AN APPROACH TO PARAMETER SENSITIVITY ANALYSES IN MODEL ASSESSMENT

by

Cecilia Sau Yen Wong

June 1978

MIT Energy Laboratory
Working Paper No. MIT-EL 78-032WP

ACKNOWLEDGEMENTS

This research was conducted as part of the M.I.T. Energy Laboratory's Model Assessment Project with support from the Electric Power Research Institute. The author expresses her deep gratitude to Professor Fred Schweppe for his advice and help in preparing this paper.

CONTENTS

Section 1: INTRODUCTION

The effect of parameter variation on system performance is important in system analysis and especially important for model assessment. The technique of parameter sensitivity analysis as, for example, that introduced into feedback system by Bode (1) and extended by Horowitz (2) is a measure of the change in some desired quantity with respect to the change in some system parameters(3). If T is the desired quantity and α is the parameter, the sensitivity function is given by

$$S_{\alpha}^{T} = \frac{\Delta T}{\Delta \alpha} \cdot \frac{\alpha}{T} .$$

Cruz (4), Perkins (5), and Morgans (6) have done extended work on linear multivariable systems.

Although such sensitivities are appropriate when taken in the context in which one intended, they are inadequate for model assessment. Parameter sensitivity analysis for model assessment should adopt an approach similar to that of building a model. The complexity of the model structure depends on the purposes of building the model. Similarly, the sensitivity of a model should depend on the criteria set for analyzing that model. It would be more appropriate to predetermine a sequence of criteria and then obtain the parameter sensitivity with respect to each criterion. A model could well be sensitive to one criterion and very insensitive to another. Rather than using the usual definitions of sensitivity, a new approach on parameter sensitivity is introduced for model assessment. This approach, named criteria sensitivity, is represented by the relationship between the percentage change in parameter vector and the minimum value of the performance indicator* of a criterion. The main idea is to present the sensitivity in two-dimensional space; one axis represents the minimum value of the performance indicator of a criterion,* and the other axis represents the percentage change in parameter vector at its nominal value. Knowing the relationship between the minimum value and the percentage change, one would get a good picture of how sensitive the model criterion is with respect to parameter change.

This report uses the above sensitivity analysis on

1. the logic of optimal plant mixes to illustrate such a new approach, and
2. the generation expansion submodel of the Regional Electricity Model, REM (7), as an example for applying such an approach to dynamic systems in general.

This report is organized as follows. Section 2 considers the static generation expansion problem (i.e., fixed year, no time dynamics). The numerical results and discussion of this static problem act as a vehicle to explain the basic ideas of this sensitivity analysis. The ideas of Section 2 are then abstracted and generalized in Section 3 to yield a more complete concept with an application to arbitrary dynamic systems. In Section 4, these general ideas are applied to the REM** study, which

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*Criterion is used as a verbal description of rules for judging the effect of parameter changes on some concerns. These concerns are represented mathematically by the performance indicator which is a scalar function.

**It is assumed that the reader is already familar with generation expansion logic in general and REM in particular.

is a dynamic model. Finally, the extension of the new theory and the relationship between parameter sensitivity and model validity are discussed in Section 5.

Section 2: STATIC GENERATION EXPANSION

For the purpose of illustrating this new approach to sensitivity analysis, let us analyze the static optimal plan mix logic by Turvey (8) for a particular region at time, say, 1975. The logic of optimal plant mix in electricity supply is to choose the plant composition for minimizing the average cost per kilowatt hour. The principal economic parameters of this cost are capital costs, operation and maintenance costs, fuel costs, and heat rates. The average cost equation defined in REM (9) is

$$\text{cost} = \frac{Pa}{\text{usage}} + Pb \qquad \text{(mills/kWh)}$$

$$Pa = \frac{\text{CHRATE} * (\text{PCAPIT} + \text{CFULK2})}{8.76 * \text{AVAFAC} * \text{DUTMAX}}$$, the usage-dependent component of production cost,

$Pb = \text{CFULM2} + \text{POAMCO}$, the usage-independent component of production cost, where

CHRATE = capital charge rate for planning
PCAPIT = predicted capital cost for plant type j
CFULK2 = cost of nuclear fuel loading
AVAFAC = availability factor
DUTMAX = maximum allowable duty cycle
CFULM2 = predicated fossil fuel cost for plant type j
POAMCO = predicated O&M cost for plant type j.

Assume there are six plant alternatives: coal-fired thermal, oil-fired thermal, light water uranium reactors; light water plutonium reactors, high-temperature gas reactors, and liquid metal fast breeder reactors. The optimal mix is to find a plant composition with minimum cost. Assume the parameter values are as given in Table 1. The optimal plan mix is OIL, COAL, and LWRU as indicated in Figure 1.

Table 1

NOMINAL VALUES OF PARAMETERS

PLANT PARAMETER	COAL	OIL	LWR U-235	LWR PU-U	HTGR	LMFBR
CHRATE	.1487	.1487	.1487	.1487	.1487	.1487
PCAPIT ($/kW)	391.3	351.6	456.1	683.1	746.0	909.5
CFULK2 ($/kW)	0	0	45.61	456.10	5.21	38.97
AVAFAC	.85	.95	.85	.70	.70	.70
DUMAX	.9	.9	.86	.86	.86	.86
Pa (mills/kWh)	8.680	6.979	11.650	32.110	21.180	26.74

CFULM2 (mills/kWh)	11.09	17.14	3.131	31.31	2.244	-.3466
POAMCO (mills/kWh)	.4797	.4797	.6396	.6396	.6396	.6396
Pb (mills/kWh)	11.57	17.62	3.771	31.950	2.883	.2930

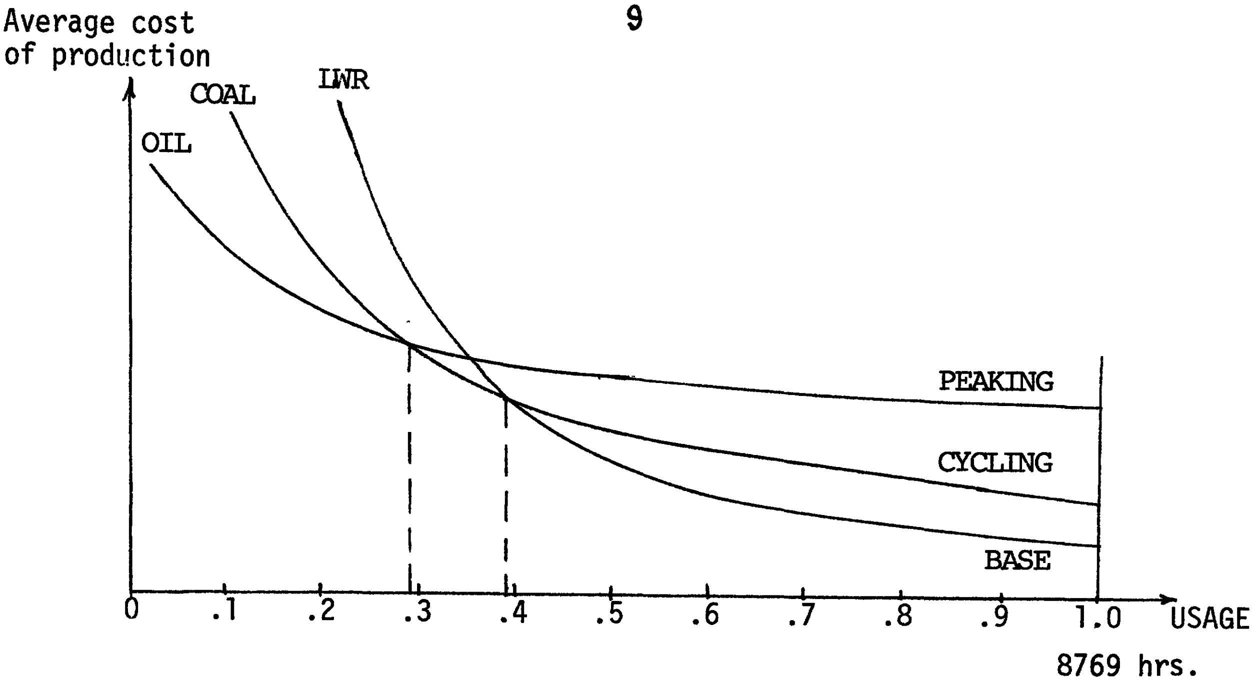

Figure 1. OPTIMAL PLANTS

If the forecasted load duration curve is as given as in Table 2, the percentage capacities for optimal plant mixes are 23% for oil, 4% for coal, and 73% for LWRU as indicated in Figure 2. If by 1985 the costs for generating electricity are the sames as those listed in Table 1, the total cost for generating electricity can be calculated by the method described in Appendix A.

Table 2
VALUES OF LOAD DURATION CURVE

USAGE	0	.01	.03	.05	.07	.1	.15	.2	.3
% CAPACITY	1	.902	.869	.847	.834	.822	.807	.790	.762
USAGE	.4	.5	.6	.7	.8	.9	.98	1.0	1.001
% CAPACITY	.720	.666	.612	.571	.535	.493	.446	.401	0

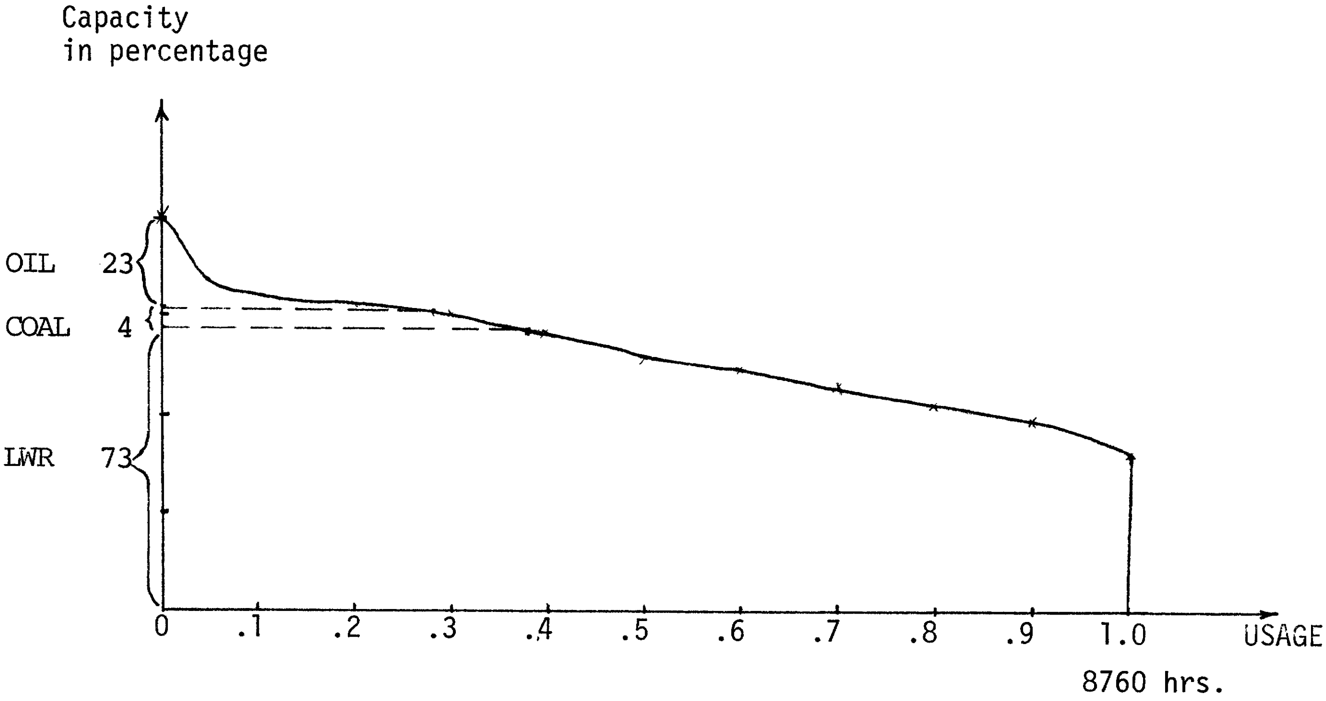

Figure 2. NOMINAL VALUES OF CAPACITY

Let us view the optimal mix logic as a black box with exogenous input parameters as listed in Table 1 (except capital charge rate, CHRATE) and outputs as the percentage capacity for various plants and total cost. Let us examine the effect of parameters changes on plant capacity and total cost.

