the original customer curve. This event has a probability of .10

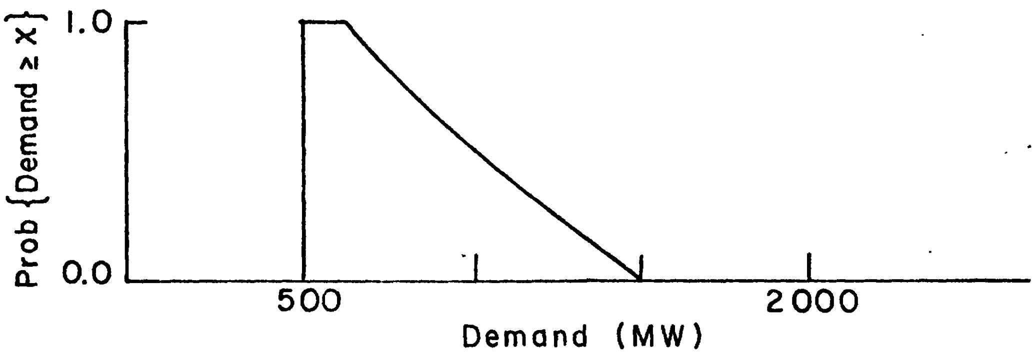


Figure 7c. If the first plant operates the second plant does not see the first 500 MW of customer demand. This event has a probability of .90

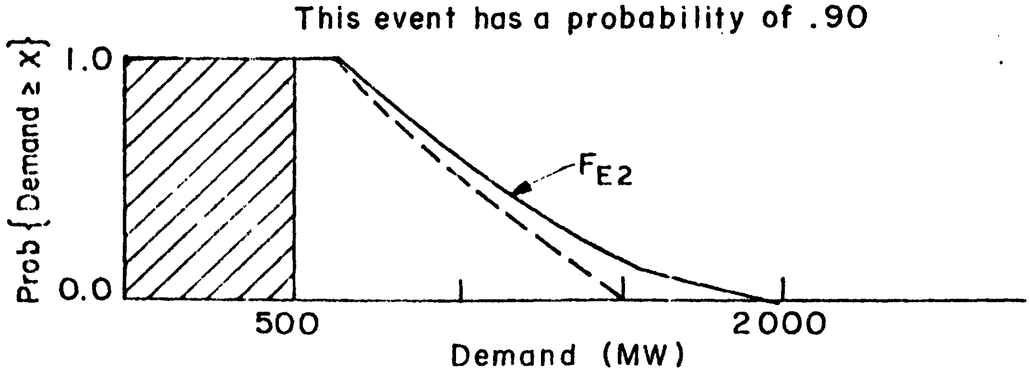


Figure 7d. The equivalent load curve for the second plant is the sum of the two curves weighted by their respective probabilities

Figure 7. Graphic illustration of convolution

curve can be used in much the same way the original load duration curve was used in the deterministic model, except that a new curve must be computed each time another unit is brought on-line.

III.B.1.b. Outage Frequency Distribution

If the power output of a plant can be modeled as a Markov chain, then there exists a mean time to failure and a mean time to repair:

m = mean time to repair

$$\lambda = 1/m \tag{10}$$

= average forced outage occurrence rate

μ = average forced outage restoral rate

and

$$q = \lambda/(\lambda+\mu)$$

$$p = \mu/(\lambda+\mu)$$

or

$$= q/(mp)$$

From the mean times to failure and repair and the outage distribution, the frequency curve for plant failures can be created using the last equation in equation set (1).

$$fq_F(D_F) = \begin{cases} p\,\lambda & \text{if } D_F = 0 \\ q\,\mu & \text{if } D_F = K. \end{cases} \tag{11}$$

From the last equation in set (10), it can be shown that $p\lambda$ equals $q\mu$. This is equivalent to stating that the frequency that the plant goes down equals the frequency that the plant goes up, i.e., it can't be brought up from a failure state more often than it fails. Combining equations (7) and (11) implies:

$$FQ_E(d) = p\lambda F_C(d) + q\mu F_C(d - K) + pFQ_C(d) + qFQ_C(d - K). \quad (12)$$

III.B.1.c. Probability and Frequency Distribution of the Equivalent Load

With these basic equations, the probabilistic analysis proceeds in much the same way as the deterministic analysis. Units are loaded starting at the left of the equivalent load duration curve. The demand on the first base-loaded unit to be brought up is the entire customer demand. There are no outages from previous units, so

$$D_{E1} = D_C \quad (13)$$

Where D_{E1} = equivalent demand on the first unit

D_C = total customer demand.

Because the two random variables, D_{E1} and D_C, are equivalent, their distribution and frequency functions are the same:

$$F_{E1}(d) = F_C(d) \quad (14)$$

$$FQ_{E1}(d) = FQ_C(d)$$

where $F_C(d)$ is the normalized load duration curve.

In the deterministic model, a unit is loaded onto the system by filling in the area under the load duration curve. The area gives the energy generated. To load a unit in the probabilistic model, the area is again filled in. The vertical axis, instead of being the percent of time that a unit operates at a given capacity, is now the probability that a unit operates at that capacity at any given time. Taking the integral over the capacity gives the expectation of the operating capacity* for the unit at any given time (see Section III.G). The expected capacity for the first unit is:

$$E(C_1) = \int_0^{K_1} F_C(x)dx \tag{15}$$

where K_1 = capacity of the first unit

C_1 = random variable describing the running capacity of the first unit.

$E(C_1)$ is the expected capacity required to meet the equivalent load, without considering the availability of the unit. The total expected energy from the first unit, taking outages into account, is:

$$M_1 = P_1 \; T \; E(C_1) \tag{16}$$

where P_1 = availability of unit one

T = total length of the time period in hours.

The capacity factor, CF, the ratio of operating capacity to nameplate capacity, is given by:

$$CF_1 = P_1 \; E(C_1) / K_1 \tag{17}$$

The expected number of times that a plant is started up can be found

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*The operating capacity is a continuous variable which takes on values between zero and the unit's capacity in response to the customer demand. This does not violate the assumption that plant outages occur in discrete blocks.

from the frequency curve by reading off the expected number of times that the loading point of the unit is crossed. Because the time scale for the frequency curves is normally in hours, this value must be multiplied by the number of hours in the time period:

$$E(N_1) = fq_{E1}(O)T \tag{18}$$

where

N_1 = number of startups for unit 1

O = loading point for unit 1.

Also, in practice, the frequency curve is stored in its cumulative form, so its derivative at the loading point must be computed.

