A GENERALIZED LOGIT FORMULATION OF INDIVIDUAL CHOICE

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MIT Energy Laboratory Working Paper No.
MIT-EL-79-010WP

February 1979

The Photovoltaics Project under which this work was completed is funded by the Department of Energy under contract number EX-76-A-01-2295, Task Order number 37.

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ABSTRACT

Probability models of individual choice consist of two components: a formulation of random utility and the stochastic specification of that utility. Usually separable direct random utility is assumed. With Weibull error terms, logit analysis results. However, logit analysis suffers from the "assumed" "independence of irrelevant alternatives". It is the contention of this paper that these difficulties result from the usual restrictive utility formulation. A more general indirect random utility formulation is introduced. Estimates of the resulting generalized logit and the more restrictive logit models are presented. Hypothesis testing is reported which rejects the restrictive utility formulations which dominate the literature.

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* This paper reflects the work done by the author for the Energy Research and Development Administration for the purpose of developing a broader model of residential energy demand and of assessing the market potential of solar photovoltaics.

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INTRODUCTION

Probability models and models of individual choice have become extremely popular in the recent past, particularly in the analysis of choices among alternative energy sources. The models of individual choice have focused upon micro decisions of individuals among discrete alternatives. More generally, probability models have been applied to aggregate data and are assumed to reflect the aggregation of individual decisions among discrete alternatives [1, 2, 3, 4, 5, 6, 9, 12, 13 and 19]. While notions of individual choice form the basis for the more aggregated probability models, alternative techniques are utilized in estimation--maximum likelihood estimates are obtained for the individual choice models, while regression techniques are utilized for the aggregated data (where replication is assumed).

For the models of individual choice, logit and probit analyses have been utilized most frequently. In the case of binary choice, the probit and logit formulations yield essentially the same results in most applications to date.[1] In the multi-choice extension, logit analysis has been used most frequently because of the ease of computation. The use of probit analysis for n choices [n > 2] is computationally difficult because in order to obtain likelihood estimates, evaluation of n - 1 multivariate normal distributions is required. While several authors [11] claim that current computer software makes the analysis of up to five alternatives possible, probit analysis still requires substantially more computational effort than logit analysis.

In light of such computational burdens, it might seem curious that probit would be used at all. One reason, of course, is the much discussed logit assumption of the "independence of irrelevant alternatives".[2] This assumption need not be a drawback. For example, in the case of evaluating a new alternative when that new alternative is sufficiently different in attribute space from all existing alternatives, the underlying assumptions of logit analysis seem reasonable and the ease with which the new alternative is built into the model is desirable. However, when a new alternative is

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[1]The reason is that most uses of probit have assumed the independence of alternative choices. See Hausman and Wise [11].

[2]As Hausman and Wise point out, it would be more descriptive to label this property the "independence of relevant alternatives". [11], p. 3.

very similar to an existing alternative, the implied consequences of the logit model are unacceptable.[1] Furthermore, as discussed below, the use of logit formulation in conjunction with the usual treatment of random utility as separable generates misspecification problems.

It is the contention of this paper that some of the difficulties that arise in using logit analysis (which are invariably linked to the "independence of irrelevant alternatives") are due to the specific utility formulation utilized in the analysis of discrete choice, in addition to the assumption about the form of the distribution of the error terms. A more general specification of utility will avoid the difficulties of the more restrictive formulation and also permit statistical tests of the validity of that same restrictive specification. By avoiding the difficulties confronted in the traditional application of logit analysis, the more general logit formulation developed here will hopefully permit continued use of logit in many simulation contexts,[2] thereby avoiding the more onerous computational burdens of using the more theoretically elegant probit analysis.

Section 1.0 below provides an overview of the standard analytic techniques in the literature. The usual models of choice utilizing separable utility and the stochastic assumptions underlying probit and logit are introduced. The treatment of conditional logit as a regression problem is also examined. In Section 2.0 a more general model of indirect utility is introduced and a general multinomial logit specification is developed. The generalized logit specification is estimated in Section 3.0 in an interfuel substitution context for energy demand.[3] Hypothesis testing regarding the validity of the more restrictive utility model is conducted. Furthermore, the likelihood estimates are heuristically compared to those resulting from treating conditional logit as a regression problem.

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[1] The red bus/blue bus problem. See Ibid.

[2] However, both the general logit and probit analyses continue to have difficulties in dealing with new technologies or new choices.

[3] This paper reflects work done by the author for the Energy Research and Development Administration for the purpose of developing a broader model of residential energy demand and of assessing the market potential of solar photovoltaics.

1.0 OVERVIEW OF STANDARD ANALYTIC TECHNIQUES

The analysis of the individual choice has utilized two sets of tools: (1) a random utility formulation, and (2) assumptions regarding the error distribution in the utility formulation. The standard utility formulation is that the utility of alternative j to individual i is:

$$U_{ij} = \overline{U}_{ij}(X_j, a_i) + \varepsilon(X_j, a_i) \tag{1}$$

$$= Z_{ij}\overline{\beta} + \varepsilon_{ij}$$

where U_{ij} is the utility of alternative j to individual i; X_j is a vector of attributes of the alternative j; a_i is a vector of characteristics of individual i; $\overline{U}_{ij}$ is the "average" or "representative" utility of an "average" individual, and $\varepsilon(X_j, a_i)$ is a random error term representing purely random behavior, measurement error, and/or unobserved characteristics of the individual and/or the alternative. Letting Z_{ij} represent combinations of X_j and a_i, then $\overline{U}_{ij}(X_j, a_i) = Z_{ij}\overline{\beta}$ where $\overline{\beta}$ is assumed constant over the entire population (i.e., homogeneous tastes).

Given utility function (1), the probability that an individual chooses alternative k is:

$$\begin{aligned} P_{ik} &= \Pr[U_{ik} > U_{ij}, \text{ for all } j \neq k] \qquad (2) \\ &= \Pr[Z_{ik}\overline{\beta} + \varepsilon_{ik} > Z_{ij}\overline{\beta} + \varepsilon_{ij}, \text{ for all } j \neq k] \\ &= \Pr[\varepsilon_{ij} - \varepsilon_{ik} < (Z_{ik} - Z_{ij})\overline{\beta}, \text{ for all } j \neq k] \\ &= \Pr[\eta_{jk} < (Z_{ik} - Z_{ij})\overline{\beta}, \text{ for all } j \neq k] \end{aligned}$$

where

$$\eta_{jk} = \varepsilon_{ij} - \varepsilon_{ik}.$$

The analysis of individual choice rests entirely on equations (1) and (2). Once the form of the utility function and the distribution f (ε) of the ε_{ij} are specified, one need only estimate the unknown parameters of $\overline{U}_{ij}$ and $f(\varepsilon)$.

