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by

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The Impact of Uncertainty on the
Effect of Rate of Return Regulation
Remains Highly Uncertain

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1. Introduction

One of the most prominent theoretical results of the literature on public utility regulation is the Averch-Johnson (A-J) effect.¹ It states that a profit maximizing monopoly firm, which faces a rate of return constraint on its input of capital, will tend to produce its output inefficiently by using too much capital -- relative to other inputs. This is predicted to occur if the allowed rate of return s is smaller than the unconstrained monopoly rate s_0 and larger than the cost of capital i . Furthermore, the smaller $s > i$, the larger K , the capital employed by the regulated firm, will be.² Empirical tests have on balance tended to favor the A-J results, but so far methodologically they have been unsatisfactory.³ Hence, this largely remains an area of theory. Recent theoretical advance has been made by introducing more realistic assumptions into the model. A special, not uncontroversial route⁴ was taken by Peles and Stein (P-S).⁵ They recently announced results partly contrary to A-J for the case of the regulated monopolist facing uncertainty of a specific kind. This comes as a surprise. For, if the allowed rate of return is the only lever through which licensed profit maximizing monopoly firms are being regulated, then the A-J results are intuitively appealing. Constraining the monopolist's allowed rate of

¹See Averch and Johnson (1962), Baumol and Klevorick (1970), Bailey (1973).

²See e.g., Bailey (1973).

³The results of Spann (1974), Courville (1974), Peterson (1975) and Boyes (1976) are convincingly attacked by McKay (1976).

⁴See e.g. Baron and Taggart (1978) or Newbery (1978).

⁵See P-S (1976).

return s below the unconstrained monopoly rate s_0 , but letting it exceed the cost of capital i , induces such firms to maximize capital input K subject to the rate of return constraint. The lower the allowed rate of return $s > i$ the larger the maximal K will be at which the constraint is satisfied. P-S's propositions deserve a more thorough discussion both because of the peculiarity of their assumptions and the striking result, which, however, in general is incorrect.

P-S have to build a model different from the one used by A-J in order to introduce and treat the influence of a stochastic environment. A-J combine the two inputs capital and labor in a neoclassical production function. Input and output quantities which maximize profits subject to the regulatory constraint, are determined simultaneously. In the P-S model, however, capital input is defined to be a scale of plant which the firm has to decide upon ex ante before the state of nature is known. After the state of nature has been revealed, the firm chooses ex post the labor input which maximizes its quasi-rents subject to the rate of return constraint.

P-S do not systematically introduce the mechanism by which the stochastic environment influences the quasi-rents. They assume that the quasi-rent R , which is a function of plant scale (capital) K and the actual state of nature u , is a well-behaved function $R(K,u)$ in both variables. This procedure facilitates the investigation of the influence of the rate of return regulation on the capital employed.

It is assumed that the firm is risk-neutral, hence tries to maximize the difference between the expected quasi-rents $E[R(K,\cdot)]$ (which after integration depends on K only) and the total cost of capital iK . A first consequence of this approach is the fact that the nature

of uncertainty, i.e., the probability distribution of u , does not play any role for the plant size of the unregulated monopolist as long as the uncertainty is a mere disturbance of quasi-rents, i.e.

$$E[R(K, \cdot)] = R(K, 0),$$

where 0 is the state of nature which does not influence the quasi-rent.⁶

"The nature of uncertainty is irrelevant for the risk neutral unregulated monopolist." (P-S, p. 280)

Within their model P-S try to investigate the effect of two types of uncertainty. Uncertainty is of the additive type, if

$$R(K, u) = R(K) + u$$

with $E[u] = 0$ and of the multiplicative type, if

$$R(K, u) = R(K) \cdot u$$

with $E[u] = 1$. Thus, in the additive case, uncertainty influences maximal quasi-rents independent of the size of K , whereas in the multiplicative case the effect of uncertainty increases with K . At a first glance, implied independence in the additive case seems to be less realistic.

What can the reasons and effects of an uncertain disturbance on quasi-rents be? P-S (p. 279) assume that there is uncertainty about the demand price, i.e., $p = f(q, u)$. Furthermore, in their model there is no uncertainty with respect to the technology, factor prices, type of regulation and the allowed rate of return. Hence, uncertainty can only enter via demand. But $R(K, u)$ is the maximum difference between two functions,

⁶Newbery (1978) criticises this point, but overlooks that for its derivation P-S have assumed uncertainty to be functionally related to R and not just to $x(p)$.

only one of which is related to demand. That means that only the additive case is compatible with the stochastic model behind the quasi-rents function. And it is surprising that P-S claim their important Anti-A-J conclusion just for the case, where uncertainty has a multiplicative effect on maximal quasi-rents.