The equivalent demand on the second unit to be brought up is the customer demand plus the demand due to the outages of the first unit:

$$D_{E2} = D_C + D_{F1} \tag{19}$$

Because of the way the equivalent load is defined, the loading point of the second unit on the equivalent load duration curve is the same whether or not the first unit fails. If the first unit fails, it creates a demand, K_1, so the second unit is loaded when the equivalent demand is K_1. If the first unit does not fail, there is no demand due to outage. The first unit supplies the demand until the demand exceeds K_1, at which point the second unit is loaded. The loading point, U, for the rth unit is just the sum of the capacities of the previously loaded units:

$$U_r = \sum_{i=1}^{r-1} K_i$$

and $$U_1 = 0 \qquad (20)$$

Equation (9) gives the equivalent load curve for D_{E2}:

$$F_{E2}(d) = p_1 F_{E1}(d) + q_1 F_{E1}(d - K_1) \qquad (21)$$

Equation (12) gives the equivalent frequency curve:

$$FQ_{E2}(d) = \lambda_1 p_1 F_{E1}(d) + \mu_1 q_1 F_{E1}(d-k_1) + p_1 FQ_{E1}(d) + q_1 FQ_{E1}(d-k_1) \qquad (22)$$

This equation cannot be written in a recursive formula similar to equation (21). However, FQ_E can be broken into parts that can be stored recursively:

$$F1_{E2}(d) = \lambda_1 p_1 F_{E1}(d) + \mu_1 q_1 F_{E1}(d - x) \qquad (23)$$

$$F2_{E2}(d) = p_1 FQ_{E1}(d) + q_1 FQ_{E1}(d - x)$$

Then,

$$FQ_{E2}(d) = F1_{E2}(d) + F2_{E2}(d). \qquad (24)$$

Having found the equivalent load curve for the second unit, the expected capacity, capacity factor, energy generated, and number of startups can be obtained:

$$E(C_2) = \int_{U_2}^{U_3} F_{E2}(x)dx$$

$$CF_2 = P_2 \quad E(C_2) \quad K_2 \tag{25}$$

$$M_2 = P_2 \quad T \quad E(C_2)$$

$$E(N_2) = fq_{E2}(U_2) \quad T$$

For the third unit, the equivalent load is given by:

$$D_{E3} = D_C + D_{F1} + D_{F2} \tag{26}$$

Using the definition of D_{E2} in equation (20):

$$D_{E3} = D_{E2} + D_{F2} \;.$$

Then,

$$F_{E3}\,(d) = P_2 F_{E2}\,(d) + q_2\, F_{E2}\,(d-K_2)$$

$$E(C_3) = \int_{U_3}^{U_4} F_{E3}\,(x)\,dx \tag{27}$$

$$CF_3 = P_3 \quad E(C_3) \quad K_3$$

$$M_3 = P_3 \quad T \quad E(C_3)$$

$$E(N_3) = fq_{E3}(U_3)\;T.$$

In general,

$$D_{Er} = D_C + \sum_{i=1}^{r-1} D_{Fi} = D_{Er-1} + D_{Fr-1}$$

$$F_{Er} = P_{r-1} F_{Er-1} (d) + q_{r-1} F_{Er-1} (d - K_{r-1})$$

$$E(C_r) = \int_{U_r}^{U_{r+1}} F_{Er} (x) dx \qquad (28)$$

$$CF_r = P_r E(C_r) K_r$$

$$M_r = P_r T E(C_r)$$

$$E(N_r) = fq_{Er}(U_r) T$$

where r = loading order of the plant.

III.B.2 Multiple Increment Algorithm

III.B.2.a. Outage Probability Distribution

In the derivation of the equation for the equivalent load, it was assumed that units would always be brought to full capacity. However, units are often brought up to full load in stages or increments. If each increment has a discrete probability of failing, then the probability mass function is given by:

$$P_F (d) = \begin{cases} P & \text{if } d = 0 \\ q_j & \text{if } d = K_j, j = 1, \ldots J \end{cases}$$

and $$p + \sum_{j=1}^{J} q_j = 1 \qquad (29)$$

where $$K_j = \sum_{i=1}^{j} k_i$$

J is the number of increments of the unit and K_J is the total capacity. K_j is the capacity up to and including the increment j. With this set of definitions:

$$q_j = \text{Probability [power outages exactly equal } K_j]. \tag{30}$$

The probability function could also be expressed for each increment of capacity. Assuming that each plant increment has an underlying distribution that describes its failure rate, define:

$$\hat{q}_j = \text{Probability [increment j fails, independent of the rest of the plant]} \tag{31}$$

and $\hat{p}_j + \hat{q}_j = 1$.

Then with the first increment loaded, the probabilities of outages are given by:

$$\Pr\,[\text{outage} = 0] = \hat{p}_1 \tag{32}$$

$$\Pr\,[\text{outage} = k_1] = \hat{q}_1.$$

The second increment can generate only if the first increment has not failed. Taking this dependence into account, the probabilities of outage levels are given by:

$$p\,[\text{outage} = 0] = \hat{p}_1\hat{p}_2 \equiv p_1 \tag{33}$$

$$p\,[\text{outage} = k_2] = \hat{p}_1\hat{q}_1 \equiv q_2$$

$$p\,[\text{outage} = k_1 + k_2] = \hat{q}_1 \equiv q_1$$

where p_1, q_1, and q_2 are analogous to the probabilities defined in equation (29). In general, the outage probabilities are given by:

$$p\,[\text{outage} = 0] = \hat{p}_1\hat{p}_2\hat{p}_3, \ldots, \hat{p}_n \qquad \equiv p_n$$

$$p\,[\text{outage} = k_n] = \hat{p}_1\hat{p}_2\hat{p}_3, \ldots, \hat{p}_{n-1}\hat{q}_n \qquad \equiv q_n$$

$$p\,[\text{outage} = k_{n-1} + k_n] = \hat{p}_1\hat{p}_2\hat{p}_3, \ldots, \hat{p}_{n-2}\hat{q}_{n-1} \qquad \equiv q_{n-1}$$

$$\vdots \qquad (35)$$

$$p\,[\text{outage} = \sum^{n} k_1] = \hat{q}_1 \equiv q_1.$$

In this paper, the dependent probabilities p and q_j, will be used. The example at the end of section III.B shows the distinction between the dependent and independent probabilities.

III.B.2.b Outage Frequency Distribution

Each increment described in equation (29) has an equivalent forced outage occurrence rate given by:

$$t_j = \text{mean time to a failure of magnitude } K_j \qquad (35)$$

$$\mu_j = \text{average restoral rate for } K_j = 1/t_j$$

Then, the frequency distribution becomes:

$$fq_F(d) = \begin{cases} p\,\lambda & \text{if } d = 0 \\ q_j\,\mu_j & \text{if } d = K_j,\ j = 1, \ldots, J \end{cases} \qquad (36)$$

where K and q are defined in equation (29).

III.B.2.c. Probability and Frequency Distribution of the Equivalent Load

Before finding the new distribution for the equivalent load, it is necessary to examine its definition:

$$D_{Er} = D_C + \sum_{i=1}^{r-1} D_{Fi}. \tag{37}$$

Included in the demand due to forced outages are outages of increments of plant r that are lower in the loading order. However, if a lower increment of a unit fails, then higher increments will not be available. Therefore, a lower increment cannot place an outage demand on a higher increment. To account for this, the demand due to forced outage of earlier increments is removed from the curve before the increment is added. The equivalent demand on increment j of unit r is given by:

$$D_{Er_j} = D_{Er-1} + D_{Fr-1} - \sum_{i=1}^{j-1} D_{Fr_i} \tag{38}$$

To compute the distribution of this random variable, it is easier to consider the system with all of the increments of unit r convolved into the equivalent load. Because the order in which random variables are convolved does not affect the final distribution, to include increment j of a unit, one can assume that the j-1 increments were the last ones added to the system. Combining equations (6) and (29), the equivalent load duration curve for increment j is:

$$F_{Er_j}(d) = p_r F_{Er}(d) + \sum_{i=1}^{j-1} q_{r_i} F_{Er}(d-K_{r_i}). \qquad (39)$$

The frequency convolution algorithm for multiple increments is more complicated than the one for probability convolution. Combining equations (7), (29), and (36), the multiple frequency distribution becomes:

$$FQ_{ER_j}(d) = \lambda_r p_r F_{Er}(d) + \sum_{i=1}^{j-1} \mu_{r_i} q_{r_i} F_{Er}(d - K_i) + p_r FQ_{Er}(d) + \sum_{i=1}^{j-1} q_{r_i} FQ_{Er}(d - K_i). \qquad (40)$$

Given the equivalent load and frequency curves for the increment j, to load increment j, the outages of the j-1 previous increments have to be deconvolved. Rearranging equation (39) gives:

$$F_{Er}(d) = \frac{1}{p_r} [F_{Er_j}(d) - \sum_{i=1}^{j-1} q_{r_i} F_{Er}(d - K_{r_i})] \qquad (41)$$

$F_{Erj}(d)$ is the equivalent load curve for the j^{th} increment to be loaded. Equation (41) is used to remove all of the outages of plant r from this curve. Points of the curve can be evaluated even though $F_{E\ r-1}$ appears on both sides of the equation. The curve is evaluated starting at d = 0. Since $F_E(d)$ always has a value of one for a negative number (i.e., the load is always greater than zero), the right hand side can be evaluated. Through an iterative process, the entire curve can be constructed from left to right.