Equation (2) specified P_{ik} in terms of $\overline{\beta}$ and the parameters of $f(\varepsilon)$.

For any individual i,

$$\sum_{k=1}^{N} P_{ik}=1,$$

where N is the number of alternatives facing i. If the P_{ik} are assumed to be drawn from a multinominal distribution, the likelihood function for the observed choices of M individuals[1] is

$$L = \prod_{i}^{M} \prod_{k}^{N} P_{ik}^{X_{ik}} \tag{3}$$

where $X_{ik} = 1$ if individual i chose alternative k and $X_{ik} = 0$ otherwise. P_{ik} are determined in equation (2) by the assumptions regarding $\overline{U}_{ij}$ and $f(\varepsilon)$.

I do not develop the details of alternative assumptions regarding $f(\varepsilon)$ and $\overline{U}_{ij}$ in the literature. However, let me cite some of the properties of three alternative assumptions.

1.1 Homogeneous Tastes, Separable Random Utility and Weibull Distribution for ε_{ij}

These are the usual assumptions underlying logit analysis.[2] In this case, β is assumed constant across the population. Furthermore, the difference

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[1]Assuming all individuals face the same choices. Hausman and Wise generalize this in [11].

[2]See Domencich and McFadden [8]; Baughman and Joskow [4]; Hausman [10]; and Theil [16].

$\eta_{jk} = \varepsilon_{ij} - \varepsilon_{ik}$ is distributed as a logistic distribution[1] or from (2)

$$P_{ik} = \Pr[\eta_{jk} < (Z_{ik} - Z_{ij})\,\overline{\beta},\ \text{for all } j \neq k] = \frac{e^{Z_{ik}\,\overline{\beta}}}{\sum_{j}^{N} e^{Z_{ij}\,\overline{\beta}}} \qquad (4)$$

$$= \frac{1}{\sum_{j}^{N} e^{(Z_{ij} - Z_{ik})\,\overline{\beta}}}$$

Thus homogeneous tastes and the assumed Weibull distribution generate P_{ik} of the form equation (4). Using (4) in the likelihood function (3) will generate logit estimates of $\overline{\beta}$.

Equation (4) also forms the basis for the regression analysis using conditional logit. Assuming all individuals are alike (dropping the i subscript), and that all individuals in the sample face the same alternatives, we have the usual log odds equation

$$\log\left(\frac{P_k}{P_j}\right) = (Z_k - Z_j)\,\overline{\beta} \qquad (5)$$

where experiment with replication is possible for the Z_k and Z_j and log (P_k/P_j) is a continuous variable.[2]

In equations (4) and (5), it is clear that the characteristics in Z_k and Z_j (Z_{ij} is the vector of combinations of X_j and a_i) which are the same

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[1]See Domencich and McFadden [8], pp. 62-65; and McFadden [14].

[2]Domencich and McFadden [8].

across alternatives, will cancel out. For example, if a conditional logit formulation is utilized to model the demand for a fuel in residential heating and Z_k summarizes the characteristics of oil (price, capital cost, etc.) and Z_j summarizes the same characteristics of natural gas, then the inclusion of personal characteristics and the characteristics of alternative fuels will cancel out in the equation. In other words, the log odds ratio of choice probabilities in choosing oil over natural gas is independent of all other alternatives. Thus, the comparison of one alternative with another is purely a binary comparison, no matter how many alternatives exist and no matter how similar such alternatives are to either of the two alternatives being considered.

It is this characteristic that is referred to as the "independence of irrelevant alternatives". As mentioned above, it can cause difficulties. It generates difficulties when new alternatives are added to the choice set. This is the "blue bus/red bus" problem.[1] The problem causes particular difficulties when new technologies or techniques are being considered when the new technique is similar to one already in use.

This "independence of irrelevant alternatives" also causes specification difficulties. This problem is found in a number of demand analyses utilizing conditional logit. The reason is that this "independence" implies that in the use of conditional logit for the estimation of price elasticities in demand models, all cross-price elasticities with respect to a given price change are restricted to be identical.[2]

This fact can be demonstrated through a model specifying total fuel demand (TOT) as

$$TOT = G(P_{INDEX}, X_1) \tag{6}$$

where P_{INDEX} is a price index (value weighted sum of individual fuel prices) and X_1 is a vector of exogeneous macroeconomic variables.

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[1] See Hausman and Wise [11]; and Domencich and McFadden [8].

[2] For the full development of this criticism, see Hausman [10]. The assumed constant cross-elasticities are found in [3] and [4].

The relative share equations (log odds equations)[1] are

$$\frac{S_i}{S_n} = F_i\left(\frac{P_i}{P_n}, X_{2i}\right) \qquad i = 1, \ldots, n-1 \tag{7A}$$

where (P_i/P_n) is the ratio of relative fuel prices,[2] X_{2i} are exogeneous and, of course, F_i is the usual exponential formulation. Alternative fuel prices are not included because of the "independence of irrelevant alternatives", i.e., they would cancel out. The usual restriction,

$$\sum_{i=1}^{n} S_i = 1 \text{ is assumed to hold.}$$

Using the fact that $F_i = S_i/S_n = Q_i/TOT/Q_n/TOT = Q_i/Q_n$ (where Q_i is the amount of fuel i consumed), we have $Q_i = Q_n F_i$ and the cross-elasticity of demand for Q_i with respect to P_j is

$$e_{Q_i P_j} = \frac{\partial Q_i}{\partial P_j}\,\frac{P_j}{Q_i} = \frac{P_j}{Q_i}\left(\frac{\partial Q_n}{\partial P_j} F_i + Q_n \frac{\partial F_i}{\partial P_j}\right) \tag{7B}$$

$$= \frac{P_j}{Q_n}\,\frac{\partial Q_n}{\partial P_j} + \frac{P_j}{F_i}\,\frac{\partial F_i}{\partial P_j} .$$

Clearly, given the formulation of (7A),

$$\frac{\partial F_i}{\partial P_j}\,\frac{P_j}{F_i} = 0, \text{ for all } j \neq i, n.$$

Hence,

$$\frac{\partial Q_i}{\partial P_k}\,\frac{P_k}{Q_i} = \frac{\partial Q_n}{\partial P_k}\,\frac{P_k}{Q_n}$$

for $k \neq i, n$; the cross-elasticities of Q_i and Q_n with respect to P_k are

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[1] See Hausman [10].

