

FREQUENCY AND INTENSITY
RESOLUTION IN AUDITION

by

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ABSTRACT

Precise measurements of intensity and frequency resolution were made as functions of sound intensity for 1000 hz tone bursts of 500 msec duration with four subjects. The resolution measurements were obtained from two-interval, two-alternative forced-choice and small-range, absolute identification experiments.

The data indicate intensity resolution is roughly constant from 10 to 36 db SPL, and improves linearly with intensity in db SPL above this intensity. Frequency resolution improves continuously, by about a factor of 2, as intensity increases from 10 to 72 db SPL. The details of the intensity resolution - frequency resolution relationship are not predicted by current models of auditory mechanisms.

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I. BACKGROUND

I.1 Introduction:

The human auditory system has only limited intensity and frequency resolution. Auditory psychophysicists have long studied these limitations independently in terms of the smallest intensity difference and the smallest frequency difference an observer can "reliably detect". A wide variety of psychophysical procedures have been employed to measure these so-called just-noticeable — differences, JND_I (the smallest discriminable intensity ratio in decibels) and JND_f (the smallest discriminable frequency increment in hertz); and although the precise value of the JND's depend greatly on the method and criterion used, some simple generalizations are commonly given to describe the results. Intensity resolution is described in terms of Weber's Law, a constant JND_I for intensities sufficiently above absolute threshold. For the case of frequency resolution, excluding high frequencies where there is some controversy in the literature, the same description applies at a particular frequency; the JND_f is constant as a function of intensity. In addition, relations between intensity and frequency resolution have been proposed by various investigators of auditory mechanisms. There exists, however, a considerable lack of data necessary to test the various

hypotheses.

This research is concerned with closely examining the two phenomena of intensity and frequency resolution as functions of intensity and the interrelations between them. Careful consideration is given to the question of how one ought to measure resolution. This point has often received insufficient attention, and as a result much of the available psychophysical data can be questioned with respect to the paradigms and data processing techniques used.

1.2 Intensity resolution:

Psychophysical intensity resolution is commonly described in terms of Weber's Law, a constant JND_I at intensities great enough so that absolute threshold effects can be ignored. The data from several studies tend to support Weber's Law, except for the continued improvement of resolution, a decreasing of the JND_I , as intensity increases (see for example Riesz¹).

According to the preliminary theory of intensity resolution², an observer's sensitivity to a small change ΔI of intensity at overall intensity I is given by:

$$d' = K_I(I) \log_{10} \left(\frac{I + \Delta I}{I} \right) \quad (1)$$

for all small range experiments; where d' is the common sensitivity measure from statistical decision theory,

$K_I(I)$ is a constant independent of ΔI , and $10 \log_{10} \left(\frac{I + \Delta I}{I} \right)$ is the decibel (db) difference between the two sound intensities³. The range R , in db, for the experiment is simply the db difference between the extreme stimuli:

$$R_{db} = 10 \log_{10} (I_{max} / I_{min}) \quad (2)$$

Note that Equation 1 is entirely consistent with Weber's Law when K_I is intensity independent, for then a constant ΔI_{db} implies a constant d' and therefore constant resolution. Aside from this "global" issue of resolution changes with sound intensity, Equation 1 predicts "local" results. In particular, at a fixed nominal intensity d' should grow linearly with $\log_{10} \left(\frac{I + \Delta I}{I} \right)$. Thus if we perform, at one intensity, a set of discrimination tests with various intensity increments and measure sensitivity in each test, we expect the ratio $d' / \Delta I_{db}$ to be independent of ΔI_{db} . The sensitivity function $d'(\Delta I_{db})$ is linear with slope:

$$\delta'_I(I) = K_I(I) / 10 \quad (3)$$

This quantity $\delta'_I(I)$ is the overall intensity resolution measure at that intensity. To the extent that Weber's Law is valid, δ'_I will be independent of I . δ'_I characterizes the entire psychometric function for the particular intensity, except for bias effects. Additionally, note that data from all test increments can be easily incorporated into the estimation of δ'_I . For a common

definition of the JND_I , it is easily shown⁴:

$$\delta'_I(I) \cdot JND_I \approx 1 \quad (4)$$

Let us now turn to the available literature to examine the relation between δ'_I and intensity. Braida's⁵ measurements indicate that above 36 db SPL, for tone bursts of 1000 hz and 500 msec duration, δ'_I increases linearly with the sensation level of the sounds. Thus the discriminability of a 1 db increment at 40 db SL is approximately equal to that of a $\frac{1}{2}$ db increment at 80 db SL. With respect to the usual scale of effects associated with Weber's Law, this improvement in resolution is small. Yet, along with Equation 4 it implies that $1/JND_I$, a measure of resolution, grows linearly with sensation level. Figures 1a and 1b present the classic data of Riesz⁶ in these terms of $1/JND_I$ and sensation level. Seven frequencies are considered, and the results are in good agreement with Braida's suggestion: the data are well fit by straight lines intersecting the 0 db SL points?⁷ McGill and Goldberg⁸ have also recognized that intensity resolution improves as a function of intensity. They conducted intensity discrimination tests at 1000 hz for tone bursts of 150 msec using a one - interval, two - alternative forced - choice paradigm (1I - 2AFC). Figure 2 shows their results in terms of δ'_I ⁹. Although there is large variability in the data,

some generalizations are possible. Above about 30 db SL resolution improves with sensation level (for two of the three subjects) in agreement with Braida's findings. Below this intensity, resolution does not deteriorate but appears to remain about constant (Weber's Law is operative) in complete contradiction to the older results of Riesz. McGill and Goldberg were themselves confused by these results and they note that resolution in the 5 to 15 db SL range exceeded that in the 15 to 25 db SL range. These results are supported by the data of Cambell and Lasky¹⁰ who used 1000 hz tones of 20 msec duration. In Figure 3 we present average results from two groups of six subjects (two subjects common to both) used in this study expressed in terms of δ'_I . The data are in excellent agreement with our previous conclusions except possibly at very high intensities (above 80 db SL).

In addition to the global issue of the dependence of δ'_I on intensity, there is the local resolution issue of the predicted local constancy of δ'_I , or equivalently, the linearity of d' verses ΔI_{db} . The question of what experimental procedure is best for measuring δ'_I is also related to the local resolution issue. Durlach and Braida¹² suggest that in those which there are no memory limitations, sensitivities should be comparable for a wide variety of psychophysical procedures. In particular, results from small - range absolute identification

tests (one - interval, n alternative forced - choice, 1I - nAFC) should be equivalent to those of two - interval, two - alternative forced - choice fixed level discrimination tests (2I - 2AFC). The optimal choice of procedure then hinges on other issues. The identification paradigm for example offers some advantages over the discrimination format. It is a more efficient way to test groups of subjects having differing sensitivities, and it generates empirical operating - characteristics while the 2I - 2AFC paradigm yields only one point on the characteristic for each ΔI_{db} . Yet, the only empirical study testing the predicted equivalence of sensitivities for the two paradigms is that of Pynn¹³. In studying discrimination (2I - 2AFC) and identification (10 stimuli, $\frac{1}{4}$ db between adjacent stimuli) at 70 db SPL for 1000 hz tone bursts Pynn found reasonable agreement between the two paradigms with an average (four subjects) $\delta'_I = 1.3$ in absolute identification and $\delta'_I = 1.4$ in discrimination. The linearity of d' verses ΔI_{db} was also examined, and agreement with the theory was good for both procedures. In his resolution measurements, Braida¹⁴ used the 10 stimuli, $2\frac{1}{4}$ db range identification format and his data additionally support the linear relation. However, he considered only the identification paradigm and at that only down to 36 db SPL. There are no data presently available testing the discrimination - identifi-

cation equivalence or the linearity of d' verses ΔI_{db} below 36 db SPL.

1.3 Frequency resolution:

The subject of pure - tone frequency resolution verses intensity has received relatively little attention in the recent psychophysical literature. From older studies, where intensities were varied, we conclude that at 1000 hz the JND_f decreases rapidly as intensity increases from threshold to 30 db SL, and then the JND_f remains constant, Weber's Law is valid (see for example Shower and Biddulph or Figure 4 ahead, extracted from Siebert¹⁴).

Paralleling the intensity resolution measure δ'_I from the work of Durlach and Braida, we similarly define δ'_f , sensitivity per hertz, as a measure of frequency resolution:

$$d' = K_f(I) \log_{10} \left(\frac{f + \Delta f}{f} \right) \quad (5)$$

$$\text{for } \frac{\Delta f}{f} \ll 1; \quad d' \approx \frac{K_f(I)}{\log_e 10} \cdot \frac{\Delta f}{f} \quad (6)$$

$$\delta'_f(I) \triangleq \frac{d'(f, \Delta f, I_{db})}{f} \quad (7)$$

where Δf and f are in hertz¹⁵. The definition of δ'_f in Equation 7 implies a relation analogous to Equation 4¹⁶:

$$\delta'_f(I) \cdot JND_f \approx 1 \quad (8)$$

The arguments favoring δ'_I over the JND_I as a resolution

measure also apply here for the frequency case.

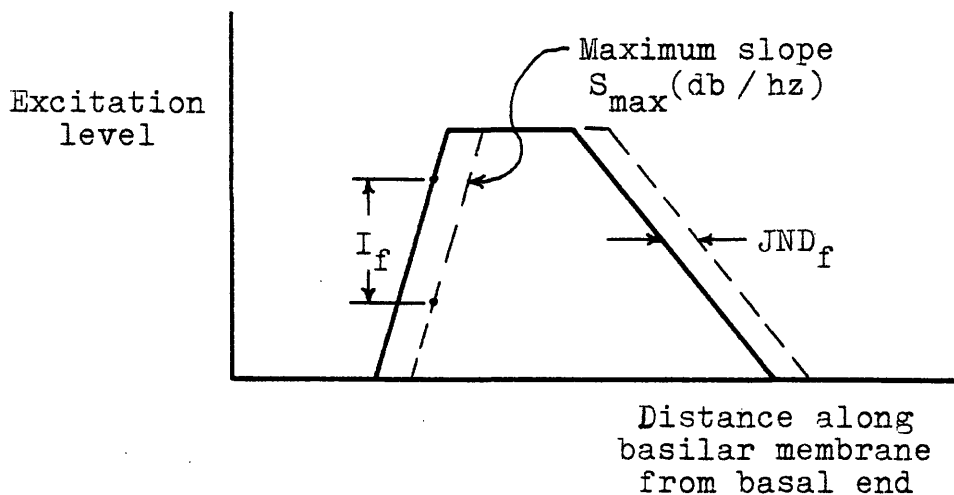
As indicated earlier there is little data in the recent psychophysical literature measuring the dependence of frequency resolution, δ'_f , on sound intensity. At a fixed intensity, d' is predicted to be linear in Δf , but no available data test this hypothesis. Furthermore, we cannot determine whether small-range absolute identification tests yield sensitivities equivalent to those from the 2I-2AFC format as no small-range identification experiments have been done in frequency.

I.4 Relations between frequency and intensity resolution:

In 1942 Makita¹⁷ postulated that frequency discrimination is accomplished by detecting a change in the amplitude of the excitation pattern at some place along the basilar membrane. In 1948 Gold and Pumphrey¹⁸ made a similar suggestion.

The literature contains other instances where relations between frequency and intensity resolution have been hypothesized. Usually, (stochastic) central processes are said to determine intensity resolution, while the peripheral (deterministic) filtering properties of the basilar membrane and the associated excitation patterns limit frequency resolution. The amplitude JND thus sets the amount by which the exci-

tation pattern must change in a frequency resolution experiment for discrimination to be possible. Such models therefore predict that the JND_f equals, or is greater than, the JND_I divided by the steepest slope, S_{max} , of the excitation pattern or peripheral filter:



$$S_{max} \cdot JND_f = \Delta I_f \geq JND_I \quad (9)$$

where ΔI_f , as shown, represents the maximum amplitude shift due to the just-noticeable change in frequency¹⁹.

A common procedure used to obtain estimates of the maximum filter slope involves the measurement of masking of tones by narrow bands of noise. In 1950, Schaefer, Gales, Shewmaker, and Thompson²⁰ performed such measurements and modeled the masking functions they obtained with simple resonant-circuit filter characteristics. Following Gold and Pumphrey's suggestion they tested Equation 9 by using JND_f data from Shower and Biddulph²¹.

The calculated intensity shifts are in general agreement with the pertinent JND_I data from Riesz²²; see Table 1:

Table 1:

Frequency	Sensation level	JND_f	I_f Computed		JND_I
			Low freq. slope	High freq. slope	
200 hz	50 db	2.0 hz	0.5 db	1.0 db	0.7 db
800	60	2.7	0.6	0.5	0.4
3200	60	6.4	0.8	0.4	0.3

For 1000 hz and 60 db SL we estimate $S_{max} = 0.18$ db/hz (using the 800 hz data with the JND_f scaled up by 1.25). Stahl²³ has also measured such "psychophysical tuning curves" with greater control over the spectral components of the narrow-band noise masker. Using data obtained at 2000 hz, we estimate $S_{max} = 0.18$ db/hz at 1000 hz and 30 db SL. (The agreement with the estimate from the Schaefer et al study is probably fortuitous.) Ritsma, Domburg, and Donders²⁴ have measured just - noticeable shifts in: (1) the edges of high and low pass filtered white noise, and (2) the center frequency of bandpass filtered white noise and bandpass filtered periodic pulse trains. The tests were performed at 2000 hz and 30 db SL. The cut-off slope of the external electronic filter was the experimental parameter. As this slope was increased the difference limens in all four test situations decreased until the slope exceeded

35 db per critical band. They conclude the internal peripheral filter has a maximum slope of 35 db per critical band and translating back to 1000 hz yields $S_{\max} = 0.18$ db/hz at 30 db SL in agreement with Stahl. However, a word of caution is in order as Sachs's²⁵ measurements of two-tone inhibition in auditory-nerve fibers suggest that narrow-band noise masking experiments may reflect inhibitory effects as well as excitatory filtering.

In 1968 Siebert²⁶ formulated a mathematical model describing auditory - nerve activity in response to tone stimuli. The model characterizes transformations of stimuli in the peripheral auditory system. Two important features incorporated are the inherent stochastic nature of the neural response and the nonlinear effect of auditory - nerve fiber rate saturation.

Siebert is concerned with the limitations imposed on discrimination performance by the intrinsic randomness and saturation of the neural response. He concludes that if the auditory - nerve response is processed optimally, imperfections in peripheral encoding account for the psychophysically observed limitations on frequency and intensity resolution. The model directly predicts the Weber fractions for intensity and frequency as functions of intensity above threshold:

$$\frac{\Delta A'}{A} = \frac{A_0}{A} + \frac{1}{C} \quad (10a)$$

$$\frac{\Delta f'}{f} = \frac{1}{N} \left[\frac{A_0}{A} + \frac{1}{C} \right] \quad (10b)$$

where:

- A_0 = Threshold amplitude for a sinewave of frequency f .
- A = Amplitude of sinewave of frequency f .
- $\Delta A'$ = Amplitude shift needed for constant detectability.
- f = Frequency of sinewave.
- $\Delta f'$ = Frequency shift needed for constant detectability.
- C = Constant dependent on signal duration and number of auditory - nerve fibers.
- N = Constant on the order of 14 (independent of A_0).