Once the equivalent load curve, F_{Er} is known then the equivalent frequency curve can be found:

$$FQ_{Er}(d) = \frac{1}{p_r} \Big[FQ_{Er_j}(d) - \lambda_r p_r F_{Er}(d) - \sum_{i=1}^{j-1} q_{r_i} FQ_{Er}(d - K_j) - \sum_{i=1}^{j-1} \mu_{r_i} q_{r_i} F_{Er}(d - K_j).\Big] \qquad (42)$$

Again, this curve is evaluated from left to right using the same technique that was used to create F_{Er}.

The expected energy, capacity factor, and number of startups for increment of plant r are computed using F_{Er} and FQ_{Er} in equation (28). The loading point for increment j is the current loading point and does not change becauce the other increments have been removed. Finally, all j increments are convolved back into the curves using equations (39) and (40).

It should be noted that the order of convolution does not change the distribution of the sum of random variables. The increments are considered individually in order to find the proper loading points and expected energies. However, even if a multiple value point plant is loaded in sequence, equation (40) should be used to account for partial outages. In practice, an equivalent forced outage rate, p_E, for the entire plant is frequently used instead:

$$P_E = P - \frac{\sum_{j=1}^{J} q_j k_j}{K_J} \tag{41}$$

Similarly, the equivalent forced outage occurrence rate is given by:

$$\lambda_E = \lambda - \frac{\sum_{j=1}^{J} \mu_j k_j}{K_J} \tag{42}$$

As an example, suppose that a plant has two valve points and that the first one is first in the loading order. The energy generated by the first increment is found from F_{E1}, the original customer demand, using equations (15), (16), and (17). To find the equivalent demand on the next plant, the outages of the first increment are convolved with the customer demand:

$$F_{E2} = p_1 F_{E1}(d) + q_1 F_{E1}(d - k_1) \tag{43}$$

For simplicity, assume that the second increment is next in the loading order. To account for the dependence of the second increment on the first, the first is removed from the curve:

$$F_{E1}(d) = \frac{1}{p_1}\left[F_{E2}(d) - q_1 F_{E1}(d - k_1)\right] \tag{44}$$

The energy generated by the second increment is also found from F_{E1} and the results are the same as if the first and second increments had been loaded together under the original demand curve. Normally, there are intervening plants and the second increment is loaded under a different curve than the first increment.

Using equation (40), the equivalent load curve for the third unit becomes:

$$F_{E3}(d) = p\ F_{E1}(d) + q_2 F_{E1}(d - k_2) + q_1\ F_{E1}(d - k_1 - k_2). \quad (45)$$

where p = probability both increments work

q_1 = probability that the first increment fails

q_2 = probability that just the second increment fails.

The resulting curve accounts for the dependence of the second increment on the first. If the first and second increments were independent then the resulting curve would be:

$$F_{E3}(d) = p_1 p_2 F_{E1}(d) + p_2 q_1 (d - k_1) + p_1 q_2 F_{E1}(d - k_2) + + q_1 q_2 F_{E1}(d - k_1 - k_2)$$

$$= p_2 F_{E2}(d) + q_2 F_{E2}(d_1 - k_2)\ . \quad (46)$$

III.C Reservoir Hydro-Electric Energy

The treatment of reservoir hydro using probabilistic model is similar to the treatment in the deterministic model, except that it is more difficult to reduce the capacity of earlier plants and to compute the area under the curve. At each successive loading point, a test is performed on the feasibility of bringing up the hydro unit. The total energy demand on a unit with a capacity, K_h, can be found using the current equivalent load duration curve. The total energy demand, D, on the hydro unit is given by:

$$D_h = T \int_{U_r}^{U_r+K_h} F_{Er}(x)\,dx \,. \qquad (47)$$

F_{Er} is the equivalent demand curve for the next unit. Equation (47) is used to find the energy demand if the next unit is the conventional hydro unit. If the energy demand is greater than the available hydro energy, then the unit is not loaded. Plant r in the economic order is loaded instead. If the total energy demand is less than or equal to the available hydro energy, then the reservoir hydro unit is loaded.

If thermal plants are run only at valve points, then the process is simplified because only the first loading point at which the available energy is greater than the energy demand has to be found. If, however, there is a constraint that all hydro energy must be used, then a procedure has to be followed which allows hydro plants to be operated at any loading point. To make the most efficient use of the free hydro energy, the previously loaded unit should be backed off until the total energy demand balances the hydro energy available. However, changing the capacity of the r-1st unit changes the shape of the equivalent load demand curve for the unit r. Equation (47) can be rewritten using equation (28) and changing the capacity of the last unit to K'_{r-1}:

$$D_h = T \quad p_{r-1} \int_{U'_{r-1}}^{U'_{r-1}+K_h} F_{Er-1}(x)\,dx$$

$$+ T \quad q_{r-1} \int_{U'_{r-1}}^{U'_{r-1}+K_h} F_{Er-1}(x - K'_{r-1})\,dx \qquad (48)$$

where

$$U'_{r-1} = \sum_{i=1}^{r-2} K_i + K'_{r-1}$$

If M_h is the actual energy available from conventional hydro, then K'_{r-1} must be found such that D_h equals M_h. This involves solving equation (48) for K'_{r-1} which is an argument in the limits of the integrals and in the integrand of the second integral. Rather than solving analytically, the last unit can simply be removed and then added in small steps until the energies are balanced to within a set tolerance. The reservoir hydro unit is loaded under the new curve, its expected energy and capacity factor are computed. It is convolved into the curve to give the equivalent demand on the remaining capacity of the unit that was backed off. In order to load the remaining capacity of the interrupted plant, outages of the earlier portion have to be removed using the multiple increment altorithm. The final curve is the same as if the hydro and thermal units had just been convolved in. The intervening steps were needed to find the loading point for the hydro unit and the energy generated by the interrupted plant.

III.D Storage Plants

Once the energy available for storage has been computed, its treatment is similar to reservoir hydro. However, in order to find the energy available, the following values must be computed: (1) the expected excess energy available from base loaded units, given that each unit has a probability of failing; (2) the probability that a storage unit has sufficient energy available and the generator does not fail; and (3) the expected cost of the stored energy.

A storage unit creates a demand on base loaded units; however, unlike the customer demand, the storage demand does not necessarily have to be met. Also, storage units do not impose outage demands on other units until their place in the merit order is reached and they are called on to generate. Therefore, a separate curve that includes the demand from storage units on base loaded plants is created. This curve is used only to generate information on the availability and cost of stored energy and is unecessary for the rest of the analysis.

The storage units are ranked, as they were in the deterministic model, so that the unit with the most hours of generation at full capacity is the first to be filled. The ranking of a storage plant relative to other storage plants is denoted by 'u' and is distinct from the plants ultimate place in the loading order, denoted by 'r'.