There are three parts in this section. Section 2.1 assumes that there is no existing plant, that is, all plants are built from scratch, and the plant composition would be exactly as that of the optimal mixes.

Since the assumption in Section 2.1 is not realistic, Section 2.2 takes existing plants into consideration and assumes that 64% of predicted capacity would be generated from existing plants. As a result of that, the plant capacity would not only depend on the composition of optimal mixes, but also on the composition of the existing plants. Similar analyses are done on both parts so that one can examine the corresponding results. Section 2.3 discusses the results in general terms.

Section 2.1: No Existing Systems

Case A1: Percentage Nuclear

It is assumed that one of the issues of interest in generation expansion study is the total nuclear capacity. In order to quantify the effect of parameter change on the percentage of nuclear capacity, it is necessary to define some criteria. One possible criterion is to determine how much change in parameter values can be allowed before nuclear is completely removed from the optimal mixes of generation type. Thus for this case we have:

Interest:	investigate the effect of parameter changes on the percentage nuclear capacity in optimal mixes.
Criterion:	what change is necessry to eliminate nuclear in optimal mixes?

If some of the parameters are changed simultaneously in a particular fashion as indicated in Table 3, 7% of such simultaneous change would result in eliminating nuclear capacity in optimal mixes. The percentages of capacity distribution with different magnitude change in parameters are summarized in Table 4 and plotted in Figure 3. These results indicate that if one wants to avoid the possibility of eliminating nuclear completely, one has to restrict the simultaneous change less than 7%, say, 6%. In other words, the 6% region in parameter space would guarantee that nuclear would not be eliminated completely.

Table 3

CHANGES IN PARAMETERS FOR ELIMINATING LWRU

	COAL	LWRU	
PCAPIT	-	+	
CFULK2	0	+	
AVAFAC	+	-	COAL ↓
DUTMAX	+	-	LWRU ↑
CFULM2	-	+	
POAMCO	-	+	

Table 4

PERCENTAGE OF CAPACITY DISTRIBUTION WITH SIMULTANEOUS CHANGE IN PARAMETERS FOR ELIMINATING LWRU

	NOMINAL	1%	2%	3%	5%	6%	7%
OIL	23%	22%	21%	19%	17%	14%	10%
COAL	4%	10%	16%	22%	32%	40%	90%
LWRU	73%	68%	63%	59%	51%	46%	0

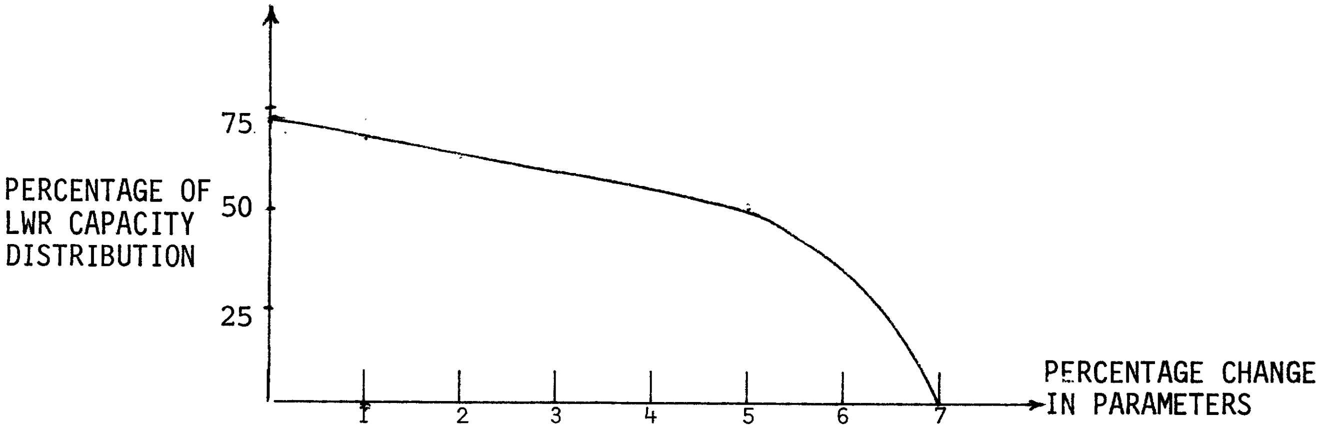

Figure 3. PERCENTAGE OF LWR CAPACITY IN DISTRIBUTION WITH SIMULTANEOUS CHANGE IN PARAMETERS

Case A2: Percentage Oil

Interest: investigate the effect of parameter changes on percentage oil capacity in optimal mixes.

Criterion: what change is necessary to eliminate oil in optimal mixes?

It has been calculated that 4% simultaneous change in parameters in a chosen direction, as listed in Table 5, would produce zero oil capacity in optimal mixes. Again, the percentage of capacity distribution with different magnitudes of simultaneous change are summarized in Table 6, and plotted in Figure 4. If one changes parameters within a 3% region at their nominal values, it is guaranteed that oil would not be eliminated completely.

Table 5
CHANGES IN PARAMETERS FOR ELIMINATING OIL

	OIL	COAL	
PCAPIT	+	-	
CFULK2	0	0	
AVAFAC	-	+	OIL ↑
DUTMAX	-	+	COAL ↓
CFULM2	+	-	
POAMCO	+	-	

Table 6
PERCENTAGE OF CAPACITY DISTRIBUTION WITH SIMULTANEOUS CHANGE IN PARAMETERS FOR ELIMINATING OIL

	NOMINAL	1%	2%	3%	4%
OIL	23%	21%	18%	13%	0
COAL	4%	8%	13%	20%	38%
LWRU	73%	71%	69%	67%	62%

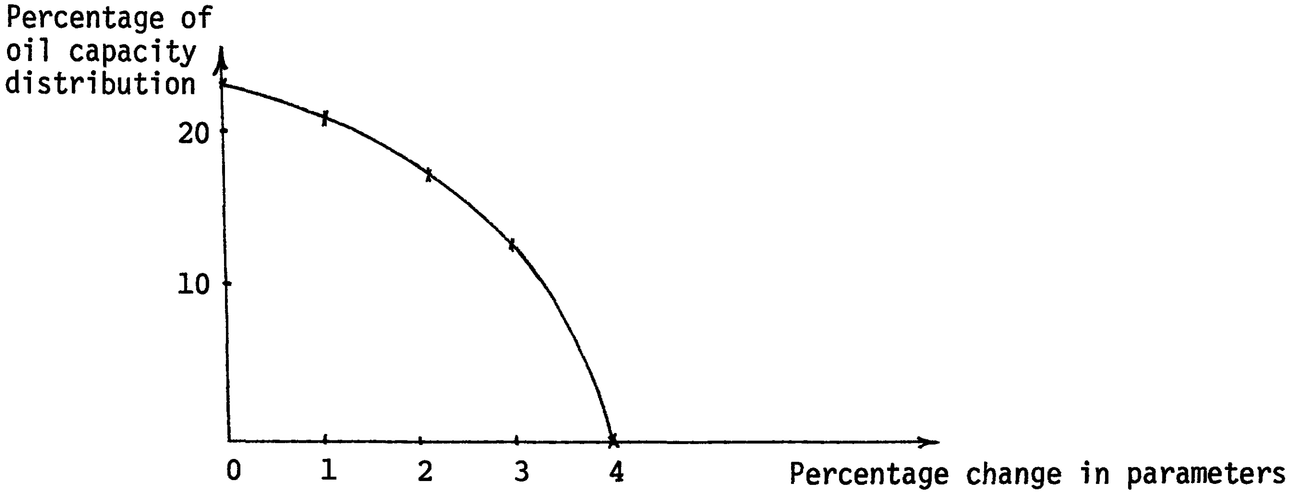

Figure 4. PERCENTAGE OF OIL CAPACITY DISTRIBUTION WITH SIMULTANEOUS CHANGES IN PARAMETERS

Case A3: Percentage Coal

Interest: investigate the effect of parameter changes on the percentage coal capacity in optimal mixes.

Criterion: what change is necessary to eliminate coal in optimal mixes?

All it takes is 1% simultaneous change in the direction indicated in Table 7 to eliminate coal completely. The distribution of percentage capacity is listed in Table 8. In this case, the guaranteed region in parameters space has to be less than 1%.

Table 7
CHANGES IN PARAMETERS FOR ELIMINATING COAL

	OIL	COAL	LWRU	
PCAPIT	-	+	-	
CFULK2	0	0	-	OIL ↓
AVAFAC	+	-	+	COAL ↑
DUTMAX	+	-	+	LWRU ↓
CFULM2	-	+	-	
POAMCO	-	+	-	

Table 8
PERCENTAGE CAPACITY DISTRIBUTION WITH SIMULTANEOUS CHANGE IN PARAMETERS FOR ELIMINATING COAL

	NOMINAL	1%
OIL	23%	25%
COAL	4%	0
LWRU	73%	75%

So far we have calculated the effect of simultaneous parameter changes on various directions. If one changes one single parameter at a time, the effect would be smaller than what we have shown in the previous tables and figures. Table 9 presents the percentage change in one single parameter for eliminating one of the plants in optimal mixes.

Table 9

PERCENTAGE CHANGE IN ONE PARAMETER RESULTS IN DIFFERENT OPTIMAL GENERATION MIXES

	COAL ↑	COAL ↓	OIL ↑	OIL ↓	LWRU ↑	LWRU ↓
OPTIMAL PLANTS	COAL,LWRU	COAL,LWRU	COAL,LWRU	OIL,LWRU	OIL,COAL	OIL,LWRU
CHRATE	*	*	*	*	*	*
PCAPIT	+4%	-20%	+25%	-5%	+46%	-8%
CFULK2	**	**	**	**	*	-80%
AVAFAC or DUTMAX	-4%	*	-20%	+6%	-32%	+8%
CFULM2	+10%	-43% (oil,coal)	*	-6%	*	-89%
POAMCO	*	*	*	*	*	*

*	small effect
**	no effect

(small effect for other plants)

Case A4: Total Cost

Interest:	investigate the effect of parameter changes on the percentage increase in total cost.
Criterion:	what change is necessary to have at most 30% increase in total cost?

Ten percent simultaneous change in a particular direction would result in a 34.5% increase in total cost. The direction change is summarized in Table 10 and the effect of parameter changes is presented in Table 11 and plotted in Figure 5. In order to guarantee that the increase in total cost is less than 30%, the region in parameter space would have to be smaller than 10%, say 9%.

