RESIDENTIAL DEMAND FOR ELECTRICITY AND GAS
IN THE SHORT RUN: AN ECONOMETRIC ANALYSIS

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ABSTRACT

Short-run residential demand equations for electricity and gas are estimated in this study. Short-run demand depends on the appliance stock in existence. Use of the appliance stock is a function of the price of fuel, income, and the weather. The major difference between this study and others explicitly using appliance stock data is that appliances are not aggregated into a single stock measure. Demand consists of the sum of the individual demands for energy for each fuel-burning appliance type. Consequently, different price, income, and weather elasticities are estimated for each use of the fuels.

The data consist of annual observations for each state for the years 1960-1975. Most of the appliance stock data were developed by Data Resources, Inc. These are supplemented by appliance data developed for use in this study. Two different methods of pooling time-series and cross-section data, the random and fixed effects models, are used, and a specification test is performed to test for consistency of the random effects model estimates.

The results are somewhat mixed. However, they do suggest directions for further research. Fairly reasonable estimates in terms of average energy consumption for each type of appliance are obtained. The aggregate price and income elasticities fall in the range found in previous work. Price elasticities appear to vary among the demands for fuel for different end uses, but the differences are not statistically significant. Income elasticities for the individual fuel uses are disappointing; they are often of the wrong sign and magnitude. The most reasonable results are obtained for the appliances which consume the most fuel. Further work most likely would benefit from aggregation of the small appliances, leaving only for

estimation the coefficients of demand for the major users of fuel and the residual aggregate appliance stock.

PREFACE

The work presented in this paper was undertaken as part of a larger project to model residential energy demand. The project is part of an analysis of the potential markets for solar photovoltaics being conducted by the MIT Energy Laboratory for the Department of Energy.

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CONTENTS

I. Introduction

The demand for energy by the residential sector of the economy is derived from the demand for the services of household appliances using that energy. As a result, ownership of appliances is requisite to energy demand and residential demand should be thought of as the result of a two-part decision-making process: the decision to purchase an appliance of a particular type and the decision to utilize it with a certain frequency or intensity. The decision to purchase an appliance with a particular set of characteristics is long-run in nature, effecting a change in the appliance stock. The decision to utilize an appliance with a certain intensity or frequency is short-run in nature, taking the appliance stock as given.

The latitude for changes in energy demand is greatest in the long run when the consumer can purchase new appliances. Characteristics of appliances, such as efficiency and fuel-consuming features, will affect the consumer's choice. Once an appliance has been purchased, the scope for altering energy demand is limited since the efficiency and other characteristics of the appliance stock are fixed. The consumer can change only the utilization of the stock by such measures as altering the frequency or duration of use of the appliances, changing thermostat control settings, or altering other factors affecting fuel consumption.[1]

Since most household appliances are durable goods with relatively long lives, changes in the energy characteristics of the appliance stock

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[1]For a more detailed discussion of the short-run and long-run components of energy demand and a critical review of models incorporating both, see Hartman [1978].

are likely to take place only gradually. Thus, in order to predict energy demand by the residential sector and the responsiveness of demand to particular policies, it is important to investigate the determinants of energy demand in the short run, when the appliance stock is held constant. The purpose of this paper is to present the results of a study on the determinants of short-run residential demand for electricity and gas.

The distinguishing characteristic of the study presented here is that an attempt is made to estimate separate coefficients for the variables affecting the demand for energy for different end uses. Instead of aggregating different appliances into a single stock measure and estimating average price and income elasticities, as is usually done,[1] demand is represented as the sum of separate demands for fuel for different uses, and price, income, and weather elasticities are allowed to differ among the different uses.

The data consist of annual observations for each state for the years 1960-1975. Most of the appliance stock data were developed by Data Resources, Inc. [1977]. It is supplemented by appliance data developed for use in this study.[2] Two different methods of pooling time-series and cross-section data, the random and fixed effects models, are used and a specification test is performed to test for consistency of the random effects model estimates.

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[1]See Fisher and Kaysen [1962], Acton, Mitchell, and Mowill [1976], Taylor, Blattenberger, and Verleger [1977], and Wills [1977].

[2]See Braid [1978].

Section II of the paper presents the short-run residential demand models for electricity and gas. Section III discusses specification and estimation issues. The data are discussed in Section IV. Section V presents the results.

II. The Model of Short-Run Demand

Although residential energy demand has been modeled by many researchers, only a few studies adequately differentiate short-run and long-run demand.[1] Since the primary distinction between the two as defined here is that the appliance stock is held constant in the short run, a short-run demand model should explicitly incorporate the stock of appliances when possible. Attention has been given to this aspect of short-run demand by Fisher and Kaysen [1962], Acton, Mitchell, and Mowill [1976], and Taylor, Blattenberger, and Verleger [1977], among others. The model of short-run residential demand for electricity and gas that is developed in this section is similar in spirit to the work of these researchers. Differences are discussed at the end of the section.

Households use fuel in order to receive the services of household appliances. It is assumed here that the demand by a household for a particular fuel consists of the sum of its demands for the fuel for each of its appliances using that fuel, i.e.

$$q_i = \sum_j q_{ij} \qquad (1)$$

where q_i is the household's total demand for fuel i and q_{ij} is the household's demand for fuel i in order to use appliance j. The appliances explicitly considered in this analysis are those used for space heating, central and room air conditioning, water heating, cooking, freezing, clothes washing and drying. An "all other" category

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[1]See Hartman [1978].

encompasses the use of electricity for lighting, refrigeration, television, dishwashers, and small electric appliances.

Since the appliance stock is assumed to be fixed in the short run, the demand for a fuel for a particular end use consists of the level of utilization of the given capital stock. Hence,

$$q_{ij} = U_{ij} \cdot APP_{ij} \tag{2}$$

where U_{ij} represents the utilization of fuel i by appliance j and APP_{ij} is the stock of appliance type j which uses fuel i. In this study APP_{ij} is used to denote the number of appliances of type j using fuel i and U_{ij} is the demand per appliance for fuel i. In the studies previously mentioned, appliances are aggregated into a single stock measure using "normal" usage or rated capacity as weights. The appliance stock is measured in energy units and U_i is the utilization rate of the appliance stock.

Since the data used in this study consist of annual observations by state, it is necessary to sum equation (1) over all households in the state to arrive at total residential demand by state. Short-run demand for electricity and gas is developed more formally below.

Short-Run Demand for Electricity

Household demand for electricity is assumed to be a linear function of price, income, and, in the case of space heating and air conditioning, of heating and cooling degree days.[1] Since electricity is sold under

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[1] Heating degree days are the number of degrees that the daily mean temperature is below 65°F. Annual heating degree days are the sum of the daily heating degree days. Cooling degree days are the number of degrees that the daily mean temperature is above 65°F. Annual cooling degree days are the sum of the daily cooling degree days.

declining block price schedules, two price variables, a marginal price and a fixed charge to represent the inframarginal blocks, are generally necessary to represent the price schedule.[1] Letting i now index households, j index appliances, and suppressing the time subscript, the demand for fuel for each end use is specified as follows:

<u>Space Heating</u>

$$QE_{ij} = \beta_{0j} + \beta_{1j}\, HD_i + \beta_{2j}\, PE_i + \beta_{3j}\, FC_i + \beta_{4j}\, Y_i + \varepsilon_{ij} \, .$$

<u>Central Air Conditioning, Room Air Conditioning</u>

$$QE_{ij} = \beta_{0j} + \beta_{1j}\, CD_i + \beta_{2j}\, PE_i + \beta_{3j}\, FC_i + \beta_{4j}\, Y_i + \varepsilon_{ij} \, .$$

<u>Freezing, Cooking, Water Heating, Clothes Washing, Clothes Drying, All Other</u>

$$QE_{ij} = \beta_{0j} + \beta_{2j}\, PE_i + \beta_{3j}\, FC_i + \beta_{4j}\, Y_i + \varepsilon_{ij}$$

where

QE_{ij}	=	the demand for electricity by household i for end use j
PE_i	=	the marginal price of electricity for household i
FC_i	=	the fixed charge facing household i
HD_i	=	heating degree days in the area in which household i lives
CD_i	=	cooling degree days in the area in which household i lives
ε_{ij}	=	a random error.

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[1] See Taylor [1975], Acton, Mitchell, and Mowill [1976], and Taylor, Blattenberger, and Verleger [1977]. The marginal price is the price per kWh of electricity on the block on which the household's last unit of demand falls. The fixed charge is the difference between the total electric bill and the charge if all units of electricity were priced at the marginal price.