[2] The P_i are not probabilities in this discussion (pp. 7-9). I use Hausman's [10] notation.

always equal.[1] Therefore, using equation (7B) for j = i and n, we have

$$\frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i} = \frac{\partial Q_n}{\partial P_i}\frac{P_i}{Q_n} + \frac{\partial F_i}{\partial P_i}\frac{P_i}{F_i} \tag{7C}$$

$$\frac{\partial Q_i}{\partial P_n}\frac{P_n}{Q_i} = \frac{\partial Q_n}{\partial P_n}\frac{P_n}{Q_n} + \frac{\partial F_i}{\partial P_n}\frac{P_n}{F_i}$$

while for $j \neq i, n$

$$\frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\partial Q_n}{\partial P_j}\frac{P_j}{Q_n} \tag{7D}$$

It should be clear that the estimates of elasticities and the constancy of cross-elasticities in (7D) depend crucially not upon the use of conditional logit, but upon the cricial formulation of demand F_i in (7A). If (7A) were formulated as

$$\frac{S_i}{S_n} = F_i(P_1 \ldots P_i, \ \ldots \ P_n, \ X_{2i})^2 \tag{7A)'$$

then (7D) would become

$$\frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\partial Q_n}{\partial P_j}\frac{P_j}{Q_n} + \frac{\partial F_i}{\partial P_j}\frac{P_j}{F_i} \tag{7D)'$$

where

$$\frac{\partial F_i}{\partial P_j} \neq 0, \text{ for all } j \neq i, n.$$

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[1]Using equation (A3), Hausman continues the derivation to

$$\frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\partial TOT}{\partial P_j}\frac{P_j}{TOT} - S_j \quad \frac{P_j}{F_j}\frac{\partial F_j}{\partial P_j} + \frac{P_j}{F_i}\frac{\partial F_i}{\partial P_j}$$

with a more detailed examination of the proposition that all cross-elasticities for a given P_j are equal. However, as in the discussion above, the derivation depends crucially on the form of demand F_i, and the fact that $\partial F_i / \partial P_j = 0$. See Hausman [10].

[2]Conditional logit estimations using this form are found in [9] and [13].

The use of (7A) rather than (7A)' generates misspecification for the following reason. Since (7A) is specified and estimated in conjunction with

$$\frac{S_i}{S_n} = F_i(\qquad), \ i = 1, \ldots n-1, \text{ subject to } \sum^{n} S_i = 1, \text{ and since}$$

$$\frac{\partial S_i}{\partial P_j} \neq 0, \text{ then } \frac{\partial S_i}{\partial P_j} \neq \frac{\partial S_n}{\partial P_j} \neq 0.$$

As a result, there is specification error in constraining

$$\frac{\partial F_i}{\partial P_j} \equiv 0,$$

which is what formulation (7A) does.

1.2 Homogeneous Tastes, Normal Distribution for ε_{ij}

These are the usual assumptions for probit analysis. In this case, equation (2) becomes[1]

$$P_{ik} = \int_{r_1=-\infty}^{V_{k1}} \cdots \int_{r_j=-\infty}^{V_{kj}} \cdots \int_{r_n=-\infty}^{V_{kn}} \phi(r;\ 0;\ \Omega)\, dr_1, \ldots dr_j \ldots dr_n \tag{8}$$

for $j \neq k$, where $V_{kj} = (Z_{ik} - Z_{ij})\,\overline{\beta}$, and $\phi(r;\ 0;\ \Omega)$ is multivariate normal with 0 mean and covariance matrix Ω evaluated at r. If the off-diagonal terms of Ω are zero, "independent" probit results; if Ω is dense, covariance probit results.

While the specification of utility (or demand) can be the same under the logit specification and the independent or covariance probit, the covariance probit formulation permits a much richer examination of individual choice because it allows for the covariance of η_{jk} (hence the ε_{ij}) in equation (2). The independent probit, by assuming $\text{COV}(\varepsilon_{ij}\varepsilon_{ik}) = 0$, has properties similar

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[1] See Domencich and McFadden [8]; and Hausman and Wise [11].

to the logit formulation. However, with three or more alternatives the behavior of the logit (and independent probit) differs from the covariance probit because the logit is based upon binary comparisons while the covariance probit is based upon an n-way comparison.

In spite of the richer stochastic specification, the use of probit is limited by the need to evaluate the integrals[1] in equation (8) when that equation is substituted into (3) to get likelihood estimates.

1.3 Heterogeneous Tastes, Normal Distribution for ε_{ij}

The analysis of heterogeneous tastes has not been pursued by many authors. Quandt [16] proposed variation in taste parameters in a binary choice model, but his stochastic specification is based upon the exponential distribution. Hausman and Wise [11] introduce random taste parameters which are incorporated into the covariance matrix Ω in equation (8).

[1] Hausman and Wise claim that n = 5 or less if currently computationally tractable. See [11].

2.0 AN ALTERNATIVE ANALYTIC TECHNIQUE

The discussion in Section 1.0 indicated three sets of assumptions regarding the theory of individual behavior (i.e., the form of $\bar{U}_{ij}$) and the stochastic nature of the analysis (i.e., the form of $f(\varepsilon)$). The use of independent or covariance probit provides the greatest flexibility for analyzing a wide range of specifications of individual choice and stochastic assumptions. However, this greater flexibility comes at increased computational complexity.

As was mentioned in the Introduction and in Section 1.0, by making some rather restrictive assumptions on the form of individual utility and $f(\varepsilon)$, conditional logit is extremely easy to use. However, it seems difficult to argue *a priori* that the ε_{ij} are distributed as Weibull, and that random utility is separable except on the grounds of the computational ease that results. Furthermore, it is precisely this set of assumptions taken together which generates the difficulties associated with the "independence of irrelevant alternatives". Since these assumptions seemed grounded in computational ease alone, alternative assumptions that avoid some of the undesirable characteristics of conditional logit would be desirable. It is the purpose of this section to introduce a more general treatment of random utility in order to avoid the difficulties associated with the "independence of irrelevant alternatives", while retaining the logit technique and its computational ease. The more general treatment of utility is not developed in a fully rigorous theoretical fashion. Instead, an indirect utility formulation is hypothesized to perform hypothesis testing upon a generalized logit model.