Table 10

CHANGES IN PARAMETERS FOR INCREASING TOTAL COST

PARAMETER	ALL PLANTS
PCAPIT	+
CFULK2	+
AVAFAC	-
DUTMAX	-
CFULM2	+
POAMCO	+

Table 11

PERCENTAGE INCREASE IN TOTAL COST WITH SIMULTANEOUS CHANGE IN PARAMETERS

SIMULTANEOUS CHANGE IN PARAMETER VECTOR	2%	4%	6%	8%	9%	10%
INCREASE IN TOTAL COST	5.9%	13.6%	19.3%	24.1%	28.8%	34.5%

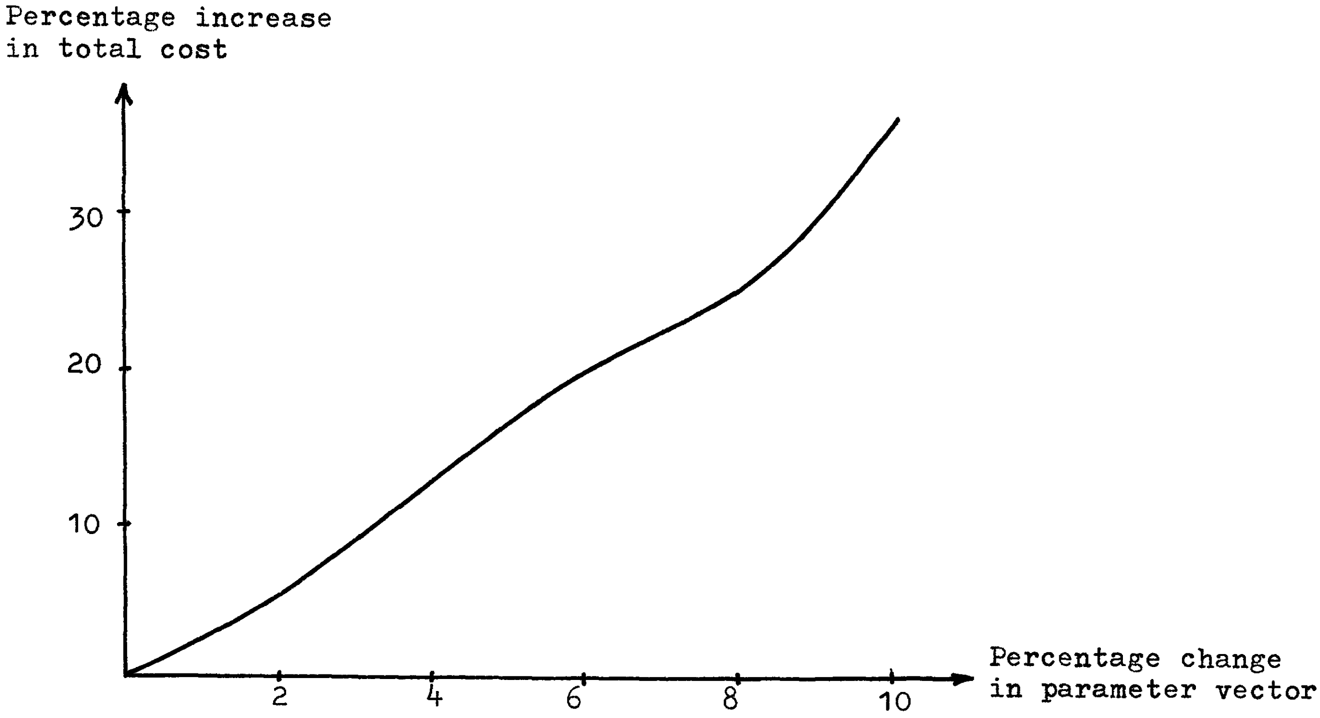

Figure 5. PERCENTAGE INCREASE IN TOTAL COST WITH SIMULTANEOUS CHANGE IN PARAMETERS

Let us summarize the results in Table 12. Column 1 shows the percentage capacity distribution at nominal values of parameters. Columns 2 to 5 show the percentage capacity distribution and increase in total cost when the parameter vector is changed in various directions according to the interests specified. The maximum size of guaranteed regions are shown in the last row. The size of maximum guaranteed region serves as a scale to measure the sensitivity so that one has a common ground to compare the sensitivity with respect to different interests.

Table 12

OPTIMAL CAPACITY DISTRIBUTION AND TOTAL COST WITH

DIFFERENT SIMULTANEOUS CHANGES IN PARAMETERS

		NOMINAL	CASE A1	CASE A2	CASE A3	CASE A4
SIMULTANEOUS CHANGE IN PARAMETER VECTOR		_	7%	4%	1%	10%
DIRECTION OF CHANGE OF PARAMETER VECTOR		_	OIL COAL ↓ LWRU ↑	OIL ↑ COAL ↓ LWRU	OIL ↓ COAL ↑ LWRU ↓	ALL ↑
CAPCITY DISTRIBUTION	OIL	23%	10%	0	25%	23%
	COAL	4%	90%	38%	0	9%
	LWRU	73%	0	62%	75%	68%
CHANGE IN TOTAL COST		-	18.2%	.16%	-2.2%	34.5%
MAXIMUM SIZE OF GUARANTEED REGION		-	6%	3%	.5%	9%

Section 2.2: Existing Systems

It would be more realistic if one takes the existing plants into consideration. If by 1985, the percentage capacity of existing plants is 64% with a combination of 20% for oil, 4% for coal, and 40% for LWRU, and if most of the costs (except those for fuels) for existing plants are the same as those listed in Table 1, one would be able to calculate the cost for generating electricity from existing plants. Also, the costs for generating electricity from new plants can be calculated if one takes the logic of building new plants as described in Appendix A.

Case B1: Percentage Nuclear

Interest: investigate the effect of parameter changes on the percentage nuclear capacity of new plants.

Criterion: what change is necessary to eliminate building any new nuclear plant?

Since the nuclear capacity of existing plants is 40%, one would build nuclear plants when the percentage in optimal mixes is above 40. Column 6 of Table 4 shows that 6% change in parameter vector would have the optimal mixes as 14% for oil, 40% for coal, and 46% for LWRU, that is, 6% of capacity is needed from new nuclear plants. In other words, it also takes 7% parameter changes to eliminate building new nuclear plants, and the corresponding guaranteed region is 6%.

Case B2: Percentage Oil

Interest: investigate the effect of parameter changes on percentage oil capacity for new plants.

Criterion: what change is necessary to eliminate building any new oil plant?

Column 2 in Table 6 shows that with 1% change in parameters, it is optimal to have 21% oil capacity. Since the existing oil capacity is 20%, 1% capacity would be needed from new oil plants. The result of 2% change in parameters shows that it is optimal to have 18% oil capacity, i.e., no new oil plant would be needed. Hence the guaranteed region is 1% change in parameter space.

Case B3: Percentage Coal

Interest: investigate the effect of parameter changes on percentage coal capacity for new plants.

Criterion: what change is necessary to eliminate building any new coal plant?

In the nominal case no new coal plant is needed because the coal capacity in optimal mixes is 4% which is the same as that of the existing capacity. Therefore no parameter change is necessary.

Case B4: Total Cost

Interest: investigate the effect of parameter changes on the percentage increase in total cost.

Criterion: what change is necessary to have at most 30% increase in total cost?

Seventeen percent simultaneous parameter changes in a particular direction would result in a 31% increase in total cost. The direction of change is the same as that indicated in Table 10, and the effects of parameter changes are presented in Table 13 and plotted in Figure 6. In order to guarantee that the increase in total cost is less than 30%, the size of the region in parameter space would have to be less than 17%, say 16%.

Table 13

EFFECT OF PARAMETER CHANGES ON TOTAL COST

SIMULTANEOUS CHANGE IN PARAMETER VECTOR	5%	10%	15%	16%	17%
INCREASE IN TOTAL COST	7%	17%	26%	28%	31%

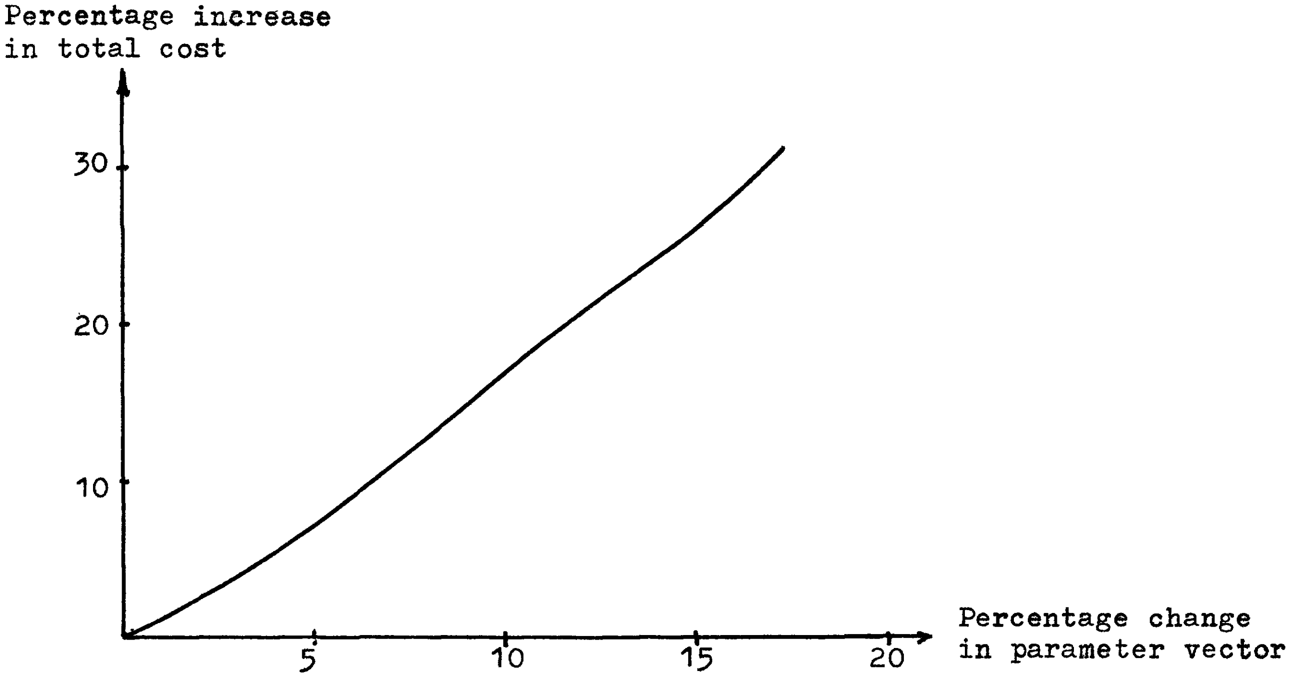

Figure 6. PERCENTAGE INCREASE IN TOTAL COST (EXISTING SYSTEMS INCLUDED) WITH SIMULTANEOUS CHANGE IN PARAMETER

Let us summarize the results of Cases B in Table 14. Column 1 shows that the percentage capacity distribution at nominal values. Columns 2, 3, and 5 show the percentage distribution and change in total cost when parameter vector are changed in various directions according to the interests specified. The sizes of the guaranteed regions are shown in the last row.

When the results from Table 14 and 12 are compared it shows that the model becomes more sensitive in cases 1 to 3, and less sensitive in case 4. The sensitivity shown in cases 1 to 3 depends not only on the direction of change in parameter vector but also on the initial capacity composition of existing plants. It is expected that the total cost would become less sensitive as indicated Column 5 of Table 14 because the cost for generating 64% capacity from existing plants depends heavily on the nominal values of parameters, and only 36% capacity comes from new plants.

