STRUCTURAL TOPOLOGY OPTIMIZATION VIA THE GENETIC ALGORITHM

by

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ABSTRACT

The genetic algorithm, a search and optimization technique based on the theory of natural selection, is applied to problems of structural topology optimization. Given a structure's boundary conditions and allowable design domain, a discretized design representation is created. The genetic algorithm then generates an optimal structure topology by evolving a population of "chromosomes," where each chromosome, after mapping into the discretized design representation, creates a potentially-optimal structure topology. Specifically, using an evolutionary, survival-of-the-fittest optimization mechanism, the genetic algorithm allows structure topologies in the population to compete against one another to serve as parent topologies. Parent topologies then pair and mate, swapping portions of their "genetic code" to create a population of child topologies of hopefully higher quality. After undergoing infrequent, random mutation, the child population replaces the original population, and the process then iterates until an optimal structure topology is located.

In this thesis, the use of the genetic algorithm in structural topology optimization is presented. After defining structural topology optimization and detailing the genetic algorithm, a review of previous research in structural optimization is provided. The implementation of this investigation's genetic algorithm-based structural topology optimization approach is then detailed, and several examples are presented: optimizations of beam cross-section topologies and cantilevered plate topologies are described, as are methods for the efficient use of finite element analysis in a genetic algorithm-based search. The optimization of finely-discretized design domains is then examined. The genetic algorithm's ability to generate design families, as well as designs combining high structural performance with high manufacturability, is then demonstrated. Structures created using this investigation's genetic algorithm-based approach are then compared to those obtained utilizing homogenization-based techniques. Finally, potential future work in suggested.

Thesis Advisor: Professor Mark J. Jakiela

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To my parents

"Be bold and courageous. When you look back on your life, you'll regret the things you didn't do more than the ones you did."

H. Jackson Brown, Jr.
Author of *Life's Little Instruction Book*
Rutledge Hill Press, Nashville, Tennessee

Introduction 1

1.1 Overview

This chapter first presents the motivation behind this investigation and then details the investigation's specific objectives. A typical problem statement is then provided, after which the organization of this thesis is detailed.

1.2 Motivation

Design is an iterative process divided into stages. After first recognizing a need for a device (Shigley and Mischke, 1989) and defining a set of specifications and functional requirements, a designer must develop a conceptual design establishing the overall form of the device, optimize the conceptual design so that it best satisfies the specifications and functional requirements, and then finally detail the design to account for manufacturability and aesthetic (among others) considerations. During this process, the designer will often find that the current design is inadequate and will therefore return to an earlier phase of the design procedure to account for the inadequacy.

Hence, the design process begins with the *formulation of functional requirements* and then continues with *conceptual design*, *optimization*, and finally *detailing* (Kirsch, 1993). Again, design is very much an iterative process, in that a designer will often return to earlier stages of the process to account for observed inadequacies in a design.

Of the four main stages of the design process, the conceptual design stage is typically regarded as critical, for it is an early phase of the design process yet establishes much of the device's overall design. Consequently, because design revisions become increasingly expensive in subsequent phases of the design process, design decisions made in the conceptual design stage must be thoroughly planned and carefully executed. Unfortunately, even though the conceptual design stage is one of the most critical elements of the design process and must be carefully planned and executed, few computational tools which aid a designer with conceptual design are available—the success (or failure) of conceptual design is, for the most part, left entirely to the ingenuity, intuition, and judgement of the designer (Kirsch, 1993).

In an attempt to aid the designer with the conceptual design stage, this investigation develops a computer-based system which automates conceptual design by modeling a conceptual design problem as an optimization problem and then using a numerical optimization procedure to search the "space" of possible designs for the optimal design.

As no single computer-based system is applicable to all conceptual design problems, this investigation focuses on developing a system which automates the conceptual design of load-bearing structures. Specifically, this investigation examines the automated generation of optimal topologies for load-bearing structures, *i.e.*, structural topology optimization. While structural topology optimization is a very specific conceptual design problem, the optimization algorithm used by this investigation to perform structural topology optimization, the genetic algorithm (Goldberg, 1989a), is very general. In fact, the genetic algorithm, a search and optimization technique based on the theory of natural selection, is not peculiar to any particular design domain and can therefore be applied to many other classes of conceptual design problems.