The analysis of random utility undergirding most probability modeling treats utility as separable, such that for

$$U_{ij} = U_{ij}(X_j^1, \; X_j^2, \; a_i) \qquad \text{and} \qquad U_{ik} = U_{ik}(X_k^1, \; X_k^2, \; a_i)$$

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[1] Hausman and Wise claim that $n = 5$ or less is currently computationally tractable on the computer. See [11].

one can factor utility as

$$U_{ij} = \Psi[\phi(X_j^1, a_i) + \phi(X_j^2, a_i)]$$

and

$$U_{ik} = \Psi[\phi(X_k^1, a_i) + \phi(X_k^2, a_i)]$$

where X_j^1 and X_k^1 are those characteristics of alternatives j and k which differ; X_j^2 and X_k^2 are those characteristics of alternatives j and k which remain the same for both choices; and a_i is the vector of socioeconomic characteristic of individual i. Given this assumed separability, j is chosen over k if

$$U_{ij} \geq U_{ik}$$

which is equivalent to

$$\phi(X_j^1, a_i) \geq \phi(X_k^1, a_i).$$ [1]

It is precisely this separability into the set of characteristics that differs between two alternatives and everything else that imposes the binary comparison in conditional logit even when there are n alternatives. Suppose, instead, we defined U_{ij} in equation (1) in a more general indirect utility formulation:

$$\begin{aligned} U_{ij} &= U_{ij}(X_1, X_2, \ldots X_j, \ldots X_N, a_i) \qquad (9) \\ &= \overline{U}_{ij}(X_1, \ldots X_N, a_i) + \varepsilon(X_1, \ldots, X_N, a_i) \end{aligned}$$

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[1] See Domencich and McFadden [8], Chapter 3.

In this specification the utility to individual i in consuming j depends not only upon the price and non-price characteristics of j, but rather on an N-way comparison with all alternatives. For example, the utility of using oil heat to home owner i does not depend strictly[1] upon the cost and cleanliness characteristics of oil alone, but upon the comparison of the characteristics with those of natural gas, electricity, coal, etc.

This formulation is more general than the usual indirect utility formulation, where indirect utility ν is defined as $\nu(P, y) = \text{MAX}\ U(X)$, s.t. $p.x = y$ and where U is the traditional direct utility formulation and $px = y$ is the budget constraint.[2] Under that formulation only the prices and characteristics of the goods chosen by the consumer appear in υ[3]; in other words, the coefficients of fuels not chosen would be zero in U_{ij} in (9). Of course it is a testable hypothesis whether this traditional indirect utility formulation with the zero constraints on the cross price terms is appropriate; the hypothesis is tested in Section 3.0.

Letting $X = (X_1, \ldots X_N)$ and assuming all individuals are identical (homogenous tastes), (9) becomes

$$U_{ij} = U_j = U_j(X, a_i) + \varepsilon(X, a_i) \tag{9A}$$

$$= U_j(X, a_i) + \varepsilon_j .$$

Likewise, equation (2) becomes

$$\begin{aligned} P_{ik} = P_k &= \Pr[U_k > U_j, \text{for all } j \neq k] \qquad (10) \\ &= \Pr\ [U_k(X, a_i) + \varepsilon_k > U_j(X, a_i) + \varepsilon_j, \text{ for all } j \neq k] \\ &= \Pr\ [\varepsilon_j - \varepsilon_k < U_k(X, a_i) - U_j(X, a_i), \text{ for all } j \neq k] \\ &= \Pr\ [\eta_{jk} < U_k(X, a_i) - U_j(X, a_i), \text{ for all } j \neq k] \end{aligned}$$

where $\eta_{jk} = \varepsilon_j - \varepsilon_k$.

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[1] Domencich and McFadden refer to this characteristic of the separable utility formulation as "strict utility". See [8], pp. 78-80.

[2] This has been pointed out to me by Ralph Braid. The notational and theoretical development of the indirect utility is found in Hal Varian, "Lecture Notes in Micro-Economic Theory," Chapter [illegible] s. Marshallean demands for fuel

[3] That is, Marshallean demands for f [illegible] appliance i is zero, $\partial\nu/\partial P_i = 0$.

Equation (10) is a more general formulation of (2). Stochastic assumptions regarding η_{jk} (i.e., $f(\varepsilon)$) will indicate whether the use of logit or probit is relevant. Suppose we assume the ε_j are distributed Weibull, then equation (4) becomes:

$$P_{ik} = P_k = \frac{e^{U_k(X,\ a)}}{\sum_{j}^{N} e^{U_j(X, a)}} \tag{11}$$

and substituting the P_{ik} into (3) will yield likelihood estimates once the form of the U_j are specified. Suppose we specify the U_k very generally as: $U_k(X,\ a) = Z\ \beta^k$, where Z is the vector of combinations of X and a. In this formulation of the utility of each choice depends in a different way (β^k) upon the vector Z. Then equation (11) becomes the usual expression for general multinominal logit:

$$P_k = \frac{e^{Z\ \beta^k}}{\sum_{j=1}^{N} e^{Z\ \beta^j}} \tag{11A}$$

where the β^j, $j = 1 \ldots N$ can be estimated using equation (3).[1]

Utilizing equation (11A) for all k and likelihood equation (3), one can test the validity of the constraining assumptions of more traditional separable utility formulations as follows. Assume there exist three alternatives: X^1, X^2, and X^3, each defined by two characteristics:

$$\text{(e.g., } X_1^1 \text{ and } X_2^1\text{).}$$

Assume likewise that the individual characteristic vector, a, consists of component. Then, in equation (1), let

$$Z_{ij} = (X_1^j,\ X_2^j,\ a_i),\ j = 1, \ldots, 3;$$

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[1]For regression form we have: $\log\ (P_k/P_j) = Z\ \beta^k - Z\ \beta^j = Z(\beta^k - \beta^j)$. (11B)

and $\beta = (\beta_1, \beta_2, \beta_3)$. Given a sample of individual choices among X^1, X^2, and X^3, and personal characteristics a, likelihood estimates (i.e., equations (4) and (3)), or regression estimates (equation (5)), will yield:

$$\bar{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{0}).$$

Using the same sample information, let

$$X = (X_1^1, X_2^1, X_1^2, X_2^2, X_1^3, X_2^3) \text{ and } Z = (X, a).$$

Generate likelihood estimates (equations (11A) and (3)), or regression estimates (11B),

$$\hat{\beta}^j = (\hat{\beta}_1^j \ldots \hat{\beta}_7^j), \ j = 1 \ldots 3.$$

Using a likelihood ratio test[1], test

$$H_0: [\beta_3^1 = \beta_4^1 = \beta_5^1 = \beta_6^1 = \beta_7^1 = \beta_1^2 = \beta_2^2 = \beta_5^2 = \beta_6^2 = \beta_7^2 = \beta_1^3 = \beta_2^3 = \beta_3^3$$

$$= \beta_4^3 = \beta_7^3 = 0 \quad \text{and} \quad \beta_1^1 = \beta_3^2 = \beta_5^3 \quad \text{and} \quad \beta_2^1 = \beta_4^2 = \beta_6^3].$$

If one can reject H_0 or some subset of it, one can reject the separable utility assumption. Incidentally, when written in the form of H_0, it becomes clear how severe the assumption of separable utility is.

If the full generalized logit model proves appropriate, the more general equations (11A) and (11B) will eliminate some of the undesirable characteristics of the "independence of irrelevant alternatives."[2] For example,

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[1] The actual likelihood ratio test would be somewhat more complicated since the β^j are identified only to a normalization.