The category "all other" includes the demand for electricity for lighting, refrigeration, television, dishwashers, and small appliances. No attempt is made to separate these uses of electricity either because data do not exist, electricity cnsumption is very small, or saturation is virtually 100 percent and the variable would be collinear with the number of households in the state which appears in the final form of the model.

To arrive at total residential demand by state, the individual demands are summed over end uses (j) and households (i).

$$QE = \sum_i \sum_j QE_{ij} = \sum_i (\beta_{01} + \beta_{11}HD_i + \beta_{21}PE_i + \beta_{31}FC_i + \beta_{41}Y_i)APP_{i1}$$

$$+ \sum_i \sum_{j=2}^{3} (\beta_{0j} + \beta_{1j}CD_i + \beta_{2j}PE_i + \beta_{3j}FC_i + \beta_{4j}Y_i)APP_{ij}$$

$$+ \sum_i \sum_{j=4}^{8} (\beta_{0j} + \beta_{2j}PE_i + \beta_{3j}FC_i + \beta_{4j}Y_i)APP_{ij}$$

$$+ \sum_i (\beta_{09} + \beta_{29}PE_i + \beta_{39}FC_i + \beta_{49}Y_i) + \sum_i \sum_j \varepsilon_{ij}APP_{ij}$$

where APP_{ij} = 1 if household i owns appliance j

= 0 if household i does not own appliance j

and the index j

= 1 for space heating

= 2,3 for central and room air conditioning, respectively

= 4-8 for cooking, water heating, clothes washing, clothes drying, and freezing, respectively

= 9 for all other uses.

To arrive at the model which is to be estimated, state averages are substituted for the price variables, income, and heating and cooling degree days.

$$QE = (\beta_{01} + \beta_{11}\overline{HD} + \beta_{21}\overline{PE} \quad \beta_{31}\overline{FC} + \beta_{41}\overline{Y})E_1 \tag{3}$$

$$+ \sum_{j=2}^{3} (\beta_{0j} + \beta_{1j}\overline{CD} + \beta_{2j}\overline{PE} + \beta_{3j}\overline{FC} + \beta_{4j}\overline{Y})E_j$$

$$+ \sum_{j=4}^{8} (\beta_{0j} + \beta_{2j}\overline{PE} + \beta_{3j}\overline{FC} + \beta_{4j}\overline{Y})E_j$$

$$+ (\beta_{09} + \beta_{29}\overline{PE} + \beta_{39}\overline{FC} + \beta_{49}\overline{Y})HS + \sum_i \sum_j \varepsilon_{ij} APP_{ij}$$

where

HS = the number of households in the state

E_j = the stock of appliance j in the state

and averages are denoted by bars over their variable names.

Short-Run Demand for Gas

Short-run demand for gas by state is derived in an analogous fashion.

$$QG = (\beta_{01} + \beta_{11}\overline{HD} + \beta_{21}\overline{PG} \quad \beta_{41}\overline{Y})G_1 \tag{4}$$

$$+ \sum_{j=4,5,7} (\beta_{0j} + \beta_{2j}\overline{PG} + \beta_{4j}\overline{Y})G_j + \sum_i \sum_{j=1,4,5,7} \varepsilon_{ij} APP_{ij}$$

where

QG = gas sales

PG = the average price of gas

G_j = the stock of gas appliance j.

Unlike the model of electricity demand, there is no "all other" category for the gas equation because the use of other gas appliances is limited.[1] The four end uses specified here, space heating (1), cooking (4), water heating (5), and clothes drying (7), comprise almost all of the uses for which residential demand for gas exists. The specification also differs from that of electricity since the average price of gas is used due to the lack of a marginal price series for gas. The biases that this creates, along with other estimation difficulties, are discussed in Section III.

Comparison with Other Models of Short-Run Demand

Several differences between the demand model specified here and those of other studies are worth examining. As mentioned previously, the major difference between this study and most others is that in this study, appliances are not aggregated into a single stock measure. The advantages and disadvantages of this approach are discussed in Section III. Studies using a single stock measure of appliances are those by Acton, Mitchell, and Mowill [1976] and Taylor, Blattenberger, and Verleger [1977]. Acton, Mitchell, and Mowill also disaggregate space heating and air conditioning from other uses of electricity. Fisher and Kaysen [1962] model short-run demand using a single stock measure of

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[1]Other residential uses of natural gas are in gas air conditioners, gas refrigerators, swimming pool heaters, gas-log fireplaces, and gas lights.

appliances, but concern about the quality of the appliance data leads them to make a simplifying assumption which allows them to estimate short-run demand without explicitly using their stock measure.

Another difference in the approach taken here is that the prices of other fuels are not included in each demand equation; only the own price is included. The reason that other fuel prices are not included is that once a household obtains a certain appliance stock, there appears to be little room for fuel substitution. Portable space heaters can be substituted for oil and gas space heating, electric frying pans and other small appliances can be substituted for gas cooking, but there are not many other ways in which one fuel may be substituted for another without changing the major appliance stock. Support for this approach is obtained from the study by Taylor, Blattenberger, and Verleger [1977], which generally finds the price of gas to be insignificantly different from zero.[1] Acton, Mitchell, and Mowill [1976] find the price of gas to be significant, but in their study it is serving as a proxy for changes in the appliance stock, and hence represents long-run as well as short-run effects of the price of gas on electricity demand. Other studies, such as the one by Mount, Chapman, and Tyrrell [1973],

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[1]In one of their short-run demand equations the price of gas appears to be significant, but since there is evidence of serial correlation, the standard errors may be biased downward.

also find the price of gas to be significantly different from zero in the electricity demand equation, but they model demand using a Koyck lag adjustment process, thus combining the long-run with the short-run.

Another difference between the model specified here and others is that short-run demand is assumed to adjust immediately to new levels of fuel prices, income, the weather, and appliance stocks. Acton, Mitchell, and Mowill also model short-run demand without an adjustment mechanism. An alternative assumption often made in the literature is that demand responds with a geometric lag to changes in the independent variables. Most researchers making this assumption are attempting to estimate long-run as well as short-run demand, in which case some kind of lag is reasonable as consumers adjust their appliance stocks, but Taylor, Blattenberger, and Verleger model short-run demand this way as well. Aside from the theoretical issue of specification, an estimation difficulty arises. Models incorporating a geometric lag are estimated with a lagged endogenous variable on the right-hand side, which leads to inconsistency of the estimates if serial correlation is present and the appropriate econometric techniques are not applied.

Other similarities and differences worth noting are the specification of the price of electricity and the level of aggregation. Like Acton, Mitchell, and Mowill [1976], Taylor, Blattenberger, and Verleger [1977], and Wills [1977], the marginal price for electricity is used along with a fixed charge. Acton, Mitchell, and Mowill drop the fixed charge variable from their specification because they finds its coefficient to be near zero and frequently of the wrong sign and statistically insignificant.

Wills restricted the coefficient of the fixed charge variable to equal its theoretical value by subtracting it from income. Only Taylor, Blattenberger, and Verleger find the coefficient on the fixed charge variable to be negative and significantly different from zero, but it is of the wrong magnitude.[1]

The data in this study consist of annual observations by state, and, as such, are highly aggregated, introducing possible aggregation biases. The level of aggregation is the same as that in most studies in this field of research. Recently, however, researchers are using more disaggregated data which presumably are of better quality and are less likely to cause aggregation biases. Examples of studies in which the unit of analysis is aggregated at a lower level than a state are Acton, Mitchell, and Mowill [1976] and Wills [1977].

III. Specification and Estimation Issues

In this section a number of issues regarding the specification and estimation of short-run energy demand equations are raised and discussed. These issues arise in most of the work in this area and researchers have dealt with them in a variety of ways. One issue is the representation of the declining block structure of rates under which most electricity and gas is sold. This problem has received much attention in the literature. The discussion below attempts to summarize the difficulties and the solutions adopted by previous researchers. Another issue, and one that has been dealt with by only a few researchers, is that of the simultaneous nature of supply and demand.[2]

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[1]The specification of the price variable is discussed in Section III.

[2]See Halvorsen [1973].

If this is indeed a problem, estimates arrived at without using simultaneous equation estimation techniques are inconsistent. Methods of pooling cross-section and time-series data is another issue discussed below. Most of the work in this area uses the variance components method of pooling,[1] which is referred to as the random effects model in this study. This approach is compared to the fixed effects method of pooling. Another issue also dealt with below is the practice of aggregating appliances into a single stock measure.