Hence, it seems more appropriate to ask directly: How can multiplicative uncertainty on maximal quasi-rents be interpreted? The most reasonable and important possibility that occurred to us is to relate such multiplicative uncertainty to the general price level in the economy. Both, the demand for the regulated firm's product and the wage level for its labor inputs could be similarly affected by such uncertainty.

Regulation in the P-S model takes place as an ex post regulation. After the state of nature u is revealed, the regulatory authority compares realized quasi-rent $R(K,u)$ with sK , the allowed return on capital, and takes away any exceeding surplus. The firm, as before, chooses K to maximize the difference between the expected regulated quasi-rents and the total cost of capital.

The ex post regulation can be interpreted in two ways: First, the regulatory agency may know the state of nature at the same instant the regulated firm does. But in this case the claim of Newbery (1978) may be valid that the model does not correctly describe the regulatory process. For, if the regulatory agency knows the effect of the state of nature on quasi-rents, it seems unnatural to assume that the agency would measure the allowed rate of return against actual quasi-rent $R(K,u)$ and not against expected quasi-rent $E[R(K,\cdot)]$. If then, regulators use expected quasi-rents in determining the allowed rate of return s ,

the conventional A-J results are obtained. (Newbery, 1978). However, one of the reasons why rate of return regulation is being used instead of directly prescribing maximal prices is precisely that regulatory agencies do not normally have adequate information on cost and demand conditions of regulated firms. Hence, an agency can hardly judge if any supernormal returns earned by a regulated firm are due to the exercise of monopoly power or to the state of nature u . Furthermore, such (windfall) gains may be challenged due to pressure on the regulatory agency coming from consumer interest groups (see e.g., natural gas). It may, therefore, be a realistic assumption that regulators are blind⁷ with respect to the influence of uncertainty. Indeed this is a feature, which public utilities often complain about.

Secondly, one can think of the regulatory agency taking away profits from the firm and turning them over to the consumers, after the market transactions have occurred. This is being practiced by regulatory agencies during inflationary periods in order to overcome financial problems of regulated firms. If the firms correctly anticipate this possibility, an argument can be made that they will charge prices in accordance with the state of the world actually anticipated by them, because they otherwise would lose revenue.⁸ P-S's own interpretation seems to be compatible with this second view, for they assume that there is no effective regulatory lag (P-S, p. 282, Footnote 4). This assumption

⁷Just as the market may be blind in eliminating firms that otherwise might have had a bright future.

⁸On the other hand strategic misrepresentation could enable the firm to circumvent the obligation to serve all demand at current prices.

may be crucial, because a regulatory lag could change their results if states of nature were serially uncorrelated.

Two further implicit assumptions used by P-S are that regulatory policy is not affected by the lower tail of the probability distribution of the u 's and that the firm has no means to use capital or otherwise insure itself to reduce uncertainty. Both of these points are similarly restrictive. Regulated industries can expect to get some relief from competition or even subsidies in case of adverse circumstances.

On the other hand, essentially any firm can change the effect of uncertainty by buying insurance, holding buffer stocks, using different production techniques etc. Without regulation, a risk-neutral firm will not do this, but if the upper tail of the probability distribution is cut off, these measures which in effect would cut off the lower tail, may increase the expected value. Such behavior could well be regarded as an efficiency loss due to regulation because without regulation it would have been rejected on efficiency grounds.

2. The Case Without Uncertainty

One important test any more general model has to pass is the test whether it yields the standard results in the standard case. To wit: If uncertainty degenerates to the certainty case we would like to obtain the classic A-J results. Otherwise it would be meaningless to ask whether the A-J effect is still valid after the introduction of uncertainty.

To this end we make the following assumption on the maximal quasi-rent function:

Let the maximal quasi-rent function

$$R: \mathbb{R}_+ \rightarrow \mathbb{R}_+$$

be strictly concave and twice differentiable, let

$R(0) = 0$, $R'(0) > i$, and let there exist a $\bar{K} > 0$ with

$$i\bar{K} = R(\bar{K}).$$

The last two assumptions are only made to avoid trivial pathologies.

The following results are derived in Appendix A:

1. The average quasi-rent function

$$K \mapsto \frac{R(K)}{K}$$

is strictly decreasing.

2. Hence its inverse function J , which is given by

$$J(r) := K \Leftrightarrow r = \frac{R(K)}{K},$$

is strictly decreasing, too.

3. There is a unique capital K_0 which maximizes profits $R(K) - iK$.

Let

$$s_0 = \frac{R(K_0)}{K_0}$$

be the unregulated rate of return at the profit maximizing capital stock.

We define the regulated profits for $s > i$ and $K > 0$ by

$$\pi(s, K) := \begin{cases} R(K) - iK & \text{if } R(K) \leq sK \\ sK - iK & \text{if } R(K) > sK \end{cases}$$

For fixed s , the function $\pi(s, \cdot)$ is concave, continuous, but not necessarily differentiable.