Table 14

CAPACITY DISTRIBUTION AND TOTAL COSTS WITH DIFFERENT SIMULTANEOUS CHANGES IN PARAMETERS

(Existing plant composition is 20% for oil, 4% for coal, and 40% for LWRU)

		NOMINAL	CASE B1	CASE B2	CASE B3	CASE B4
SIMULTANEOUS CHANGE IN PARAMETER VECTOR		-	7%	2%	-	17%
DIRECTION OF CHANGE OF PARAMETER VECTOR		-	OIL COAL ↓ LWRU ↑	OIL ↑ COAL ↓ LWRU	- -	ALL ↑
CAPACITY DISTRIBUTION	OIL	23%	20%	20%	-	28%
	COAL	4%	40%	11%	-	8%
	LWRU	73%	40%	69%	-	64%
CHANGE IN TOTAL COST		-	4.7%	2.7%	-	31.2%
MAXIMUM SIZE OF GUARANTEED REGION			6%	1%	-	16%

Let us closely examine the results obtained in cases 1 and 4. Column 2 in Table 14 indicates that when the parameters are changed 7% in a particular direction, the percentage capacity of LWRU decreases from 73% to 40% while the total cost increases only by 4.7%. Column 5 indicates that when the parameters are changed 17% in a different direction, the percentage capacity of LWRU decreases from 73% to 64% while the total cost increases by 31%.

Section 2.3 Generalizing the Results

Now consider the results of Sections 2.1 and 2.2. It is obvious that the model's sensitivity depends on

(1) issues of concern (i.e., percentage nuclear, total cost, etc.),

(2) the criterion used to quantify the effect of parameter changes on the specified interest, and the corresponding scalar performance indicator used to measure such an effect, and

(3) the type of parameter changes.

The issue of concern depends on the purposes of building the model and the applicaton of the model.

The choice of criterion is admittedly a difficult decision. For example, consider the criteria used in Cases A1 and B1 of "completely wiping out nuclear from the mixes." Some modelers might choose to look for parameter variations which cause at most a 50% decrease in the nuclear mixes. The corresponding performance indicator for the former is percentage nuclear capacity, and for the latter is the percentage nuclear capacity minus 50 percent nuclear capacity at nominal value. However, the results of Sections 2.1 and 2.2 emphasize that is is necessary to make such choices. In order to quantify the model's sensitivity it is essential to define the performance indicator explicitly as the measure of such effect.

The type of parameter changes used in Sections 2.1 and 2.2 are not the "usual" type. Instead of varying one parameter at a time, all of the parameters are varied simultaneously. To illustrate the difference, let us consider a very simple model:

$$x = \alpha_1 \alpha_2$$

x: output

α_1, α_2: parameters.

Let α_1^o, α_2^o and x^o be the nominal values of α_1, α_2 and x where $\alpha_1^o = \alpha_2^o = x^o = 1$. For a small change in α_1 and α_2, we have

$$\begin{aligned} x &= (\alpha_1^o + \Delta\alpha_1)(\alpha_2^o + \Delta\alpha_2) \\ &= \alpha_1^o \ \alpha_2^o + \alpha_2^o(\Delta\alpha_1) + \alpha_1^o(\Delta\alpha_2) + (\Delta\alpha_1)(\Delta\alpha_2) \\ &= x^o + \Delta\alpha_1 + \Delta\alpha_2 + \text{higher order term.} \end{aligned}$$

Thus a 5% change in either parameter α_1 or α_2 individually causes a 5% change in the output x, but a 5% simutaneous change in α_1 and α_2 can cause a change in x anywhere from 0 to 10%. Usually sensitivity studies are associated with changing parameters one at a time, however, the sensitivity analysis of this approach emphasizes the superimposed effect of simultaneous change in parameters.

The results of Sections 2.1 and 2.2 can be viewed as examples of the new approach for sensitivty anlsysis which is hereinafter called "criterion sensitivity." Thus two major factors which depend on the criteria sensitivity approach are:

(1) A sequence of predetermined criteria and their corresponding scalar functions of index performance. For example, (a) the effect of parameter changes on eliminating nuclear in case A1 of Section 2 could serve as a criterion and its corresponding performance indicator would be the percentage nuclear capacity and (b) the effect of parameter changes on a 30% (at most)

increase in total cost as in Case A4 could also be considered as one of the criterion and its corresponding performance indicator would be the 30% minus percentage increase in total cost.

(2) The definition of percentage change in parameter vector as the percentage of simultaneous change in a direction which is chosen relative to the specified performance indicator.

Section 3: CRITERIA SENSITIVITY

The "critera sensitivity" concept discussed in Section 2.3 is now expressed in a more general framework.

Let us consider a time-discrete model in the following form

$$x(n + 1) = \phi(x(n), x(n - 1), x(n - 2), \ldots, x(1) , n, \alpha)$$

where

$x(n)$: K_x dimensional state variable vector

α: K_α dimensional parameter vector (includes initial conditions

n: time variable for $n = 1. \ldots, t_1$.

Base case:

α^o: nominal parameter values

$x^o(n)$: outputs for α^o.

Perturbed case:

$\alpha = \alpha^o + \Delta\alpha$

$x(n)$ = outputs for α.

For each predetermined criterion C_i (e.g., effect of parameter change on eliminating nuclear in optimal mixes as in Case A1 of Section 2.1), a scalar function $I_i(n)$ is defined as the indicator of performance (e.g., percentage nuclear capacity in optimal mixes as in Case A1), and the behavior of the system is considered as acceptable when the indicator of performance is positive* for all time $n = 1, \ldots, t_1$.

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*The positive value of performance indicator I(n) is used in defining acceptable behavior of system in general context. For example, in Case B1 of Section 2.2, the performance indicator I(n) can either be defined as (i) % nuclear capacity - 40%, and the behavior of system is considered as acceptable if I(n) is positive, or (ii) % nuclear capacity, and the behavior of system is considered as acceptable if I(n) is above 40%.

Percentage change in parameter vector is usually taken as the percentage change of one parameter only. Different from the usual meaning, the percentage change in parameter vector for this approach is used to represent the size of the region in parameter space under examination. For example, seven percent change in parameter vector means that the region considered is within seven percent change from its nominal value α^o. Any perturbed parameter vector would be within that seven percent cuboid. Similarly, if the percentage change in parameter vector is d, the perturbed parameter vector α would lie within d percent cuboid, that is,

$$\alpha \in \Omega_\alpha(d) = \{\alpha : \alpha_j^o(1 - .01 \times d) \leq \alpha_j \leq \alpha_j^o(1 + .01 \times d)$$
$$\text{for } j = 1, \ldots, K_\alpha\}.$$

It is not necessary that all parameters have to be changed by d percent (e.g., in Case A1, all the changes in parameters are listed in Table 3 while other parameters are unchanged). Depending on the chosen direction of change of parameter vector, some components of the vector are changed and others remain unaltered. Weighted coefficients can be introduced for parameters such that the perturbed components are changed proportionally to their weights.

The direction of change of parameter vector is chosen such that the minimum value $MI_i(n, d)$ of $I_i(n)$ over all α in the region of d-percent change is obtained (e.g., in case A1, the minimum value of $I_i(n)$ over all is the percentage LWRU capacity listed in Table 4). That is,

$$MI_i(n, d) = \min_{\alpha \in \Omega_\alpha(d)} I_i(n)$$

The relationship of minimum values $MI_i(n, d)$ and percentage change d is plotted so that one can see how $MI_i(n, d)$ changes gradually (e.g., in Case A1, percentage LWRU capacity vs percentage change in parameter vector is plotted in Figure 3).

The maximum guarantee-acceptable neighborhood, GAN_i is defined as the region of maximum percentage change dm such that $MI_i(n, dm)$ is positive for all $n = 1, \ldots, t_1$ (e.g., in Case A1, maximum GAN is the 6% region). In other words, the behavior of the system is acceptable for any perturbed parameter vector within the guarantee-acceptable neighborhood GAN_i. The maximum size of GAN_i serves as a measure for the parameter sensitivity with respect to each criterion C_i.

It is beyond the scope of this paper to discuss the systematic way of identifying the maximum GAN for any type of performance functions. The procedure to identify GAN for indicator-of-performance function with monotonic* properties is now discussed.

Procedure:

(1) Define a sequence of criteria C, with corresponding indicator-of-performance functions I(n), and assign values of weighted coefficients for all parameters.

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*Monotonic property is that for any parameter α_j in parameter space; the partial derivatives of $I_i(n)$ with respect to α_j is either non-positive or non-negative for all n.

(2) Based on the signs of the partial derivative of $I_i(n)$ with respect to each parameter α_j, choose a direction D_i change of parameter vector such that it would give a largest decrease in $I_i(n)$ for all n.

(3) If $I_i(n)$ can be expressed explicitly in terms of percentage change d, obtain the maximum guarantee-acceptable neighborhood GAN_i analytically. Otherwise, one could do the following.

(4) Calculate the size* of probable GAN by using the upper and lower bounds for endogenous variables $y_j(n)$ for all $j = 1, \ldots, K_y$.

(5) Find the maximum static guarantee-acceptable neighborhood by using the nominal values of endougenous variables, and

(6) Find the maximum dynamic guarantee-acceptable neighborhood by changing the input values of exogenous parameters in a particular direction with certain step size until $I_i(n)$ is no longer positive for some time n.

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*The size indicates the least known percentage change needed for driving $I_i(n)$ to negative.

Section 4: APPLICATION TO REM

In order to demonstrate how to apply criteria sensitivity analysis on a dynamic model, let us use the generation and expansion (G&E) submodel of REM as an example. REM contains nine regions corresonding to the nine census* regions of the United States. Within each region and at any time n, the generation and expansion (G&E) submodel is used to choose the optimal mixes of eight plant types** with hydroelectric capacity supplied exogenously.

The corresponding subroutine of G&E of the FORTRAN version of REM is OPPLAND. The optimal capacity mixes of the eight plant types are computed within the subroutine. OPPLAND takes variables listed in Table 1 of Section 2 as input variables and produces outputs as optimal capacity mixes. As indicated in Figure 8, these variables of each plant type are exogenous except CHRATE, which is determined by other parts of REM, is endogenous. The new installed capacity of each plant type is computed by using logic similar to that described in Section 2. The values of optimal capacity and existing capacity of each plant type are used for such computation.

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*The nine census regions are: (1) New England, (2) Middle Atlantic, (3) East North Central, (4) West North Central, (5) South Atlantic, (6) East South Central, (7) West South Central, (8) Mountain, and (9) Pacific.

**The eight plants are: (1) coal-fired power plant, (2) natural gas-fired power plant, (3) oil-fired power plant, (4) light water uranium rector, (5) light water plutonium reactor, (6) high-temperature gas reactor, (7) liquid metal fast breeder, and (8) gas turbines and internal combustion as peaking units.