Each storage unit has the following characteristics:

D_{su} = demand for storage by unit u

$$P_s(D_{su}) = \begin{cases} p_{cu} \text{ if } D_{su} = KC_u \\ q_{cu} \text{ if } D_{su} = 0 \end{cases} \quad (49)$$

where

q_{cu} = probability that the charging cycle of unit u fails

KC_u = changing capacity of unit u.

Note that the storage units impose demand on the base loaded units when they work. This is the reverse of generating units that impose outage demands when they fail. (See equation (8).)

IIID.1 Energy Supplied to Storage

The demand imposed by storage units can be modeled as an increase in the customer demand. The equivalent augmented demand, D', can be defined as:

$$D'_{Er} = D_{Er} + KC_u \qquad (50)$$

The distribution of KC_u is given in (49) and the distribution of D_{Er} is given by F_{Er}, the equivalent load curve for unit r. Convolving these distribution results in the distrubiton for the augmented demand:

$$F'_{ru} = q_{cu}F_{Er}(d) + p_{cu}F_{Er}(d - KC_u) \qquad (51)$$

The expected capacity available for charging storage unit u from base load plant r is the area between F_{Er} and F_{ru}' as shown in figure 8. This can be written as

$$E(C'_{ru}) = \int_{d_{min}}^{U_{r+1}} F'_{ru}(x)dx - \int_{d_{min}}^{U_{r+1}} F_{Er}(x)dx \qquad (52)$$

where

d_{min} = minimum demand. This is the first point where p[demand $\leq x \leq 1.0$].

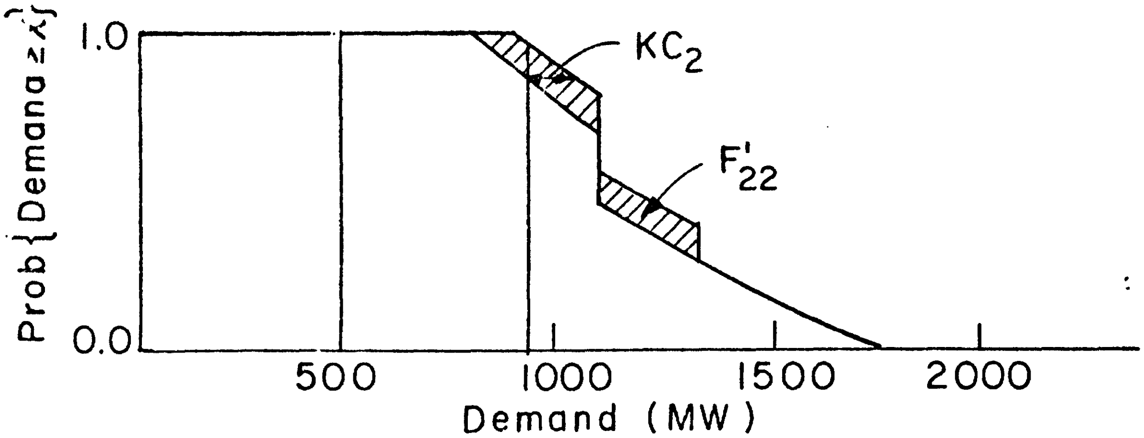

Figure 8d. Loading of second storage unit onto the augmented demand curve

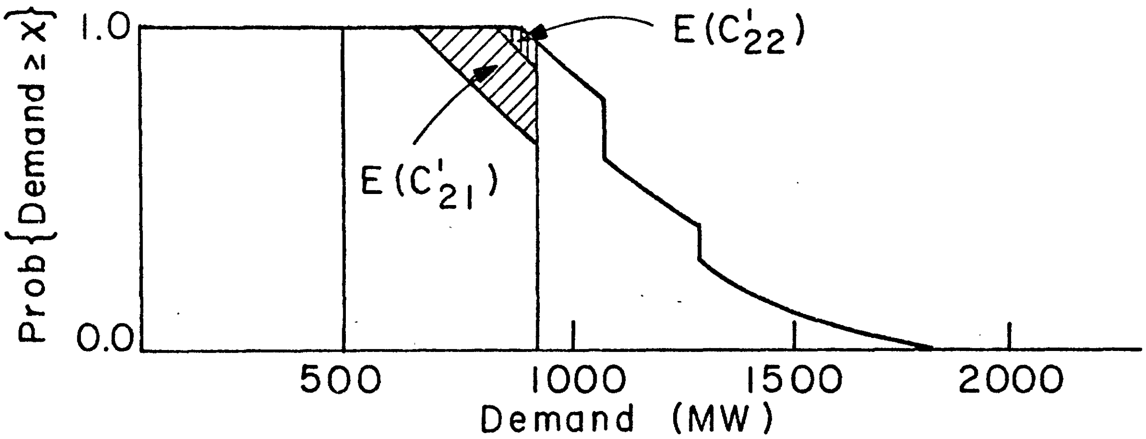

Figure 8e. The area above the second unit is the expected capacity available for each of the storage units

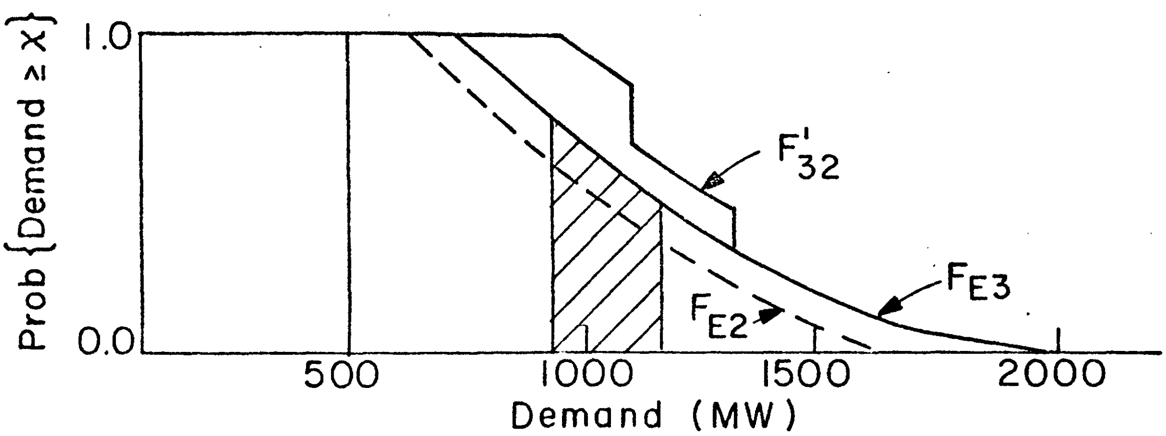

Figure 8f. Loading of the third unit. A new equivalent demand curve, F_{E3}, and a new augmented demand curve, F'_{32}, are created

Figure 8. Storage demand on base loaded units

C'_{ru} = excess capacity available from unit r for storage unit u.

Combining equations (51) and (52):

$$E(C'_{ru}) = p_{cu} \int_{d_{min}}^{U_{r+1}} \left[F_{Er}(x - KC_u) - F_{Er}(x)\right] dx. \quad (53)$$

This capacity is available with the probability of p_r, the availability of the base load unit. The expected energy available to the storage unit is the expected capacity multipled by the time length and the availability of the base load plant.

$$a_{ru} = p_r \; E(C'_{ru}) \quad T \quad (54)$$

where

a_{ru} = expected energy available from plant r for storage unit u.