Since for each $s > i$ the capital can be chosen such that regulated profit is positive, we can define a function

$$K:]i, \infty] \rightarrow \mathbb{R}$$

where $K(s)$ is the capital which maximizes the regulated profits $\pi(s, \cdot)$ if the allowed rate of return equals s .

The function $K(\cdot)$ enjoys the main properties which it should according to A-J. More precisely (see Appendix A for a sketch of the proof):

$$K(s) = \begin{cases} J(s) & \text{if } s \leq s_0 \\ K_0 & \text{if } s > s_0. \end{cases}$$

Hence, $K(\cdot)$ is continuous and strictly decreasing for $s < s_0$. In particular, for all s_1, s_2 with $i < s_1 < s_2 < s_0$ we have $K(s_1) > K(s_2) > K_0$ and

$$K(s) \rightarrow K_0 \quad \text{for } s \rightarrow \infty.$$

3. The Stochastic Case

There are some results which can be obtained for a general maximal quasi-rents function $R(K, u)$ (see Appendix B). However, results of the A-J type need more structure. Following P-S we concentrate on two special cases: uncertainty is of the

additive type if $R(K, u) = R(K) + u$;

multiplicative type if $R(K, u) = R(K) \cdot u$,

where $R(\cdot)$ is a maximal quasi-rent function as treated in the preceding section and u is a real valued random variable with

$$E[u] = 0$$

in the additive case and

$$E[u] = 1$$

in the multiplicative case.

3.1 The Additive Case

We assume that uncertainty is described by a density function f which is

positive on all of \mathbb{R} , and has an expected value

$\int_{-\infty}^{+\infty} u f(u) du$ of zero.

Let F be its distribution function. The expected regulated profit for $s > i$, $K > 0$ is given as

$$\pi(s,K) = \int_{\{u|R(K) + u \leq sK\}} (R(K) + u - iK) f(u) du + \int_{\{u|R(K) + u > sK\}} (sK - iK) f(u) du$$

This can be rewritten as

$$= \int_{-\infty}^{sK-R(K)} (R(K) - sK + u) f(u) du + (s-i)K.$$

1. As a first result we get: For $s=i$ the right hand term disappears whereas the left hand term clearly is negative (remember that f is strictly positive). As a consequence for small s and fixed K the regulated firm cannot expect to make a profit. It is true that, for sufficiently small $s > i$, the regulated firm cannot make a profit for any positive capital -- a noted difference to the deterministic case.⁹

We confine our attention to the set S of rate of returns at which the firm could make a profit. Clearly, S is an interval $]\bar{s}, \infty]$ where the case $\bar{s} = \infty$ corresponds to no regulation. \bar{s} depends on R and f .

For convenience denote

$$h(s,K) := sK - R(K).$$

The expected profit $\pi(s,K)$ is a differentiable function. Its first and second partial derivatives with respect to K are (see Appendix B)

$$\pi_2(s,K) = (s-i) + (R'(K) - s) F(h(s,K))$$

$$\pi_{22}(s,K) = R''(K)F(h(s,K)) - (R'(K) - s)^2 f(h(s,K)).$$

Using these formulas one can state that

2. For fixed s , expected profit $\pi(s, \cdot)$ is a concave function.

⁹The proof of this is somewhat elaborate. The authors will gladly supply a copy of it upon request.

3. For each $s \in S$, there exists a capital stock, $K(s)$, which maximizes expected profits given the allowed rate of return, s .
4. The function $K(\cdot)$ is differentiable. We have $K(s) > K_0$ for $s \in S$ and
$$K(s) \rightarrow K_0 \text{ for } s \rightarrow \infty.$$
5. The implicit function theorem gives enough information on $K'(s)$ to infer that we cannot expect a monotone behavior for $K(\cdot)$; i.e., $s_1 > s_2 \Rightarrow K(s_1) \leq K(s_2)$ is not true.
6. P-S, however, already formulated a weak statement on the behavior of $K(\cdot)$. This statement, which is readily proven with the formula for $\pi_2(s, K)$ is:

$$\pi_2(s, K_0) = (s-i)(1 - F(h(s, K_0))) > 0 \text{ for } s > i.$$

Together with the concavity of $\pi(s, \cdot)$ this implies that starting from K_0 , the firm could follow the gradient to arrive at $K(s)$.

The results under (1) and (5) show that not all of the A-J type results on rate-of-return-regulation carry over from the deterministic case to uncertainty.