Siebert stresses that the crux of these results lies not in the exact form of the equations²⁷, but in the parallel nature of the two. Thus plots of $\Delta A'/A$ and $\Delta f'/f$ verses A/A_0 should be parallel and separated by a factor N , on the order of 14. Figure 4 presents some measured JND's for frequency and intensity testing this prediction. The dashed lines indicate the theoretical functions, Equations 10a and 10b. Generally the data support the theory except that the proportionality constant, N , is somewhat larger than expected.

As opposed to examining $\Delta A'/A$ and $\Delta f'/f$ we prefer to use the resolution measures δ'_I and δ'_f . The frequency case follows immediately as $\Delta f' \triangleq \text{JND}_f \approx (\delta'_f)^{-1}$ by Equation 8. For amplitude shifts we have:

$$\text{JND}_I \triangleq 20 \log_{10} \left(1 + \frac{\Delta A'}{A} \right) \quad (11)$$

and in general there is no reduction we can make. How-

ever, excluding near threshold intensities, the JND_I is small enough to permit a linear approximation to the logarithm:

$$\log_{10}\left(1 + \frac{\Delta A'}{A}\right) = \frac{1}{\log_e 10} \frac{\Delta A'}{A} \quad (12)$$

with not too large an error (e.g. for $JND_I = 1$ db, % error = 6%). Combining Equations 4, 10, 11, and 12 gives:

$$\frac{\Delta A'}{A} \approx \frac{2.3}{20} \delta'_I(I) \quad (13)$$

$$\frac{\Delta f'/f}{\Delta A'/A} = \frac{1}{N} \approx \frac{20}{2.3 f} \cdot \frac{\delta'_I(I)}{\delta'_f(I)} \quad (14)$$

Thus, Siebert's prediction of proportional Weber fractions for frequency and intensity transforms, at a particular frequency, to:

$$\frac{\delta'_I(I)}{\delta'_f(I)} \approx D = \frac{2.3 f}{20 N} \quad (15)$$

where D is constant, independent of intensity.

II. RESEARCH PROGRAM

II.1 General description:

Measurements of frequency and intensity resolution as functions of intensity for 1000 hz tone bursts have been performed on the same subjects using the same equipment and psychophysical methods. The data allow examination of: $\delta'_I(I)$, $\delta'_f(I)$, the local linearity of d' with stimulus increment (in db or hz), and relations between $\delta'_I(I)$ and $\delta'_f(I)$. The absolute thresholds at 1000 hz were also measured.

Both discrimination tests (2I-2AFC) and small-range identification tests (1I-9AFC) were performed in frequency and intensity to test the predicted equivalence of sensitivities for the two paradigms. In addition, learning controls were employed to permit an evaluation of the relative paradigm efficiencies.

All experiments were conducted monaurally. Five intensities were studied: 10, 18, 36, 54, and 72 db SPL. Visual feedback was given on each trial in all tests. Three students, ages 17 to 21, plus the author, age 21, all with normal hearing, served as subjects.

II.2 2I-2AFC Experiments:

Three intensity increments and three frequency increments were used at each intensity, with at least

six runs of seventy-five trials being taken for each increment. Thus, a minimum of $3 \times 6 \times 75$ or 1350 trials were performed at each intensity for frequency and intensity discrimination. The increments at a particular sound pressure level were selected to span the psychometric function from approximately 60 to 90% correct responses. In all intensity discrimination tests the two stimuli were symmetrically spaced, in db, about the nominal level; i.e. $S_1 = (I_{db} + \frac{\Delta I_{db}}{2}, 1000 \text{ Hz})$, and $S_2 = (I_{db} - \frac{\Delta I_{db}}{2}, 1000 \text{ Hz})$. For the frequency discrimination tests, $S_1 = (I_{db}, 1000 \text{ Hz})$ and $S_2 = (I_{db}, 1000 + \Delta f \text{ Hz})$. On each trial the subject was presented both stimuli in temporal order, either S_1, S_2 or S_2, S_1 with equal a priori probabilities for each ordering. His task was simply to judge the stimulus ordering on each trial. Preceding each discrimination test the two stimuli were presented alternately, with marking lights, to allow the subjects to familiarize themselves with the stimuli.

The two tone bursts on each trial were 500 msec long and the interstimulus time was 250 msec. A 25 msec onset and offset time was employed to prevent possible "click" transient cues. The answer period was 1½ seconds, after which a lamp on each subject's response box indicated the correct answer for that trial. The next trial started ½ second later; thus the total time for each trial was ¾ seconds. This brisk cycling was acceptable to the

subjects and no stimulus marker lights were needed.

Each subject's absolute detection performance was determined using the 2I-2AFC paradigm. The two stimuli were a tone burst of intensity I_{db} , and silence; i.e. $S_1 = (I_{db}, 1000 \text{ Hz})$, and $S_2 = 0$. Warning lights marked the stimulus intervals necessitating an increase in the inter-stimulus time to 500 msec. The other timing parameters and feedback were as in the discrimination tests. For each subject detection tests were performed at three stimulus intensities (chosen to span the psychometric function from approximately 60 to 90% correct responses), and six seventy-five trial runs were performed for each intensity. Preceding the threshold tests, signal bursts were presented at a moderate intensity and then were gradually attenuated to the test level. The subjects were run individually for these tests.

II.3 1I-9AFC Experiments:

We conducted ten small-range, absolute identification experiments. In five of the experiments the stimuli were identical except for intensity; in the remaining five the stimuli differed only in frequency. The experiments in each group of five differed only with respect to overall intensity. A minimum of eight runs, seventy-five trials each, were performed in each experiment; and the following shows, as an example, the stimuli used at 72

db SPL:

<u>Intensity Experiment</u>	<u>Frequency Experiment</u>
$\Delta I = \frac{1}{4}$ db Range = 2 db	$\Delta f = 1$ hz Range = 8 hz
S ₁ = 73 db, 1000 hz	S ₁ = 72 db, 1008 hz
S ₂ = 72 $\frac{3}{4}$ db, 1000 hz	S ₂ = 72 db, 1007 hz
S ₃ = 72 $\frac{1}{2}$ db, 1000 hz	S ₃ = 72 db, 1006 hz
S ₄ = 72 $\frac{1}{4}$ db, 1000 hz	S ₄ = 72 db, 1005 hz
S ₅ = 72 db, 1000 hz	S ₅ = 72 db, 1004 hz
S ₆ = 71 $\frac{3}{4}$ db, 1000 hz	S ₆ = 72 db, 1003 hz
S ₇ = 71 $\frac{1}{2}$ db, 1000 hz	S ₇ = 72 db, 1002 hz
S ₈ = 71 $\frac{1}{4}$ db, 1000 hz	S ₈ = 72 db, 1001 hz
S ₉ = 71 db, 1000 hz	S ₉ = 72 db, 1000 hz

Preceding each run the stimuli were presented once, sequentially, in synchrony with the digital display used for feedback during these tests.

The single tone burst on each trial was, as in the 2I-2AFC tests, 500 msec in duration with a 25 msec rise and fall time on the electronic switching gate. The answer period was about 4 seconds; and for the next 2 seconds the correct response was given on the digital display. The total trial time was approximately 8 seconds.

II.4 Temporal sequence of experiments:

The tests on intensity resolution were completed before any frequency resolution measurements were made. On each day the nominal intensity was kept constant throughout the test session. There were four or five two hour test sessions per week. From day to day the nominal intensity used was dictated from a pre-determined

pseudo-random scheme.

The first hour of each experimental session consisted of four or five identification tests. In the second hour, nine discrimination tests were performed, three for each stimulus increment. (This schedule included 20 to 30 minutes of rest time dispersed throughout the session.) The increments were randomized from test to test to minimize short-term training effects, and to maintain the interest of the subjects. In order to obtain eight identification runs and eighteen discrimination runs for each intensity, two sessions were required.

For each subject, thresholds were measured twice, once early in the experimental program and once at the end. On each day preliminary tests were performed to determine the three intensities to be used and to allow adaptation to low intensity stimuli. Following this, nine absolute detection tests were performed, three for each intensity, with the intensities randomized from test to test.

II.5 Training:

Prior to data collection, two weeks were allocated to training. In this period discrimination and identification tests were performed at all intensities with equal time devoted to each of the two paradigms. This training concentrated largely on intensity resolution tests. The

training period terminated when all subjects appeared to have reached their asymptotic level of performance in the discrimination tests. Upon completion of the intensity resolution experiments a week was allocated to training for the frequency resolution tests.

II.6 Data processing:

For each test a paper tape was punched encoding the stimulus presented on each trial and the response of each subject. These tapes were later processed on the Communications Biophysics Laboratory PDP-4 computer. The analysis program used allowed for cummulation of confusion matrices and computed the overall sensitivity from all tests of a particular type.

For the discrimination tests the resolution measures δ'_x and δ'_f were computed by averaging the estimates of these quantities obtained from each of the three stimulus increments used at each intensity.

From the absolute identification tests the sensitivities between all adjacent stimuli were computed using the appropriate 2 by 9 stimulus-response matrix for each stimulus pair. Using the additivity property of sensitivity²⁸, the total sensitivity between the extreme stimuli, $d'_T(S_1 \text{ to } S_9)$, was then obtained by summing the sensitivities between adjacent stimuli. Dividing the range, R , in db or hz, into the total sensitivity yields

the appropriate resolution measure, δ'_I or δ'_f :

$$\delta' = \frac{d'_T(S, \text{ to } S_q)}{R}$$

For the threshold measurements we have simply indicated the sensitivities in the I - silence detection tests for each of the three intensities used.

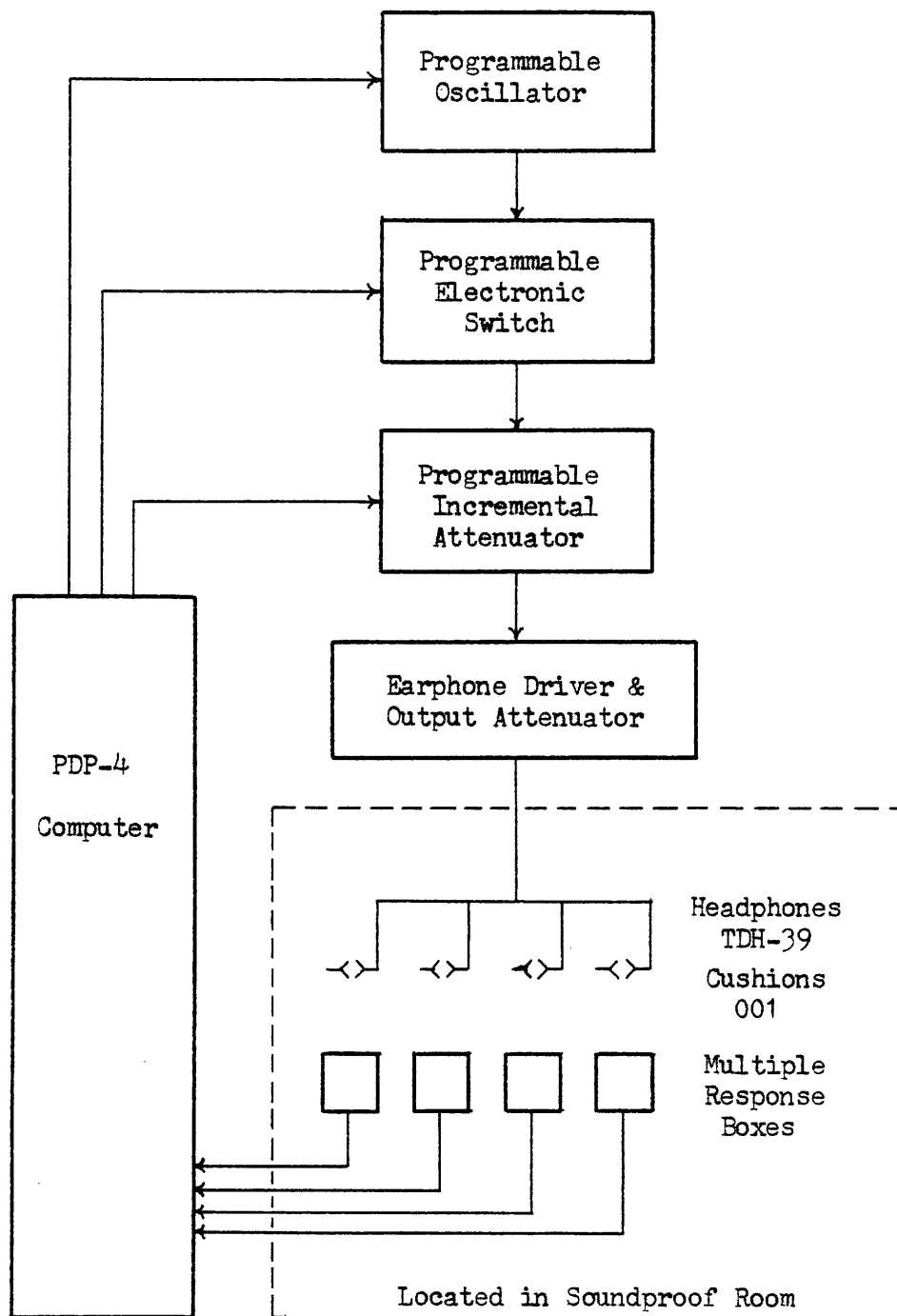
Lastly, all sensitivities are given relative to one - interval paradigm experiments. Specifically, sensitivities from two - interval discrimination and threshold tests have been divided by $\sqrt{2}$ and only the resulting one - interval equivalent values are presented.

II.7 Equipment:

All experiments were controlled by the Communications Biophysics Laboratory PDP-4 computer. The necessary programs for control and data analysis were available from the CBL program library.

The equipment used is shown schematically on the following page:

Equipment schematic:



III. RESULTS

III.1 Intensity resolution:

The sensitivities from all intensity resolution tests, discrimination - $d'(I_{db}, \Delta I_{db}, 1000 \text{ Hz})$ and identification - $d'_r(I_{db}, R_{db}, 1000 \text{ Hz})$, are given in Table 2.

As indicated earlier the resolution measures δ'_r and δ'_f are meaningful only if sensitivity grows linearly with the stimulus increment. Thus, we first consider the predicted local linearity of d' with ΔI_{db} , using the data obtained in discrimination tests. In order to include data from all subjects and all intensities, we have normalized the discrimination data from Table 2 with respect to the sensitivity for a 1 db increment at each intensity, for each subject. For example, at 72 db SPL the increments were $\frac{1}{4}$, $\frac{1}{2}$, 1 db and we divide each subject's sensitivities by his $d'(72 \text{ db}, 1 \text{ db}, 1000 \text{ Hz})$. If δ'_r is locally constant, then our normalized data should yield $\frac{1}{4}$, $\frac{1}{2}$, and 1 respectively. Table 3 shows the predicted and experimental results of this test while Figure 5 displays the results in the form of a scatter diagram. It is clear that there is relatively little variability in the data; and, there is no consistent deviation from linearity. Thus, in agreement with the findings of Pynn and Braida²⁹, our data strongly support:

$$d' = \delta'_r \cdot \Delta I_{db} \quad (16)$$

where δ'_x is locally constant³⁰.