Specification of the Price Variable

Specification of the price variable presents a problem for the estimation of demand equations for electricity and gas because of the declining block nature of rate structures. Typically, a customer is charged a fixed fee plus a price p_1 for the first x_1 units of demand, a lower price p_2 for the next x_2 units of demand and so on. Various researchers have discussed the problems created by this type of pricing schedule and have shown that the rate schedule generally can be represented by two terms: a marginal price, which is the price per kWh of consumption for the block on which the last unit of demand by the customer falls, and a fixed charge, which represents the inframarginal blocks of consumption.[2] The fixed charge is the difference between the

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[1] The variance components method of pooling time series and cross section data was used by Balestra & Nerlove [1966].

[2] See Taylor [1975], Acton, Mitchell and Mowill [1976], and Taylor, Blattenberger, and Verleger [1977].

total bill and the charge the customer would have faced had all units of electricity been priced at the marginal rate. The elasticity of demand with respect to this variable should be equal to the product of the income elasticity and the budget share commanded by the fixed charge,[1] but of the opposite sign. Basically, the reason is that changes in the prices of the infra-marginal blocks do not change the marginal price, (hence the price consumers are using to equate the ratios of marginal utilities to prices of goods in their consumption decisions) but do affect the real income of the consumer. If the prices of the inframarginal units of demand rise, the consumer has less real income and hence should react as if real income has fallen.

Due to the efforts of Taylor, Blattenberger and Verleger [1977], there exists a time-series and cross-section data base containing state-averaged marginal and fixed charge prices for electricity. Their data is used in this study. Unfortunately, since no similar series exists for gas, the average price of gas is used instead of the marginal price and a fixed fee.

Use of the average price instead of the marginal price and the fixed charge variable produces inconsistent estimates. The bias in the price coefficient due to the errors in variables problem is toward zero. Berndt [1978] has shown recently that the bias caused by omitting the fixed charge variable is negligible.

Simultaneity

The demand for electricity is part of a simultaneous system of

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1 The mean inframarginal budget share by state over the 1960-72 period is about .01.

equations which in addition to demand includes supply and the rate-setting process. It is difficult to tell how much of a problem simultaneity creates without examining it explicitly. Considering the complexity of the regulatory process and supply characteristics, price changes or differences may be much more related to factors other than quantity. In any case, an instrumental variable estimator ought to be used to insure consistent estimates. Mount, Chapman and Tyrrell [1973], used instrumental variables to estimate their demand model, and found that the estimates were very close to those achieved by ordinary least squares. Houthakker, Verleger and Sheehan [1974], also used an instrumental variable estimator, but their standard errors were large. Halvorsen [1973], who explicitly modeled the supply side, achieved essentially the same estimates of the demand parameters with two-stage least squares as with ordinary least squares. The study here does not use instrumental variables.

Aggregation of Appliance Stocks

Most studies using appliance stocks aggregate the different kinds of appliances into one stock measure using as weights the "normal" usage or rated capacity of the appliances. Several difficulties arise with this approach which are avoided by specifications (3) and (4). First, it is very likely that the demands for energy for different end uses have different elasticities. When appliances of different types are aggregated, this kind of information, which may be very useful, is lost. Second, if elasticities for different end uses vary, average elasticities will depend on the particular appliance configuration of the household or of the state. Thus, it is inappropriate to use estimates of average elasticities to project future demand if the appliance mix is changing.

Along the same lines, an elasticity estimated by pooling state aggregated appliance data may not be a very good estimate of the elasticity for an individual state if the appliance-mix of the state is different from the typical state appliance configuration.

One advantage of aggregating appliance stocks is that the demand equation can be divided by the appliance stock and the utilization rate of the stock can be estimated instead of the demand. The advantage of putting the appliance stock on the left hand side of the equation arises if there are measurement errors in the appliance stock, which there undoubtedly are. If a variable measured with error is on the left hand side of an equation, no problem in terms of estimation is created unless the measurement error is correlated with the right hand side variables. In the disaggregated model discussed in Section II, the appliance stocks are on the right hand side of the equation. If they are measured with error, the estimated coefficients are inconsistent and generally biased toward zero.

Another problem with the estimation of separate elasticities for different end uses is that some of the variables are highly collinear, making it difficult to obtain precise estimates of the parameters. In addition, degrees of freedom are lost when more coefficients are being estimated.

Pooling Time-Series and Cross-Section Data

When pooling time-series and cross-section data, the specification should allow for differences which might exist either across the units of observation or across time. In this study the concern was that there might be some part of demand associated with each state that the right hand side variables could not explain. Suppose that the error term,

η_{it}, consists of two parts: an individual component, α_i, and a random error, ε_{it}. If the expectation of α_i, along with expectation of ε_{it}, is zero and if it is uncorrelated with the right hand side variables, ordinary least squares leads to consistent, but inefficient results, since the covariance matrix does not obey the least squares assumptions. Instead, it takes the form:

$$E(\eta\eta') = I_N \otimes (\sigma_\varepsilon^2 I_T + \sigma_\alpha^2 J_T) ,$$

where η is a stacked vector of state time-series errors, N is the number of states, T is the number of years and J_T is a TXT matrix of ones, if the following assumptions hold:

$$E(\alpha_i) = E(\varepsilon_{it}) = 0 \quad \text{for all i and t}$$

$$E(\varepsilon_{it}\varepsilon_{it'}) = E(\varepsilon_{it}\varepsilon_{jt}) = E(\varepsilon_{it}\alpha_i) = E(\varepsilon_i\varepsilon_j) = 0 \quad \text{for all } i \neq j \text{ and } t \neq t'$$

$$\text{var}(\alpha_i^2) = \sigma_\alpha^2 \quad \text{for all i}$$

$$\text{var}(\varepsilon_{it}) = \sigma_\varepsilon^2 \quad \text{for all i and t .}$$

This specification is called a variance components or a random effects model. To attain consistency the critical assumption is that the individual state effect, α_i, is uncorrelated with the right hand side variables.[1]

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1 See Maddala [1971].

The efficient estimation technique for this model is generalized least squares. Since $E(\eta\eta')$ has to be estimated, the coefficients are only consistent and not unbiased. A two-step procedure is available which yields asymptotically efficient estimates.[1] Estimates of σ_α^2 and σ_ε^2 can be obtained from the ordinary least squares residuals by the formulas[2]:

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{(N-1)(T-1)} \sum_i \sum_t (\hat{\varepsilon}_{it} - \frac{1}{T}\hat{\varepsilon}_{i.} - \frac{1}{N}\hat{\varepsilon}_{.t})^2$$

$$\hat{\sigma}_\alpha^2 = \frac{1}{T}(\sum_i \frac{\hat{\varepsilon}_{i.}^2}{T(N-1)} - \hat{\sigma}_\varepsilon^2) .$$

If the data is then transformed according to the formula

$$\tilde{X}_{it} = X_{it} - \gamma X_{i.}$$

where $\tilde{X}_{it}$ is the transformed variable, and

$$\gamma = 1 \pm \sqrt{\frac{\hat{\sigma}_\alpha^2}{\hat{\sigma}_\alpha^2 + T\hat{\sigma}_\varepsilon^2}} ,$$

ordinary least squares can be performed on the transformed variables to obtain asymptotically efficient estimates.

An important assumption underlying the use of the random effects model is that the individual state effect α_i is uncorrelated with the right hand side variables. If this assumption is not true, the random effects model leads to inconsistent estimates, as does ordinary least squares. However, a fixed effects model, in which individual state dummy variables are estimated, yields consistent estimates because the error term in this model consists only of ε_{it}, which is assumed to be uncorrelated with the right hand side variables. If the assumptions of

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1 See Wallace and Hussain [1969].

2 Analysis of variance notation is being used, e.g. $\varepsilon_{i.} = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it}$.

the random effects model hold, however, the fixed effects model is not efficient. The fixed effects model can be estimated by transforming the observations into deviations from state means over time and using ordinary least squares on the transformed variables.

It is sometimes difficult to decide which approach is appropriate.[1] The efficiency of the random effects model is desirable, but not at the cost of losing consistency. A specification test developed by Hausman [1976] can be used to test whether the assumptions of the random effects model hold. The basis for the test is that if the assumptions of the random effects model hold, both the random and fixed effects models produce consistent estimates. Under the null hypothesis of no misspecification, Hausman has shown that the statistic

$$m = \frac{\hat{q}' V(\hat{q})^{-1} \hat{q}}{K}$$

is distributed as F(K,T-K), where $\hat{q} = \hat{\beta}_{FE} - \hat{\beta}_{RE}$, the difference between the fixed and random effects estimates, $V(\hat{q}) = V(\hat{\beta}_{FE}) - V(\hat{\beta}_{RE})$, and K is the number of coefficients. In forming $V(\hat{q})$, the estimate of σ^2 from the fixed effects model should be used in order to insure that the estimate of σ^2 is independent of $\hat{q}$ so that m is distributed as F.