3.2 The Multiplicative Case

We assume this time that uncertainty is described by a density function f such that

$$f(u) \text{ is positive, whenever } u \text{ is positive, it is continuous and} \\ \int_{-\infty}^{+\infty} u f(u) du = 1.$$

The notation will be kept as close to the additive case as possible.

$$R(K, u) := R(K) \cdot u$$

and

$$h(s, K) := \frac{sK}{R(K)} .$$

Furthermore, for

$$\lambda \in \mathbb{R}$$

we define

$$\alpha(\lambda) := \int_{-\infty}^{\lambda} u f(u) du .$$

As before for $s \geq i$, $K > 0$

$$\pi(s, K) = \int_{-\infty}^{h(s, K)} (R(K) \cdot u - sK) f(u) du + (s-i)K$$

- (1) From this it follows that for each fixed K expected regulated profits must be negative for small s , and it can be shown that the set S of regulated rate of returns which allow a positive profit is a ray $]\bar{s}, \infty]$ with $\bar{s} > i$.

Choosing a slightly different representation for the regulated profits we get

$$\pi(s, K) = [s(1 - F(h(s, K))) - i]K + R(K) \cdot \alpha(h(s, K))$$

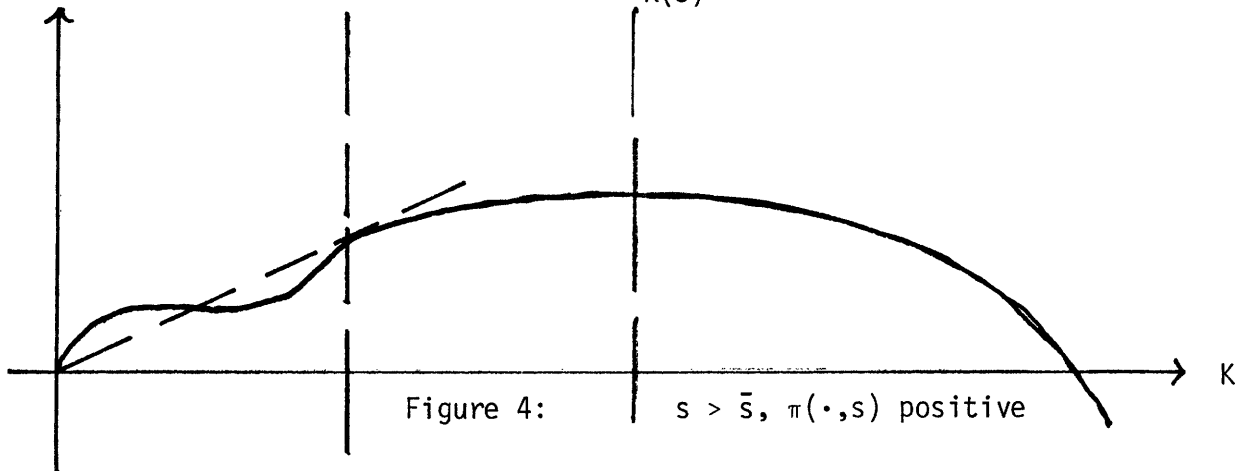
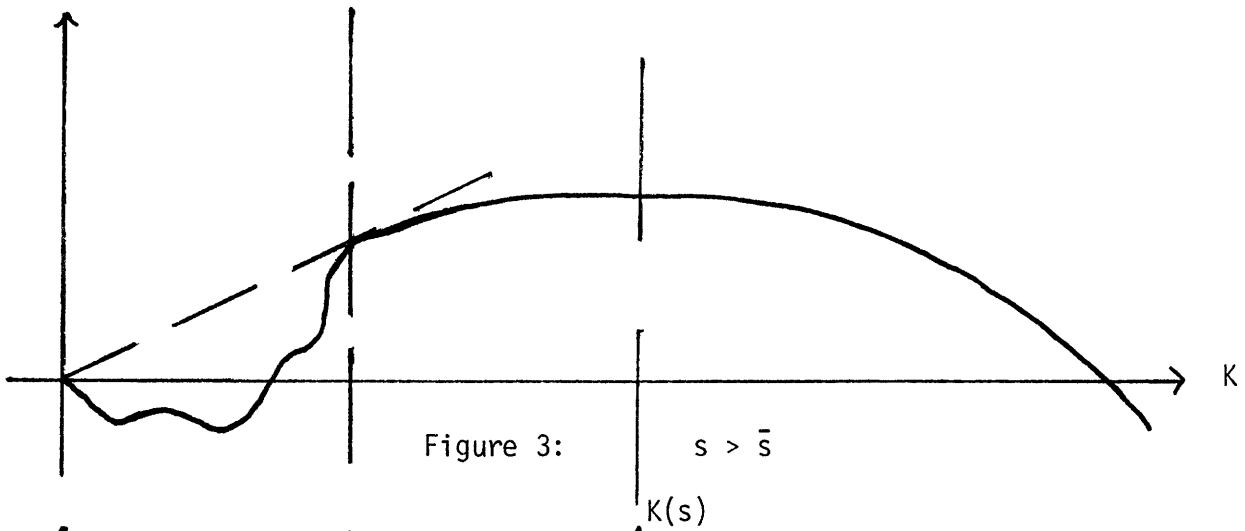
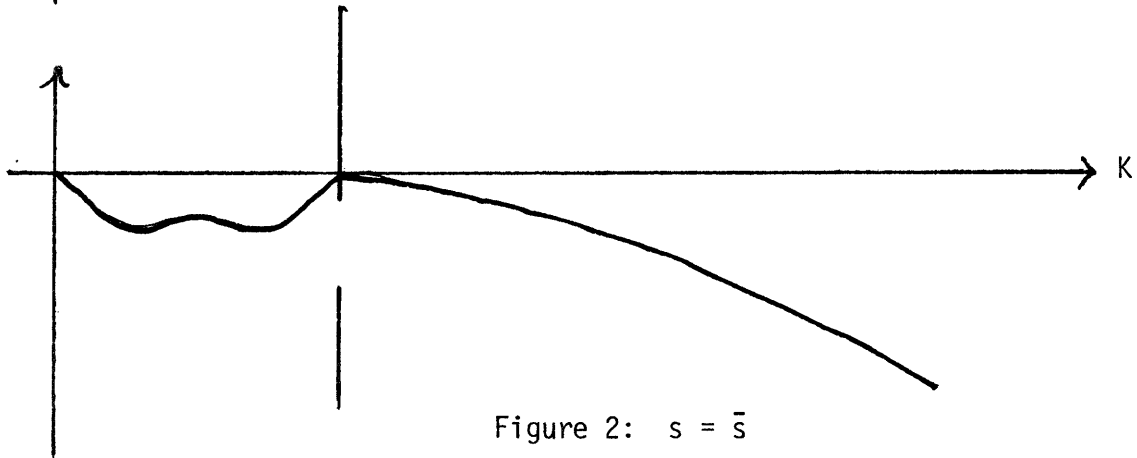
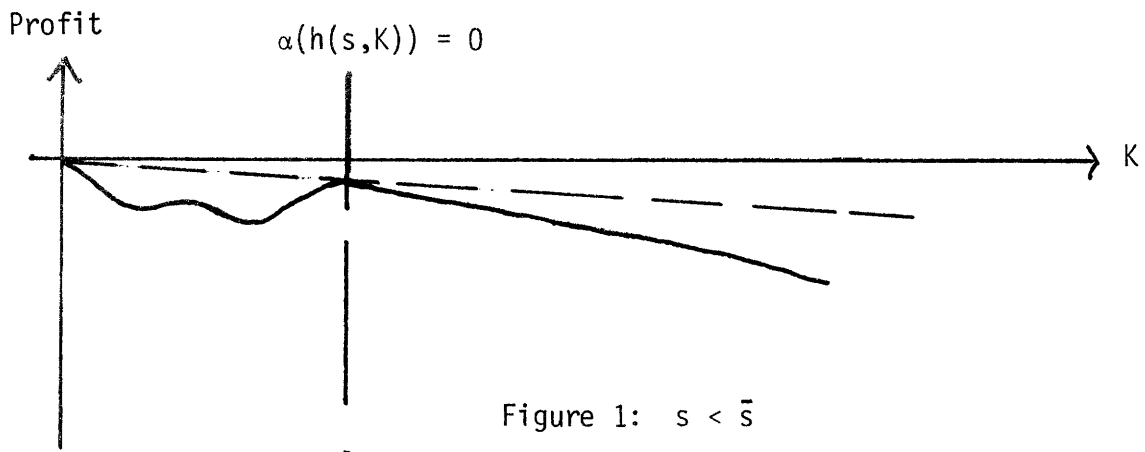
and from this the derivatives