Equations (50) through (53) imply that the storage demand is constant through time. However, the storage unit has a limited capacity and once the reservoir is full, the demand stops. Due to inefficiencies, the storage unit consumes more energy than its rated size, so pumping stops when the area between the curves equals the total energy requirement, Z_u:

$$Z_u = z_u/e_u \quad (55)$$

where

Z_u = total energy required by storage unit u

z_u = size of storage reservoir u

e_u = efficiency of storage unit u.

The equivalent augmented demand curve can be written as:

$$F'_{ru}(d) = q_{cu}F_{Er}(d) + p_{cu}F_{Er}(d - KC_u) \qquad \text{for } d \leq d'_{u+1}$$

$$\text{and } F'_{ru}(d) = F_{Er}(d) \qquad \text{for } d > d'_{u+1} \qquad (56)$$

where d'_u is determine such that:

$$Z_u = T \quad p_{cu} \int_{d_{min}}^{d'_u} [F_{Er}(x - KC_u) - F_{Er}(x)]dx. \qquad (57)$$

The resulting curve is shown in Figure 8c. This same curve could have been derived by adding the capacity of the storage unit to the original customer demand. However, the demand level at which storage starts and stops depends on the capacities and outage rates of earlier plants, so it is not possible to predict ahead of time when the storage demand will occur.

If there are additional storage plants, then they must also be added to the augmented demand curve. If the first storage plant were to fail, then the base load plant would supply the second storage plant instead.

$$F'_{ru+1}(d) = q_{cu+1}F'_{ru}(d) + p_{cu+1}F'_{ru}(d - KC_{u+1}) \qquad \text{for } d \leq d'_{u+1}$$

$$\text{and} \quad F'_{ru+1}(d) = F'_{ru}(d) \qquad \text{for } d > d'_{u+1} \qquad (58)$$

where d' is determine such that:

$$Z_{u+1} = T \quad p_{cu+1} \int_{d_{min}}^{d'_u} [F_{Er}(x - KC_u) - F_{Er}(x)]dx. \qquad (59)$$

All storage units are loaded using equations (58) and (59).

The expected capacity available from base plant r for the first storage unit is given by:

$$E(C'_{r1}) = p_{c1} \int_{U_r}^{U_{r+1}} \left[F_{Er}(x - KC_1) - F_{Er}(x)\right] dx. \qquad (60)$$

The expected energy available is:

$$a_{r1} = p_r \; T \; E(C'_{r1}) \qquad (61)$$

and the expected cost is:

$$c_{r1} = a_{r1} \; c_r \qquad (62)$$

To find the expected capacity available to the second storage unit, the equivalent loading point is increased by KC_1, the capacity of the first storage unit. Then,

$$E(C'_{r2}) = p_{c1} \int_{U_r+KC_1}^{U_{r+1}+KC_1} \left[F'_{r2}(x - KC_1) - F'_{r2}(x)\right] dx.$$

$$a_{r2} = p_r \; T \; E(C'_{r2}) \qquad (63)$$

and the expected cost is:

$$c_{r2} = a_{r2} \; c_r .$$

Equation (63) is repeated until the expected energy supplied to each of the storage plants by plant is found.

The next base plant in the loading order must supply whatever energy the first one could not due to outages, insufficient capacity, or insufficient energy. For the first storage plant, the augmented demand curve is given by

$$F'_{r+1,1}(d) = p_r F'_{r1}(d) + q_r F'_{r1}(d - K_r) \tag{64}$$

and the expected capacity is:

$$E(C'_{r+1,1}) = q_{r+1} \int_{U_{r+1}}^{U_{r+2}} F'_{r+1}(x) - F'_{r+1}(x - K_r)\, dx. \tag{65}$$

Equations (64) and (65) are much simpler than equations (58) and (60) because the demand due to storage is already included in F'_{r1}. The only additional factor that must be included is the probability that the first base unit fails and that the second must supply the additional energy to the storage unit.

In general for the first base load plant with excess energy

$$F_{ru+1}(d) = q_{cu} F'_{ru}(d) + p_{cu} F'_{ru}(d - KC_u). \tag{66}$$

$$E(C'_{ru}) = p_{cu} \int_{U_r+KC_{u-1}}^{U_{r+1}+KC_{u-1}} [F_{ru}(x - KC_u) - {}'F_{ru}(x)]dx. \tag{67}$$

$$a_{ru} = p_r \, T \, E(C'_{ru})$$

$$c_{ru} = c_r \quad a_{ru}$$

Finally, the expected energy and its cost for each storage plant are computed.

$$A_u = (\sum_r a_{ru}) \cdot e_u \qquad (68)$$

$$c_u = (\sum_r a_{ru}\, C_r \qquad)/A_u$$

The total expected energy and cost for the storage units cannot be found until all the base units have been loaded. There is an implicit assumption that the storage units will not be used before the base load units.

III.D.2 Energy Supplied by Storage

The expected energy cost for storage unit, u, as computed in equation (67) dictates the minimum spot it will have in the economic loading order. However, because the storage plant has limited energy, its use may be postponed until all its energy can be dispatched at full generating capacity. The argument is analogous to that used for reservoir hydro because the objective is again to minimize the use of expensive fuels.

When a storage plant is loaded as a generator in position r in the loading order, it has the following characteristics:

q_r	=	probability that the generator part of the cycle fails	
K_r	=	capacity of the generator	
A_r	=	expected energy available	(69)
c_r	=	expected cost per unit energy	
λr	=	average forced outage occurrence rate	

In theory the average forced outage occurence rate should be modified to include the effects of the outages of base load units and other storage plants. However, these effects are negligible and difficult to compute, so they are ignored.

III.E Time-Dependent Power Plants

The electrical output of some types of plants such as solar and wind, vary with the weather and the time of day. These plants cannot be modeled as any of the plants discussed above. In the original model, there was an implicit assumption that if all the other plants failed, a peaking unit could generate 100% of the time. There is also an assumption that a plant can produce at its full capacity at any time. Obviously neither of these assumptions is true for time and weather-dependent generation. However, one other attribute of these plants is that their marginal cost is zero because they use free resources as fuel.

If time-dependent plants have a marginal cost of zero then they will be used whenever they are available. Under this assumption, time-dependent plants can be modeled as modifying the net hourly load on the rest of the central station plants. The net hourly load will have a distribution that includes the uncertainty in the demand and the output of the generators.

Let

$P_C[D_C = x \mid t, \vec{w}_t] =$ Probability that the customer demand = x given the time and the weather

(70)

$P_G[G_i = x \mid t, \vec{w}_t] =$ Probability that generator i has capacity = x given the time and the weather

where $\vec{w}_t =$ vector of meteorological measurements representing the weather at time t (e.g. solar insolation, wind speed, temperature).

Note that the probability of failure of a generator will be assumed to be independent of time. For example, a solar generator producing anywhere from zero to its rated capacity will always have a forced outage rate of q.

If there is only one weather-dependent generator, then the reduced load on the utility is the random variable, D_R:

$$D_R = D_C - G_i \tag{71}$$

Using convolution:

$$P_N[D_N = y|t, \vec{w}_t] = \sum_x P_C [D_C = y - x \mid t, \vec{w}_t] \; P_G [G_i = x|t, \vec{w}_t]. \qquad (72)$$

For more than one time-dependent generator, a distribution for the net output of the units is found using convolution in an iterative process. Using the abbreviation '*' for convolution:

$$P_{C+G}(y) = \int_x P_C(x)P_G(y - x) \qquad (73)$$

$$\equiv P_C * P_G(y),$$

Then

$$P_{G_T}(y|t,\vec{w}_t) = P_{G1} * P_{G2} * \ldots P_{Gm}(y|t,\vec{w}_t) \qquad (74)$$

And

$$P_R(y|t,\vec{w}_t) = P_C * P_{GT}(y|t,\vec{w}_t). \qquad (75)$$

where

n = total number of time-dependent plants

G_T = total generation from the plants (random variable)

D_R = reduced demand in time t.