1 See Hausman [1976] for a brief discussion of the issue and further references.

Heteroskedasticity

It is clear from the model that heteroskedasticity exists since the error term is

$$\sum_i \sum_j \varepsilon_{ij} APP_{ij} \ .$$

If ε_i and ε_j for $i \neq j$ are independent, and ε_i is distributed $N(0,\sigma_\varepsilon^2)$, then the error term is distributed $N(0,\sum_i \sum_j \sigma_{ij}^2 APP_{ij})$. A simple procedure, although not quite correct, was adopted. The electricity observations were weighted by dividing them by the square root of the number of households in the state and the gas observations were divided by the square root of the number of gas space heating customers in the state. The results did not appear to be very sensitive to the procedure used.[1]

IV. The Data

Electric appliance stocks and the stock of gas-heated houses were obtained from the study by Data Resources, Inc. (DRI) [1977]. Gas appliance stock data for water heaters, ranges and clothes dryers were developed as a part of this study. A detailed review of the methodology used to develop the electric appliance stock data by DRI and the approach used to develop the gas appliance stock data is contained in Braid [1978]. Concern over the quality of the appliance stock data led to the development of an alternative stock series for stocks other than space

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[1] In fact, the ordinary least squares residuals show only slight heteroskedasticity. Taylor, Blattenberger and Verleger [1977] also report little indication of heteroskedasticity.

heating and air conditioning. The alternative series is also discussed in Braid [1978]. It is developed by trending saturation rates obtained from census data for each appliance and for each state between the years 1960 and 1970. The series thus obtained is then adjusted to insure that state stocks sum to the national stock for each year.

Other data were obtained from the following sources. Average marginal and fixed charge electricity price data were obtained from DRI. The data were constructed by taking a customer-weighted average over different rate schedules within a state of the marginal and fixed charge prices for the average level of kWh consumption. Gas revenues and sales by state were taken from Gas Facts and the average price was calculated by dividing revenues by sales. Electricity sales came from the Edison Electric Institute Statistical Yearbook. Personal income was taken from the Survey of Current Business. Average heating and cooling degree day data was developed by taking a population weighted average of heating and cooling degree days of major population centers. Heating and cooling degree data was obtained from the National Oceanic and Atmospheric Administration. Prices and income were deflated by the consumer price index and the cross-section index developed by Anderson [1973].

V. The Results

The electricity demand equation (3) was estimated using annual state data for the period 1960-1972. Four states (Alaska, Hawaii, Virginia and Maryland) were excluded because one or more variables were missing. The gas demand equation (4) was estimated using annual data for all states for the period 1960-1975. The results of the estimation are discussed in this section. The first part of the discussion describes the results obtained by ordinary least squares. The second part compares the ordinary least squares results with results obtained using random and fixed effects models. The third part discusses the differences found in the results using the two alternative sets of data.

<u>Ordinary Least Squares Results</u>

Several kinds of information are useful in evaluating the results of the estimation of equations (3) and (4). The signs of the estimated coefficients can be compared to prior beliefs, i.e. the price coefficients are expected to be negative and the income and degree day coefficients are expected to be positive. Tests of the significance of individual coefficients or groups of coefficients can be made. It is also useful to calculate the estimated average energy use per appliance from the estimated equations. These estimates can then be compared with estimates, both econometric and engineering, from other studies in order to determine how reasonable the results are. Finally, the elasticities of demand can be examined for their plausibility and compared to elasticities from other studies.

Table 1 presents the results of estimating the electricity demand equation (3) by ordinary least squares. First, judging by the signs and significance of the coefficients, the results are mixed. The coefficients of the degree day variables are all positive as expected and highly significant. Five of the eight price coefficients are negative but only three of the five (space heating, room air conditioning, and water heating) are significantly different from zero. These three significant coefficients, however, are for major uses of electricity. The positive price coefficients are all insignificant. Five of the eight income coefficients are positive as expected, but only two of them (clothes washing and drying) are significantly different from zero. None of the negative coefficients on income are significant. The results for the fixed charge variable are not as good. Only three coefficients (space heating, water heating and clothes washing) are negative as expected and only two of these (water heating and clothes washing) are significantly different from zero. None of the positive coefficients are significantly different from zero. In addition, the magnitudes of the fixed charge coefficients are not equal to their theoretical values.[1]

Although many of the fixed charge coefficients are insignificantly different from zero, the hypothesis that the fixed charge coefficients are all equal to zero is rejected by an F test at the .01 level of significance.[2] Similarly, the hypothesis that all of the price and income coefficients are zero is rejected at the .01 level of significance.[3] The results of the restricted regression (no price or income variables) are contained in column (1) of Table A2 in the appendix.

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[1] To compare the coefficients, the coefficient on the fixed charge variable must be divided by 1000 to put the variable in the same unit (thousands of dollars) as the income variable.

[2] The statistic m is distributed approximately as F $(8,\infty)$, which has a critical value of 2.51 at the .01 level of significance.

$$m = \frac{(SSR(\text{restricted}) - SSR(\text{unrestricted}))}{SSR\ (\text{unrestricted})} \cdot \frac{df\ (\text{numerator})}{df\ (\text{denominator})}$$

$$= \frac{(53{,}144{,}300 - 46{,}088{,}000)}{46{,}088{,}000} \cdot \frac{561}{8} = 10.74$$

where SSR is the sum of squared residuals and df is the degrees of freedom. The null hypothesis is rejected.

[3] The statistic m is distributed approximately as $F(21,\infty)$, which has a critical value of about 1.88 at the .01 level of significance.

$$m = \frac{(87{,}640{,}600 - 46{,}088{,}000)}{46{,}088{,}000} \cdot \frac{561}{21} = 24.09.$$

The null hypothesis is rejected.

Table 1

Coefficient Estimates for the Short Run Demand for Electricity-DRI Data

(Standard errors in parentheses)

OLS

Space Heating	
C	19,449.3
	(5,415.42)
HD	2.16623
	(.224396)
PE	-3,831.58
	(1,208.72)
FC	-224.075
	(345.854)
Y	-1.1296
	(3.97153)
kWh/appliance/year[1]	12,782
Central Air Conditioning	
C	3,483.31
	(6,462.41)
CD	.792216
	(.327277)
PE	-1,133.63
	(1,516.02)
FC	49.2886
	(464.642)
Y	.088769
	(.405312)
kWh/appliance/year	3,312
Room Air Conditioning	
C	7,564.65
	(2,870.32)
CD	.873652
	(.182517)
PE	- 2,281.23
	(577.580)
FC	7.86344
	(198.558)
Y	-.235154
	(.164602)
kWh/appliance/year	2,333

[1] calculated at the means of the independent variables.

Table 1 (continued)

<u>Freezing</u>	
C	1,575.96
	(3,431.46)
PE	-208.117
	(723.623)
FC	191.148
	(259.049)
Y	.0454039
	(.231395)
kWh/appliance/year	2,273
<u>Cooking</u>	
C	574.209
	(2,557.91)
PE	382.234
	(531.623)
FC	367.721
	(237.569)
Y	-.211413
	(.168337)
kWh/appliance/year	649
<u>Water Heating</u>	
C	9,041.07
	(2,983.00)
PE	-1,613.53
	(596.213)
FC	-1,017.65
	(233.839)
Y	.0537846
	(.191141)
kWh/appliance/year	3,033
<u>Clothes Washing</u>	
C	-1,286.04
	(919.255)
PE	201.612
	(192.311)
FC	-168.705
	(83.3908)
Y	.173596
	(.0580157)
kWh/appliance/year	57

Table 1 (continued)

Clothes Drying	
C	-6,118.59
	(2,825.09)
PE	849.349
	(692.973)
FC	352.696
	(254.135)
Y	.755601
	(.176472)
kWh/appliance/year	1,806
Other	1,398.97
	(101.713)
Intercept	-19.149
	(21.0868)
R^2	.9936
mean of the dependent variable (millions of kWh)	2654.95
Standard error of the regression	286.624
Sum of squared residuals	46,088,000

The estimates of average kWh consumption for each appliance in Table 1 can be compared to estimates from other sources which appear in Table 2. In general, the estimates compare quite favorably. With few exceptions the relative magnitudes of energy consumption by the appliances are correct. Although for some uses estimates of average demand are off by as much as 1,000 kWh per year, considering the difficulties inherent in the study, the results are quite reasonable. The estimates are superior to those of the restricted regression (no price or income variables) which are found in column (1) of Table A2 in the appendix.

Table 3 presents the estimated elasticities of demand for electricity with respect to prices, income and degree days for the different end uses. The elasticities are calculated at the means of the independent variables using for quantity the average use estimated by the equation. The aggregate elasticity is calculated by weighting the individual elasticities by an estimate of the proportion of total demand consumed by each appliance.