Having established this result, the δ'_x estimates from each intensity increment are tabulated in Table 4. The overall $\delta'_x(I)$ measures and the resolution at each intensity averaged over subjects, $\bar{\delta}'_x(I)$, are also included in Table 4 and are plotted in the form of intensity resolution functions in Figure 6³¹. (Note that only the discrimination data has been used.)

III.2 Frequency resolution:

The frequency resolution results are characterized by greater intrasubject variability in sensitivity, and greater variability in the individual test results with the same stimuli. Thus, some discrimination tests were conducted individually, and some tests were repeated as well. At first performance appeared to be improving with time, but this hypothesis was later discarded. Consequently all data obtained has been used in computing our results³².

Table 5 lists the sensitivities from the frequency resolution tests: discrimination - $d'(1000 \text{ Hz}, \Delta f, I_{db})$ and identification - $d'_r(1000 \text{ Hz}, R \text{ Hz}, I_{db})$. (Subject MM withdrew from the experimental program early and only discrimination tests were performed.)

First, we consider the predicted local linearity of d' with Δf , using the data obtained in the discrimi-

nation tests. Two normalizations are presented: (1) with respect to the sensitivity obtained with a 2 hz increment at each intensity for each subject, Table 6 and Figure 7, and (2) with respect to the sensitivity from the maximum frequency increment at each intensity for each subject, Table 7 and Figure 8. Both presentations are characterized by greater variability than for the intensity discrimination data, but d' appears to grow linearly with Δf . There seem to be no consistent deviations with subject or intensity.

We therefore compute the resolution at each intensity by averaging the δ'_f estimates from each increment. These δ'_f estimates, the resulting $\delta'_f(I)$ resolution measures, and the average resolution at each intensity, $\overline{\delta'_f}(I)$, are all presented in Table 8. The individual and average resolution measures are plotted as frequency resolution functions in Figure 9³³. (Note that as in the intensity case, only the data obtained in 2I-2AFC tests has been used.)

III.3 Threshold measurements:

The experimental plan included three absolute detection tests at each of three intensities on each of two days, one early in the experimental program and one at the end. For two subjects, SR and WMR, the test results obtained on the two days were consistent; but the remaining two subjects, MR and MM, exhibited absolute

threshold improvement of about 5 db in the test performed at the end of the experimental program. A third set of tests were performed on MR with results that were in agreement with the second set of tests. For MM this was not possible; however, we believe the improved data are indicative of the thresholds for both MR and MM. By the second testing the subjects were more familiar with the detection paradigm and generally more experienced. Consequently, the data from the initial tests for MR and MM are not included in our results.

The absolute threshold data are presented in Table 9. The data are graphed in Figure 10 as absolute detection functions. Table 9 also includes an estimate of ξ'_I for each subject obtained by linearizing those data. The reported "thresholds" are the integer intensities which correspond to unit absolute sensitivity. The average threshold was -2.2 db SPL and the standard deviation is 2.7 db.

IV. DISCUSSION

IV.1 Absolute identification:

Our results suggest that sensitivities from small-range, absolute identification tests and discrimination tests are unequal. Table 10 shows a comparison of our discrimination and identification results in terms of δ'_I for intensity resolution tests and δ'_f for frequency resolution measurements. In all cases subjects exhibited poorer sensitivity (lower δ') in the identification paradigm than in 2I-2AFC in contradiction to the prediction of Durlach and Braida³⁴.

As indicated earlier (see I.2) Pynn³⁵ has studied intensity discrimination (2I-2AFC) and identification (1I-10 AFC, R = 2¼ db) at 70 db SPL and found $\bar{\delta}'_I = 1.42$ in discrimination and $\bar{\delta}'_I = 1.29$ in absolute identification. For comparison consider our data at 72 db SPL; $\bar{\delta}'_I = 1.39$ in absolute identification, but $\bar{\delta}'_I = 2.19$ in discrimination. While the two studies have similar identification results, our subjects were significantly better in the 2I-2AFC task.

In our experimental program equal time was allotted for training in the 2I-2AFC and 1I-9AFC paradigms. Training terminated when stable performance was achieved in the 2I-2AFC paradigm, and this may have allowed insufficient practice with identification. Our

subjects seemed to be less motivated in the identification tests. Slight drifts in the overall level of the acoustic signal reaching the eardrum resulting from earphone movement during the tests would degrade small-range, one - interval sensitivity, but not two - interval tests and may account for part of the observed identification - discrimination discrepancy³⁶.

We believe absolute identification sensitivities are thus not indicative of the fundamental resolution properties of the auditory system. Consequently we have included only the discrimination measurements for our intensity and frequency resolution functions, Figures 6 and 9, and the absolute identification results will not be considered further in this report.

IV.2 Intensity resolution:

As the question of the local constancy of δ'_I has already been dealt with (see III.1), we consider here only the global issue, the form of $\delta'_I(I)$. Our intensity resolution functions of Figure 6 agree with the previous results discussed in I.2. Specifically, $\delta'_I(I)$ is constant from 10 to 36 db SPL and grows linearly with intensity in db SPL above 36 db SPL. The linearly improving resolution in the high intensity range is consistent with Braida's³⁷ findings. The constancy of δ'_I at low intensities implies that Weber's Law is valid here and

not at high intensities as is usually stated. One set of data obtained from WMR indicate constant resolution, $\delta'_I \approx 1$, extending down to approximately 4 db SL.

Figure 11 presents our average intensity resolution function, $\bar{\delta}'_I(I)$, together with the findings of McGill and Goldberg³⁸ (see I.2 and Figure 2 for details of their tests). The McGill and Goldberg resolution function was computed by averaging data from the three subjects they report and scaling up the results by a factor of two. The agreement with our results is good. The poorer sensitivities for their subjects may result from the 8 second intertrial time, one - interval paradigm used in their tests.

In Figure 12 we compare our results to those of Cambell and Lasky³⁹ (see I.2 and Figure 3 for details of their tests). Their data have been scaled up by a factor of five. Again there is excellent agreement between the two studies including the slight peak in resolution at about 20 db SPL. All four of our subjects demonstrated better resolution at 18 db SPL than at 36 db SPL, and the average improvement was 20%.

IV.3 Frequency resolution:

Siebert⁴⁰ has compiled a selection of older psychophysical results on frequency resolution where intensities were varied (see Figure 4). Our results generally agree

with those data. Frequency resolution (see Figure 9) improves with increasing intensity up to about 54 db SPL. Correspondingly, the JND_f 's (see Figure A.2) first decrease and then approach about 2 hz (four subject average) at high intensities. The rate of improvement in resolution with intensity is smaller than that reported previously. The data in Figure 4 indicate that resolution improves rapidly as intensity increases up to 30 db SL and then remains constant.

IV.4 Relations between frequency and intensity resolution:

A primary objective of this research was to explore the relationship between frequency and intensity resolution as a function of intensity. The main prediction of interest is Siebert's "proportional resolution hypothesis:

$$\frac{\Delta f'/f}{\Delta A'/A} = \frac{1}{N} \quad (14)$$

which we have transformed to:

$$\frac{\delta'_I(I)}{\delta'_f(I)} \simeq D = \frac{2.3 f}{20 N} \quad (15)$$

The linear approximation for the $\log_e(1 + \Delta A'/A)$ used to obtain Equation 15 (see I.4) is accurate with our data since the JND_I 's are small. (The largest error is 7% for MR at 36 db SPL where $\delta'_I = 0.86$ and $JND_I = 1.26$ db.)

As a first test of Equation 15 we present our

average intensity and frequency resolution functions in Figure 13. The two are neither strongly similar nor strongly different. Intensity resolution, and to a lesser extent frequency resolution, improves essentially linearly with intensity in db SPL above 36 db SPL. On the other hand, intensity resolution is constant from 10 to 36 db SPL, but frequency resolution decreases at small intensities.

Figure 14 presents $\delta'_I(I)/\delta'_f(I)$ for each subject individually. For each subject and the average of all four the ratio clearly depends on intensity, contradicting Siebert's prediction. Subject MM is closest to the predicted constancy of $\delta'_I(I)/\delta'_f(I)$, but his deviations, while smaller, are similar to those of the remaining subjects. The greatest intrasubject variability in the ratio occurs at 36 db SPL. This results from large intrasubject frequency resolution differences. For the sake of completeness, we compute the average N (averaged over subjects and intensities) to be 37 which is consistent with Siebert's observations, $N \approx 40$ in Figure 4.

Our data can also be used in conjunction with the various peripheral filter models and Equation 9:

$$S_{\max} \cdot \text{JND}_f = \Delta I_f \geq \text{JND}_I \quad (9)$$

From other studies (see I.4) estimates of the maximum filter slope, S_{\max} , are available at 30 and 60 db SL.

Our average JND's at these intensities are obtained from Figures A.1 and A.2. Table 11 shows the relevant computations testing Equation 9 and we observe the final two

Table 11:

Sensation Level	JND_f	S_{max}	ΔI_f	JND_I
30 db	3.0 hz	0.18 db/hz	0.54 db	0.94 db
60	1.9	0.18	0.34	0.57

columns are of the same range, yet the JND_I 's are larger than the corresponding amplitude shifts produced. This discrepancy is not surprising, especially in light of the questions raised as a result of Sachs's⁴² inhibition measurements.

IV.5 Absolute thresholds:

The auditory threshold is now generally regarded as a statistical quantity. There is no single intensity I_0 such that for $I < I_0$ sounds are inaudible while for $I > I_0$ sounds are always heard. To the extent that detectability increases rapidly from zero over a small intensity range, a phenomenon like the classical threshold may be said to exist. If, however, the detectability transition were very gradual, the concept of a threshold would be far less meaningful.

Our absolute detection functions (see Figure 10)

indicate that the transition proceeds reasonably rapidly. An intensity range of 4 db accounted for a change of about 2 in sensitivity (in three of the four subjects). In other words, a 4 db intensity change increased two-interval detection performance from 64% correct to 96% correct.

From the preliminary theory of intensity resolution of Durlach and Braida⁴³ a simple relationship is predicted between the absolute detection function, $d'(I,0)$, and the intensity resolution function:

$$\frac{d}{dI} [d'(I,0)] = \delta'_I(I) \quad (17)$$

That is, the slope of the absolute detection function is the resolution at that intensity. Empirically this relationship can be tested only near threshold for $d'(I,0)$ rapidly grows too large for practical measurement.

Below the lowest intensity used for threshold measurement, resolution is surely quite small. From our intensity resolution findings it appears that δ'_I stays at 1 down to about 4 db SL. Thus, there is a region of about 6 db over which $\delta'_I(I)$ changes from near 0 to 1. From Equation 17 it is clear the detection function must accelerate rapidly in this resolution transition region; but its slope, $\delta'_I(I)$, should not exceed 1 as this would imply better resolution than at

higher intensities. The slopes of our detection functions are all less than 1; specifically, $\delta'_I \approx \frac{1}{2}$ for three subjects and $\frac{1}{4}$ for MR (see Table 9).

This analysis suggests a way to define the threshold. Using a detectability criterion, a reasonable but arbitrary definition of threshold is that intensity corresponding to unit absolute sensitivity (or equivalently, about 75% correct in a 2I-2AFC detection test). Viewing the threshold in terms of changing resolution may be more meaningful. Resolution stays constant with $\delta'_I = 1$ until it is suddenly reduced to near zero within about a 6 db range of intensity. The threshold is simply a single intensity label for this transition region.

V. CONCLUSIONS

Our principal results are:

(1) At a nominal intensity I :

$$d'(I_{db}, \Delta I_{db}, f) = \delta'_x(I) \cdot \Delta I_{db} \quad (16)$$

$$d'(f, \Delta f, I_{db}) = \delta'_f(I) \cdot \Delta f \quad (18)$$

Except for bias effects, this linearity of sensitivity with stimulus increment enables complete characterization of the psychometric function by the single parameter $\delta'_x(I)$ or $\delta'_f(I)$, a measure of the observer's resolution at that intensity.

(2) Sensitivities in small-range absolute identification tests, in intensity and frequency, were always less than those in corresponding discrimination tests.

(3) Intensity resolution results were more uniform than those from frequency resolution tests.

(4) Intensity resolution is constant and thus, Weber's Law is valid, from near threshold to about 36 db SPL. For higher intensities resolution improves essentially linearly with intensity so that $\delta'_x(72 \text{ db SPL}) \approx 2 \delta'_x(36 \text{ db SPL})$.

(5) Frequency resolution changes smoothly by about a factor of 2 from 10 to 72 db SPL, though resolution appears to improve only slightly above 54 db SPL. Classical data show the changes in resolution to be essentially completed by the 30 db SL level, and the deterioration in resolution below this intensity is sharper than we

observed.

(6) The detailed dependence of δ'_x/δ'_f on intensity is more complicated than current models suggest, including Siebert's stochastic auditory - nerve model.

TABLES

(Tables 1 and 11 in text.)

Table 2:

Intensity resolution sensitivities:

discrimination - $d'(I_{db}, \Delta I_{db}, 1000 \text{ Hz})$ identification - $d'_T(I_{db}, R_{db}, 1000 \text{ Hz})$

I db SPL	ΔI_{db}	# trials	Subject			
			WMR	MR	SR	MM
72	1	450	1.90	2.14	2.33	1.93
	$\frac{1}{2}$	450	0.90	1.02	1.84	1.00
	$\frac{1}{4}$	450	0.45	0.45	0.66	0.56
	R = 2	675	3.29	2.23	3.09	2.44
54	1	450	1.25	1.37	2.28	2.20
	$\frac{3}{4}$	450	0.92	1.01	1.57	1.41
	$\frac{1}{2}$	450	0.59	0.61	0.91	1.03
	R = 2	600	1.26	1.56	1.82	1.73
36	2	450	1.81	1.66	2.26	2.28
	1	450	0.85	0.81	1.14	1.07
	$\frac{1}{2}$	450	0.49	0.47	0.61	0.60
	R = 4	600	1.67	1.88	2.72	2.82
18	$1\frac{1}{2}$	450	1.54	1.64	1.78	1.70
	1	450	1.07	1.08	1.82	1.28
	$\frac{1}{2}$	450	0.55	0.55	0.54	0.71
	R = 4	600	3.10	2.62	2.43	2.92
10	$1\frac{1}{2}$	450	1.47	1.27	1.97	1.56
	1	450	0.96	1.00	1.38	0.87
	$\frac{1}{2}$	450	0.41	0.51	0.79	0.27
	R = 4	675	2.33	2.59	3.35	2.71

Table 3:

Linearity test; d' verses ΔI_{db} .