This distribution of D_R gives the probability of each possible net load for a particular hour. It includes, for example, the probability of the load level when all generators fail, and the probability of the load level when they all operate.

Once the distributions of the hourly net load are known, these distributions can be combined to form distributions for longer periods of time. The process of combining curves is done in a manner similar to that used in forming the standard load duration curve. First, all of the occurrences of a particular load level are grouped, their probabilities are summed, and then the sum is weighted by the total number of hourly density functions:

$$P_R(x) = \frac{\sum\limits_t P_R(x|t, \vec{w}_t)}{T} \tag{76}$$

The distribution P_R, gives the probability that the net load, R, equals any particular value. From this distribution, the load duration curve that gives the probability that the load exceeds a particular value can be found:

$$F_R(x) = \text{Prob } [R \geq x] = \int_{y=x}^{\infty} P_R(y). \tag{77}$$

The reduced curve can then be used instead of the original customer curve for the rest of the analysis.

The analysis can be simplified greatly if the load and generator characteristics are assumed to be uniform for certain blocks of time. For example, one third of the days are sunny and the load and generator output are the same for all sunny days.

A new load frequency curve must also be computed. The original equation (7) for the load frequency curve was:

$$FQ_E(d) = \int_0^d [fq_F(x)F_C(d - x) + f_F(x)FQ_C(d - x)]dx. \tag{78}$$

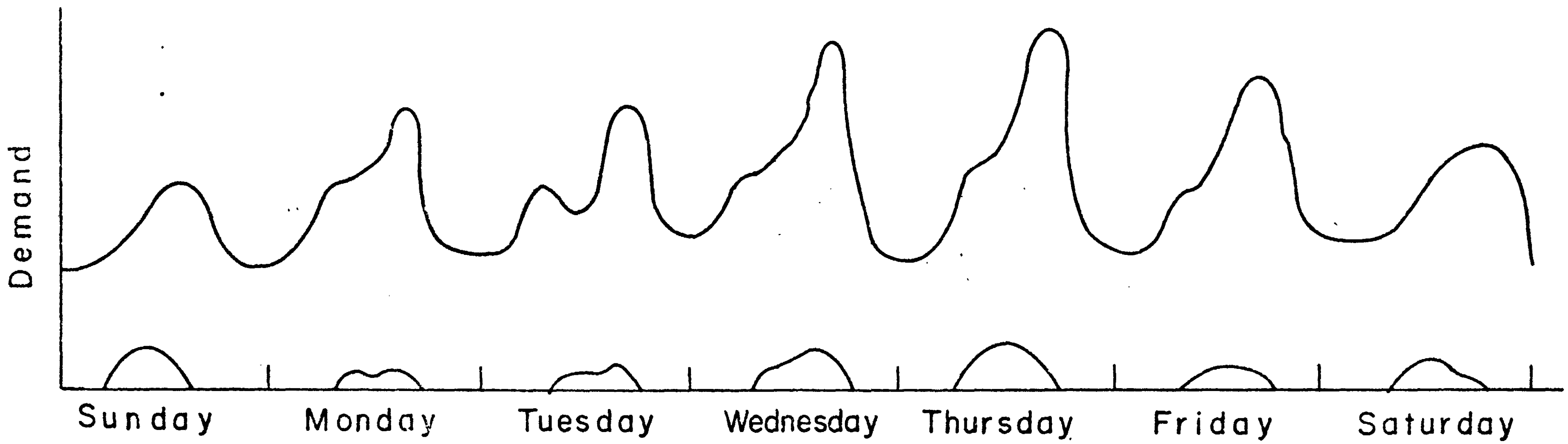

Figure 9a. Weekly time dependent demand curve. The output of a solar generator is plotted at the bottom of the curve

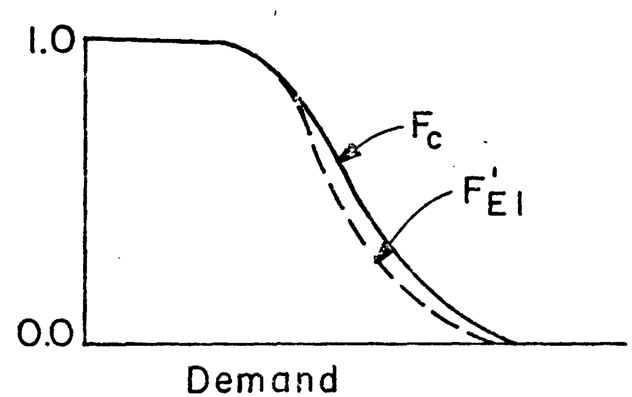

Figure 9b. Equivalent load curve. F_c is the load duration curve created from Figure 9a. F_{EI} is the equivalent demand curve after the solar generation has been subtracted from the curve, assuming that it has a forced outage rate of .05

Figure 9. Equivalent load curve for time dependent generation

This formula is modified to use the hourly load density functions, rather than the cumulative duration curves.

$$fq_R(d|t, \vec{w}_t) = \int_0^d [fq_G(x) f_C(d - x|t, \vec{w}_t) + f_G(x) fq_C(d - x|t, \vec{w}_t)]dx \qquad (79)$$

where the customer load depends on the time, but the outage rate and mean time to failure of the generator itself are assumed to be independent of time. The cumulative frequency curve is given by:

$$FQ_R(d) = \frac{\sum_t fq_R(x\ t, \vec{w}_t)dx}{T} \qquad (80)$$

The reduced load frequency curve replaces the original customer load frequency curve for the rest of the analysis.

III.F Loss of Load Probability

After the last unit has been loaded, the final curve is the equivalent load curve for the entire system. Since the loss of load probability is defined to be the percent of time that the customer demand cannot be met, its value can be read directly from the final curve. The energy demand that cannot be supplied is given by:

$$\bar{E}_n = T \int_{U_{n+1}}^{\infty} F_{En}(x)\ dx \qquad (81)$$

where n = number of plants.

The loss-of-load probability is given by:

$$LOLP = F_{En}(U_{n+1}) \tag{82}$$

where U_{n+1} is the total installed capacity of the system. Figure 10 shows the final system configuration.

The expected loss of load frequency is given by:

$$LOLF = FQ_{En}(U_{n+1}) \tag{83}$$

and from equation (1), the expected loss of load duration is

$$LOLD = \frac{LOLP}{LOLF} . \tag{84}$$

One other measure of the reliability of a power system is its loss of energy probability, LOEP. The LOEP is not really a probability, but an expected value for the fraction of the original demand that cannot be met. It is defined as:

$$LOEP = \frac{\int_{U_{n+1}}^{\infty} F_{En}(x)dx}{\int_{0}^{d_{max}} F_c(x)dx} \tag{85}$$