Although some of the elasticities are either of the wrong sign or clearly of the wrong magnitude, others are reasonable. For instance, the price elasticity for space heating is -.55 while the price elasticity for freezing is only -.17. The relative magnitude of these estimates is in the range one would expect. In general, households can vary more easily their consumption of fuel for space heating than they can for freezing. It is difficult to judge the individual elasticities in the absence of other individual estimates. Most of them are in the inelastic range which is in accord with prior expectations. The heating degree day elasticity of .88 is thought to be reasonable since engineering models assume an elasticity equal to 1.0.[1]

In terms of the aggregate elasticities, the estimates compare favorably with those from other studies. The aggregate short-run price elasticity is -.19 and the aggregate short-run income elasticity is .09. Elasticities from other studies appear in Table 4. Acton, Mitchell and Mowill find the short-run price elasticity to be -.35. Fisher and Kaysen produced estimates ranging from -.16 to -.25. Mount, Chapman, and Tyrrell estimated the short-run price elasticity to be in the neighborhood of -.14 to -.36. Other estimates are lower. The estimates of the short-run income elasticity range from almost zero up to .40.

Appliances are not aggregated in this study because of the belief that price and income elasticities vary for the different end uses. This hypothesis can be tested. The null hypothesis that the price

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1 Lehman and Sebenius [1977], p. 1.

elasticities are equal cannot be rejected.[1] This is not a surprising result since the estimates are not very precise. The conclusion that the elasticities are in fact equal probably should not be drawn. Future work might experiment with different degrees of appliance aggregation and estimate only a few elasticities instead of one for every appliance.

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[1] To test the hypothesis that the price elasticities are equal the statistic $m = c'\hat{\eta}/(\hat{\sigma}_\varepsilon^2 c'(X'X)^{-1}c)^{\frac{1}{2}}$ was calculated where c is a vector with four elements equal to 1 and four equal to -1, $\hat{\eta}$ is the vector of price elasticity estimates, and $\hat{\sigma}_\varepsilon^2(X'X)^{-1}$ is an estimate of the variance-covariance matrix of $\hat{\eta}$. The variance of $\hat{\eta}_j$ is set equal to $(\overline{PE}/\hat{\overline{Q}}_j)^2$ var $(\hat{\beta}_{2j})$. The covariance of $\hat{\eta}_j$ and $\hat{\eta}_{j'}$ is set equal to $(\overline{PE}/\hat{\overline{Q}}_j)(\overline{PE}/\hat{\overline{Q}}_{j'})$cov $(\hat{\beta}_{2j}, \hat{\beta}_{2j'})$. Since the $\hat{\overline{Q}}_j$ are also stochastic, the true variance-covariance matrix for $\hat{\eta}$ is much more complicated, involving many more terms and variances and covariances. Rather than calculate the true covariance matrix the statistic was calculated using two separate estimates of the $\hat{\overline{Q}}_j$, those from this study and those made by Dole, in order to obtain an indication of the sensitivity of the results of the test to the assumptions made about the $\hat{\overline{Q}}_j$. Using the $\hat{\overline{Q}}_j$ estimated in this study, m = 6.0/6.05 = 1.01.

Since m is distributed as t and the critical value of t at the .05 level of significance is about 1.96, the null hypothesis that m is equal to 0 cannot be rejected. The value for m calculated using Dole's estimates for the $\hat{\overline{Q}}_j$ is only .18, so the same conclusion holds with his estimates.

Table 2
Average Consumption Estimates for Electric Appliances
(kWh/yr/unit)

	Edison Electric Institute	Stanford Research Institute	Dole
Central Heating	_	14,153	13,662
Central Air Conditioning	-	3,600	3,565
Room Air Conditioner	-	1,375	1,370
Freezer	1190-1820	1,478	1,395
Range	700-730	1,180	1,200
Water Heater	4470-4810	4,490	4,350
Clothes Washer	100	100	-
Clothes Dryer	990	990	1,000

Table 3

Elasticities of the Short-Run Demand for Electricity[1]

Price elasticities	
Space Heating	-.55
Central air conditioning	-.62
Room air conditioning	-1.78
Freezing	-.17
Cooking	1.07
Water Heating	-.97
Clothes Washing	6.46
Clothes Drying	.86
Aggregate[2]	-.19
Income elasticities	
Space heating	-.80
Central air conditioning	.24
Room air conditioning	-.91
Freezing	.18
Cooking	-2.94
Water heating	.16
Clothes washing	27.45
Clothes drying	3.77
Aggregate	.09
Heating degree day elasticities	
Space heating	.88
Aggregate	.11
Cooling degree day elasticities	
Central air conditioning	.28
Room air conditioning	.44
Aggregate	.04

Table 3 (continued)

<u>Fixed Charge elasticities</u>	
Space heating	-.06
Central air conditioning	.05
Room air conditioning	.01
Freezing	.29
Cooking	1.98
Water heating	-1.17
Clothes washing	-10.33
Clothes drying	.69
Aggregate	-.22

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1 Individual appliance elasticities are calculated at the means of the independent variables using the estimated average kWh consumption of the appliance. For example, the price elasticity of demand for space heating is $\frac{\partial Q}{\partial P} \cdot \frac{P}{Q_{ij}} = -3{,}832 \times \frac{1.823}{12{,}782} = -.55$.

2 Aggregate elasticities are calculated by weighting each individual elasticity by the percentage of total residential demand for electricity estimated to be used for that end use. The weights were calculated by multiplying the number of appliances of each type by the estimate of average use made by Dole [1975]. The following table shows the calculation of the weights.

Weights Used to Aggregate Elasticities

	Fuel Consumption per Appliance	Average State Appliance Stock	Average State Fuel Consumption	Percent of Total
Central Heating	13,662	66,678.3	9.1096×10^8	.12
Central air conditioning.	3,565	88,476.9	3.1542×10^8	.04
Room air conditioning	1,370	318,087	4.3578×10^8	.06
Freezing	1,395	324,783	4.5307×10^8	.06
Cooking	1,200	480,437	5.7652×10^8	.08
Water heating	4,350	301,634	1.3121×10^9	.17
Clothes washing	100	698,573	6.98573×10^7	.01
Clothes drying	1,000	280,440	2.8044×10^8	.04
Other[3]	2,600	1,280,290	3.3288×10^9	.43
			76.82947×10^8	

[3] The sum of estimated annual use for lighting (1000 kWh), color television (500 kWh) and refrigeration (1,100 kWh) taken from estimates found in Dole [1975].

Table 4

Estimates of Elasticities from other Studies

	Short-Run Price Elasticity	Short-Run Income Elasticity
Acton, Mitchell & Mowill	-.35	.40
Fisher and Kaysen	-.16 to -.25	.07 to .33
Mount, Chapman, and Tyrrell	-.14 to -.36	.02 to .10
Taylor, Blattenberger and Verleger	-.05 to -.54	.0004 to .38
Wills	-.08	.32
Houthakker, Verleger, and Sheehan	-.03 to -.09	.13 to .15

The results of the estimation of the demand for gas can be analyzed in the same manner as the results for electricity. Table 5 presents the ordinary least squares results for equation (4). All four price coefficients are negative, and, except for water heating, they are significantly different from zero. Only two of the four income coefficients (space heating and water heating) are positive and of the two, only the income coefficient for space heating is significant. Of the negative income coefficients, the one for cooking is significantly different from zero. As in the case of electricity, the hypothesis that the price and income coefficients are zero is rejected by an F test at the .01 level of significance.[1] The results of the restricted regression appear in column (2) of Table A2 in the appendix.

Table 6 contains estimates from two sources of fuel consumption by gas appliance. Comparing these estimates with the ones in Table 5, it can be seen that the estimates in this study for space heating and cooking are within a reasonable range, but the estimates for water heating and especially clothes drying are too high.

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[1] The statistic m is distributed approximately as $F(8,\infty)$ which has a critical value of 2.51 of the .01 level of significance.

$$m = \frac{(120{,}642 - 103{,}066)}{103{,}066} \cdot \frac{736}{8} = 15.69.$$

The null hypothesis is rejected.

Table 5

Coefficient Estimates for The Short-Run Residential Demand for Gas-Trended Data (standard errors in parentheses)

Space Heating	
C	-48.6689 (150.710)
HD	.142483 (.00389035)
PG	-5.80559 (3.53814)
Y	.0208748 (.0141358)
Therms/appliance/year[1]	815
Cooking	
C	882.809 (212.212)
PG	-10.5303 (5.94445)
Y	-.0651795 (.0164323)
Therms/appliance/year	126
Water Heating	
C	120.794 (217.963)
PG	-1.67961 (5.49556)
Y	.0338782 (.0185614)
Therms/appliance/year	426

[1] Calculated at the means of the independent variables.