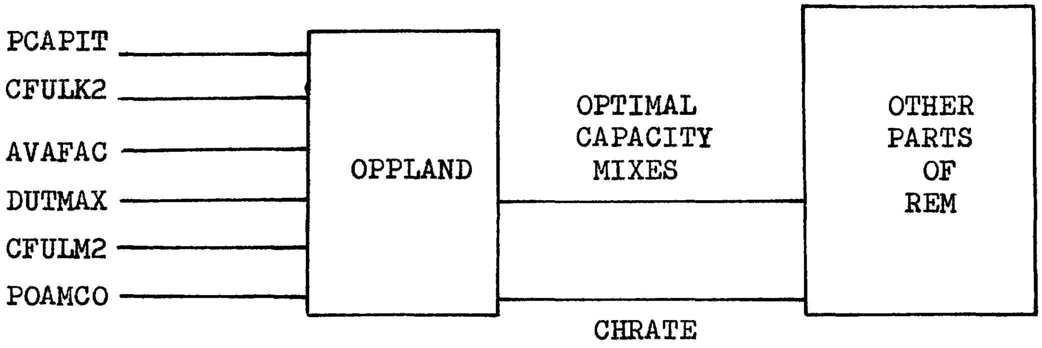

Figure 8. **INPUTS AND OUTPUTS OF OPPLAND**

(Generation and Expansion Submodel)

If one is interested in investigating the effect of parameter changes on the percentage installed nuclear capacity, one would define the criterion as to what change is necessary to eliminate new construction of nuclear capacity. In other words, one would like to do criteria sensitivity analysis similar to Case B1 in Section 2 for all nine regions and every year from 1975 to 1997. In order to do criteria sensitivity analysis, let us follow the procedure described in Section 3.

Step 1:

Consider

(a) the criteria as the effect of parameter change on eliminating building any new nuclear capacity and its corresponding indicator of performance function I(n) as the additional installed nuclear capacity 10 years* later.

(b) the acceptable behavior as that new nuclear capacity would not be eliminated completely from 1985 on (i.e., $I(n) > 0$ for any $n \geq 1985$), and

(c) all weighted coefficients for parameters as ones.

Step 2:

In order to choose a direction of change of parameter vector let us compute the signs of partial derivatives of additional nuclear capacity with respect to each exogenous parameter (i.e., the signs of

$$\frac{\partial I(n)}{\partial \alpha_j}$$

for n = 1975, ..., 1997, where α_j is an exogenous parameter). The direction of change of parameter vector is chosen as the opposite sign of that of the partial derivatives such that I(n) would approach zero more rapidly. The direction of change** is listed in Table 15.

Table 15

CHANGE IN PARAMETERS FOR ELIMINATING NEW INSTALLED NUCLEAR CAPACITY

	ALL FOSSILS	ALL NUCLEAR
PCAPIT	-	+
CFULK2	-	+
AVAFAC	+	-
DUTMAX	+	-
CFULM2	-	+
POAMCO	-	+

---

*The lead time for construction a nuclear plant is 10 years.

**The direction of change is chosen for decreasing the average cost of all fossils and increasing the cost of all nuclear.

Step 3:

Since the performance function I(n) cannot be expressed explicitly in terms of exogenous parameters, one has to follow steps 4 to 6.

Step 4: Calculate the size of probable GAN_i.

As it is shown in Section 2, eliminating building any new nuclear plant would be more sensitive than just eliminating nuclear among optimal mixes. If one computes the least known percentage change in parameters needed for eliminating nuclear among optimal mixes, the same least known would certainly apply to eliminate additional nuclear capacity.

In order to obtain a rough idea of what the least known percentage change is, let us examine the endogenous variable, CHRATE. It is explained in Appendix B that the increase in CHRATE would decrease optimal nuclear capacity, and conversely the decrease in CHRATE would certainly delay the elimination of nuclear among optimal mixes. Thus one can use the lower bound of CHRATE to determine the least known percentage change in parameters needed for eliminating nuclear.

By using the computer program CALPC* and the lower bound of CHRATE (i.e., .11**) the least known percentage change needed for driving $I_i(n)$ to negative is calculated and summarized in Table 16. Row 1 in Table 16 shows that in 1975 the least known percentage change is 12, 5, 2, and 4 for regions 1, 2, 3, and 5 respectively. For the rest of the

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*The documentation of CALPC is listed in Appendix C.

**The calculation of the lower bound is shown in Appendix B.

Table 16

THE LEAST KNOWN PERCENTAGE CHANGE IN PARAMETERS NEEDED FOR ELIMINATING NUCLEAR AMONG OPTIMAL MIXES

Region Time	1	2	3	4	5	6	7	8	9	All
1975	12	5	2	-	4	-	-	-	-	12
1976	15	6	4	-	6	2	2	-	3	15
1977	17	7	4	-	7	3	2	-	3	17
1978	19	7	5	-	7	4	2	-	3	19
1979	19	7	5	-	8	4	2	-	4	19
1980	20	7	5	-	8	4	-	-	4	20
1981	20	7	5	-	8	4	-	-	4	20
1982	20	7	5	-	8	4	-	-	4	20
1983	21	7	5	-	8	4	-	-	4	21
1984	21	7	5	-	8	4	-	-	3	21
1985	22	7	5	-	8	4	-	-	3	22
1986	22	7	5	-	8	4	-	-	3	22
1987	22	7	5	-	8	4	-	-	3	22
1988	22	22	5	-	7	4	-	-	3	22
1989	21	21	5	-	7	4	-	-	2	21
1990	21	21	5	-	7	4	-	-	2	21
1991	20	20	4	-	7	3	-	-	-	20
1992	20	20	4	-	20	3	-	-	-	20
1993	20	20	4	-	20	3	-	-	-	20
1994	20	20	4		20	3				20
1995	20	20	20	-	20	3	-	-	-	20
1996	20	20	19	-	20	19	-	-	-	20
1997	19	19	20	-	19	19	-	-	-	20

1	New England	6.	East S. Central
2	Middle Atlantic	7.	West S. Central
3	East N. Central	8.	Mountain
4	West N. Central	9.	Pacific
5	South Atlantic	All	Total U.S.

Note: - means nuclear is not among the optimal in nominal case.

regions, there is no change in parameter vector because nuclear is not among the optimals in the nominal case. The last column shows the least known percentage change needed for eliminating nuclear among optimals for the entire United States. The largest number, 22, indicates that if one changes the parameter vector in the direction specified as in Table 15, 22% would definitely eliminate nuclear among optimals from 1975 to 1997. Hence, the same number, 22%, of parameter change would certainly eliminate any additional nuclear capacity from 1985 to 1997. Twenty-two percent is the size of the probable GAN.

Step 5: Compute static results.

The static analysis for one region at one year in Case A1 of Section 2 is obtained by using the nominal value of CHRATE. With the help of the computer program CALPC, one can easily calculate the static minimum percentage change in parameters for eliminating nuclear among optimals for all regions and at every year from 1975 to 1997. The results are summarized in Table 17. Row 1 shows that in 1975, it takes 8%, 3%, 1%, and 2% for eliminating nuclear among optimals for regions 1, 2, 3, and 5 respectively. The last column shows the minimum percentage needed for such elimination for the entire United States. Thus the same percentage would definitely work for eliminating additional nuclear capacity 10 years from the corresponding time.

Step 6: Compute dynamic results.

The dynamic results shown in Table 18 are obtained by changing the values of exogenous parameters* within OPPLAND only for the entire period

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*The changes are listed in Appendix D.

TABLE 17 STATIC MINIMUM PERCENTAGE CHANGE IN PARAMETERS FOR ELIMINATING NUCLEAR AMONG OPTIMALS

Region Time	1	2	3	4	5	6	7	8	9	All
1975	8	3	1	-	2	-	-	-	-	8
1976	12	5	2	-	4	-	1	-	2	12
1977	14	5	3	-	5	2	1	-	2	14
1978	15	5	3	-	6	2	1	-	2	15
1979	15	5	4	-	6	2	1	-	2	15
1980	16	5	4	-	6	3	-	-	2	16
1981	16	6	4	-	6	3	-	-	2	16
1982	17	6	4	-	6	3	-	-	2	17
1983	17	6	4	-	6	3	-	-	2	17
1984	17	6	4	-	6	3	-	-	2	17
1985	18	6	4	-	6	3	-	-	2	18
1986	18	6	4	-	6	3	-	-	1	18
1987	18	6	4	-	6	3	-	-	1	18
1988	18	18	4	-	6	3	-	-	1	18
1989	18	17	4	-	6	3	-	-	1	18
1990	18	17	4	-	6	3	-	-	1	18
1991	17	17	3	-	5	3	-	-	-	17
1992	17	16	3	-	17	3	-	-	-	17
1993	17	16	4	-	17	3	-	-	-	17
1994	17	16	3	-	17	3	-	-	-	17
1995	17	16	17	-	17	3	-	-	-	17
1996	17	16	17	-	17	17	-	-	-	17
1997	17	16	17	-	17	17	-	-	-	17

1 New England
2 Middle Atlantic
3 East N. Central
4 West N. Central
5 South Atlantic
6. East S. Central
7. West S. Central
8. Mountain
9. Pacific
All Total U.S.

Note: - means nuclear is not among the optimal in nominal case.

		1	2	3	5	6	7	9	All
Minimum Percentage Change in Parameter for Eliminating Nuclear Among Optimal from 1975		9	6	4	6	3	1	2	9
Percentage Installed Capacity at 1995	Nominal	57	50	55	51	46	11	33	34
	With Corresponding Change	22	17	17	16	13	6	10	12
Guaranteed Acceptable Neighborhood (GAN)		8%	5%	3%	5%	2%	.5%	1%	8%

TABLE 18 DYNAMIC RESULTS WITH DIFFERENT SIMULTANEOUS CHANGES IN PARAMETERS

1. New England
2. Middle Atlantic
3. East N. Central
5. South Atlantic
6. East S. Central
7. West S. Central
9. Pacific

All Total U.S.

1975 to 1997 and then running the whole REM model without changing exogenous parameter variables in other REM subroutines. Since some of the parameters changed in OPPLAND are also used in other subroutines, the results must be interpreted accordingly, i.e. as the sensitivity of changes in OPPLAND on REM when OPPLAND is imbedded in the overall REM structure. Ideally these results would be compared with similar sensitivity studies done when the parameter values are changed simultaneously throughout all of REM. However, this second set of tests were not made so the differences in the two approaches are not known.

The direction of change is the same as that listed in Table 15, and the step size is .01. Row 1 shows that the dynamic minimum percentage of change in parameter vector for eliminating nuclear among optimals for the entire period of 1975 to 1997. It is the same minimum for eliminating additional nuclear capacity from 1985 to 1997.* The New England region, which relies heavily on nuclear, requires 9% change for such elimination, while the West South Central region requires the least change, that is, 1%. Rows 2 and 3 show the percentage installed nuclear capacity in 1995. With 9% change in New England, the nuclear capacity drops from 57% to 22% and for the entire United States it drops from 34% to 12%. With 1% change in the West South Central region, the percentage installed nuclear capacity drops from 11% to 6%. The maximum size of guarantee-acceptable neighborhood (GAN) for each region is shown in the last row. For any change of parameters within a GAN it is guaranteed

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*The model is valid till 1997.