where U_{n+1} = total installed capacity

d_{max} = peak customer demand

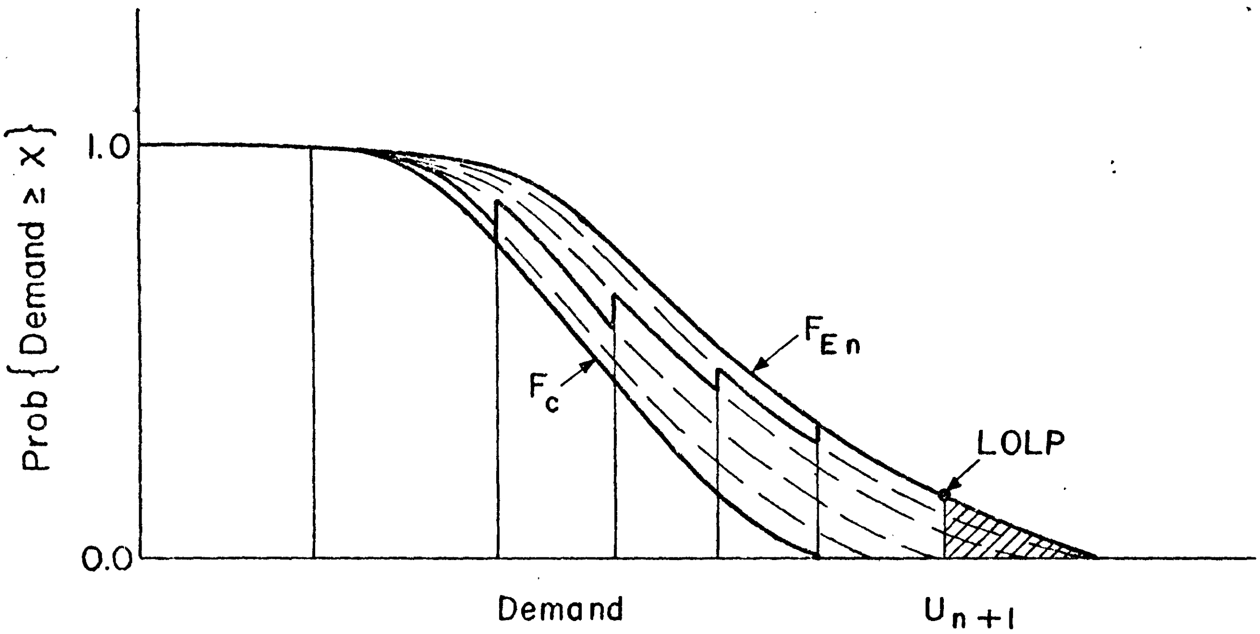

Figure 10a. Final equivalent demand curve

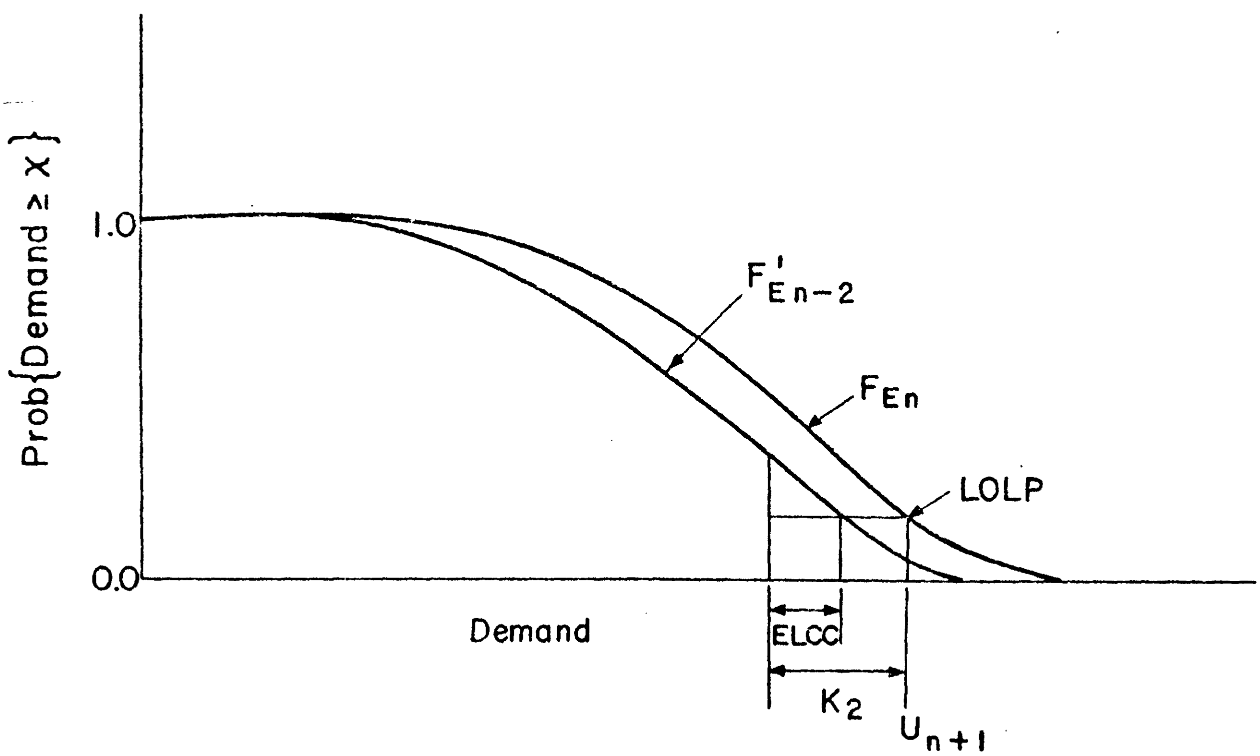

Figure 10b. Equivalent load carrying capability for plant 2

Figure 10. Final System Configuration

III.G Expected Value of the Operative Capacity

In the deterministic model, the energy is found from the area under the load duration curve. In the probabilistic model, this integral must be reinterpreted because the vertical axis does not have the dimension of time. To make the reinterpretation, a new random variable, the operating capacity of unit r, is defined:

$$C_r = D_{Er} - U_r \tag{86}$$

where U_r is the loading point of unit r. The reverse cumulative distribution of C_r is:

$$F_{cr}(S) = Pr \ [C_r > S]. \tag{87}$$

By definition of C_r, $F_{cr}(S)$ is given by:

$$\begin{aligned} F_{cr}(S) &= Pr[D_{Er} \geq S + U_r] \\ &= F_{Er} \ (S + U_r) \end{aligned} \tag{88}$$

The expected value of a non-negative random variable is defined as:

$$E \ (x) = \int_0^{x \ max} y \ G_x(y) \ dy \tag{89}$$

Integrating equation (89) by parts and using the fact that $G_x(x_{max}) = 1$ gives:

$$E(x) = \int_0^{x_{max}} F_x(y)\, dy \,. \tag{90}$$

Using equations (88) and (90) the expectation of C_r is:

$$\begin{aligned} E(C_r) &= \int_o^{K_r} F_{cr}(y)\, dy \\ &= \int_0^{K_r} F_{Er}\,(y + U_r)\, dy \qquad (91) \\ &= \int_{U_r}^{U_{r+1}} F_{Er}(x)\, dx \end{aligned}$$

This is just the area under the equivalent load duration curve. Therefore, the calculations for the energy produced by a unit appear the same, although the interpretation of the variables is quite different. In the probabilistic model, the expected operating capacity at any given time is computed and then multiplied by the length of the time period.

III.H Storage Dispatch Strategy

Assume that plants are loaded in order of increasing operating cost, that all storage units are loaded at the point where their costs become competitive, and that the capacity of the storage units is reduced so that a unit generates for the maximum length of time at the point where it is first competitive.

Now suppose that the operating capacity of the storage unit is increased. Because the energy remains constant, the hours of operation must be reduced. This means that the storage unit must be moved up in the loading order (see figure 4). Moving the storage unit creates two effects. One is that the plant directly below the storage unit must generate longer to make up for the hours that the storage unit is no longer supplying. The other is to decrease the capacity requirements on units higher in the loading order.