[2] Much empirical work in this area has constrained cross-elasticities or has been forced to assume that the change in a common variable will not affect the log odds ratio for two given choices. For example, in a case of fuel shares,

$$\partial \log(S_{GAS}/S_{ELECTRICITY}/\partial Z) \equiv 0$$

where Z is a common variable such as the income or price of oil. This assumption is too severe and the generalized multinomial formation avoids it.

the fuel demand model variation in equation (7A) resulted from a separable utility assumption. If we utilize the generalized multinomial logit formulation (11A), then the fuel share equation becomes (7A)' and the constant cross-elasticities are eliminated, as is the specification error that

$$\frac{\partial S_i}{\partial P_j} \cong 0 \quad \text{when} \quad \frac{\partial S_j}{\partial P_j} \neq 0 \quad \text{and} \quad \sum_{i=1}^{n} S_i = 1.^1$$

where again S_j is the fuel share of fuel j, and P_j is a fuel price (pp.7-9 above).

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[1]This can be stated more precisely as follows: Let Y be a polychotimous random variable described by the set of M multinomial probabilities. $Pr(Y = y_i) = P_i$ where

$\sum^{M} P_i = 1$, where $0 \leq P_i \leq$ for all i. If we relate probability of choosing fuel i to a set of exogenous variables Z through the functional form ϕ_i, where Z includes all fuel prices (own and other), then

$$Pr(Y = y_i | Z) = P_i = \frac{e^{\phi_i(Z)}}{\sum_{r=1}^{M} e^{\phi_r(Z)}}$$

Likewise,

$$Pr(Y = y_j | Z) = P_i = \frac{e^{\phi_j(Z)}}{\sum_{r=1}^{M} e^{\phi_r(Z)}} .$$

Using these specifications for P_i and P_j, we get $\frac{P_i}{P_j} = \frac{e^{\phi_i(Z)}}{e^{\phi_j(Z)}}$ and

$$\log \frac{P_i}{P_j} = \phi_i(Z) - \phi_j(Z).$$

If $\phi_i(Z) = \alpha_i Z$ for all i, then we have

$$\log \frac{P_i}{P_j} = \alpha_i Z - \alpha_j Z \tag{*}$$

$$= (\alpha_{i1} - \alpha_{j1})Z_1 + (\alpha_{i2} - \alpha_{j2})Z_2 + \dots (\alpha_{iL} - \alpha_{jL})Z_L$$

$$= \beta_1 Z_1 + \beta_2 Z_2 + \dots \beta_1 Z_L$$

where again Z includes all fuel prices. Clearly, if we replace fuel share data for probability data, we obtain equation (7A)' not equation (7A). Only if the price coefficients in (*) or (7A)' are $\cong 0$ will the equational form reduce to (7A). There is no reason to believe that is the case.

3.0 SOME EMPIRICAL RESULTS

The purpose of the empirical work reported here is limited in scope. I intend to first test the hypotheses that alternatives to the full generalized multinomial logit formulation are appropriate choice model specifications. Second, I intend to heuristically compare the likelihood estimates (resulting from treating the data as individual micro-decisions) with regression estimates (resulting from aggregating micro data to the state level and estimating the traditional conditional logit formulations using share data.

The likelihood and regression specifications are estimated for choice models for fuel demand. The models are applied to the household choice among electricity, gas and oil for home heating. As explained more fully below, in the micro individual choice models the probability of choosing a particular fuel is related to the operating costs and capital costs (of the relevant fuel burning appliances) of the alternative fuels and such socioeconomic/demographic characteristics as the availability of gas and whether the individual consumer lives in an urban or rural setting. In the regression framework the log odds ratio of aggregated decisions is related to these same exogenous variables.

The data consists of the independent variables for both urban and rural areas in each of the 50 states, plus the District of Columbia for 1960 and 1970 (Discussion in Table 3-1). Because an annual time series of cross section was not available, the likelihood and regression estimates are from static rather than the more desirable dynamic (lagged endogenous) specification.[1]

The regression analyses utilize state data for total households using gas, oil or electricity to generate share estimates (Sg, So and Se, respectively). The individual choice model treats each individual decision. Given the large size of the data array, a number of truncated samples are used. In particular, the number of households in each state is reduced by 10^{-5}, 10^{-4}, and 10^{-2} in various estimations. In other words, for the 10^{-5} truncation in a given state, if 4,000,000 households chose gas, 2,567,000 electricity and 5,272,000 oil, the estimation treats this as 40, 26 and 53 households, respectively.[2] Likelihood

[1] For a discussion of the alternatives, see Hartman and Hollyer [9].

[2] The regression analyses utilize TSP. The Likelihood analyses utilize a program developed by Charles Manski and is described in "The Conditional/Polytomous Logit Program: Instructions for Use", an unpublished mimeo (Carnegie-Mellen University, 1974). Clearly in the likelihood formulation not all of the data is micro data since state per capita income is used for all households in that state.

TABLE 3-1: DATA SERIES AND SOURCES

Pe	User cost of electricity in $\$/(10^6$ BTU) for a given state, calculated from the average cost of the first 250 kWh/mo. consumed. Typical Electric Bills, 1960, 1970, Federal Power Commission, Washington, DC.
Pg	User cost of natural gas in $\$/(10^6$ BTU) for a given state, averaged consumer cost. Gas Facts 1961, 1971, American Gas Association, Arlington, VA.
Po	User cost of oil in $\$/(10^6$ BTU) for a given state, derived from American Petroleum Institutes, Petroleum Facts and Figures, 1971 edition. Wholesale prices multiplied by retail markup of 54% in 1960 and 78% in 1970. Markups are the difference between the average Bureau of Labor Statistics price and API's.
CAP_e	The annual amortization and maintenance costs of an average electric heating syste, not including heat pump but including direct electric and electric furnace system[1] in the given region of the U.S.
CAP_g	The annual amortization and maintenance costs of an average gas heating system[1] in the given region of the U.S.
CAP_o	The annual amortization and maintenance costs of an average oil heating system[1] in the given region of the U.S.
PCI	State per capita incomes from U.S. Department of Commerce, Bureau of Economic Analysis, Survey of Current Business, August 1976, Vol. 56, No. 8.
TEMP	A variable proxying the severity of climatic conditions in each state. It is annual heating degree days for each state averaged over 1931-1960. State weighted average degree days were calculated on an SMSA basis by percent of a state's population residing in SMSA's for which heating degree day data were available. The data on heating degree days by region came from the ASHRAE 1973 Systems Handbook in Chapter 43, Energy Estimating Methods. The data were provided for U.S. Cities from a publication of the U.S. Weather Bureau, Monthly Normals of Temperature, Precipitation, and Heating Degree Days, 1962, and are the period 1931 to 1960, inclusive. The yearly totals are based on 65° F.
AV	Availability index computed as the number of distribution main miles per state resident, multiplied by 100. Source--Gas Facts, American Gas Association, 1961 and 1971 editions; U.S. Census of Population, 1960, 1970.
RU	Rural-urban dummy variable assuming a value of 1 for the rural segment of the sample.