$$\pi_2(s, K) = s(1 - F(h(s, K))) - i + R'(K) \alpha(h(s, K))$$

$$\pi_{22}(s, K) = R''(K) \cdot \alpha(h(s, K)) - f(h(s, K)) \frac{s^2}{R^3(K)} \cdot [R'(K)K - R(K)]^2.$$

For the shape of $\pi(s, \cdot)$ we can thus derive the following:

- (a) If $s \notin S$ and $s < \bar{s}$, $\pi(s, \cdot)$ is always negative and above some \bar{K} it is concave and falling. (See figure 1.)
- (b) If $s \notin S$ and $s = \bar{s}$, the maximal profit is zero at a capital K where $\alpha(h(s, K)) = 0$ and from there on it is concave and falling. (See figure 2.)
- (c) If $s \in S$ the function can be positive or negative for small K , depending on $\pi_2(s, 0) > 0$ or not. (See figures 3 and 4.)
As long as $\pi_2(s, \cdot)$ is negative its behavior is unpredictable. However, it intersects the abscissa with a positive slope and is monotonically increasing until $K(s)$ is reached.



For K exceeding \bar{K} , defined by

$$\alpha(h(s, \bar{K})) = 0,$$

$\pi(s, \cdot)$ is concave. We always have that

$$\bar{K} < K(s).$$

\bar{K} also can be characterized by

$$\frac{\pi(s, \bar{K})}{\bar{K}} = \pi_2(s, \bar{K})$$

(marginal profit equals average profit for given s). At \bar{K} the average quasi-rent is maximal. Hence, the capital $K(s)$, which maximizes regulated profit, is always bigger than the capital which maximizes average regulated profits.

From the statements (a), (b) and (c) one can derive the following rule for the regulated firm not knowing the shape of $\pi(s, \cdot)$:

If at a K profit is positive, follow the gradient.

If profit is negative and $\alpha(h(s, K)) \geq 0$, follow the gradient.

If profit is negative and $\alpha(h(s, K)) < 0$, increase K until $\alpha(h(s, K)) = 0$. If profits then become positive, follow the gradient, otherwise ask for a rate increase or abandon the market.

One can further show that the function $K(s)$ which assigns the profit maximizing capital to a rate of return s is well defined on S , is differentiable, and fulfils

$$K(s) \rightarrow K_0 \text{ for } s \rightarrow \infty.$$

Now we are in a position to discuss the "Anti-A-J-theorem" found in P-S. They state that as long as the regulatory constraint does not drive the firm out of business "the closer the regulated rate of return is to the cost of capital (but $s > i$), the smaller will be the optimum

ex ante scale of plant relative to that chosen by the unregulated monopolist." (P-J, p. 278). In our language this reads:

For small enough $s_1, s_2 \in S, s_1 < s_2$, the inequality $K(s_1) < K(s_2) < K_0$ holds.

P-S conclude part of this statement (precisely: For small enough $s_1 > i$ with $\pi(s_1, K_0) > 0$ we have $K(s_1) < K_0$) by looking at $\pi_2(s, K_0)$. They find that $\pi_2(i, K_0) < 0$ and conclude that $\pi_2(s, K_0) < 0$ for s small enough. From this they gather that $K(s)$ must be left of K_0 , or $K(s) < K_0$. There are, however, two flaws in this argument: First, since profit $\pi(i, K_0) < 0$ we cannot conclude that $\pi_2(s, K_0) < 0$ for any s with $\pi(s, K_0) > 0$.

Secondly, even if $\pi_2(s, K_0) < 0$, we cannot deduce that $K(s) < K_0$ since $\pi_2(s, \cdot)$ is not a concave function. It has some "concave parts", but we do not know in advance whether K_0 lies in that part of the domain of $\pi_2(s, \cdot)$.

There is however a condition which would ensure $\pi_2(s, K_0) < 0$ and $K(s) < K_0$. Since

$$\pi_2(s, K_0) = \frac{\pi(s, K)}{K} + [R'(K) - \frac{R(K)}{K}] \cdot \alpha(\frac{sK}{R(K)})$$

$\pi(s, K_0) \leq 0$, and $\alpha(\frac{sK_0}{R(K_0)}) < 0$ yield $\pi_2(s, K_0) < 0$, and $\pi_{22}(s, K_0) < 0$ as well.

Even when P-S were wrong in stating a general Anti-A-J-Theorem, their intuition, however, was correct to the extent that it is impossible to prove an A-J-theorem either.¹⁰ The function $K(\cdot)$ may have many forms.

¹⁰The example on page 286 which illustrates the Anti-A-J result cannot, however, be accepted as proper evidence since it does not comply with the assumptions made in P-S. First the probability distribution does not have a density function. Secondly, even if one considers this a minor defect, one observes that there are no quasi-rents functions R which fulfill the condition of the examples that

All of the shapes shown in figure 5, and many more, are possible. The reader might be interested to know that these various shapes are obtained by just changing the density function of the probability distribution of u . To be more specific:

In all three cases $R : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is defined by $R(K) = \sqrt{K}$. The density¹¹ $f : \mathbb{R} \rightarrow \mathbb{R}_+$ is defined for case

$$(1) \text{ by } f(u) := \begin{cases} \frac{1}{4}u + \frac{1}{2} & \text{for } -2 \leq u \leq 0 \\ -\frac{1}{4}u + \frac{1}{2} & \text{for } 0 \leq u \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

$$(2) \text{ by } f(u) := \begin{cases} 1 & \text{for } |u| \leq 1 \text{ and} \\ 0 & \text{otherwise} \end{cases}$$

$$(3) \text{ by } f(u) := \begin{cases} \frac{1}{e} \cdot \frac{18}{25} \cdot e^{\frac{6}{5}u} & \text{for } u \leq \frac{5}{6} \\ \frac{1}{2} \cdot \sqrt{\frac{5}{6}} u^{-\frac{5}{2}} & \text{for } u \geq \frac{5}{6} \end{cases}$$

Line 1 is the shape which occurred most often in examples we treated numerically. In this case the Anti-A-J effect prevails for a subset of S .

$$P\{u \mid sK \leq R(K) + u\} = \frac{D}{2}$$

respectively

$$P\{u \mid sK \leq R(K)(1+u)\} = \frac{D}{2}$$

independently of s and .