TABLE 19. DATA FOR TOTAL UNITED STATES STATISTICS IN NOMINAL CASE AND WITH 9% CHANGE IN PARAMETERS

		1975	1980	1985	1990	1995
Installed	Coal	195.38	253.48	323.25	398.90	429.82
		(38.1%)	(41.4%)	(43.3%)	(43.3%)	(40.2%)
Capacity	Oil	49.34	47.41	43.26	38.30	32.84
		(9.6%)	(7.8%)	(5.8%)	(4.2%)	(3.1%)
(GW)	LWRU	43.13	81.81	140.26	246.90	361.52
		(8.4%)	(13.4%)	(18.8%)	(26.8%)	(33.8%)
	Total	512.76	611.69	746.75	921.37	1069.38
Electric Demand (MMWH)		1878.424	2433.05	3106.831	3819.220	4673.125
Cost of Electricity (mills/kWh)		26.704	35.296	46.417	60.714	79.576
Capital Investment (Bills $)		152.663	246.929	419.999	756.158	1232.469

TABLE 19a NOMINAL CASE

		1975	1980	1985	1990	1995
Installed	Coal	195.38	253.48	325.58	506.58	657.40
		(38.1%)	(41.4%)	(43.6%)	(55.5%)	(63.2%)
Capacity	Oil	49.34	47.55	43.38	38.30	32.63
		(9.6%)	(7.8%)	(5.8%)	(4.2%)	(3.1%)
(GW)	LWRU	43.13	81.81	137.74	131.02	120.63
		(8.4%)	(13.4%)	(18.5%)	(14.4%)	(11.6%)
	Total	512.76	611.69	746.53	912.85	1039.57
Electric Demand (MMWH)		1878.424	2432.986	3085.968	3712.119	4402.711
Cost of Electricity (mills/kWh)		26.704	35.213	47.243	64.110	86.293
Capital Investment (Bills $)		152.663	246.924	412.467	669.841	1031.688

TABLE 19b 9% CHANGE IN PARAMETERS

that nuclear would not be totally eliminated druing the period of 1975 to 1997 and it is also guaranteed that some new nuclear capacity would be installed between 1985 and 1997.

Table 19 shows some of the data generated by REM for total United States statistics; Table 19a is for nominal case and Table 19b is for the case with 9% change in parameters. By comparing the corresponding data in 1995 in both cases, one notices that coal takes the place of LWRU when nuclear is eliminated completely from optimals. That is, LWRU capacity drops from 33.8% to 11.6% while coal capacity increases from 40.2% to 63.2%, yet the total capacity only decreases by 2%, that is from 1069.38 to 1039.57 GW. The electric demand drops from 4673.125 to 4402.711 mMWh, that is, a 5.8% drop. Since the cost for generating electricity from each plant type is the same as that in the nominal case, a switch from LWRU to coal would result in an 8.4% increase in the cost of electricity, that is, the increase from 79.576 to 86.293 mills/kWh shown in the last column. Capital investment has the largest percentage change, that is, a 16.3% drop. This is because coal plants need less capital than LWRU plants.

Table 20 shows the effect of total United States* installed nuclear capacity with different magnitudes of simultaneous change in parameters for eliminating new construction of nuclear capacity. Data from the first three rows (i.e., from 1975 to 1985) show that there is hardly any difference from that in the nominal case. It is because 10 years is the

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*The effect on regional installed nuclear capacity is shown in Appendix E.

Installed Nuclear Capacity	Change in Parameters					
	Nominal	2%	4%	6%	8%	9%
1975	43.134	43.134	43.134	43.134	43.134	43.134
	(8.4%)	(8.4%)	(8.4%)	(8.4%)	(8.4%)	(8.4%)
1980	81.806	81.806	81.806	81.806	81.806	81.806
	(13.4%)	(13.4%)	(13.4%)	(13.4%)	(13.4%)	(13.4%)
1985	140.256	138.137	138.137	137.801	137.737	137.737
	(18.6%)	(18.5%)	(18.5%)	(18.5%)	(18.5%)	(18.5%)
1990	246.904	198.963	171.915	136.876	131.018	131.018
	(26.8%)	(21.7%)	(18.8%)	(15.0%)	(14.4%)	(14.4%)
1995	361.523	297.115	209.708	132.512	126.441	120.626
	(33.8%)	(28.1%)	(19.9%)	(12.7%)	(12.1%)	(11.6%)

Table 20. TOTAL U.S. INSTALLED NUCLEAR CAPACITY WITH SIMULTANEOUS CHANGE IN PARAMETERS FOR ELIMINATING NEW CONSTRUCTION OF NUCLEAR PLANT

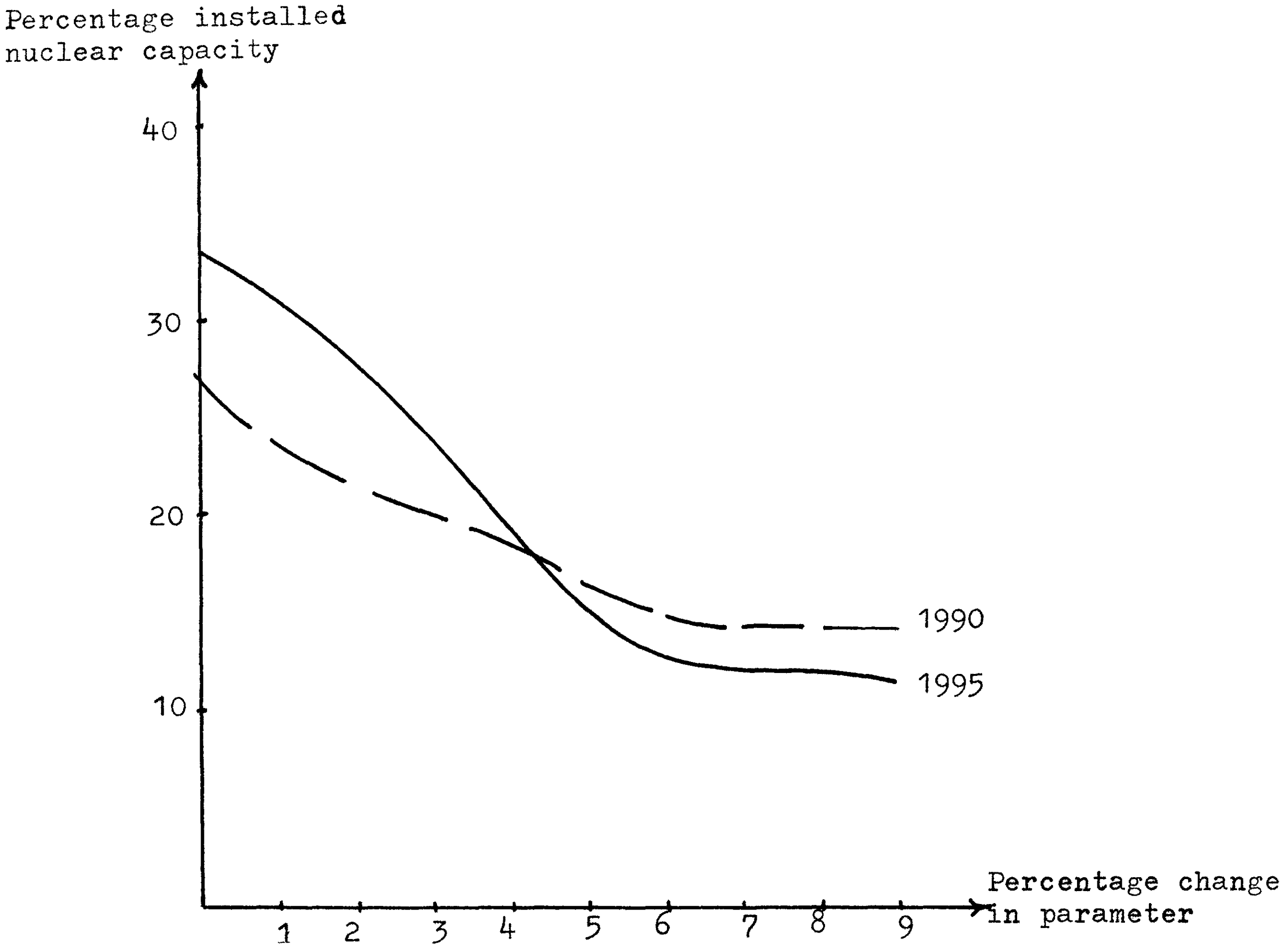

Figure 9. PERCENTAGE INSTALLED NUCLEAR CAPACITY WITH DIFFERENT SIMULTANEOUS CHANGES IN PARAMETER

lead time for nuclear construction and the model REM is designed in such a way that once the construction is in the pipeline, one cannot stop the inflow of new constructed capacity. The last two rows show that the effect becomes more significant by 1990. The percentage installed nuclear capacity is plotted against the percentage change in parameters as in Figure 9. By comparing these two curves, one notices that the one at 1995 has a much sharper decrease than that at 1990. It implies that the effect of eliminating new construction of nuclear capacity would gradually magnify as time goes by.

Section 5: DISCUSSION AND EXTENSIONS

The implications of the results of Sections 2 and 4 relative to model validation are now discussed. It is inappropriate to try to cover all aspects of model validation (or even to try to explicitly define model validity). However, some comments on the relationship between validity and sensitivity are given. It should be emphasized that sensitivity studies can only show that a model is invalid; they cannot show that it is valid.

The concepts discussed in Section 3 can be generalized in the context of model validation by phrasing them in terms of parameter and output spaces.

Define three spaces as follows:

Parameter space: Space whose coordinates are the parameters being varied during the sensitivity studies.

Output space: Space whose coordinates are the model outputs of concern.

Differential output space: Space whose coordinates are the changes in model outputs caused by a particular policy perturbation of concern.

In each space, define a region as follows:

Parameter uncertainty region: Region (or set) in parameter space in which "true" parameter values are believed to lie.

Output uncertainty region: Region (or set) in output space resulting from parameters varying within the parameter uncertainty region.

Differential output uncertainty region: Region (or set) in differential output space resulting from parameter varying within the parameter uncertainty region.

The model maps the parameter uncertainty region into the two output regions. Define two types of validity as follows:

Base case validity: Numerical values of outputs are valid forecasts = predictions of what will happen under conditions hypothesized for the base case.

Policy perturbation validity: Direction and magnitude of change in outputs caused by policy perturbation are valid.

Using the above definitions it can be said that a model exhibits

- Base case invalidity if the output uncertainty region is too large.
- Policy perturbation invalidity if the change in outputs caused by policy perturbation lie within the output uncertainty region (i.e., effect of noise is larger than that of signal).
- Policy perturbation invalidity if the differential output uncertainty region is too large.

The relative advantages/disadvantages of the two types of policy perturbation invalidity will not be discussed here. It is important to note that none of these statements are the same thing as saying "a highly sensitive model is invalid." In fact, a model could also be found to be invalid if it is too insensitive to certain parameters.

The results of Sections 2 and 4 are essentially stated in terms of output space. However, it is clear that the ideas generalize readily (in a conceptual sense) from output space to differential output space.

Relative to REM itself, the numerical results of Sections 2 and 4 show that REM can be "very sensitive" to certain types of parameter changes if certain types of criteria are used. It is felt that these sensitivities have the potential for invalidating REM for certain types of studies. However, no conclusions on REM's validity are being made or implied here. Such conclusions can only be made in the context of a particular application. Furthermore, it must be reemphasized that the numerical results were done on a version of REM that was subsequently modified.

This paper initiates a starting point to develop new approaches in parameter sensitivity for model assessment. There is no doubt that this criteria sensitivity theory can be extended to a wide-range area.

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APPENDIX A

1. Calculate the total cost for generating electricity from new plants only (i.e., every plant is built from scratch).

Given:

(1) average cost equation

$$AC_i = \frac{Pa_i}{u} + Pb_i \qquad i = 1, 2, \ldots, n$$

where AC - average cost (mill/kWh)

Pa - usage factor dependent component of production cost (mill/kWh)

Pb - usage factor independent component of production cost (mill/kWh)

u - usage factor (dimensionless);

(2) percentage capacity distribution among n types of plants as $p_1, p_2, p_3, \ldots, p_n$ and their corresponding maximum usage factor and area under duration curves as $u_1, u_2, \ldots, u_n$ and $a_1, a_2, \ldots, a_n$ correspondingly.