Table 5 (continued)

Clothes Drying	
C	925.032 (549.485)
PG	-29.9698 (17.6206)
Y	-.0214319 (.0423308)
Therms/appliance/year	351
Intercept	-1,545.51 (717.834)
R^2	.9855
Mean of dependent variable	15.2850
Standard error of the regression	11.8337
Sum of squared residuals	103,066

Table 6

Average Consumption Estimates for Gas Appliances (therms/yr/unit)

	(1) Dole	(2)[1]
Central Heating	1040	880
Range	95	138
Water Heater	260	288
Clothes Dryer	43	40

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[1] C.W. Behrens, "AHAM offers Energy Saving Aids to the Public," Appliance Manufacturer, October 1974, as reported in Dole [1975].

The elasticity estimates are shown in Table 7. The estimates are calculated using the estimates of average use from this study. The price elasticities of demand for gas for cooking and clothes drying appear to be much higher than those for space heating and water heating. The aggregate price elasticity of -.15 seems reasonable for a short-run price elasticity. The positive income elasticities are .23 for space heating and .72 for water heating. The others are clearly unreasonable. The degree day elasticity is .91, which, as in the case of electricity, is close to the value of 1 which is used in engineering models.[1]

The null hypothesis that the price elasticities are equal was tested using the same method explained in the discussion of the electricity results. The value for m calculated using the $\hat{\bar{Q}}_j$ estimated in this study is 2.59, which would allow the hypothesis that the price elasticities are equal to be rejected since the critical value of the t statistic is approximately 1.96 at the .05 level of significance. Using Dole's estimates of the $\hat{\bar{Q}}_j$, however, the value of m falls to .17, so the results are very sensitive to the assumptions made about the $\hat{\bar{Q}}_j$ and no conclusions should be drawn from this test. Again, it is not surprising that the hypothesis that the price elasticities are equal cannot be rejected because of the lack of precision in the estimates and the conclusion should not be drawn that the price elasticities are in fact equal.

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1 Lehman and Sebenius [1977], p.1.

Table 7

Short-Run Elasticities of the Demand for Gas[1]

Price Elasticities	
Space Heating	-.09
Cooking	-1.02
Water Heating	-.05
Clothes Drying	-1.05
Aggregate[2]	-.15
Income Elasticities	
Space Heating	.23
Cooking	-4.66
Water Heating	.72
Clothes Drying	-.55
Aggregate	.02
Heating Degree Day Elasticities	
Space Heating	.91
Aggregate	.68

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1 Individual elasticities are calculated as described in footnote (1) of Table 3.

2 Aggregate elasticities are calculated using the weights shown in the following table.

Weights Used to Calculate Aggregate Elasticities

	Average Fuel Consumption per Appliance[3]	Average State Appliance Stock	Average Fuel Consumption per State	Percent of Total
Space Heating	1040	650,053	6.7606×10^8	.75
Cooking	95	598,943	5.6900×10^7	.06
Water Heating	260	647,049	1.6823×10^8	.19
Clothes Drying	43	124,621	5.3587×10^6	.01
			9.065×10^8	

3 The estimates used are from Dole [1975].

<u>Comparison of Methods of Pooling Cross-Section and Time-Series Data</u>

As discussed in Section III, the appropriate method of pooling cross-section and time-series data depends on the assumptions of the model. If the individual effect associated with each state is a random variable with an expected value of zero and if it is uncorrelated with the right hand side variables, the random effects model produces consistent and efficient estimates. OLS produces consistent estimates of the parameters, but not efficient ones, and the estimates of the standard errors are biased. If, however, the individual element associated with each state is correlated with the right hand side variables, both OLS and the random effects model produce inconsistent estimates. The fixed effects model still produces consistent estimates.

The results of estimating the electricity demand equation (3) by OLS, a fixed effects model and a random effects model appear in Table 8. The elasticities are contained in Table 9. The fixed charge variables have been excluded from the regressions.[1] Some of the estimates, particularly those of the fixed effects model, differ substantially from the estimates made using the other models. This is not surprising. As Maddala [1971] has shown, the random effects estimator for β can be written as

$$\hat{\beta} = \frac{W_{xy} + \theta B_{xy}}{W_{xx} + \theta B_{xx}}$$

where $$W_{xx} = \sum_t (X_{it} - \overline{X}_{i.})^2$$

[1] If the fixed charge variables belong in the regression, which, theoretically, they do, leaving them out introduces specification error. However, as shown by Berndt [1978], the resulting bias is small.

$$W_{xy} = \sum_{t} (X_{it} - \overline{X}_{i.})(Y_{it} - \overline{Y}_{i.})$$

$$B_{xx} = \sum_{i}\sum_{t} (X_{it} - \overline{X})^2 - W_{xx}$$

$$B_{xy} = \sum_{i}\sum_{t} (X_{it} - \overline{X})(Y_{it} - \overline{Y}) - W_{xy}$$

$$\theta = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T\sigma_\alpha^2}$$

W refers to within groups and B refers to between groups. The ordinary least squares estimator corresponds to $\theta = 1$ and the fixed effects estimator corresponds to $\theta = 0$. The fixed effects estimator eliminates a large portion of the total variation in the data since it eliminates the between group variation, which is much larger than the within group variation. Since θ in the random effects model for electricity is estimated to be .667, the random effects estimates are not very different from the OLS estimates. The hypothesis that the state dummy variables are all equal to 0 is rejected by an F test at the .01 level of significance.[1] Similarly, the hypothesis that the variance of the random component in the random effects model is zero is rejected at the .01 level with a χ^2 test.[2]

---

1 The statistic m is distributed approximately as F (49,∞), which has a critical value of about 1.59 at the .01 level of significance.

$$m = \frac{(53{,}144{,}300 - 21{,}080{,}100)}{21{,}080{,}100} \frac{520}{49} = 16.14.$$

The null hypothesis is rejected.

2 The statistic m is distributed as χ_1^2 which has a critical value of 6.63 at the .01 level of significance.

$$m = -2\log\frac{L_R}{L_U} = -2(-4408.89) + 2(-4334.39) = 149.$$

The null hypothesis is rejected.

Table 8

Comparison of Short-Run Electricity Demand Results with Different Pooling Techniques
(Standard errors in parentheses)

	(1) OLS	(2) Fixed Effects	(3) Random Effects
Space Heating			
C	17,872.7	6,403.51	18,502.7
	(4,574.20)	(6,239.07)	(4,770.76)
HD	1.96513	1.72969	1.85439
	(.213195)	(.215803)	(.223802)
PE	-5,220.26	-2,309.76	-5,405.54
	(1,107.98)	(1,236.27)	(1,156.14)
Y	-.993086	-.0691287	-.964652
	(.394075)	(.472662)	(.395688)
kWh/appliance/year[1]	9,639	10,576	9,611
Central Air Conditioning			
C	446.909	-127,105	-442.191
	(5,933.09)	(5,178.45)	(5,911.53)
CD	1.15168	.979084	1.01472
	(.341979)	(.302834)	(.346164)
PE	17.0104	1,730.17	748.918
	(1,441.06)	(1,240.48)	(1,431.19)
Y	.182645	.0110619	.225335
	(.424352)	(.370403)	(.418362)
Kwh/appliance/year	3,405	4,268	4,137
Room Air Conditioning			
C	3,510.09	-1,022.23	3,198.93
	(2,393.45)	(2,111.13)	(2,402.11)
CD	.853756	.840579	.79841
	(.179396)	(.210960)	(.191889)
PE	-1,678.31	-900.394	-1,591.42
	(546.981)	(493.802)	(550.181)
Y	.0589242	.286982	.0687321
	(.156012)	(.139120)	(.155263)
kWh/appliance/year	1,977	903	1,848

---

[1] Calculated at the means of the independent variables.