[1]For a full discussion of the assumptions and parameters underlying the amortization, see J. G. Delene, "A Regional Comparison of Energy Resource Use and Cost to Consumers of Alternate Residential Heating Systems," Oak Ridge National Laboratory, ORNL-TM-4689, November 1974, Table 13. [10]. The Delene estimates are regionalized by techniques discussed in Hartman and Hollyer [9].

estimates for the 10^{-5} truncation are discussed for explicit hypothesis testing (Table 3-2). However, likelihood estimates for 10^{-2} and 10^{-4} truncations are presented for comparison (Table 3-3); the corresponding hypothesis testing results for these truncations are indicated.

To refresh the reader's mind, the forms of the generalized multinomial logit, the strict choice formulation, mixed generalized logit and the regression form of conditional logit are given below.

GENERALIZED MULTINOMIAL LOGIT

$$P_i = \frac{e^{\beta_i x}}{\sum_j e^{\beta_j x}} = \frac{1}{1 + \sum_{j \neq 1} e^{(\beta_j - \beta_i)x}} \tag{12A}$$

STRICT CHOICE FORMULATION

$$P_i = \frac{e^{\beta x_i}}{\sum_j e^{\beta x_j}} = \frac{1}{1 + \sum_{j \neq 1} e^{\beta(x_j - x_i)}} \tag{12B}$$

MIXED GENERALIZED LOGIT

$$P_i = \frac{e^{(\beta y_i + \alpha_i x)}}{\sum_j e^{(\beta y_j + \alpha_i x)}} = \frac{1}{1 + \sum_{j \neq i} e^{\beta(y_j - y_i) + (\alpha_j - \alpha_i)x}} \tag{12C}$$

REGRESSION FORM

$$\frac{S_i}{S_j} = \frac{e^{\beta_i x}}{e^{\beta_i x}} = e^{(\beta_i - \beta_j)x} \tag{12D}$$

$$\frac{S_i}{S_j} = \frac{e^{\beta x_i}}{e^{\beta x_j}} = e^{\beta(x_i - x_j)}$$

where P_i is the probability an individual chooses fuel i; x and y are vectors of independent variables; and S_i is the share of total fuel choices that are for fuel i.

TABLE 3-2

LIKELIHOOD RESULTS (FOR 10^{-5} TRUNCATION)

		Strict Choice Formulation	Mixed Generalized Logit	Generalized Logit 1	Generalized Logit 2
Pg	α			-4.0566 (-5.88)	-3.9608 (-3.41)
Po	α	-.65921 (-12.96)	-1.1816 (-6.22)	0.61472 (0.29)	1.9104 (0.33)
Pe	α			1.0144 (4.67)	2.031 (3.07)
CAP_g	α			0.51269 (1.69	0.64808 (0.94)
CAP_o	α	-.013691 (- 3.15)	0.12673 (0.49)	-7.0435 (-2.61)	-1.3872 (-2.69)
CAP_e	α			0.31822 (1.67)	0.94896 (1.41)
PCI	α		-0.0017722 (-5.21)		-0.0008782(-1.24)
AV	α		0.0047287 (1.28)		0.0089439(0.96)
TEMP	α		0.0005316 (3.15)		0.0010449(2.15)
RU	α		-1.6683 (-2.26)		-1.4735 (1.80)
Pg	γ			-0.1518 (-0.23)	-1.3992 (-1.22)
Po	γ		-1.1816 (-6.22)	-1.3981 (-0.67)	1.25 (0.22)
Pe	γ			-0.84942 (3.54)	1.5682 (2.35)
CAP_g	γ			0.84283 (2.55)	1.1112 (1.60
CAP_o	γ		0.012673 (0.49)	-1.0418 (-3.47)	-1.7806 (-3.36)
CAP_e	γ			0.37852 (1.99)	0.95064 (1.42)
PCI	γ		-0.0014497 (-4.04)		-0.0008572(-1.21)
AV	γ		-0.010534 (-2.74)		-0.0073102(-0.77)
TEMP	γ		0.00094706 (5.47)		0.001467 (3.02)
RU	γ		0.60392 (0.82)		0.98438 (1.20)
Log L		-530.83	-367.3	-422.96	-331.2
2 Log λ*		399.26 (0.995)	72.20 (0.995)	183.52 (0.995)	

*λ is the likelihood ratio for (Ho: the relevant model is appropriate rather than generalized logit 2). All likelihood ratio tests are significant at the 99.5% level in parentheses.

**Asymptotic t statistics for (Ho: parameter = 0) in parentheses.

TABLE 3-3

LIKELIHOOD RESULTS FOR ALTERNATIVE TRUNCATIONS

STRICT CHOICE MODEL

	10^{-5} TRUNCATION		10^{-4} TRUNCATION		10^{-2} TRUNCATION	
β1	-0.65923	(12.9)	-0.46977	(54.21)	-0.44003	(554.7)
β2	-0.013691	(3.15)	-0.018156	(15.62)	-0.018243	(160.1)
Log L =	-530.83		-7214.4		-759580.0	

FULL GENERALIZED LOGIT

	10^{-5} TRUNCATION		10^{-2} TRUNCATION	
γ1	-3.9608	(3.41)	-1.8827	(117.7)
γ2	1.9104	(0.34)	-0.38104	(11.45)
γ3	2.0310	(3.07)	0.91498	(172.6)
γ4	0.64808	(0.94)	0.32534	(43.7)
γ5	-1.3872	(2.70)	-0.31671	(52.8)
γ6	0.94896	(1.42)	0.016194	(7.86)
γ7	-0.00087872	(1.24)	-0.00022957	(33.7)
γ8	-0.0010449	(2.15)	0.00027746	(73.1
γ9	-1.4735	(1.79)	-1.502	(136.4)
γ10	0.0089439	(0.96)	0.0034578	(45.5)
α1	-1.3992	(1.21)	0.11567	(7.51)
α2	1.2500	(0.22)	-1.4562	(41.9)
α3	1.5682	(2.35)	0.6034	(109.8)
α4	1.1121	(1.61)	0.56037	(67.1)
α5	-1.7806	(3.33)	-0.50359	(70.4)
α6	0.95064	(1.41)	0.011995	(5.71)
α7	-0.0008572	(1.20)	-0.00018383	(27.0)
α8	0.0014671	(3.02)	0.00057377	(151.5)
α9	0.98438	(1.20)	0.32206	(29.3)
α10	-0.0073102	(0.77)	-0.0086169	(105.5)
Log L =	-331.2		-563340.0	

NOTE: t statistics for (Ho: Parameter = 0) in parentheses.