Then

$$\text{Total Cost} = \sum_{i=1}^{n} C_i$$

where

$$C_i = (\frac{Pa_i}{u} + Pb_i) * a_i \qquad (\text{mill unit} = \frac{\text{mill}}{\text{kWh}} * \text{unit kWh})$$

a_i = area (unit kWh).

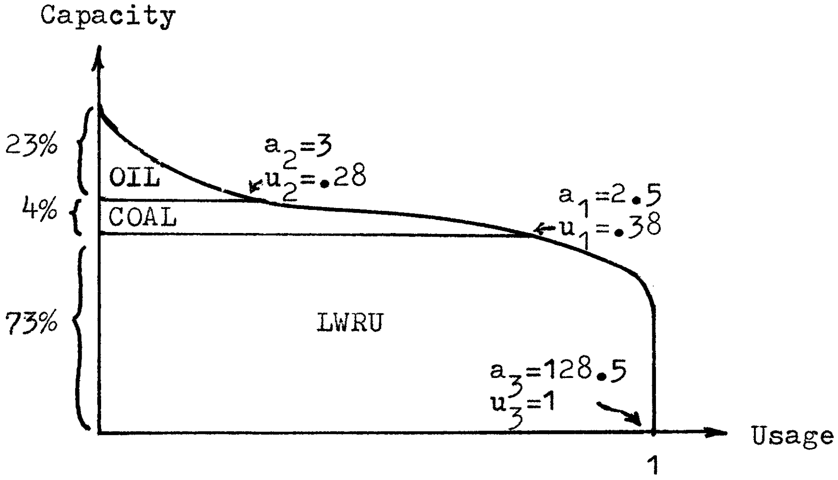
Example A1: Nominal Case in Section 2.

Given:

(1)

	COAL	OIL	LWRU
Pa	8.68	6.979	11.65
Pb	11.57	17.62	3.771

(2)



Then cost for generating electricity from

Coal:

$$\left(\frac{8.68}{.38} + 11.57\right) * 2.5 = 86.03$$

Oil:

$$\left(\frac{6.979}{.28} + 17.62\right) * 3 = 127.64$$

LWRU:

$$(11.65 + 3.771) * 128.5 = 1996.18$$

Total Cost = 2179.85 mill unit

2. Calculate the total cost for generating electricity from both existing and new plants.

Given:

(1) average cost equation

$$AC_i' = \frac{Pa_i'}{u} + Pb_i' \qquad \text{for existing plants}$$

$$AC_i'' = \frac{Pa_i''}{u} + Pb_i'' \qquad \text{for new plants}$$

$i = 1, 2, \ldots, n$

(2) optimal percentage distribution among n plant types as $p_1, p_2, \ldots, p_n$

(3) percentage distribution among existing plants as p_1', $p_2', \ldots, p_n'$

Then the total cost can be calculated as follows

(a) sum up the optimal and existing capacity into two groups: fossil and nuclear

(b) subtract existing capacity from optimal capacity, i.e.,

$$\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} p_i'$$

(c) what is left in each group would be distributed proportionally to the positive differences $(p_i - p_i')$ among plant types in each group

(d) sum up percentage capacity for each plant type as q_1, q_2, ..., q_n; find their corresponding maximum usage factor w_1, w_2, ..., w_n, and compute their corresponding area under duration curve as a_1, a_2, ..., a_n for each plant type

(e)

$$\text{total cost} = \sum_{i=1}^{n} C_i' + C_i''$$

where

$$c_i' = (\frac{Pa_i'}{w_i} + Pb_i') * a_i * \frac{P_i'}{q_i} \quad \text{for existing plants}$$

$$c_i'' = (\frac{Pa_i''}{w_i} + pb_i'') * a_i * (\frac{q_i - P_i'}{q_i}) \quad \text{for new plants}$$

percentage increase in total cost equals

$$\frac{\text{Total Cost - Total Cost in Nominal Case}}{\text{Total Cost in Nominal Case}} \times 100\%$$

Example A2: Calculate the total cost for Case B1 (Section 2) with 7% change in parameters

Given:

(1)

	COAL	OIL	LWRU
Pa'	8.68	6.978	11.725
Pb'	10.76	17.62	4.035
Pa"	7.05	6.979	14.413
Pb"	10.76	17.62	4.035

(2)	Optimal capacity	90%	10%	0%
(3)	Existing capacity	4%	20%	40%

Then the total cost can be calculated as follows

(a)

	FOSSIL	NUCLEAR
Optimal capacity	100%	0%
Existing capacity	24%	40%

(b)

$$\sum_{i=1}^{3} p_i - \sum_{i=1}^{3} p_i' = 100\% - 64\% = 36\%$$

(c)

	COAL	OIL	LWRU
Optimal capacity	90%	10%	0%
Existing capacity	4%	20%	40%
Difference	86%	-10%	-40%

(d)

		COAL	OIL	LWRU
	Existing capacity	4%	20%	40%
	New capacity	36%	0%	0%
q_i	Total capacity	40%	20%	40%
w_i	Usage factor	1	.15	1
a_i	Area	51.5	1.5	80

(e) cost C_i' from existing plants

COAL: $(8.68 + 10.76) * 5.15 * (4/40) = 100.12$

OIL: $(\frac{6.979}{.15} + 17.62) * 1.5 * (20/20) = 96.22$

LWRU: (11.725 + 4.035) * 80 * (40/40) = 1260.77

cost C_i" from new plants

COAL: (7.05 + 10.76) * 51.5 * (36/40) = 825.49

Total Cost = 2282.60 mill unit

percentage increase in total cost equals

$$\frac{2282.60 - 2179.85^*}{2179.85} * 100\% = 4.7\%$$

---

*2179.85 is the total cost at nominal value calculated in example B1. The percentage increases in total cost with different simultaneous changes in parameters are indicated in the second last row of Table 14, Section 2.

APPENDIX B

1. Let a be the multiplicative change in CHRATE, i.e.

new CHRATE = a * nominal value of CHRATE.

For the nominal case, the average cost equation is given as:

$$\tilde{C}_i = \frac{Pa_i}{U} + Pb_i \qquad \text{for } i = 1, 2, \ldots, n$$

and

$$\tilde{u}\,(=\text{USEVAL}) = \frac{Pa_j - Pa_k}{Pa_k - Pb_j} \qquad \text{for } j \neq k.$$

If a ≠ 1, then the new average cost equation is:

$$C_i = \frac{Pa_i * a}{U} + Pb_i$$

and the corresponding

$$\text{USEVAL} = \frac{Pa_j - Pa_k}{Pb_k - Pb_j} * a$$

a > 1 ⟹ SEVAL shifts to the right as indicated in Figure B; and

a < 1 ⟹ USEVAL shifts to the left.

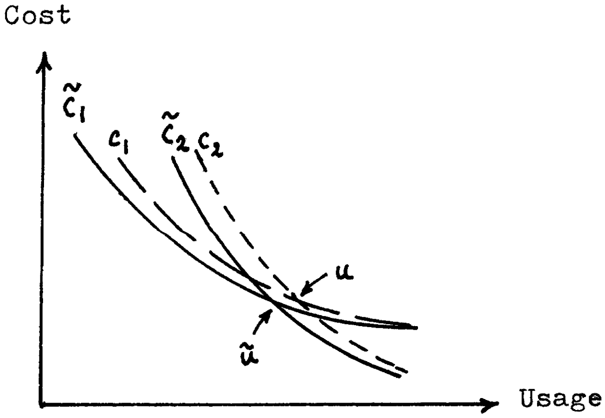


Figure B.

If the USEVAL is the intercepting point between fossil and nuclear, and both plants are among optimal mixes, it implies that the increase in CHRATE would decrease the optimal nuclear capacity, and conversely the decrease in CHRATE would increase the optimal nuclear capacity.

2. Calculate the lower bound of CHRATE

Capital charge rate CHRATE in REM is defined as:

$$\text{CHRATE} = \frac{1}{L} + \frac{D * \text{DINTN} + \dfrac{(TE * Re + PS * \text{PINTN})}{1 - \text{TAXINC}}}{D + TE + PS}$$

where

D = total debt capital

DINTN = .085, interest rate on new debt capital

TE = total equity capital

Re = .14, regulated return on equity

PS = total preferred stock capital

PINTN = .085, interest rate on new preferred stock

TAXINC = .30, taxing rate

L = 40, lifetime of plant

Lower bound of CHRATE

$$\lim_{\substack{D \to 1 \\ TE \to 0 \\ PS \to 0}} \quad \frac{1}{40} + \frac{D * .085 + \dfrac{(TE \times .14 + PS * .085)}{1 - .30}}{D + TE + PS}$$

$$= 1/40 + .085$$

$$= .11$$

APPENDIX C

DOCUMENTATION OF PROGRAM CALPC

```
C     PROGRAM CALPC
C     THIS PROGRAM IS USED FOR STATIC ANALYSIS:
C     TO CALCULATE
C     (1)  THE APPROXIMATED LEAST KNOWN PERCENTAGE CHANGE IN PARAMETER
C          VECTOR NEEDED FOR
C          ELIMINATING NUCLEAR AMONG OPTIMAL MIXES,
C          (i.e., THE SIZE FOR PROBABLE GUARANTEED ACCEPTABLE NEIGHBORHOOD
C          GAN)
C     (2)  THE MINIMUM PERCENTAGE CHANGE IN PARAMETER VECTOR FOR ELIMINA-
C          TING NUCLEAR AMONG OPTIMAL MIXES.
C
           DIMENSION PA(9), PB(9), USEVAL(12), IPLANT(10)
           DIMENSION DM(9, 54), PCM(54), ~~DD(50)~~, XX(50).
C
C     INITIALIZATION
C
C     NSC  = 1 IS USED FOR CALCULATING THE SIZE OF PROBABLE GAN
        NSC     = 0
        RTN     = 1975
        RTE     = 1998
        IDP     = 50
C     LOOP THROUGH TIME
        DO 580 NCT  = 1, 54
C     READ DATA
```

```
    100 IF (RTIME.GT.RTE) GO TO 900
         IF (RTIME.EQ.RTE. AND IREG. EQ. 9) GO TO 600
         READ (8, 10) (PA(I), I = 1, 8), CHRTE, NUS, NUSMO, KYEAR,
        *1REG, RTIME
         READ (8, 11) (PB(I), I = 1, 8), (USEVAL(I), I =1, NUS)
         READ (8, 12) (IPLANT (I), I = NUSMO)
         IF (KYEAR.EQ.2.OR.RTIME.LT.RTN) GO TO 100
C   USE LOWER BOUND .11 OF CHRTE FOR FINDING THE SIZE OF PROBABLE
C        GAN
         IF (NSC.NE.1) GO TO 300
         DO 190 J = 1, 8
    190  PA(J) = (PA(J)/CHRTE) * .11
C
C   MAIN PROGRAM
C
C   OBTAIN NUCLEAR PLANT TYPE IN OPTIMAL MIXES
    300  INU = 0
         INUP = 0
         IPID = 0
         DO 310 I = 1, NUSMO
         IF (IPLANT(I).LT.4.OR.IPLANT(I).GT.7) GO TO 310
         IPID = I
         INU = IPLANT(IPID)
         INUP = IPLANT(IPID - 1)
         GO TO 400
```

```
      310 CONTINUE
C     CALCULATE THE PERCENTAGE CHANGE FOR EACH REGION AT EVERY TIME
C
C     IF NUCLEAR IS NOT AMONG THE OPTIMAL MIXES, SKIP THE CALCULATION
      400  IF (IPID.EQ.0) GO TO 580
C     OTHERWISE INCREASE THE COSTS FOR NUCLEAR PLANT AND DECREASE COSTS
C     FOR FOSSIL WITH STEP SIZE 1 PERCENT
        DO 430 I = 1, IDP
        D = .01 * (I -1)
        PAIN = PA(INUP) * (1. - D)/(1. + D)**2)
        PAJN = PA(INU) * (1. + D)/(1. - D)**2)
        PBIN = PA(INUP) * (1. - D)
        PBJN = PB(INU) * (1. + D)
C       CALCULATE USEVAL
        XINT = (PAIN - PAJN)/(PBJN - PBIN)
        XX(I) = XINT
C       IF USEVAL ≥ 1, STOP LOOPING
        IF (XINT.GE.1) GO TO 500
        430 CONTINUE
C       RECORD THE PERCENTAGE CHANGE FOR EACH REGION AT EVERY TIME
        500 DM(1REG,NCT) = D
C       RECORD THE PERCENTAGE CHANGE FOR U.S. TOTAL
        DO 510 I = 1, 9
```

```
      510 IF (PCM(NCT).LT.DM(I, NCT)) PCM(NCT) = DM(I, NCT)
C           IF (RTIME.EQ.RTE.AND.IREG.EQ.9) GO TO 600
      580 CONTINUE
C     WRITE THE MATRIX OF PERCENTAGE CHANGE FOR EVERY REGION AT EVERY TIME
      600 DO 610 I = 1, NCT
        RTIME = RTN + (I - 1) * .5)
        WRITE (3, 22) RTIME, (DM(J, I), J = 1, 9) PCM(I)
      610 CONTINUE
C     FORMAT STATEMENT
      10 FORMAT (8(G11.4, 1X), 3X, F8.5, 3I3, I7, F8.1)
      11 FORMAT (8(G11.4, 1X), 3X, 4F8.5)
      12 FORMAT (99X, 4I8)
      22 FORMAT (F.10.1, 10F6.2)
C     900 STOP
      END
```