Further, the figure 1 on p. 283, used in this connection is somewhat misleading. In the case of no uncertainty a maximal quasi-rents function R with a maximum cannot be derived from a model with a production function having positive marginal productivity of capital. This is seen by a straightforward application of the first order conditions which we get from profit maximization and the maximization of the quasi-rents function. Because of the condition $E[R(K,u)] = R(K,0)$ the same reasoning applies in the case of uncertainty.

¹¹We are well aware of the fact that, strictly speaking, the first two of the following density functions do not fulfill all our descriptions on f . This, however, is only a minor annoyance since a slight disturbance of f (which is difficult to handle numerically) would remedy this without destroying the shape of $K(\cdot)$.

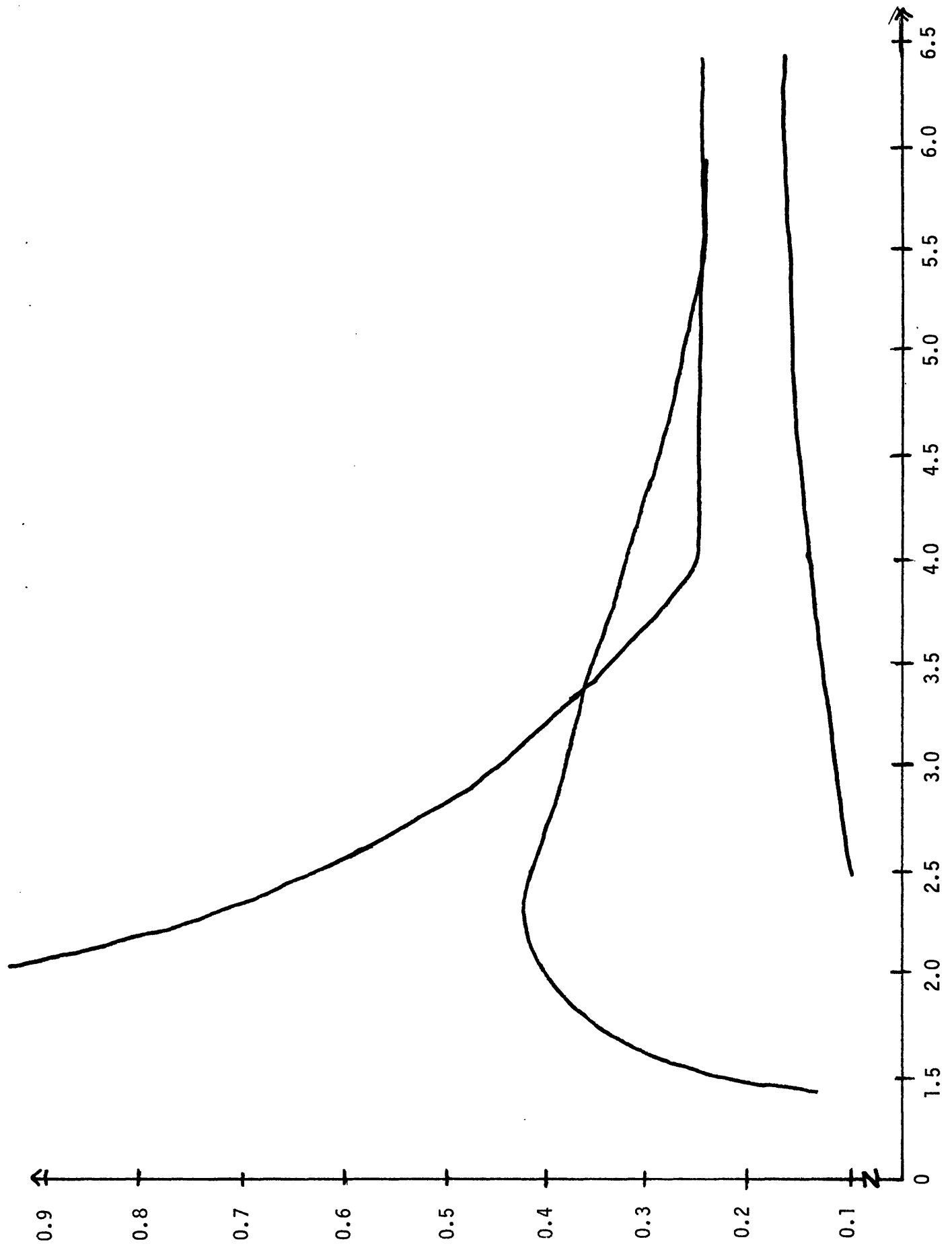


Figure 5

Line 3 represents a shape of $K(\cdot)$ which shows Anti-A-J behavior for all of S. One would expect this kind of behavior, if an Anti-A-J theorem would be true.