Entries are $\frac{d'(I_{db}, \Delta I_{db}, 1000 \text{ hz})}{d'(I_{db}, 1 \text{ db}, 1000 \text{ hz})}$:

I db SPL	ΔI_{db}	Subject				Average	Theory
		WMR	MR	SR	MM		
72	1	1.00	1.00	1.00	1.00	1.00	1.00
	$\frac{1}{2}$	0.47	0.48	0.79	0.52	0.57	0.50
	$\frac{1}{4}$	0.24	0.21	0.28	0.29	0.26	0.25
54	1	1.00	1.00	1.00	1.00	1.00	1.00
	$\frac{3}{4}$	0.74	0.74	0.69	0.64	0.70	0.75
	$\frac{1}{2}$	0.47	0.45	0.40	0.47	0.45	0.50
36	2	2.13	2.05	1.98	2.13	2.07	2.00
	1	1.00	1.00	1.00	1.00	1.00	1.00
	$\frac{1}{2}$	0.58	0.58	0.54	0.56	0.57	0.50
18	$\frac{1}{2}$	1.44	1.52	0.98	1.33	1.32	1.50
	1	1.00	1.00	1.00	1.00	1.00	1.00
	$\frac{1}{2}$	0.51	0.51	0.30	0.55	0.47	0.50
10	$\frac{1}{2}$	1.53	1.27	1.43	1.79	1.51	1.50
	1	1.00	1.00	1.00	1.00	1.00	1.00
	$\frac{1}{2}$	0.43	0.51	0.57	0.31	0.46	0.50

Table 4:

Intensity resolution; $\delta'_I(I)$ estimates, $\delta'_I(I)$, and $\bar{\delta}'_I(I)$:

I db SPL	ΔI_{db}	Subject				Average
		WMR	MR	SR	MM	
72	1	1.90	2.14	2.33	1.93	
	$\frac{1}{2}$	1.80	2.04	3.68	2.00	
	$\frac{1}{4}$	1.80	1.80	2.64	2.24	
	$\delta'_I(72)$	1.83	1.99	2.88	2.06	2.19
54	1	1.25	1.37	2.28	2.20	
	$\frac{3}{4}$	1.23	1.34	2.09	1.88	
	$\frac{1}{2}$	1.18	1.22	1.82	2.06	
	$\delta'_I(54)$	1.22	1.31	2.07	2.05	1.66
36	2	0.91	0.83	1.13	1.14	
	1	0.85	0.81	1.14	1.07	
	$\frac{1}{2}$	0.98	0.94	1.32	1.20	
	$\delta'_I(36)$	0.92	0.86	1.20	1.14	1.03
18	$1\frac{1}{2}$	1.03	1.09	1.19	1.14	
	1	1.07	1.08	1.82	1.28	
	$\frac{1}{2}$	1.10	1.10	1.08	1.42	
	$\delta'_I(18)$	1.07	1.09	1.36	1.28	1.20
10	$1\frac{1}{2}$	0.98	0.84	1.31	1.04	
	1	0.96	1.00	1.38	0.87	
	$\frac{1}{2}$	0.82	1.02	1.58	0.54	
	$\delta'_I(10)$	0.92	0.95	1.42	0.82	1.03

Table 5:

Frequency resolution sensitivities:

discrimination - d' (1000 hz, Δf , I_{db})
 identification - d'_T (1000 hz, R_{hz} , I_{db})

I db SPL	Δf	Subject							
		WMR		MR		SR		MM	
		#	d'	#	d'	#	d'	#	d'
72	4							300	2.14
	3	1125	1.48	450	1.67			450	1.30
	2	1425	0.79	600	1.08	675	1.74	450	0.91
	1	1200	0.45	600	0.48	675	0.87	525	0.39
	$\frac{1}{2}$					525	0.55		
	R = 8	750	2.24	675	1.98	750	2.26		
54	3	750	1.27	450	1.59	375	2.34	450	1.52
	2	750	0.81	450	1.09	600	1.70	450	0.87
	$1\frac{1}{2}$					450	1.38		
	1	675	0.38	450	0.57	675	0.66	450	0.47
	R = 8	600	2.48	600	1.94	600	1.91		
36	4	525	1.35	450	1.78			525	1.39
	3	525	0.80	525	0.85	600	2.14	450	0.85
	2	450	0.54	525	0.67	750	1.25	450	0.53
	1					600	0.66		
	R = 12	675	3.04	675	2.01	600	2.64		
18	4	975	1.14	675	1.25	900	1.63	450	1.39
	3	900	0.91	675	0.66	525	1.33	525	0.76
	2	675	0.54	525	0.53	900	0.64	450	0.53
		R = 12	600	2.39	600	1.67	600	1.94	
10	6	450	1.42	450	2.14	450	2.14		
	4	450	0.93	525	1.17	150	1.19		
	2	525	0.58	525	0.53	300	0.65		
		R = 16	675	1.95	600	2.38			

Table 6:Linearity test; d' verses Δf .

Entries are $\frac{d'(1000 \text{ hz}, \Delta f, I_{db})}{d'(1000 \text{ hz}, 2 \text{ hz}, I_{db})}$:

I db SPL	Δf	Subject				Average	Theory
		WMR	MR	SR	MM		
72	4				2.35	2.35	2.00
	3	1.87	1.54		1.43	1.61	1.50
	2	1.00	1.00	1.00	1.00	1.00	1.00
	1	0.57	0.44	0.50	0.42	0.48	0.50
	$\frac{1}{2}$			0.32		0.32	0.25
54	3	1.57	1.47	1.37	1.74	1.56	1.50
	2	1.00	1.00	1.00	1.00	1.00	1.00
	$1\frac{1}{2}$			0.81		0.81	0.75
	1	0.47	0.52	0.39	0.54	0.48	0.50
36	4	2.48	2.66		2.60	2.58	2.00
	3	1.48	1.27	1.72	1.59	1.52	1.50
	2	1.00	1.00	1.00	1.00	1.00	1.00
	1			0.53		0.53	0.50
18	4	2.10	2.37	2.54	2.60	2.40	2.00
	3	1.67	1.24	2.07	1.42	1.60	1.50
	2	1.00	1.00	1.00	1.00	1.00	1.00
10	6	2.43	4.08	3.29		3.27	3.00
	4	1.60	2.22	1.83		1.88	2.00
	2	1.00	1.00	1.00		1.00	1.00

Table 7:

(Part 1)

Linearity test; d' verses Δf .

Entries are $\frac{d'(1000 \text{ Hz}, \Delta f, I_{db})}{d'(1000 \text{ Hz}, \Delta f_{MAX}, I_{db})}$:

I db SPL	Δf	Subject		Theory
		WMR	MR	
72	3	1.00	1.00	1.00
	2	0.53	0.65	0.67
	1	0.30	0.29	0.33
54	3	1.00	1.00	1.00
	2	0.64	0.68	0.67
	1	0.30	0.35	0.33
36	4	1.00	1.00	1.00
	3	0.60	0.48	0.75
	2	0.40	0.38	0.50
18	4	1.00	1.00	1.00
	3	0.80	0.53	0.75
	2	0.48	0.42	0.50
10	6	1.00	1.00	1.00
	4	0.66	0.55	0.67
	2	0.41	0.25	0.33

Table 7:

(Part 2)

Linearity test; d' verses Δf .

Entries are $\frac{d'(1000 \text{ Hz}, \Delta f, I_{db})}{d'(1000 \text{ Hz}, \Delta f_{MAX}, I_{db})}$:

I db SPL	Δf	SR	Subject		
			Theory	MM	Theory
72	4			1.00	1.00
	3			0.61	0.75
	2	1.00	1.00	0.43	0.50
	1	0.50	0.50	0.18	0.25
	$\frac{1}{2}$	0.32	0.25		
54	3	1.00	1.00	1.00	1.00
	2	0.73	0.67	0.57	0.67
	$1\frac{1}{2}$	0.60	0.50		
	1	0.28	0.33	0.31	0.33
36	4			1.00	1.00
	3	1.00	1.00	0.61	0.75
	2	0.58	0.67	0.39	0.50
	1	0.31	0.33		
18	4	1.00	1.00	1.00	1.00
	3	0.81	0.75	0.54	0.75
	2	0.39	0.50	0.39	0.50
10	6	1.00	1.00		
	4	0.55	0.67		
	2	0.30	0.33		

Table 8:

Frequency resolution; $\delta'_f(I)$ estimates, $\delta_f(I)$, and $\bar{\delta}_f(I)$:

I db SPL	Δf	Subject				Average
		WMR	MR	SR	MM	
72	4				0.54	
	3	0.49	0.56		0.43	
	2	0.39	0.54	0.87	0.46	
	1	0.45	0.48	0.87	0.39	
	$\frac{1}{2}$			1.10		
	$\delta'_f(72)$	0.44	0.53	0.95	0.46	0.60
54	3	0.42	0.53	0.78	0.51	
	2	0.40	0.54	0.85	0.43	
	$1\frac{1}{2}$			0.92		
	1	0.38	0.57	0.66	0.47	
	$\delta'_f(54)$	0.40	0.55	0.80	0.47	0.53
36	4	0.34	0.44		0.35	
	3	0.27	0.28	0.72	0.28	
	2	0.27	0.33	0.62	0.27	
	1			0.66		
	$\delta'_f(36)$	0.29	0.34	0.67	0.30	0.40
18	4	0.29	0.31	0.41	0.35	
	3	0.30	0.22	0.41	0.25	
	2	0.27	0.26	0.32	0.27	
	$\delta'_f(18)$	0.29	0.26	0.38	0.29	0.31
10	6	0.24	0.36	0.36		
	4	0.23	0.29	0.30		
	2	0.29	0.26	0.33		
	$\delta'_f(10)$	0.25	0.30	0.33		0.29

Table 9:

Absolute detection sensitivities:

Subject	I db SPL	# trials	$d'(I,0)$	δ_I^*	Threshold**
WMR	+ 1	450	2.66	0.53	- 2
	- 1	450	1.50		
	- 3	450	0.52		
MR	- 2	375	1.34	0.20	- 4
	- 4	375	1.06		
	- 6	375	0.59		
SR	- 2	450	2.28	0.44	- 4
	- 4	450	1.09		
	- 6	450	0.61		
MM	+ 4	225	2.32	0.44	+ 1
	+ 2	225	1.21		
	0	225	0.55		

* Obtained by linearizing the absolute detection function.

** Integer intensity corresponding to unit absolute sensitivity.

Table 10:

δ 's from discrimination and identification tests:

I db SPL	Format	Subject				Average
		WMR	MR	SR	MM	
Intensity experiments						
72	Disc	1.83	1.99	2.88	2.06	2.19
	AI	1.65	1.12	1.55	1.22	1.39
54	Disc	1.22	1.31	2.07	2.05	1.66
	AI	0.63	0.78	0.91	0.87	0.80
36	Disc	0.92	0.86	1.20	1.14	1.03
	AI	0.42	0.47	0.68	0.71	0.57
18	Disc	1.07	1.09	1.36	1.28	1.20
	AI	0.78	0.66	0.61	0.73	0.70
10	Disc	0.92	0.95	1.42	0.82	1.03
	AI	0.58	0.65	0.84	0.68	0.69
Frequency experiments						
72	Disc	0.44	0.53	0.95	(0.46)	0.60
	AI	0.28	0.25	0.28		0.27
54	Disc	0.40	0.55	0.80	(0.47)	0.53
	AI	0.31	0.24	0.24		0.26
36	Disc	0.29	0.34	0.67	(0.30)	0.40
	AI	0.25	0.17	0.22		0.21
18	Disc	0.29	0.26	0.38	(0.29)	0.31
	AI	0.20	0.14	0.16		0.17
10	Disc	0.25	0.30	(0.33)		0.29
	AI	0.12	0.15			0.14

FIGURES

Figure 1a:

Intensity Resolution -
data from Riesz

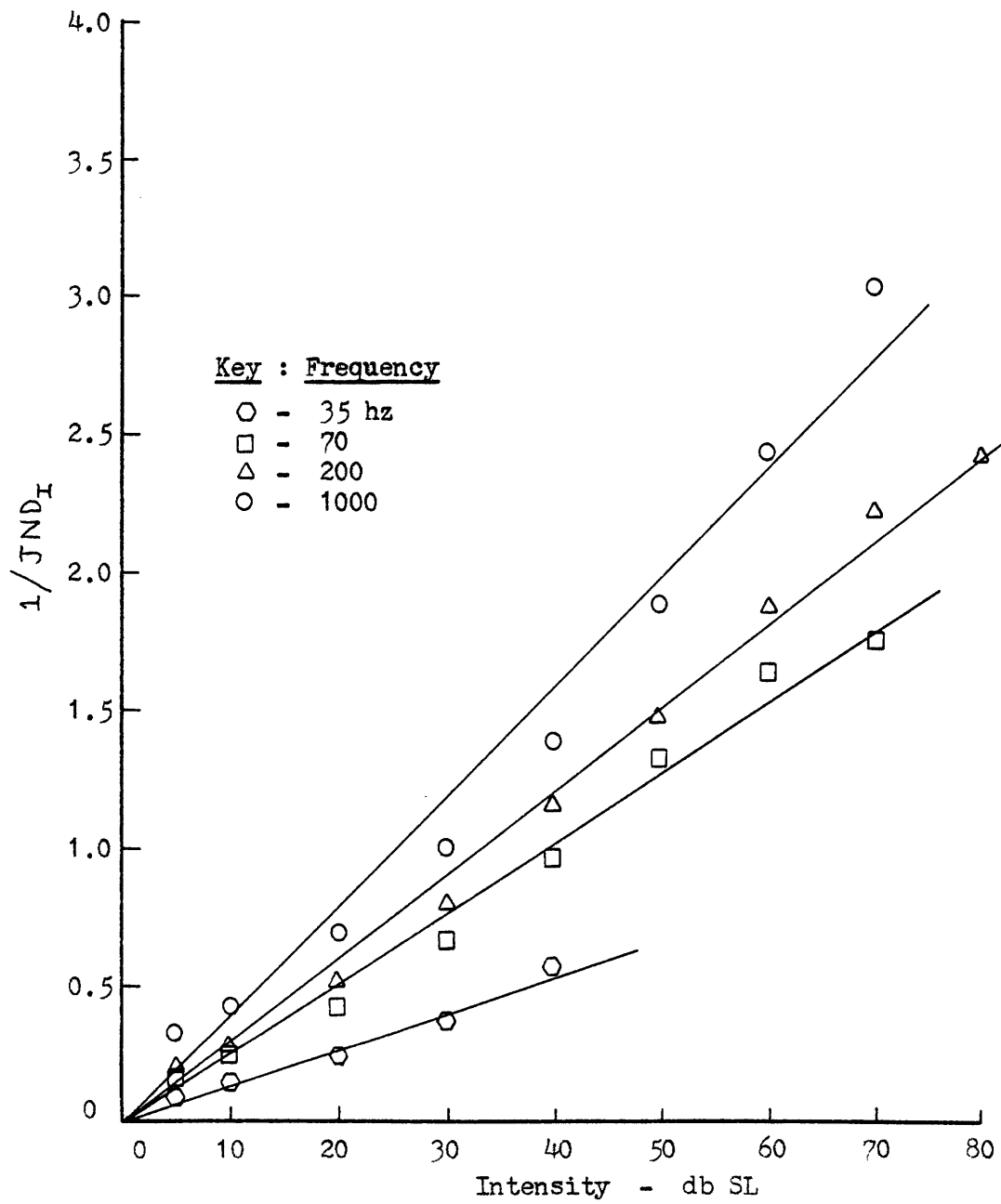


Figure 1b:

Intensity Resolution -
data from Riesz

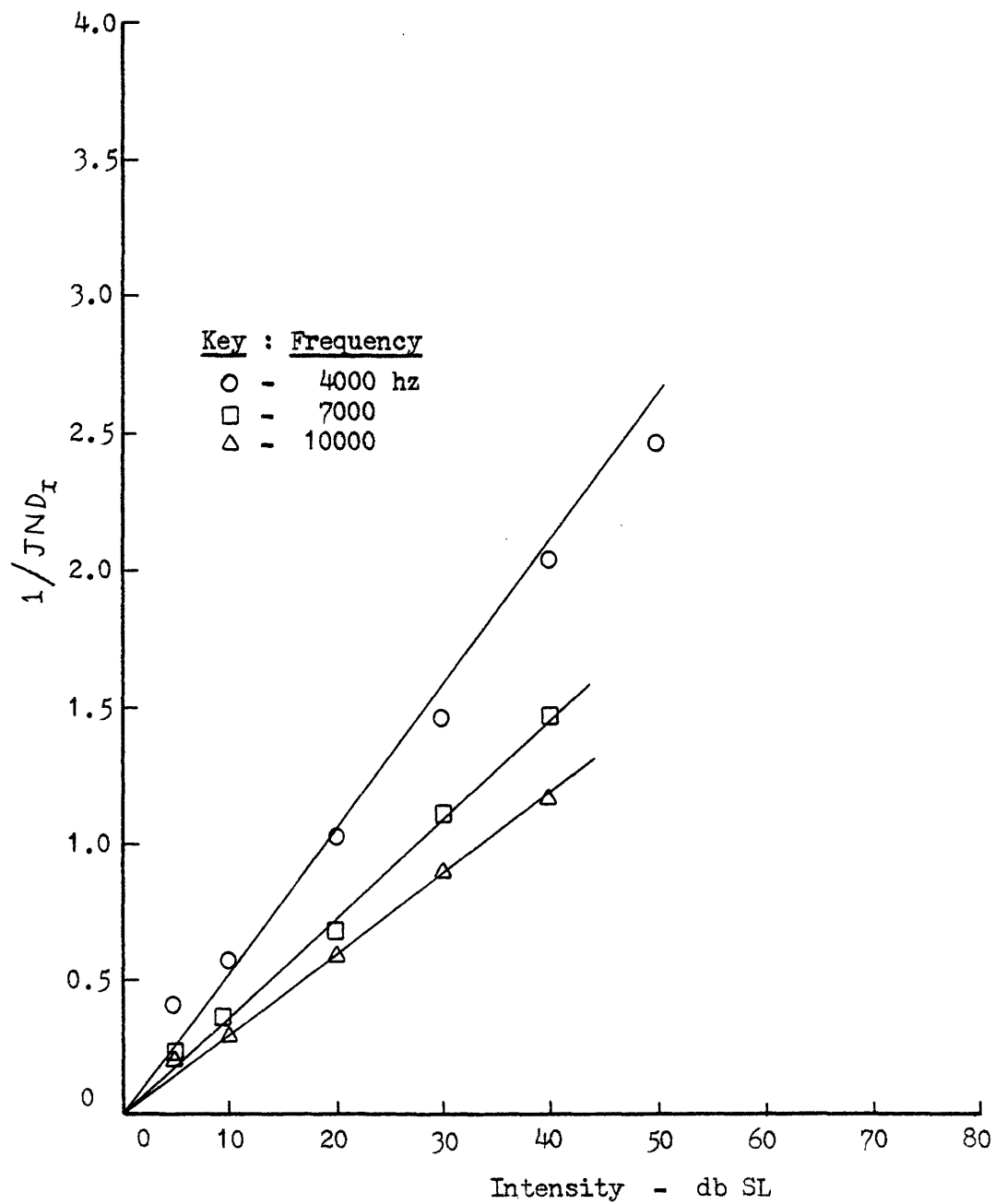


Figure 2:

Intensity Resolution -

data from McGill and Goldberg

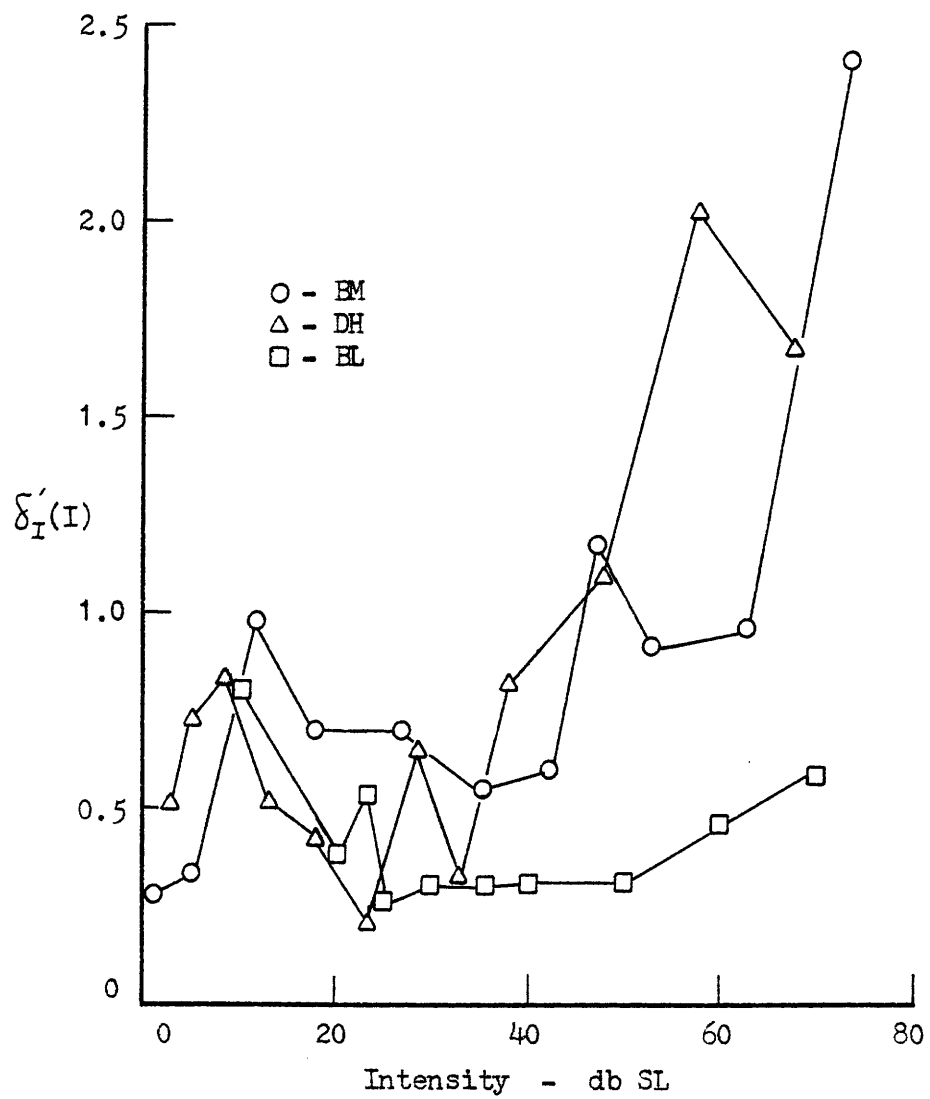


Figure 3:

Intensity Resolution -

data from Cambell and Lasky

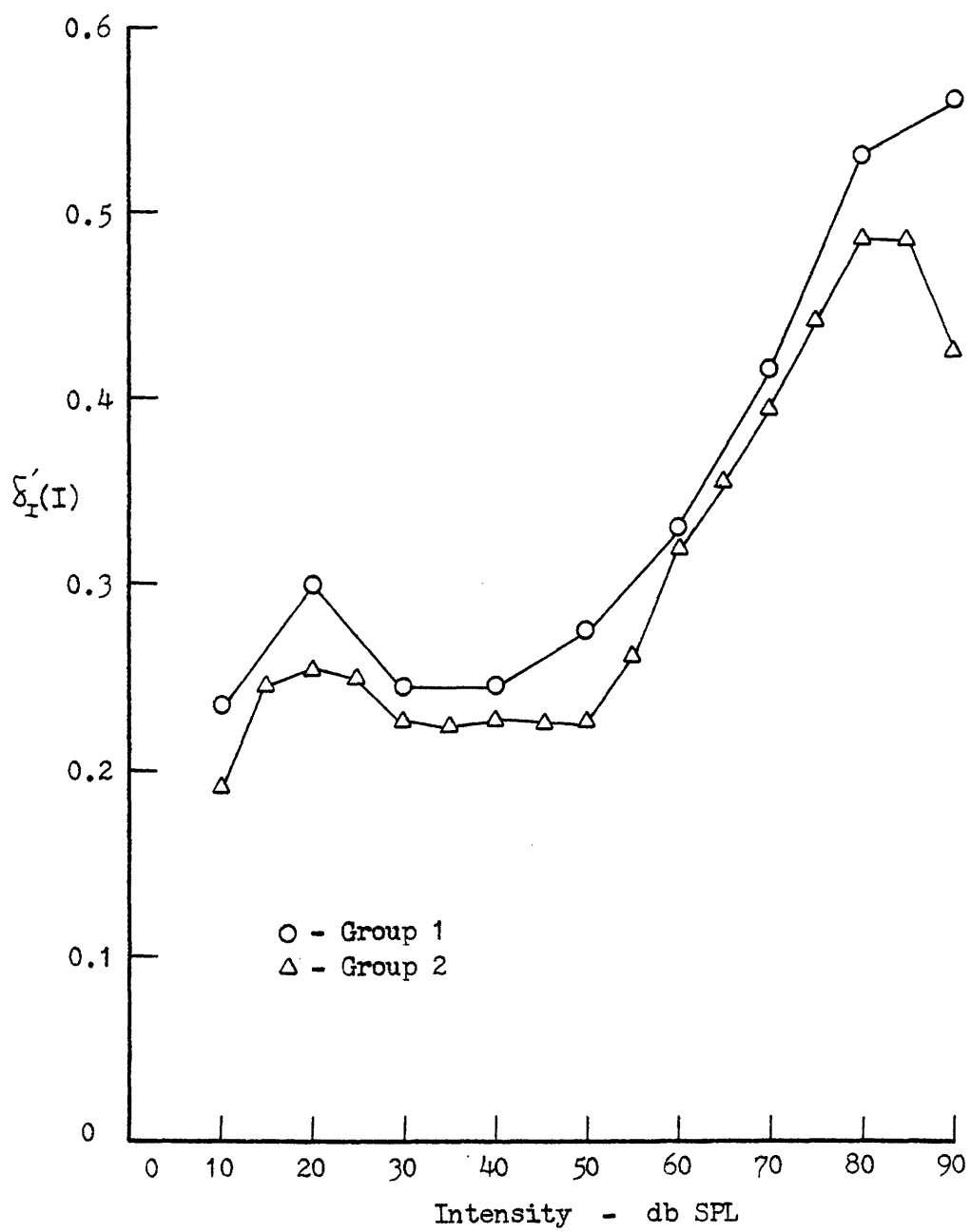


Figure 4:

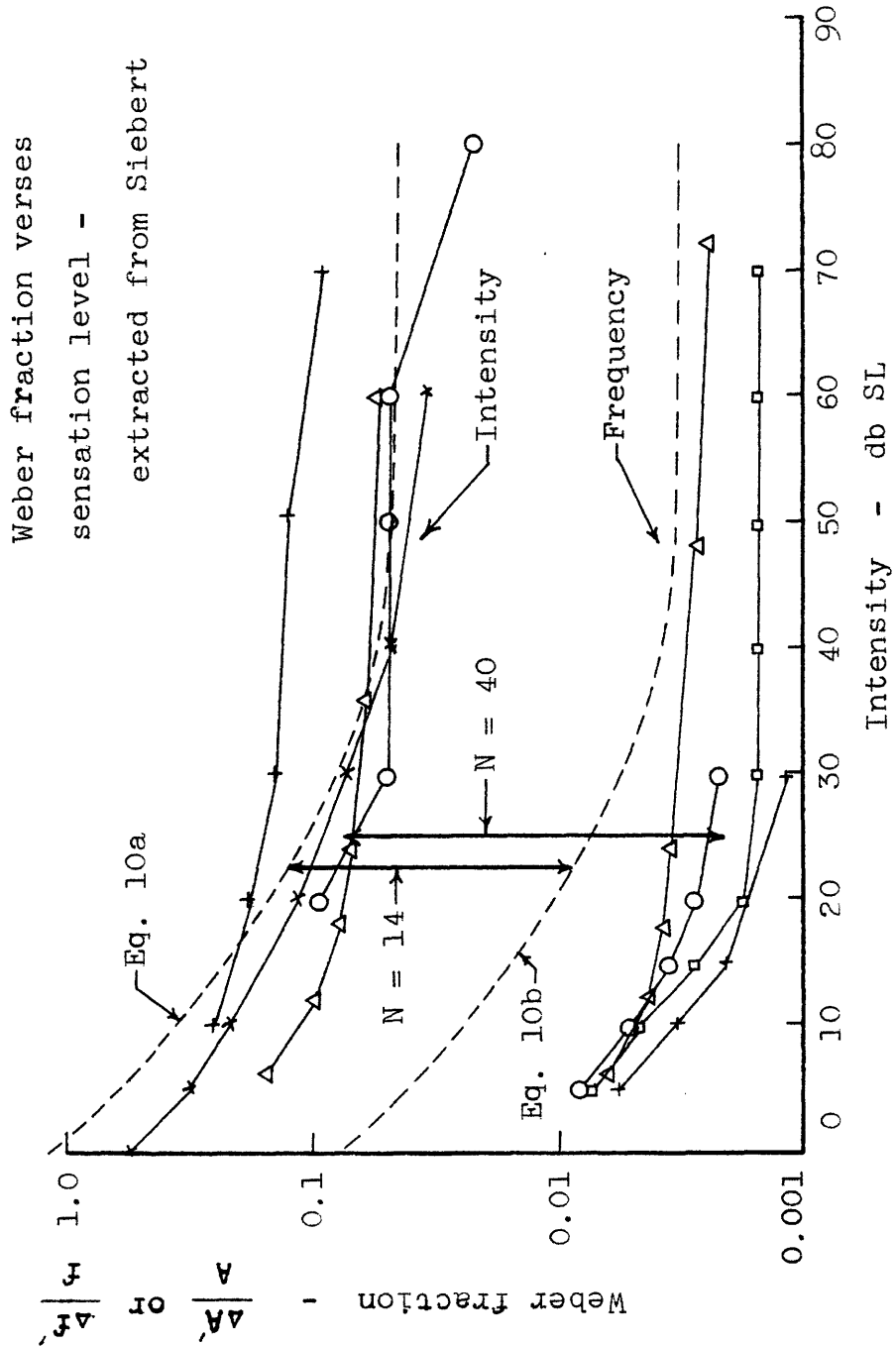


Figure 5:

Scatter diagram testing linearity of d' verses ΔI_{db} -

$$\frac{d'(I_{db}, \Delta I_{db}, 1000 \text{ Hz})}{d'(I_{db}, 1 \text{ db}, 1000 \text{ Hz})}$$