In the generalized logit form, x was seen above (pp. 12-15) to include the characteristics of all the options plus the characteristics of the individual. Resulting parameter estimates are obtained for $(\beta_j - \beta_i)$ for all $j \neq i$; that is, the β_j are identified to a normalization.

In the strict choice model, x includes only the characteristics which differ across choices (pp. 3-4 above). The remaining characteristics drop out since β is estimated for $(x_j - x_i)$.

The mixed generalized formulation permits the estimation of the effects of a common set of characteristics (x) across choices in addition to the effects of characteristics which vary (y). In the latter case, β is estimated, in the former $(\alpha_j - \alpha_i)$ are estimated.[1]

The regression forms merely take aggregated observations of the generalized logit or strict choice forms and estimates the log odds ratio as a linear function of x.

In Table 3-2, likelihood estimates are presented for two generalized logit formulations (1 and 2), for a mixed generalized logit formulation and for a strict choice formulation for the 10^{-5} truncation.

The full generalized logit (2) specification utilizes x = (Pg, Po, Pe, CAP_g, CAP_o, CAP_e, PCI, AV, TEMP and RU). Generalized logit 1 utilizes x = (Pg, Po, Pe, CAP_g, CAP_o, and CAP_e). In both specifications β_e (for electricity) is normalized to zero. Hence, in Table 3-2, $\alpha = \beta_g - \beta_e = \beta_g$, and $\gamma = \beta_o - \beta_e = \beta_o$, where β_g and β_o are the parameter vectors for gas and oil in (12A). Given the fact that in the generalized logit form the parameters are identified to a normalization only, it is impossible to interpret the signs of the estimated values in any meaningful sense. The signs of the commonly estimated parameters are identical for generalized logit 1 and 2, except the insignificant $\hat{\gamma}_2$. The likelihood ratio test of Ho: generalized logit 1 is the correct specification, can be rejected at the 99.5% level.

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[1] This corresponds in principle to the regression specification form in Baughman and Joskow [4].

Mixed generalized logit permits the estimation of $\beta_g - \beta_e$ and $\beta_o - \beta_e$ for PCI, AV, TEMP and RU, while imposing a common price and capital cost effect.

Finally, in the strict choice model, a common price and capital cost effect is estimated--β_1 and β_2 in the notation of (pp. 2-4). In the strict choice formulation the parameters are identified and examination of (12B) will indicate that the priors on β_1 and β_2 are less than 0. Both estimated coefficients in the strict choice formulation are less than 0 and significant. In the mixed generalized logit form the common price effect is negative and significant. The capital cost effect is positive but not significantly different from zero. The signs of $\alpha_7 - \alpha_{10}$ and $\gamma_7 - \gamma_{10}$ in the mixed generalized logit are the same as generalized logit 2. However, they are more significant. One cannot accept the hypothesis that either the strict choice or mixed generalized logit formulation is the appropriate one when compared with the full generalized logit 2. Hence, based upon the evidence here, the separable utility formulation underlying much of the choice modeling in the literature must be rejected for energy demand for alternative fuels in home heating.[1]

The effects of alternative truncations upon parameter estimates and the likelihood ratio hypothesis tests given above are indicated in Table 3-3. Clearly as the truncation is diminished from 10^{-5} to 10^{-2}, the amount of information in the sample is expanded considerably. In the strict choice model, the coefficient signs all remain the same. However, the coefficient estimates are refined to very precise point estimates. The asymptotic t statistics are at unheard levels of 160 to 550. In the full generalized logit model, the 10^{-2} truncation produces consistently highly significant parameter estimates.

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[1]In this static formulation, of course. The reader should note that the mixed generalized logit is one form of traditional indirect utility formulation (pp. 12-13 above) with cross-price and cross capital cost terms constrained to be zero. This traditional indirect utility form is rejected.

Furthermore, the rejection of the alternative utility specifications (full generalized 1, mixed generalized (i.e., traditional indirect utility formulation[1]) and strict choice) is even more resounding in the 10^{-2} truncation. For example, from Table 3-3 (Ho: the strict choice model is the appropriate formulation) can be rejected above the 99.99% level, with $-2 \log \lambda = 392500$.[1]

Regression forms (12D) are estimated for the analog to generalized logit 2 and the strict choice formulation. The results are given in Table 3-4. OLS and WLS estimates are given, as are the maximum likelihood estimates from Tables 3-2 and 3-3. The parameter estimates for the strict choice formulation are quite similar for OLS, WLS and ML. However, for the generalized logit 2 the parameter estimates from lines (1, 3 and 5) and (2, 4 and 6) differ considerably at times. In some cases the signs reverse while both parameter estimates are significant.

Table 3-4 presents an interesting set of results. Based upon the strict choice formulation, it would appear going from the micro, individual-choice, likelihood model to the aggregated regression model does lead to similar estimates of β (see 12B and 12D). Thus, neither aggregation and regression assumptions nor the likelihood assumption of stochastic independence of individual choices appear to interfere with estimating the parameters of choice inherent in the strict choice model. However, with the full generalized logit 2, no such similarity of parameters (row by row comparison of rows 1, 3 and 5, and 2, 4 and 6) appears, particularly for the 10^{-5} truncation. In the 10^{-2} truncation more information is utilized and the maximum likelihood estimates are quite significant and compare well with the WLS estimates (rows 7 and 8, and 3 and 4). The parameter estimates which differ the most are those for fuel choices (Pg, Po and Pe).

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[1] Clearly this hypothesis testing and the likelihood estimation assumes independence on the part of each household decision. Such independence is not present given the existence of state and regional supply effects. I have not assessed the significance of the lack of assumed independence upon my results.