APPENDIX D

Changes made in subroutine OPPLAND:

After line NBJ00390 (i.e., IF(KYEAR.EQ.2.AND.K.EQ.4) J = 8) insert statements as follows --

```
     IF (KYEAR.NE.3.OR.J.GE.8.OR.RTIME.LT.1975.0) GO TO 50
C    PC IS PERCENTAGE CHANGE
       PC = .01
       IS = 0
C    DECREASE COST FOR FOSSIL AND INCREASE COST FOR NUCLEAR
       IF (J.LE.3) IS = -1
       IF (J.GE.4.OR.J.LE.7) IS = 1
       D = PC * IS
       PA(J) = CHRATE(IREG) * ((PCAPIT(J, KYEAR))  * (1. + d)
      *   + CFLUK2(J) * (1. + d))/(8.76 * AVAFAC(J) * (1 - d) * DUTMAX(J)
      *  *(1. - D))
       PB(J) = (CFULM2(J, KYEAR)+ POAMCO (J, KYEAR)) * (1 + d)
       GO TO 1


NBJ 00400 50 PA(J) = CHRATE (IREG) ...
```

APPENDIX E

APPENDIX E

1% Change

INSTALLED CAPACITY IN GW	1 New England	2	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	6.765	10.517	8.139	4.069	2.035	3.391	43.134
Total Capacity	19.151	69.964	89.567	78.502	42.017	77.849	70.469	512.757
1980 LWRU Capacity	7.937	12.857	15.044	19.299	5.600	4.675	8.690	81.806
Total Capacity	22.958	71.604	102.240	96.949	50.726	100.567	78.331	611.689
1985 LWRU Capacity	11.202	17.110	28.130	31.744	13.490	10.696	14.511	138.256
Total Capacity	26.755	73.318	110.245	104.558	74.620	107.701	103.700	746.440
1990 LWRU Capacity	19.725	30.301	50.350	52.347	28.884	10.283	26.101	228.763
Total Capacity	36.270	83.144	125.906	136.041	89.174	139.789	121.111	923.159
1995 LWRU Capacity	26.909	45.912	79.331	86.440	39.559	9.563	36.728	334.322
Total Capacity	47.538	92.083	145.652	171.618	95.325	160.466	139.320	1061.765

TABLE E 1. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 1% CHANGE IN PARAMETERS

	2% Change						
INSTALLED CAPACITY IN GW	1	2	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	6.765	10.517	8.139	4.069	-	3.391	43.134
Total Capacity	19.151	69.964	89.567	78.502	42.017	-	70.469	572.757
1980 LWRU Capacity	7.937	12.857	15.044	19.299	5.600	-	8.690	81.806
Total Capacity	22.958	71.604	102.240	96.949	50.726	-	78.331	611.689
1985 LWRU Capacity	11.202	17.110	28.130	31.625	13.490	-	14.511	138.137
Total Capacity	26.755	73.318	110.244	104.585	74.620	-	103.700	746.476
1990 LWRU Capacity	19.725	30.301	46.114	52.228	15.683	-	13.855	198.963
Total Capacity	36.272	83.143	126.639	136.004	83.509	-	118.622	915.735
1995 LWRU Capacity	26.914	45.924	74.247	86.330	31.461	-	12.796	297.115
Total Capacity	47.643	92.177	144.430	171.935	94.617	-	134.862	1056.955

TABLE E 2. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 2% CHANGE IN PARAMETERS

	3% Change							
INSTALLED CAPACITY IN GW	1	2	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	6.765	10.517	8.139	4.069	-	-	43.134
Total Capacity	19.151	69.964	89.567	78.502	42.017	-	-	512.757
1980 LWRU Capacity	7.937	12.857	15.044	19.299	5.600	-	-	81.806
Total Capacity	22.958	71.604	102.240	96.949	50.726	-	-	611.689
1985 LWRU Capacity	11.202	17.110	28.130	31.625	13.490	-	-	138.137
Total Capacity	26.755	73.318	110.244	104.585	74.620	-	-	476.475
1990 LWRU Capacity	19.725	30.301	33.031	52.728	12.903	-	-	183.101
Total Capacity	36.272	83.143	125.474	136.004	81.080	-	-	912.146
1995 LWRU Capacity	26.914	45.926	66.075	80.807	11.946	-	-	263.907
Total Capacity	47.683	92.271	149.887	174.826	89.325	-	-	1060.595

TABLE E 3. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 3% CHANGE IN PARAMETERS

4%

9%

INSTALLED CAPACITY IN GW	1	2	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	6.765	10.517	8.139	-	-	-	43.134
Total Capacity	19.151	69.964	89.567	78.502	-	-	-	512.757
1980 LWRU Capacity	7.937	12.857	15.044	19.299	-	-	-	81.806
Total Capacity	22.958	71.604	102.240	96.949	-	-	-	611.689
1985 LWRU Capacity	11.202	17.110	28.130	31.625	-	-	-	138.137
Total Capacity	26.755	73.318	110.244	104.584	-	-	-	746.474
1990 LWRU Capacity	18.706	30.301	26.732	48.361	-	-	-	171.915
Total Capacity	35.249	83.143	126.727	135.871	-	-	-	912.273
1995 LWRU Capacity	25.218	42.280	24.600	73.424	-	-	-	209.708
Total Capacity	50.156	90.164	142.654	172.678	-	-	-	1052.128

TABLE E 4. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 4% CHANGE IN PARAMETERS

	5% Change							9%
INSTALLED CAPACITY IN GW	.1	2.	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	6.765	-	8.139	-	-	-	43.134
Total Capacity	19.151	69.964	-	78.502	-	-	-	512.757
1980 LWRU Capacity	7.937	12.857	-	19.299	-	-	-	81.306
Total Capacity	22.958	71.604	-	96.949	-	-	-	611.689
1985 LWRU Capacity	10.802	17.110	-	31.625	-	-	-	137.737
Total Capacity	26.816	73.316	-	104.583	-	-	-	746.527
1990 LWRU Capacity	17.288	16.121	-	39.168	-	-	-	147.123
Total Capacity	34.829	81.132	-	137.858	-	-	-	910.250
1995 LWRU Capacity	23.109	34.626	-	63.169	-	-	-	189.689
Total Capacity	42.306	92.778	-	166.037	-	-	-	1045.643

TABLE E 5. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 5% CHANGE IN PARAMETERS

	6% change						
INSTALLED CAPACITY IN GW	1	2	3	5	6	7	9.	ALL
1975 LWRU Capacity	4.316	6.765	-	8.139	-	-	-	43.134
Total Capacity	19.151	69.964	-	78.502	-	-	-	512.757
1980 LWRU Capacity	7.937	12.857	-	19.299	-	-	-	81.806
Total Capacity	22.958	71.604	-	96.949	-	-	-	611.689
1985 LWRU Capacity	10.866	17.110	-	31.625	-	-	-	137.801
Total Capacity	26.819	73.316	-	104.583	-	-	-	746.529
1990 LWRU Capacity	16.040	16.121	-	30.168	-	-	-	136.876
Total Capacity	34.287	81.130	-	138.831	-	-	-	912.288
1995 LWRU Capacity	21.182	14.706	-	27.838	-	-	-	132.512
Total Capacity	46.379	87.494	-	169.774	-	-	-	1043.657

TABLE E 6. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 6% CHANGE IN PARAMETERS

8%

INSTALLED CAPACITY IN GW	1	2	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	–	–	–	–	–	–	52.268
Total Capacity	19.151	–	–	–	–	–	–	529.179
1980 LWRU Capacity	7.937	–	–	–	–	–	–	81.806
Total Capacity	22.958	–	–	–	–	–	–	611.689
1985 LWRU Capacity	10.802	–	–	–	–	–	–	137.737
Total Capacity	26.816	–	–	–	–	–	–	746.527
1990 LWRU Capacity	10.182	–	–	–	–	–	–	131.018
Total Capacity	34.856	–	–	–	–	–	–	912.853
1995 LWRU Capacity	15.111	–	–	–	–	–	–	126.441
Total Capacity	43.835	–	–	–	–	–	–	1041.393

TABLE E 7. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 8% CHANGE IN PARAMETERS

	9%		9%					9%
INSTALLED CAPACITY IN GW	1	2	3	5	6	7	9	ALL
1975 LWRU Capacity	4.316	-	-	-	-	-	-	43.134
Total Capacity	19.151	-	-	-	-	-	-	512.757
1980 LWRU Capacity	7.937	-	-	-	-	-	-	81.806
Total Capacity	22.958	-	-	-	-	-	-	611.689
1985 LWRU Capacity	10.802	-	-	-	-	-	-	137.737
Total Capacity	26.816	-	-	-	-	-	-	746.527
1990 LWRU Capacity	10.182	-	-	-	-	-	-	139.018
Total Capacity	34.856	-	-	-	-	-	-	912.853
1995 LWRU Capacity	9.296	-	-	-	-	-	-	120.626
Total Capacity	42.027	-	-	-	-	-	-	1039.573

TABLE E 8. INSTALLED NUCLEAR AND TOTAL CAPACITY OF EACH REGION WITH 9% CHANGE IN PARAMETERS.