1.3 Objective

The general objective of this investigation is to help determine the utility of genetic algorithms in conceptual design, while the specific objective is to use the genetic algorithm to perform structural topology optimization. In particular, this investigation intends to accomplish the following:

- Develop a genetic algorithm-based approach for the structural topology optimization of continuum structures, where the optimal distribution of material and void within a discretized design domain is found.

- Apply this genetic algorithm-based structural topology optimization approach to a variety of examples in an attempt to establish the feasibility, demonstrate the capabilities, and increase the performance of the approach.

- Compare the abilities and limitations of this genetic algorithm-based structural topology optimization approach to those of other techniques which have also been devised for the automated generation of optimal structure topologies.

1.4 Problem Statement

This genetic algorithm-based structural topology optimization approach is typically applied to problems of the following form:

> Given a design domain representing the region which a structure may occupy, as well as an applied load and a set of support points (Figure 1.1a), generate the structure topology which exhibits maximum structural performance while satisfying a set of constraints (Figure 1.1b).

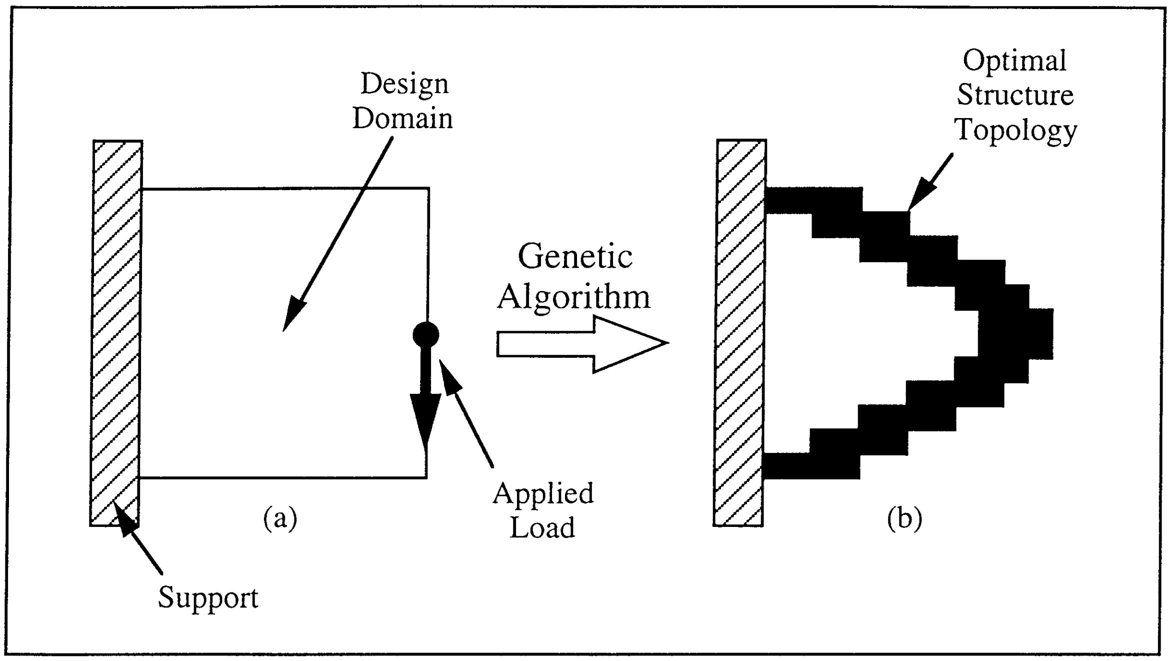


Figure 1.1: A schematic representation of genetic algorithm-based structural topology optimization. Both the example design domain and optimal topology are hypothetical.

Hence, using a given design domain, applied load, and set of support points, the genetic algorithm must find the distribution of material and void within the design domain (*i.e.*, it must find the structure topology) which provides optimal structural performance while satisfying a set of constraints. The genetic algorithm generates this optimal structure topology through an evolutionary process, where a population of structure topologies is evolved until an optimal structure topology is created. In this evolutionary, survival-of-the-fittest optimization mechanism, structure topologies in a parent generation compete against one another to serve as parent designs. Parent designs then pair and mate, swapping portions of their topological attributes to create a generation of child designs. After undergoing random mutation, the child generation replaces the parent generation, and the entire process then iterates until an optimal structure topology is located.