TABLE 3-4: COMPARISON OF REGRESSION AND LIKELIHOOD RESULTS

GENERALIZED LOGIT	Pg	Po	Pe	CAP_g	CAP_o	CAP_e	PCI	AV	TEMP	RU	Log L	R^2	D.W.	TECHNIQUE
Log (Sg/Se)	-0.6040 (-3.62)	-1.0460 (-2.48)	0.4608 (7.77)	-0.3928 (-3.00)	.2956 (2.58)	.07584 (3.86)	-.00049 (-5.54)	.006608 (7.29)	.000118 (2.80)	-1.7682 (-12.05)	-293.9	.75	1.79	OLS
Log (So/Se)	1.1464 (5.83)	-1.5649 (-3.15)	0.2062 (2.95)	0.1263 (0.82)	-.1809 (-1.34)	.11416 (4.93)	-.00054 (-5.15)	-.005516 (-5.16)	.000338 (6.79)	.2058 (1.19)	-327.4	.76	1.39	OLS
Log (Sg/Se)	-1.6265 (-10.90)	-0.0749 (-0.214)	0.9003 (17.72)	0.2536 (3.57)	-.2732 (-4.81)	.02990 (1.47)	-.00012 (-1.81)	.003634 5.25	.000226 (6.52)	-1.3434 (-12.61)	-494.7	.87	1.78	WLS
Log (So/Se)	0.5423 (3.24)	-2.6711 (-5.77)	0.4266 (6.74)	0.5573 (5.50)	-.4526 (-5.09)	-.02345 (-0.97)	-.00017 (-2.27)	-.007661 (-7.98)	.000533 (14.32)	.2613 (2.17)	-505.2	.76	1.22	WLS
P_{gas} (gas/elec.)	-3.9608 (-3.41)	1.9104 0.33	2.031 (3.07)	0.6480 (0.94)	-1.3872 (-2.69)	.94896 (1.41)	-.00087 (-1.24)	.008943 (0.96)	.001044 (2.15)	-1.4735 (-1.80)	-331.2			ML 10^{-5} TRUNCATION
P_{oil} (oil/elec.)	-1.3992 (-1.22)	1.25 (0.22)	1.5682 (2.35)	1.1112 (1.60)	-1.7806 (-3.36)	.95064 (1.42)	-.00085 (-1.21)	-.007310 (-.77)	.001467 (3.02)	.9843 (1.20)				
P_{gas} (gas/elec.)	-1.8827 (-117.7)	-0.3810 (11.45)	0.9149 (172.6)	0.3253 (43.7)	-.3167 (-52.8)	.01619 (7.86)	-.00022 (-33.7)	.003457 (45.5)	.000277 (73.1)	-1.502 (-136.4)	-56334			ML 10^{-2} TRUNCATION
P_{oil} (oil/elec.)	0.1156 (7.51)	-1.4562 (-41.9)	0.6034 (109.8)	0.5603 (67.1)	-.5039 (-70.4)	.01199 (5.71)	-.00018 (-27.0)	-.008616 (-105.5)	.000573 (151.5)	.3220 (29.3)				
STRICT CHOICE FORMULATION														
Log (S_i/S_j)	-.48576 (-18.5)	-.051784 (-6.41)									-845.2	.28	1.93	OLS
Log (S_i/S_j)	-.56380 (-13.8)	-.025913 (-2.55)									-1611.8	.76	.96	WLS
P_i	-.65921 (-12.91)	-.013691 (-3.15)									-530.8			ML, 10^{-5} TRUNCATION
P_i	-.46977 (-54.21)	-.018156 (-15.62)									-7214.4			ML, 10^{-4} TRUNCATION
P_i	-.44003 (-554.7)	-.018243 (-160.1)									-759580			ML, 10^{-2} TRUNCATION

NOTES: ML = Maximum Likelihood; OLS = Ordinary Least Squares; and WLS = Weighted Least Squares.

This discussion for Table 3-4 is heuristic at best. What is required is a rigorous analytic examination of aggregation effects upon the individual choice model to the regression form. Furthermore, an analysis of severity of the effects of actual non-independence of household fuel choice by state upon the likelihood estimates is required. Finally, a comparison of the likelihood estimates of the regression form and the logit form (either strict choice or generalized logit) is also required. Such research is currently underway.

BIBLIOGRAPHY

1. Anderson, Kent P., The Price Elasticity of Residential Energy Use, The Rand Paper Series #P-5180 (February 1974).

2. Anderson, Kent P., Residential Demand for Electricity: Econometric Estimates for California and the United States, Rand Corporation, R-905-NSF (January 1972).

3. Anderson, Kent P., Residential Energy Use: An Econometric Analysis, Rand Corporation, R-1297-NSF (October 1973).

4. Baughman, M. and Joskow, P., "Interfuel Substitution in the Consumption of Energy in the United States, Part I: The Residential and Commercial Sectors", MIT Energy Laboratory Report No. NIT-EL74-002.

5. Baughman, M. and Joskow, P., A Regionalized Electricity Model", MIT Energy Laboratory Report No. 75-005, December 1974.

6. Baughman, M. and Joskow, P., "Energy Consumption and Fuel Choice by Residential and Commercial Consumers in the U.S.," Energy Systems and Policy, Vol. 1, No. 4.

7. Delene, J. G., A Regional Comparison of Energy Resource Use and Cost to Consumers of Alternate Residential Heating Systems, Oak Ridge National Laboratory, #W-7405-ENG-26 (November 1974).

8. Domencich, J. and McFadden, D., Urban Travel Demand, A Behavioral Analysis, North Holland/American Elseview, 1976.

9. Hartman, R. and Hollyer, M., "An Examiantion of the Use of Probability Modeling for the Analysis of Interfuel Substitution in Residential Fuel Demand," MIT Energy Laboratory Working Paper No. MIT-EL-77-018WP (July 1977).

10. Hausman, J., "Project Independence Report: An Appraisal of U.S. Energy Needs Up to 1985," The Bell Journal (Autumn 1975), pp. 517-551.

11. Hausman, J. and Wise, D., "A Conditional Probit Model for Qualitative Choice: Discrete Heterogenous Preferences," forthcoming in Econometrica.

12. Joskow, P., and Baughman, M., "The Future of the U.S. Nuclear Energy Industry," The Bell Journal of Economics, Spring 1976, pp. 3-33.

13. Lin, W., Hurst, E. and Cohn, S., Fuel Choices in the Household Sector, Oak Ridge National Laboratory, #W-7405-ENG-26 (October 1976).

14. McFadden, D., "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka, Frontiers in Econometrics (Academic Press, New York).

15. Pindyck, R. and Rubinfeld, D., Econometric Models and Economic Forecasts, McGraw-Hill, New York (1976).

BIBLIOGRAPHY (concluded)

16. Quandt, E. R., "Estimation of Model Splits," Transportation Research, Vol. 2, pp. 41-50.

17. Theil, H., "A Multinominal Extension of the Linear Logit Model," International Economic Review, Vol. 10, No. 3 (October 1969), pp. 251-259.

18. Theil, H., "On the Estimation of Relationships Involving Qualitative Variables," American Journal of Sociology, Vol. 76 (1970), pp. 103-154.

19. Wilson, J. W., "Residential Demand for Electricity," The Quarterly Review of Economics and Business, 11:1 (Spring 1971), pp. 7-19.

20. Zellner, A. and Lee, J. H., "Joint Estimation of Relationships Involving Discrete Random Variables," Econometrica, Vol. 33 (April 1975), pp. 382-394.