Line 2, however, shows that it is not possible to prove the occurrence of an Anti-A-J behavior even for only a small subset of S. Here the firm follows the conventional A-J proposition.

Summarizing: P-S raise important questions about the generality of the A-J results. Their statement on the case, where uncertainty is additive can be confirmed, but their strong Anti-A-J conclusion on the multiplicative case should be considered with some care, unless additional assumptions on R and f are imposed.

Appendix A

From the mean value theorem we get the fundamental inequality

$$R(K) - KR'(K) > 0 \quad \text{for all } K > 0.$$

From this one infers immediately that the function $K \mapsto \frac{R(K)}{K}$ is strictly falling. Hence, the regulated profit can be rewritten as

$$\pi(s, K) = \begin{cases} (s-i)K & \text{if } K < J(s) \\ R(K) - iK & \text{if } K \geq J(s). \end{cases}$$

Let $s > i$ be fixed. We get $\frac{R(K)}{K} < i < s$ for $K \geq \bar{K}$, and (regulated) profit is negative for large K . Thus, there is a profit maximizing capital which is unique because of strict concavity. The exact value of the maximizing capital depends on the slope of $\pi(s, \cdot)$ in $K = J(s)$.

If $R'(J(s)) - i \leq 0$ (i.e. $\Leftrightarrow J(s) \geq K_0 \Leftrightarrow s \leq \frac{R(K_0)}{K_0} = s_0$)
then $K(s) = J(s)$.

If $R'(J(s)) - i > 0$, or $s > s_0$,
then $K(s) = K_0$.

Since $J(\cdot)$ is continuous and falling to $J(s_0) = K_0$, all is proven.

Appendix B

Calculations of the Derivatives

It is more interesting for the reader if we offer the calculation for a more general stochastic quasi-rent function rather than considering the additive and multiplicative cases separately. Furthermore, the reader gets an idea to what extent the results might generalize to the non-additive and non-multiplicative cases.

We recall

$$\pi(s,K) := \int_{\{u | R(K,u) \leq sK\}} (R(K,u) - sK) f(u) du + (s - i)K$$

We assume now that for each $K \geq 0$ the function $u \mapsto R(K,u)$ is strictly monotonic. Hence we can define the function h by $h(s,K) = u \Leftrightarrow R(K,u) = sK$; i.e. $h(s,K)$ is the breakeven realization of the state of the world.

Moreover, if the function $R(\cdot, \cdot)$ is continuously differentiable then is h . This follows from the implicit function theorem. For a reference see Dieudonne, p. 270. The reader will realize that all these assumptions are fulfilled in the additive as well as in the multiplicative case.

Having introduced the function $h(\cdot, \cdot)$ we rewrite

$$\pi(s,K) = \int_{-\infty}^{h(s,K)} (R(K,u) - sK) f(u) du + (s - i)K.$$

Calculating the partial derivative with respect to k creates a small difficulty since K appears in the upper (resp. lower) limit of integration. This can be overcome by using differential calculus of several variables. We skip the calculation and refer the reader to Dieudonne, p. 177, Problem 1.

The result is

$$\pi_2(s,K) = \int_{-\infty}^{h(s,K)} (R_1(K,u)-s) \cdot f(u) du + h_2(s,K)(R(K,h(s,K))-sK) \cdot f(h(s,K)) + (s-i)$$

Recalling that $R(K,h(s,K)) = sK$ this reduces to

$$= \int_{-\infty}^{h(s,K)} (R_1(K,u)-s) \cdot f(u) du + (s-i)$$

Iterating this procedure, we get for the second partial derivatives

$$\pi_{22}(s,K) = \int_{-\infty}^{h(s,K)} R_{11}(K,u) f(u) du + h_2(s,K)(R_1(K,h(s,K))-s) f(h(s,K))$$

and

$$\pi_{21}(s,K) = 1-F(h(s,K)) + h_1(s,K)(R_1(K,h(s,K))-s) f(h(s,K)).$$

The reader will find no difficulties to obtain the formulas (1), (2), (3) and (4) for the additive and multiplicative case by inserting the respective expressions for R , R_1 , and R_{11} , h , h_1 , and h_2 into the above equations. For completeness we give the formula for the first partial derivative:

$$\pi_1(s,K) = K \cdot (1 - F(h(s,K))).$$

In the case one does not have an explicit formula for $R(K,u)$, as in the additive and multiplicative case, one could try to use the Implicit Function Theorem for deriving expressions for h_1 and h_2 in terms of R .

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