Table 8 (continued)

	(1)	(2)	(3)
<u>Freezing</u>			
C	7,497.23	382.444	5,253.08
	(2,721.98)	(2,550.82)	(2,802.30)
PE	-818.268	-745.186	-431.261
	(701.014)	(640.294)	(715.175)
Y	-.362109	.162647	-.233030
	(.217471)	(.191832)	(.216565)
kWh/appliance/year	2,742	490	2,367
<u>Cooking</u>			
C	4,186.43	-6,975.62	3,714.34
	(2,032.45)	2,362.69	(2,165.65)
PE	153.861	1,079.69	-89.8967
	(473.333)	(539.610)	(492.611)
Y	-.367877	.498981	-.269261
	(.170211)	(.167116)	(.173593)
kWh/appliance/year	1,152	-511	1,124
<u>Water Heating</u>			
C	-14.3784	1,842.62	-129.898
	(2,436.4)	(2,512.72)	(2,510.35)
PE	-743.297	-1,000.84	-557.052
	(580.499)	(646.722)	(604.883)
Y	.468130	.0857251	.433053
	(.182784)	(.171500)	(.182267)
kWh/appliance/year	2,849	791	2,757
<u>Clothes Washing</u>			
C	-2,392.88	-3,064.43	-2,226.55
	(732.678)	(522.063	(674.353)
PE	407.666	123.166	302.686
	(179.048)	(127.025)	(164.793)
Y	.192463	.303618	.192336
	(.0578662)	(.0419126)	(.0533024)
kWh/appliance/year	85	-104	59

Table 8 (continued)

	(1)	(2)	(3)
Clothes Drying			
C	-2,279.08	17,879.4	372.678
	(2,203.36)	(2,604.02)	(2,259.80)
PE	-470.452	-3,840.09	-793.178
	(656.316)	(683.827)	(663.271)
Y	.722520	-.871266	.488647
	(.173340)	(.181780)	(.173151)
kWh/appliance/year	3,375	3,027	3,331
Other	1,331.60	3,902.19	1,456.71
	(97.5653)	(449.920)	(106.400)
Intercept	-79.4455	-	-52.39541
	(20.8979)		(25.9603)
R^2	.9928	-	-
Mean of dependent variable (millions of kWh)	2654.95	-53.6561	1753.26
Standard error of the regression	305.613	192.309	269.816
Sum of squared residuals	53,144,300	21,080,100	41,423,600

Table 9

Elasticities of the Demand for Electricity

	(1) OLS	(2) Fixed Effects	(3) Random Effects
Price elasticities			
Space Heating	-.99	-.40	-1.03
Central Air Conditioning	.01	.74	135.19
Room Air Conditioning	-1.55	-1.82	-1.57
Freezing	-.54	-2.77	-.33
Cooking	.24	-3.85	-.15
Water Heating	-.48	-2.31	-.37
Clothes Washing	8.75	-2.16	9.39
Clothes Drying	-.25	-2.31	-.43
Aggregate	-.40	-1.11	5.17
Income Elasticities			
Space Heating	.93	.06	.91
Central Air Conditioning	.48	.02	.49
Room Air Conditioning	.27	2.86	.82
Freezing	-1.19	2.99	-.89
Cooking	-2.88	-8.80	-2.16
Water Heating	1.48	.98	1.42
Clothes Washing	20.41	-26.31	29.37
Clothes Drying	1.93	-2.59	1.32
Aggregate	.38	-.55	.54
Heating Degree Day Elasticities			
Space Heating	1.06	.85	1.01
Aggregate	.13	.10	.12
Cooling Degree Day Elasticities			
Central Air Conditioning	.39	.27	.29
Room Air Conditioning	.50	1.09	.50
Aggregate	.05	.08	.04

As discussed in Section III, a specification test can be used to test the assumption of no misspecification in the random effects model. The null hypothesis of no misspecification is rejected with an F test at the .01 level of significance.[1] This result indicates that a fixed effects model is necessary for consistency. Since much of the work in this field utilizes a random effects approach it would be useful to look at the results of similar specification tests performed for other studies.[2]

Table 10 presents the results of the estimation of the demand for gas equation (4) using ordinary least squares, a fixed effects and a random effects model. The elasticities are contained in Table 11. The hypothesis that the individual state dummy variables are all equal to zero is rejected by an F test at the .01 level of significance.[3] The hypothesis that the variance of the random term in the random effects model is zero is also rejected at the .01 level of significance by a chi-square test.[4] In the case of gas, θ is estimated to be .239.

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[1] The statistic m is distributed approximately as $F(28, \infty)$.

$$m = \frac{\hat{q}' V(\hat{q})^{-1} \hat{q}}{\text{\# of coefficients}} = \frac{324.777}{28} = 11.6$$

where $\hat{q} = \hat{\beta}_{FE} - \hat{\beta}_{RE}$ and $V(\hat{q}) = (\hat{\beta}_{FE}) - V(\hat{\beta}_{RE})$.

The standard errors of $\hat{\beta}_{RE}$ have been adjusted to use the estimates of σ^2 from the fixed effects model to insure that the standard errors are consistent. The critical value at the .01 level of significance is about 1.70.

[2] Houthakker, Verleger & Sheehan [1974] report that their results using variance components and state dummies are quite similar, but no statistical test is reported.

[3] $m = \frac{(103,066-16,196)}{16,196} \cdot \frac{687}{.49} = 75.2.$

The critical value for $F(49, \infty)$ at the .01 level of significance is 1.59.

[4] $m = -2 \log L_R/L_U = -2(-4186.68)+2(-3636.75) = 1100$.

The critical value for χ_1^2 at the .01 level of significance is 6.63.

Table 10

Short-Run Residential Demand for Gas-Trended Data
(Standard errors in parentheses)

	(1) OLS	(2) Fixed Effects	(3) Random Effects
Space Heating			
C	-48.6689	804.892	562.257
	(150.710)	(111.749)	(126.858)
HD	.142483	.0931378	.118882
	(.00389035)	(.00580665)	(.00463314)
PG	-5.80559	-4.50701	-1.65961
	(3.53814)	(3.42923)	(3.79839)
Y	.0208748	-.0189052	-.0185456
	(.0141358)	(.00797443)	(.00964024)
Therms/appliance/year	815	1047	975
Cooking			
C	882.809	-795.865	3.98169
	(212.212)	(161.385)	(162.829)
PG	-10.5303	21.6947	14.575
	(5.94445)	(5.04215)	(5.40432)
Y	-.0651795	.0171251	-.0047048
	(.0164323)	(.0100359)	(.0108568)
Therms/appliance/year	126	-365	137
Water Heating			
C	120.794	230.571	238.039
	(127.963)	(146.007)	(146.893)
PG	-1.67961	-15.2165	-13.7007
	(5.49556)	(5.60274)	(4.99719)
Y	.0338782	.00729366	.00892274
	(.0185614)	(.0905341)	(.00996805)
Therms/appliance/year	426	114	156

Table 10 (continued)

	(1)	(2)	(3)
Clothes Drying			
C	925.032	1,717.92	2,831.75
	(549.485)	(335.925)	(357.229)
PG	-29.9698	-33.714	-491.046
	(17.6206	(10.362)	(11.1703)
Y	-.0214319	-.0705665	-1.49838
	(.0423308)	(.0239859)	(.0259988)
Therms/appliance/year	351	625	842
Intercept	- 1,545.51	-	-112.957
	(717.834)	-	(2,075.80)
R^2	.9855	-	-
Mean of Dependent variable	15.2850	-6.3934	-1.21349
Standard error of regression	11.8337	4.68786	5.68434
Sum of Squared residuals	103,066	14,196.4	23,781.4

Table 11

Short-Run Elasticities of the Demand for Gas

Price Elasticities	OLS	Fixed Effects	Random Effects
	(1)	(2)	(3)
Space Heating	-.09	-.05	-.02
Cooking	-1.02	.73	1.30
Water Heating	-.05	-1.64	-1.08
Clothes Drying	-1.05	.66	-7.14
Aggregate	-.15	-.30	-.21
<u>Income Elasticities</u>	(1)	(2)	(3)
Space Heating	.23	-.16	-.17
Cooking	-4.66	-.42	-.31
Water Heating	.72	.58	.52
Clothes Drying	-.55	-1.02	-16.04
Aggregate	.02	-.05	-.21
<u>Degree Day Elasticities</u>	(1)	(2)	(3)
Space Heating	.91	.46	.08
Aggregate	.68	.35	.06

As in the case of the demand for electricity, a specification test rejects the hypothesis of no misspecification in the random effects model[1], thus indicating that the OLS and random effects estimates are inconsistent.

Comparison of Two Alternative Sets of Data

Concern over an errors in variables problem due to the appliance stock data led to the development of an alternative data set which might avoid possible large errors in the original data set.[2] Section IV of this paper briefly describes the development of this alternative data set. Although confidence may be gained with the alternative data that there are no grevious errors in the stock data, quite a bit of information is not used in calculating this data. Thus, it is not clear which data set is better. Estimates were made using both sets of data in order to compare the results.

For electricity, more reasonable results were obtained using the original data than the alternative data and thus far all estimates presented have been made with the original DRI data. For comparative purposes, the results of the estimation of specification (3) using OLS with the alternative data are presented in Table 12. Estimated elasticities appear in Table 13. In terms of estimated average use and

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1 $m = \dfrac{\hat{q}'V(\hat{q})^{-1}\hat{q}}{\#\text{ of coefficients}} = \dfrac{101.951}{13} = 7.84.$

The critical value at the .01 level of significance for $F\ (13,\infty)$ is 2.18.