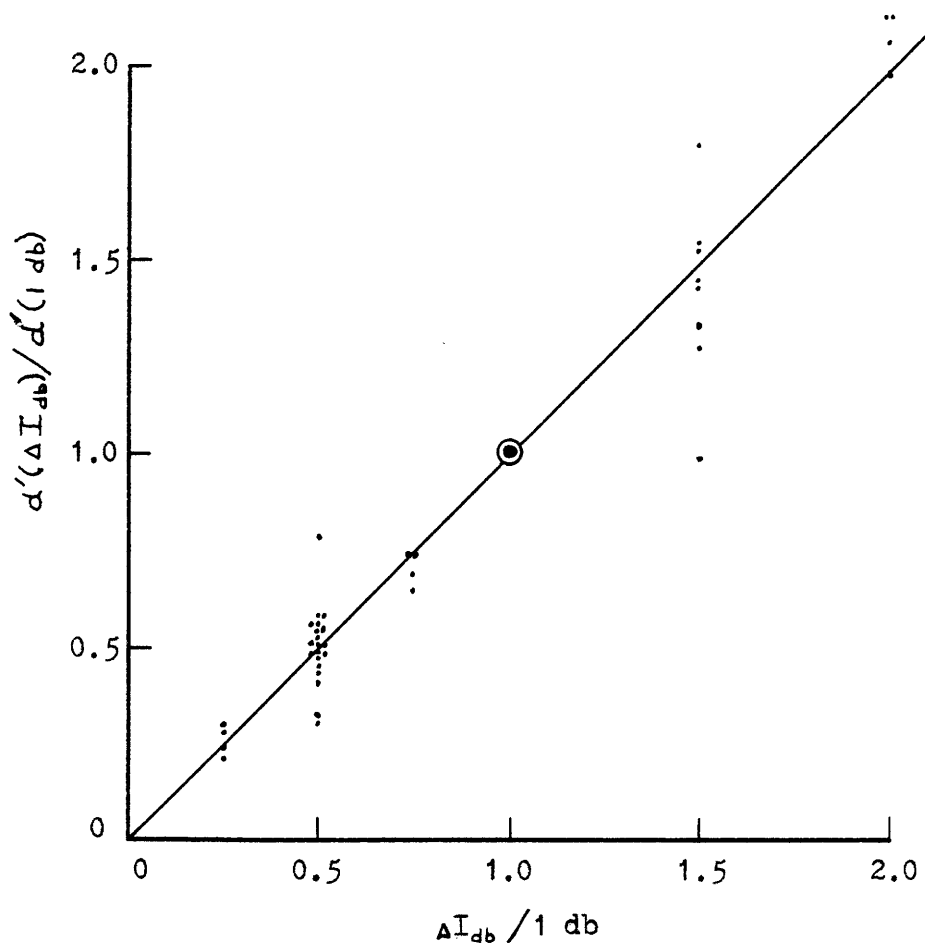


Figure 6:

Intensity Resolution Functions

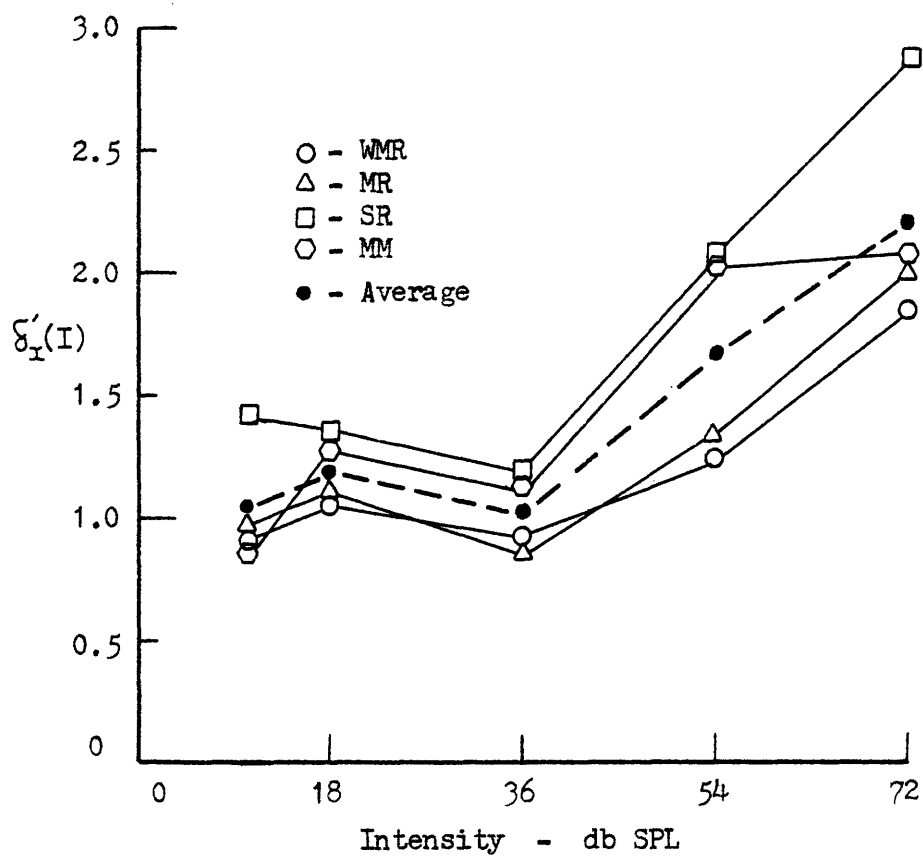


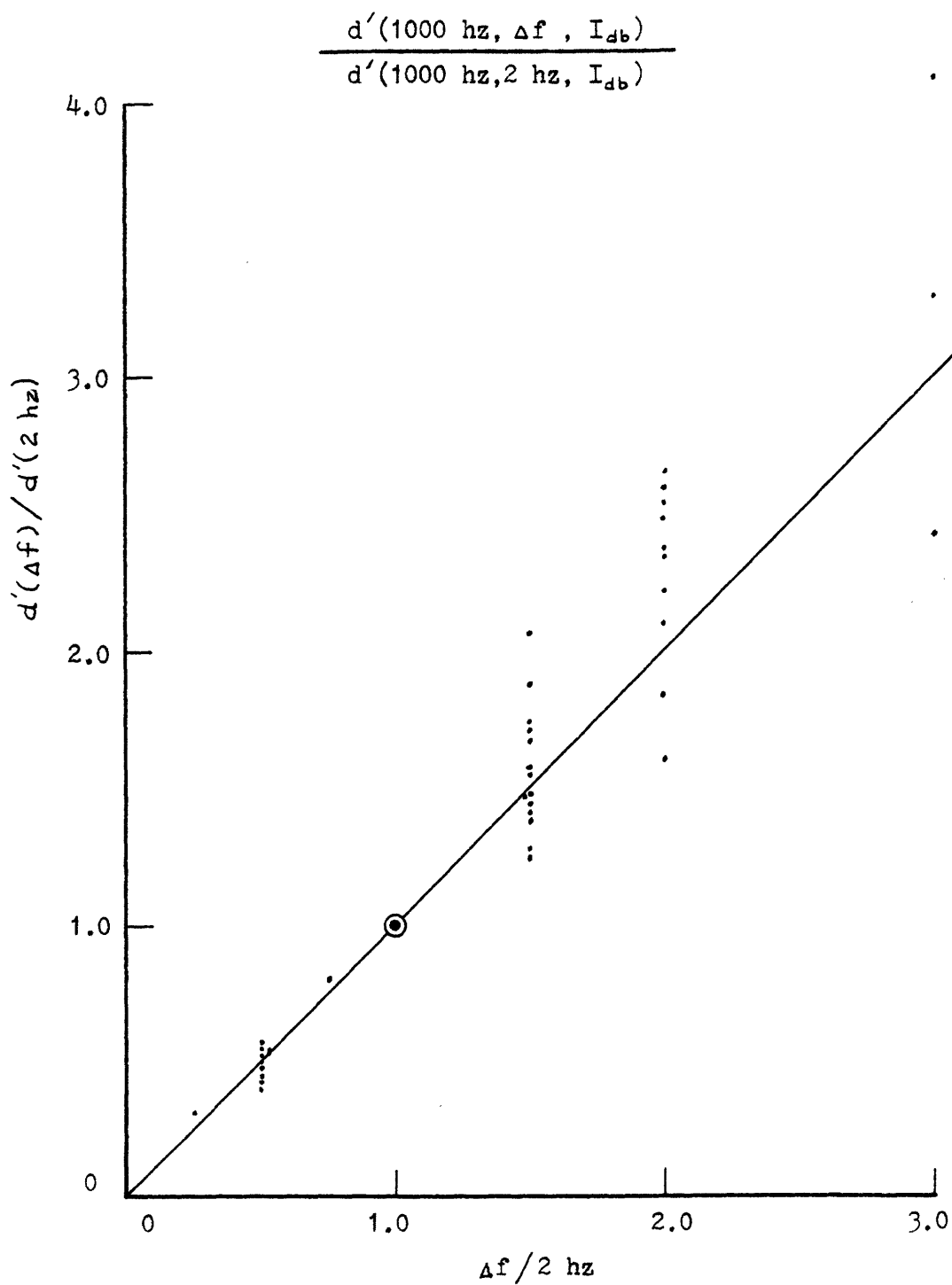
Figure 7:Scatter diagram testing linearity of d' verses Δf -

Figure 8:

Scatter diagram testing linearity of d' verses Δf -

$$\frac{d'(1000 \text{ hz}, \Delta f, I_{db})}{d'(1000 \text{ hz}, \Delta f_{MAX}, I_{db})}$$

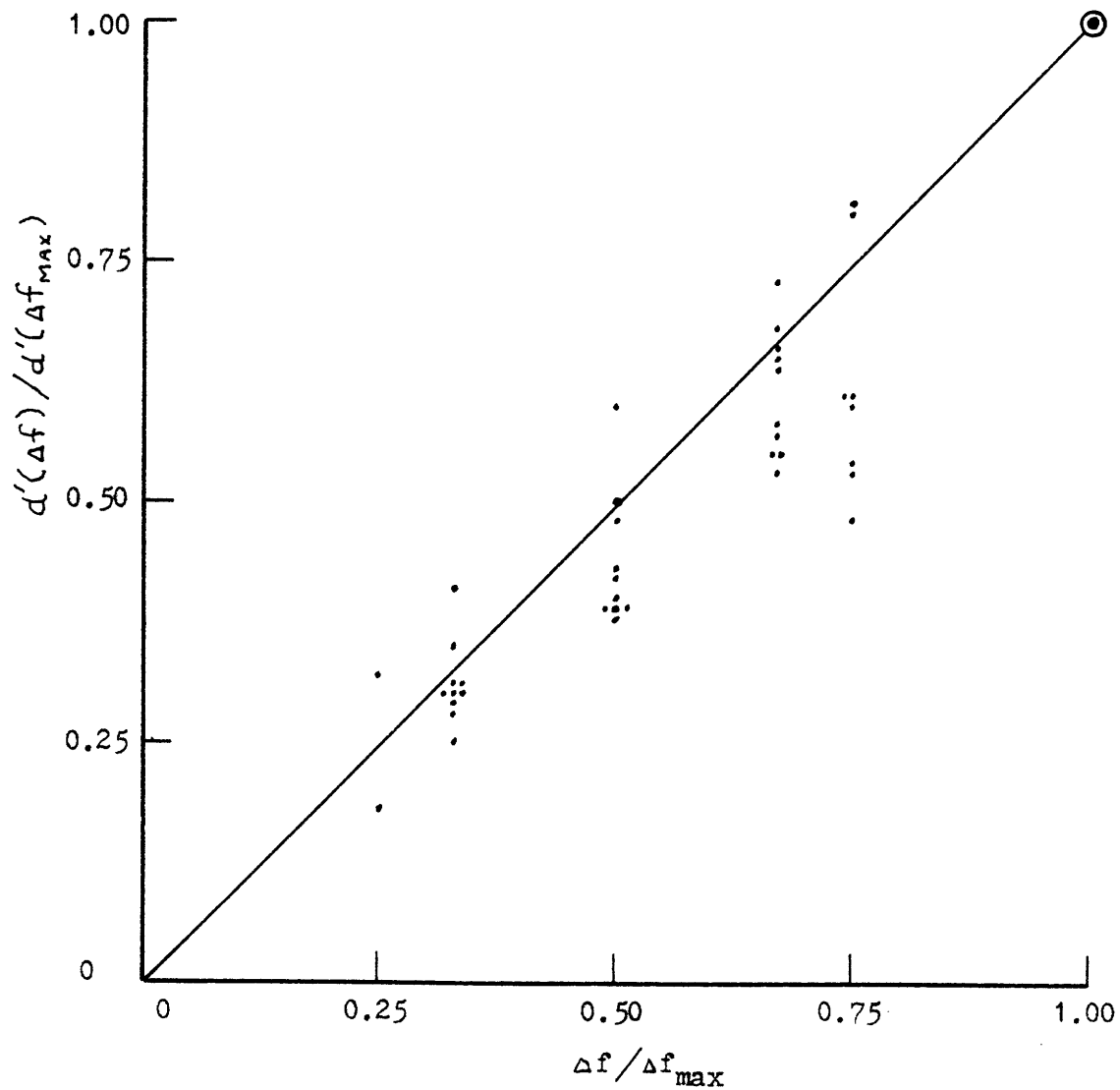


Figure 9:

Frequency Resolution Functions

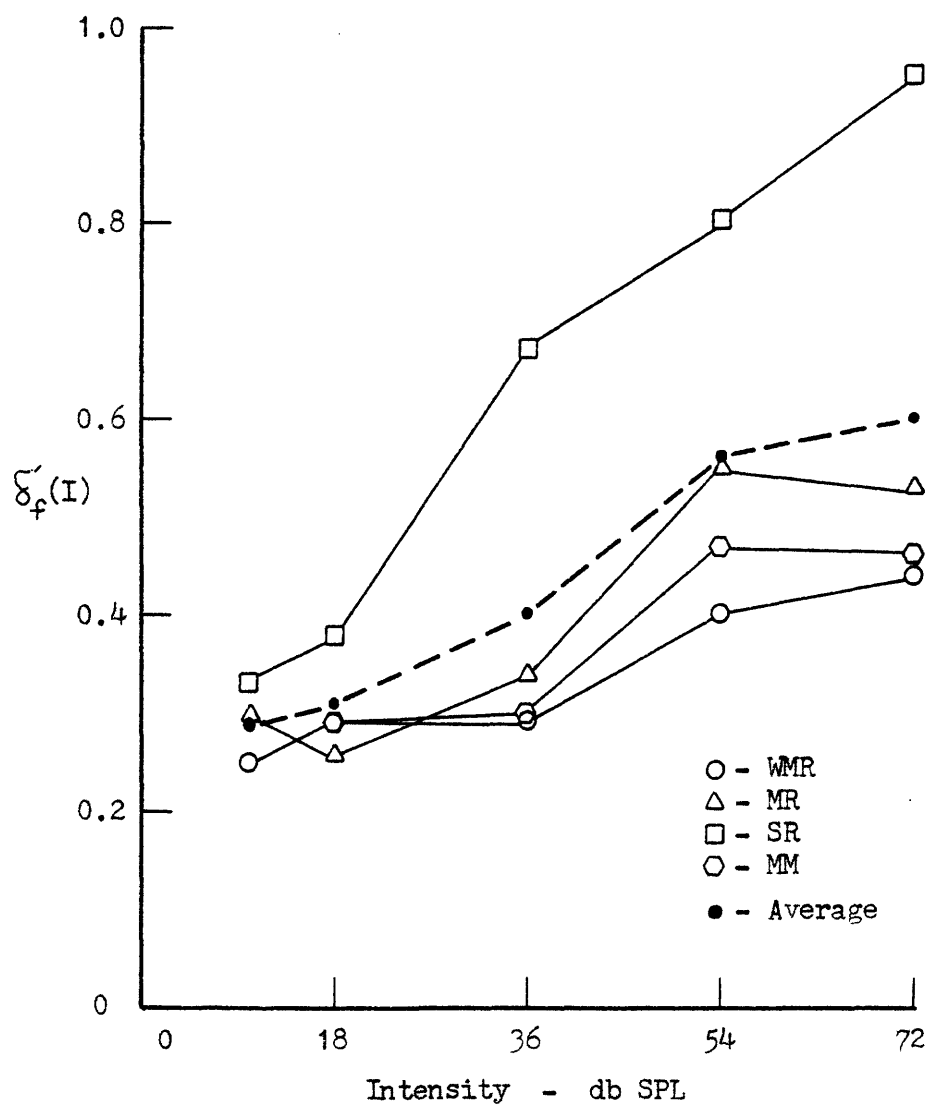


Figure 10:

Absolute Detection Functions

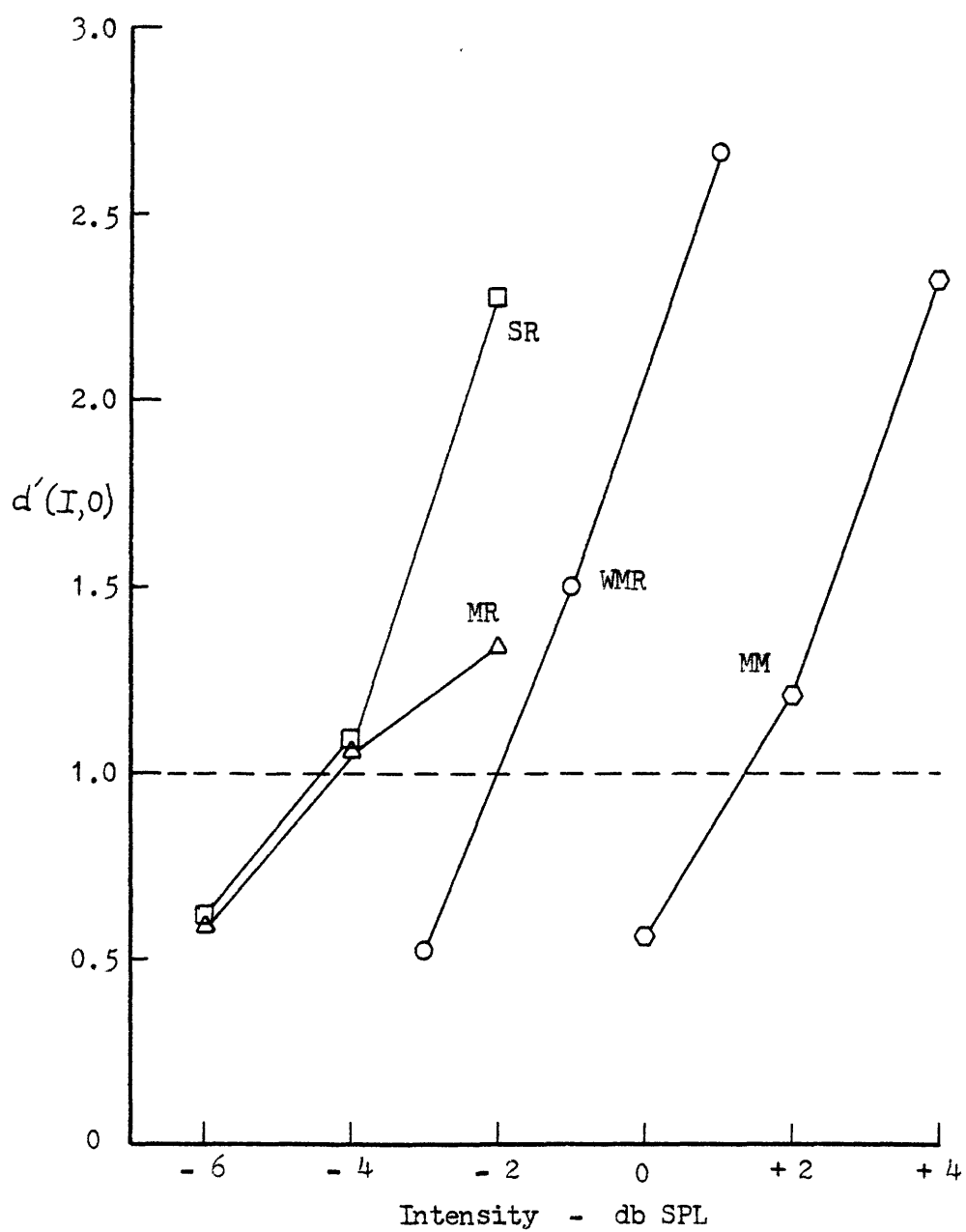


Figure 11:

Average Intensity Resolution -

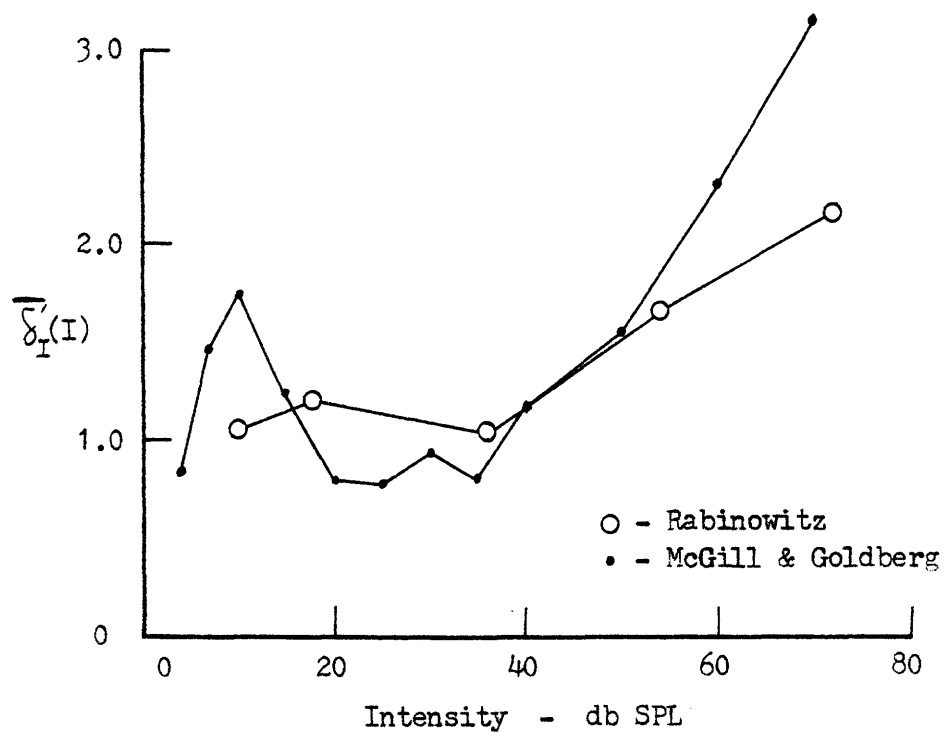
Comparison of our results to
those of McGill and Goldberg

Figure 12:

Average Intensity Resolution -

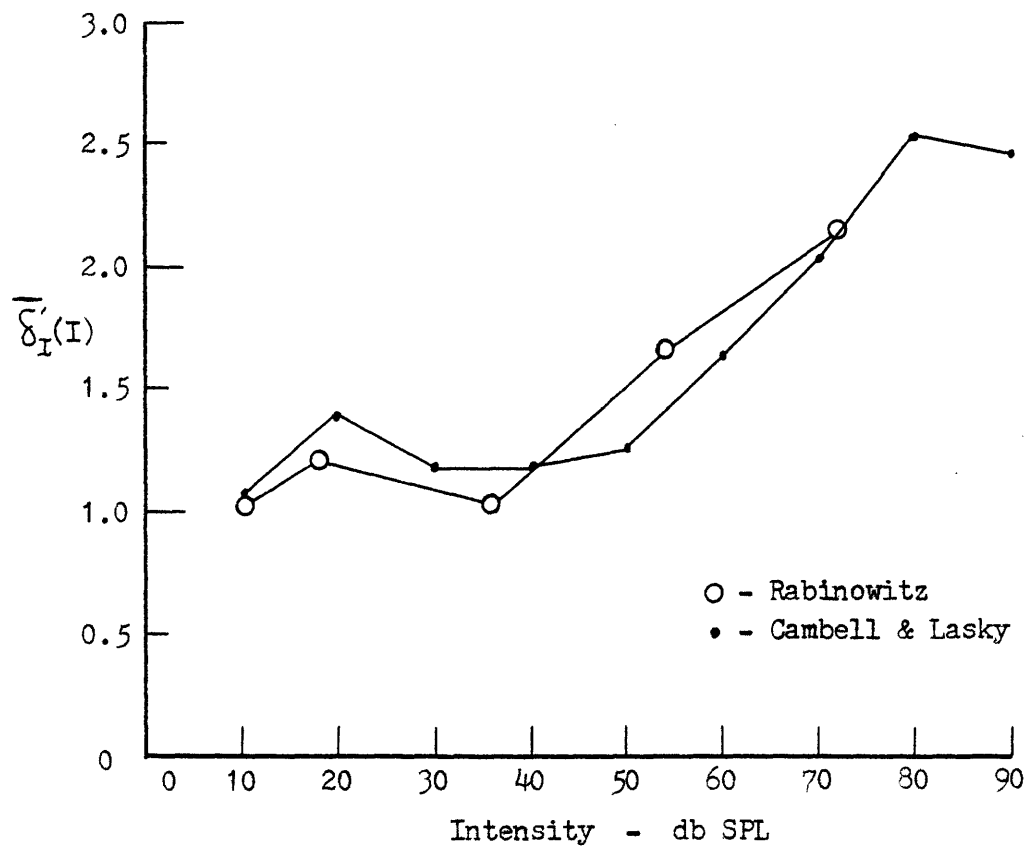
Comparison of our results to
those of Cambell and Lasky

Figure 13:

Average Intensity and Frequency Resolution Functions

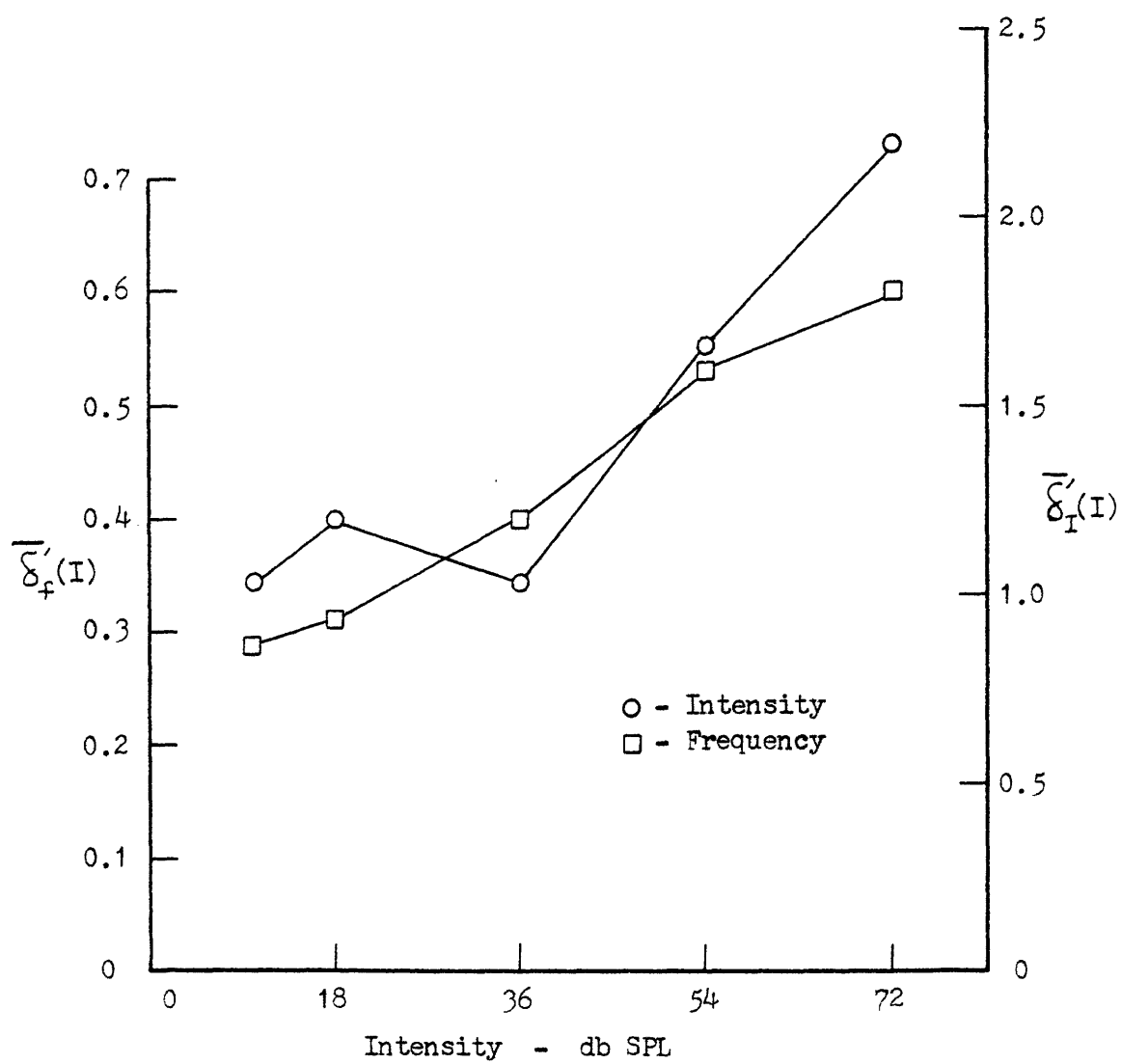
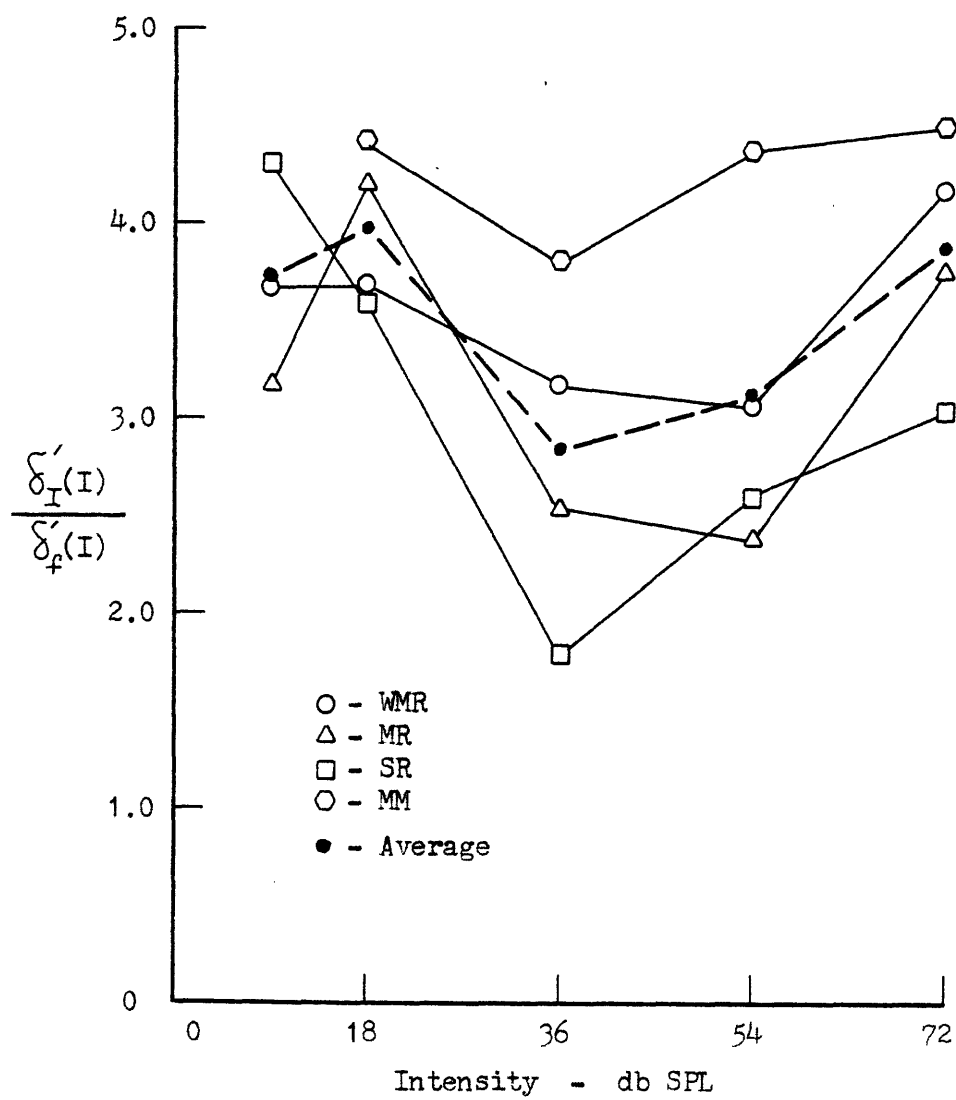