The extra cost required to make up for the loss in storage generation time is:

$$\Delta C' = \Delta H \bullet MW \bullet C_r \tag{92}$$

And the savings from decreasing the capacity requirement is:

$$\Delta C = \Delta MW \bullet (H - \Delta H)\, C_k \tag{93}$$

where

MW = original storage operating capacity

MW = increase in storage operating capacity

H = original generation hours

H = decrease in generation hours

C_r = cost of replacement generation

C_k = cost of unit(s) displaced.

Since the plants were loaded in order of increasing cost:

$$C_k \geq C_r \qquad k = r + 1, \ldots, n. \tag{94}$$

In some cases the replacement generation and the displaced capacity may be for the same unit; however, equation (94) still holds.

The stored energy remains constant:

$$E = H \bullet MW = (H - \Delta H) \quad (MW + \Delta MW) \tag{95}$$

or

$$(H - \Delta H) \bullet \Delta MW = \Delta H \quad MW \tag{96}$$

This implies:

$$\Delta C - \Delta C' = \Delta MW \bullet (H - \Delta H)\, C_k - \Delta H \quad MW \quad C_r$$

$$= \Delta H \bullet MW\, (C_k - C_r)$$

$$\geq 0 \tag{97}$$

by equations (94) and (96). Therefore, the savings are always greater than or equal to the additional cost of delaying storage generation.

Therefore, it is always advantageous to increase the operating capacity of storage as far as possible. Equation (97) states that storage should be used to replace the maximum amount of energy near the top of the customer load curve since a given amount of energy must be generated to meet the customer demand and since energy lower in the customer load curve is generated by plants with lower operating costs.

The argument given above does not carry through to the probabilistic model since there is no upper limit on the capacity required by the system to meet the peak. That is, each additional megawatt at the top of the curve reduces the loss of load probability, but, the loss of load probability never reaches zero. (There is always a finite chance that all plants will fail).

In the probabilistic case, increasing the capacity of the storage may actually increase the costs since the additional capacity places an outage demand on future units. However, the additional costs would be relatively small, and the overall effect would be to increase the reliability of the system.

III.I. Effective Load Carrying Capability

One measure of the worth of a power plant to an electric power system is its effective load carrying capability (ELCC). The load carrying capability is a function of the demand on the system, and the capacities and outage rates of all the plants. So the same plant may have quite different load carrying capability on different power systems. Basically, the load carrying capability indicates how much load the plant displaces. However, there are several alternative definitions and techniques for finding the load carrying capability, so it is necessary to give the definition along with the value. (See reference 10.)

In this paper, the effective load carrying capability of a plant is defined to be the capacity of a 100% reliable generator that can replace the plant without changing the loss of load probability. This value is equivalent to the load that can be subtracted uniformly from the demand so that the loss of load probability is the same as if the plant operates to meet the original demand.

The procedure for finding the load carrying capability involves finding the equivalent load curve for the entire system and the loss of load probability, then removing the plant in question from the equivalent load curve and finally finding the capacity that must be added to bring the loss of load probability down to its former value.

The first step is to find the equivalent load curve without plant r. Using equation (41):

$$F_{En-r}(d) = \frac{1}{p_r} [F_{En}(d) - \sum_{i=1}^{J} q_{r_i} F_{En-r}(d - K_{r_i})] \qquad (98)$$

where F_{En-r} = final equivalent load curve with plant r removed.

Then, a 100% reliable plant is added. Looking at equation (28), it can be seen that adding a 100% reliable plant to the equivalent load curve leaves it unchanged:

$$F'_{En}(d) = p' F_{En-k}(d) + q' F_{En-k}(d - K') \qquad (99)$$

If $p' = 1$ and $q' = 0$, then:

$$F'_{En}(d) = F_{En-k}(d) \tag{100}$$

where $F_{En}(d)$ = equivalent load curve with reliable plant replacing plant r.

Finally, the equivalent installed capacity, U'_n, such that the loss of load probability on the new curve equals the loss of load probability on the old curve must be found.

$$LOLP = F_{En}(U_{n+1}) = F'_{En}(U'_n). \tag{101}$$

The difference between U_{n-r}, the total installed capacity excluding plant r, and the new equivalent installed capacity, U'_n, is the effective load carrying capability:

$$ELCC_r = U'_n - U_{n-r}. \tag{102}$$

This is illustrated in figure 10.

It should be noted that the equivalent load carrying capability does not depend on the loading point of the plant. It does not measure how much load the plant actually displaces, but how much it is capable of displacing.

IV. Notation and Definitions

Each random variable, X, has a probability mass function associated with it:

$$f_X(t) = \Pr\,[t \leq X \leq t + dt]$$

and:

$$\int_{-\infty}^{\infty} f_X(t)dt = 1$$

or, if the random variable is discrete:

$$P_X(t) = \Pr\,[X = t]$$

and:

$$\sum_t P_X(t) = 1$$

The subscript on each function indicates that the function describes the random variable, X.

The cumulative distribution function (CDF) of a random variable will be noted by "G." The CDF of a random variable, X, is defined to be:

$$G_X(t) = \Pr\,[X \leq t]$$

$$= \int_{-\infty}^{t} f_X(s)ds$$

or for discrete random variables:

$$G_X(t) = \sum_{s=-\infty}^{t} P_X(s)$$

Another function of the random variable, X, can be defined to be:

$$F_x = 1 - G_x \ .$$

This function will be called the reverse cumulative probability distribution. F_x is not a standard probability function; however, it is useful here since the load duration curve is in this form. Thus:

$$F_x(t) = \Pr\,[X \geq t] = \int_t^{\infty} f_x(s)ds$$

or for discrete random variables:

$$F_x(t) = \sum_{s=t}^{\infty} P_x(s) \ .$$

The random variables and their respective reverse cumulative probability distributions as used in the probabilistic analysis are given below:

D_c	=	customer demand	$F_c(d)$	$= \Pr\,[D_c \geq d]$
D_f	=	forced outage demand	$F_f(d)$	$= \Pr\,[D_f = d]$
D_e	=	equivalent demand	$F_e(d)$	$= \Pr\,[D_c + D_f \geq d]$
	=	$D_c + D_f$		

Other variables that are input, or calculated from the functions defined above are as follows:

a_{ru}	=	expected energy available from plant r for storage unit u
A_u	=	total expected energy available for storage unit u
CF_r	=	capacity factor of plant r
C_r	=	operating capacity of plant r (MW)

C_{ru} = excess capacity available from plant r for storage unit u (MW)

c_r = cost per MWHr for plant r ($/MWH)

c_u = expected cost for the storage unit u ($/MWH)

D_h = megawatt hour demand which would be placed on the hydro unit h if it were loaded next in the loading order (MW)

D_{Er} = equivalent demand on unit r

D_{Er} = equivalent augmented demand on unit r including storage demand

e_u = pumping/generating efficiency for the uth pumped hydro plant

E_n = expected energy demanded that cannot be produced after all n plants have been loaded (MWH)

h = conventional hydro index after sorting. The hydro unit with the longest hours of generation at full capacity would have a hydro index of 1

K_r = rated capacity of plant r (MW)

KC_u = rated generating capacity of plant r (MW)

LOLP = loss of load probability

m = mean time to failure

M_r = energy generated by the rth plant (MWH)

n = number of plants on the system

N_r = number of startups for unit r

p_r = probability that plant r generates

q_{r_j} = probability that increment j of plant r fails to generate

r = loading order of plant

T = number of hours in the time period

u = pumped hydro index. The pumped hydro unit with the longest hours of generation at full capacity would have a hydro index of 1.

z_u = size of the loading point of unit r hydro reservoir u (MWH)

Z_u = energy consumed to fill hydro reservoir u (MWH)

λ = average forced outage occurrence rate

μ = average restoral rate

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