2 See Ralph Braid [1978].

elasticities the results are not as reasonable. This can be seen by comparing the estimated mean fuel consumption for each appliance with the estimates of Table 1 and Table 2 and by comparing the elasticity estimates in Table 13 with those in Tables 3 and 4. The results are quite similar in terms of signs and significance.

Table 12

Short-Run Demand for Electricity - Trended Data
(standard errors in parentheses)

	OLS
Space Heating	
C	7,035.33
	(3,528.64)
HD	1.87849
	(.154718)
PE	-3,840.68
	(966.257)
Y	.0475575
	(.293413)
kWh/appliance/year[1]	10,244
Central Air Conditioning	
C	-25,201.1
	5,256.9)
CD	2.0689
	(.254947)
P	4,423.86
	(1,195.69)
Y	1.68085
	(.362211)
kWh/appliance/year	424
Room Air Conditioning	
C	12,227.8
	(2,193.24)
CD	.665531
	(.148745)
PE	-2,204.5
	(522.311)
Y	-.707509
	(.137392)
kWh/appliance/year	2609

1 Calculated at the means of the independent variables.

Table 12 (continued)

Freezing	
C	15,940.1
	(2,472.62)
PE	-3,329.18
	(651.743)
Y	-.560838
	(.206682)
kWh/appliance/year	4817
Cooking	
C	-670.171
	(2,355.66)
PE	1,342.88
	(593.013)
Y	-.160121
	(.189464)
kWh/appliance/year	335
Water Heating	
C	9,387.83
	(2,296.37)
PE	-2,582.89
	(581.16)
Y	-.130522
	(.175544)
kWh/appliance/year	3503
Clothes Washing	
C	-4,954.72
	(1,125.6)
PE	664.656
	(262.454)
Y	.529140
	(.0888692)
kWh/appliance/year	1026

Table 12 (continued)

<u>Clothes Drying</u>	
C	-2,350.52
	(2,079.47)
PE	665.003
	(610.207)
Y	.388480
	(.160839)
kWh/appliance/year	2363
<u>All other</u>	690.972
	(148.276)
Intercept	-25.0452
	(15.3692)
R^2	-
Mean of dependent variable (millions of kWh)	177.867
Standard error of the regression	10.0654
Sum of Squared Residuals	58,963.7

Table 13

Short-Run Demand Elasticities

	Electricity-Alternative Data	Gas-Original Data
Price Elasticities		
Space Heating	-.68	-.08
Central Air Conditioning	19.02	
Room Air Conditioning	-1.54	
Freezing	-1.26	
Cooking	7.31	-1.45
Water Heating	-1.34	.21
Clothes Washing	1.18	
Clothes Drying	.51	-1.09
Aggregate	.90	-.12
Income Elasticities		
Space Heating	.04	.25
Central Air Conditioning	35.73	
Room Air Conditioning	2.44	
Freezing	-1.05	
Cooking	-4.31	-4.62
Water Heating	-.34	1.06
Clothes Washing	4.65	
Clothes Drying	1.48	-.81
Aggregate	1.34	.10
Heating Degree Day Elasticities		
Space Heating	.80	.73
Aggregate	.10	.55
Cooling Degree Day Elasticities		
Central Air Conditioning	5.69	
Room Air Conditioning	.30	
Aggregate	.25	

In the case of gas the alternative data appears to give marginally superior results in terms of estimated average usage and price elasticities, so this data set was used in the estimation shown here. Results for the original data calculated from shipments and estimated depreciation rates, as described in Braid [1978], appear in Table 14. The elasticities appear in Table 13. The results are quite similar to the ones presented earlier, as can be seen by comparing Table 14 with Table 5 and the elasticities in Table 13 with those of Table 7.

The conclusion drawn is that the alternative trended data has no major advantage over the original data, in spite of the difficulties involved in the development of the original data.

Conclusion and Some Suggestions for Further Research

The purpose of this study was to estimate short-run residential demand equations for electricity and gas using time-series and cross-section data on a specification which did not aggregate appliances into a single stock measure. The hope was that separate price and income elasticities could be obtained for individual end uses of energy by the household. The results are mixed. Fairly reasonable estimates in terms of average energy consumption for each type of appliance are obtained. Price elasticities appear to vary among the demands for fuel for different end uses, but the differences are not statistically significant. Income elasticities are disappointing. They are often of the wrong sign and of unreasonable magnitude. In addition, the coefficients of the fixed charge price variables in the electricity demand equation are generally of the wrong sign. Aggregate elasticities

Table 14

Short-Run Residential Demand for Gas-Shipment Data

	(4)
Space Heating	
C	-49.438
	(162.487)
HD	.128461
	(.00337273)
PG	-4.94082
	(3.6626)
Y	.0209577
	(.0154334)
Therms/appliance/year	753
Cooking	
C	1,195.05
	(215.446)
PG	-19.1306
	(6.25764)
Y	-.08296
	(.01678)
Therms/appliance/year	162
Water Heating	
C	133.497
	(230.593)
PG	6.79032
	(5.91575)
Y	.0458836
	(.020305)
Therms/appliance/year	390

Table 14 (continued)

	(4)
<u>Clothes Drying</u>	
C	1,837.22 (484.284)
PG	-55.2071 (16.4514)
Y	.0560875 (.0399356)
Therms/appliance/year	621
<u>Intercept</u>	-1.26557 (7.58185)
R^2	-
Mean of dependent variable	29.6116
Standard error of the regression	4.46419
Sum of Squared Residuals	14,667.7

are reasonable. Specification tests indicate that the use of a random effects model or OLS produces inconsistent estimates of the parameters and thus that a fixed effects model is necessary for consistent results. However, the results obtained using a fixed effects model are disappointing. In spite of the difficulties involved in developing an appliance stock data base for the intercensus years from either saturation rates or shipment data, the results obtained from the data are superior in the case of electricity and almost identical in the case of gas to results obtained from an alternative data set derived so as to minimize the probability of large random errors.

Future short-run work on residential demand should concentrate on the estimation of parameters of the demand for fuel for individual end uses, but little would be lost and precision would probably be gained if only major appliances were disaggregated from the rest of the appliance stock. Future work might also consider imposing constraints that individual price or income elasticities are zero. Higher quality data at a lower level of aggregation should also improve the quality of the estimates. Since appliance characteristics, such as efficiency, have changed over the years, it would also be desirable to incorporate this information into the model. Future work should also include tests of the assumptions made in pooling data.

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Appendix

Table A1

THE DATA

Abbreviation	Variable	Units	Mean
C	constant		
PE	price of electricity	¢/kWh	1.823
PG	price of natural gas	¢/therm	12.25
HD	heating degree days	^{o}F	5,207
CD	cooling degree days	^{o}F	1,166
QE	sales of electricity	m of kWh	2,655
QG	sales of natural gas	m of therms	859.62
Y	income per household	$/hsehold.	9,012
FC	fixed charge	$	3.49

Appendix

Table A2

Restricted Regressions - No Price or Income Variables

(Standard errors in parentheses)

	(1) electricity (kWh)	(2) gas (therms)
Space Heating		
C	4,294.56	282.036
	(763.438)	(24.505)
HD	3.21180	.1444484
	(.18358)	(.00381676)
Fuel consumption/appliance[1]	21,019	1,026
Central Air Conditioning		
C	-840.747	
	(971.344)	
CD	2.12806	
	(.363656)	
Fuel consumption/appliance	1,640	
Room Air Conditioning		
C	3,430.99	
	(387.108	
CD	-.0948689	
	(.1680557)	
Fuel consumption/appliance	3,320	

1 Calculated at the means of the independent variables.

Appendix

Table A2 (continued)

	(1)	(2)
<u>Freezing</u>		
Fuel consumption/appliance	3,901.99 (334.259)	
<u>Cooking</u>		
Fuel consumption/appliance	2,317.42 (287.545)	36.8577 (19.1372)
<u>Water Heating</u>		
Fuel consumption/appliance	1,288.63 (242.380	321.886 (30.9308)
<u>Clothes-Washing</u>		
Fuel consumption/appliance	189.398 (104.690)	
<u>Clothes Drying</u>		
Fuel consumption/appliance	3,289.58 (293.742)	434.097 (73.3611)
HS	653.257 (105.262)	
Intercept	-184.098 (22.9104)	-3,884.56 (636.382)
R^2	.9884	.9832
Mean of dependent variable	2654.95	15.285
Standard error of the regression	387.057	12.7339
Sum of Squared Residuals	87,640,600	120,642