Figure 14:

Ratio of Intensity to Frequency Resolution



FOOTNOTES

1. See Riesz, 1928.
2. For a complete discussion of the theory, see Durlach and Braida, 1969.
3. For sound intensities we make use of the following standard conventions:

$$\begin{array}{l} \text{Intensity in db} \\ \text{sound pressure level} \end{array} = I_{\text{db}} \text{ SPL} = 10 \log_{10} \left(\frac{I}{I_{\text{ref}}} \right)$$

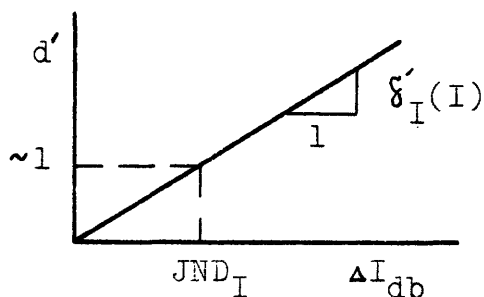
where I_{ref} corresponds to the intensity for the standard reference pressure 0.0002 dynes/cm².

$$\begin{array}{l} \text{Intensity in db} \\ \text{sensation level} \end{array} = I_{\text{db}} \text{ SL} = 10 \log_{10} (I/I_0)$$

where I_0 corresponds to the threshold of audibility at a particular frequency.

$$\begin{array}{l} \text{db increment or} \\ \text{difference between} \\ \text{two tones} \end{array} = \Delta I_{\text{db}} = \begin{cases} 10 \log_{10} \left(\frac{I + \Delta I}{I} \right) \\ 10 \log_{10} (I_2/I_1) \end{cases}$$

4. Define $JND_I = \Delta I_{\text{db}}$ that can be discriminated with probability of being correct = 75% in a 2I-2AFC test. Assuming 0 or small bias (which is usually the case for the two-interval paradigm) this criterion translates to: $JND_I = \Delta I_{\text{db}}$ such that $d' \approx 1$ for a one-interval experiment. The increment required to give $d' \approx 1$ is just $1/\delta'_I$:



$$\delta'_I(I) \cdot JND_I \approx 1$$

5. See Braida, 1969.
6. See Riesz, 1928.
7. Note that the sound intensities corresponding to threshold at the different frequencies are quite disparate. For example, threshold at 1000 Hz is

approximately 0 db SPL while at 35 hz it is about 75 db SPL.

8. See McGill and Goldberg, 1968.
9. McGill and Goldberg present their data in terms of an "energy detection model", but we have reprocessed the data to yield $\delta'_I(I)$:

$$\left. \begin{aligned} I &= 10 \log_{10} E_o \\ I_s &= 10 \log_{10} E_s \end{aligned} \right\} \text{ data given}$$

$$E_o = \text{antilog}_{10} (I/10)$$

$$E_s = \text{antilog}_{10} (I_s/10)$$

$$JND_I = 10 \log_{10} \left(\frac{E_o}{E_o - E_s} \right)$$

$$\delta'_I(I) = (JND_I)^{-1} \quad \text{from Equation 4.}$$

10. See Cambell and Lasky, 1967.
11. Cambell and Lasky present their resolution data in terms of masker levels in db, but we have reprocessed the data to yield $\delta'_I(I)$:

$$I = \text{intensity in db SPL}$$

$$ML = \text{masker level in db}$$

$$I_s = I - ML$$

$$A = \text{antilog}_{10} (I/20)$$

$$A_s = \text{antilog}_{10} (I_s / 20)$$

$$JND_I = 20 \log_{10} \left(\frac{A + A_s}{A} \right)$$

$$\delta'_I(I) = (JND_I)^{-1} \quad \text{from Equation 4.}$$

12. See Durlach and Braida, 1969.
13. See Pynn, 1968.
14. See Shower and Biddulph, 1931; and Siebert, 1968.

15. As $\Delta f/f$ is typically less than 0.01, the approximation to the logarithm used in Equation 6 is very accurate.
16. Define $JND_f = \Delta f$ for 75% correct in a 2I-2AFC test. The proof then follows directly as in footnote 4.
17. See Makita, 1942.
18. See Gold and Pumphrey, 1948.
19. To account for intensity resolution's dependence upon intensity, some of the models allow for changes in both slopes of the excitation pattern with the level of stimulation. See for example Goldstein, 1965.
20. See Schaefer, Gales, Shewmaker, and Thompson, 1950.
21. See Shower and Biddulph, 1931.
22. See Riesz, 1928.
23. See Stahl, 1969.
24. See Ritsma, Domburg, and Donders, 1968.
25. See Sachs, 1969.
26. See Siebert, 1968.
27. Note that Equation 10a for intensity resolution predicts an asymptotic Weber fraction, $1/C$, which from our earlier remarks, seems not to be the case.
28. The additivity property is one of the basic axioms of the "decision model" discussed by Durlach and Braida, 1969.
29. See Pynn, 1968; and Braida, 1969.
30. The local constancy of δ'_I further suggests one need not investigate many points on the psychometric function. What performance level yields the best estimate of δ'_I ? This question is currently being investigated theoretically.
31. Using Equation 4, the JND_I 's have been computed and are shown in the Appendix, Table A.1 and Figure A.1.

32. The important issue involved here (to which we have not found a satisfactory solution) is the conditions under which data should be excluded, if any.
33. Using Equation 8, the JND_f 's have been computed and are shown in the Appendix, Table A.1 and Figure A.2.
34. See Durlach and Braida, 1969.
35. See Pynn, 1968.
36. We are currently exploring the earphone problem and preliminary study suggests the tighter sealing circumaural earphone cushions, No. 001, are preferable to those used in this work, No. MC-162-A, which rest lightly on the outer ear. The use of insert earphones is also under consideration.
37. See Braida, 1969.
38. See McGill and Goldberg, 1968.
39. See Cambell and Lasky, 1967.
40. See Siebert, 1968.
41. See Siebert, 1968.
42. See Sachs, 1969.
43. See Durlach and Braida, 1969. Equation 17 follows directly from the additivity property of sensitivity:

$$d'(I + \Delta I, 0) - d'(I, 0) = d'(I + \Delta I, I)$$

Dividing by ΔI and taking the limit as $\Delta I \rightarrow 0$ then yields the required result.

APPENDIX

Just - Noticeable - Differences

Table A.1:

Just - noticeable - differences:

I db SPL	Subject				Average
	WMR	MR	SR	MM	
JND _I (db)					
72	0.55	0.50	0.35	0.48	0.47
54	0.82	0.76	0.48	0.49	0.64
36	1.08	1.16	0.83	0.88	0.99
18	0.93	0.92	0.74	0.78	0.84
10	1.08	1.05	0.70	1.22	1.01
JND _f (hz)					
72	2.27	1.88	1.05	2.22	1.86
54	2.50	1.82	1.25	2.13	1.92
36	3.45	2.94	1.50	3.33	2.81
18	3.45	3.84	2.63	3.45	3.34
10	4.00	3.33	3.00		3.44

Figure A.1:

Just - noticeable - differences
in Intensity

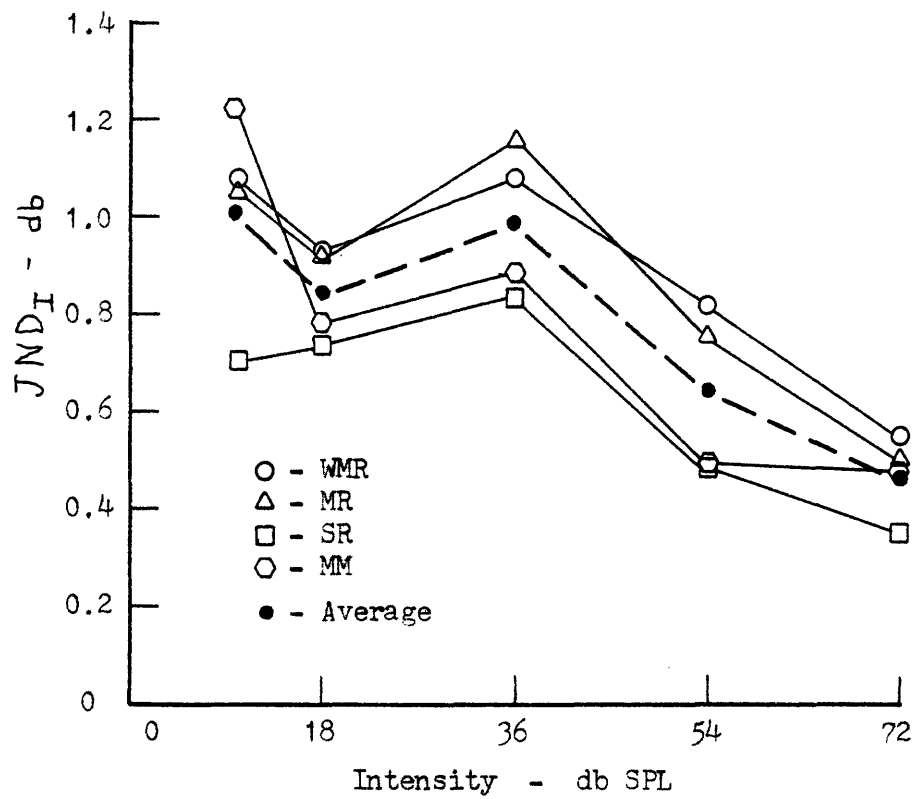
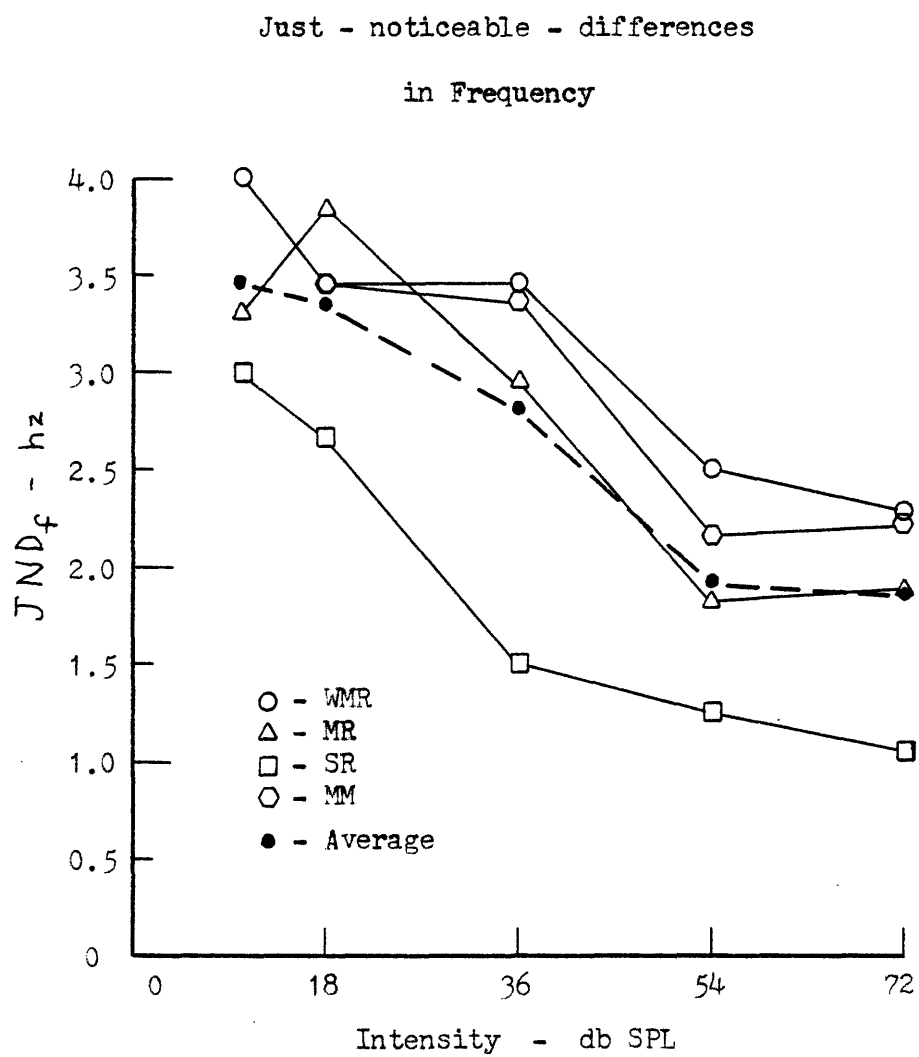


Figure A.2:

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