This investigation uses the genetic algorithm to optimize a structure's topology based on a variety of criteria. For instance, one example attempts to maximize a structure's stiffness-to-weight ratio subject to a manufacturability constraint, while another example attempts to minimize a structure's mean compliance subject to a maximum volume constraint. Note, however, that most examples addressed in this investigation perform an unconstrained maximization of stiffness-to-weight ratio.

1.5 Organization

This thesis details this investigation's development and application of a genetic algorithm-based approach to structural topology optimization.

Chapter 2, *Background*, begins by introducing and defining the general application domain of this investigation, namely structural optimization. Note that while this investigation specifically addresses structural topology optimization, the broader area of structural optimization is detailed for completeness. Following this description, the genetic algorithm is detailed.

Chapter 3, *Previous Work*, then provides a detailed review of previous research in both genetic algorithm-based and non-genetic-algorithm-based structural optimization. After first describing non-genetic-algorithm-based approaches to structural topology optimization, the chapter details genetic algorithm-based approaches to the broader area of structural optimization.

Chapter 4, *This Investigation*, begins by introducing the genetic algorithm-based structural topology optimization technique developed in this investigation and then describes the areas in which this investigation's approach extends previous work in genetic algorithm-based structural topology optimization. The chapter concludes with a detailed explanation of the implementation of this investigation's structural topology optimization approach.

Chapter 5, *Examples*, details the application of this investigation's genetic algorithm-based structural topology optimization approach to a variety of increasingly-complex example problems.

Chapter 6, *Conclusions*, overviews the contributions of this investigation and provides conclusions regarding the abilities and limitations of its genetic algorithm-based structural topology optimization approach. The chapter concludes by offering potential areas of future research.

Chapter 7, *References*, provides a list of previous work cited in this thesis.

Chapter 8, *Index*, provides lists of authors cited and subjects detailed in this thesis.

Background 2

2.1 Overview

This chapter first introduces and defines the application domain of this research, structural optimization, and then details the technique used to perform the optimization, the genetic algorithm.

2.2 Structural Optimization

2.2.1 Introduction. Structural optimization is the automated synthesis of a mechanical component based on structural considerations. In other words, structural optimization generates a mechanical component design which exhibits optimal structural performance. Falling under the broader category of design optimization, where the design of a device is optimized so that the device exhibits maximum utility subject to a given set of functional requirements and performance constraints, structural optimization is the design optimization of mechanical components where the utility, functional requirements, and/or performance constraints are structural in nature (Kumar, 1993).

Utility, in the context of structural optimization, is a measure of a component's structural performance, effectiveness, and desirability. For example, maximizing a component's utility could include maximizing stiffness, maximizing manufacturability, minimizing weight, or minimizing cost. Functional requirements, again in the context of structural optimization, are specifications which define the intended use of the component and the conditions under which the component will operate. Typical specifications include size and weight limitations, material properties, locations of support points, and locations, directions, and magnitudes of applied loads. Lastly, performance constraints are defined ranges of acceptable structural behavior. For example, constraints can specify maximum allowable stress levels, maximum acceptable weight levels, maximum deflection levels, minimum heat dissipation rates, etc.

As demonstrated above, a wide variety of properties can serve as measures of utility or as performance constraints. These properties, which all influence a component's design, are commonly referred to as *design criteria.* Design criteria typically used in structural optimizations include, but are not limited to, a component's stiffness, strength, stress levels, weight, volume, thermal properties, manufacturability, fatigue life, dynamic behavior, deflection, or cost. Structural optimization problems never simultaneously account for all design criteria—the particular design criteria addressed in a given problem's utility and constraint calculations depend upon the problem's domain and the available computational resources.

Hence, structural optimization generates a component design which exhibits maximum structural utility subject to a set of functional requirements and constraints on the component's structural behavior. Examples of structural optimization include the minimization of the volume of a cantilevered plate in tension subject to a maximum deflection constraint, or the maximization of the stiffness of a simply-supported beam under pure bending subject to a maximum mass constraint.

2.2.2 Structural Optimization Techniques. Either analytical or numerical techniques can be used to solve structural optimization problems (Kirsch, 1993). Analytical structural optimization techniques, pioneered by Mitchell (Mitchell, 1904), typically employ mathematical approaches such as calculus or variational methods to find optimum structure designs. Equations describe a component's design, and an optimum design is found when systems of equations defining the conditions for optimality are solved analytically. While analytical techniques do provide an exact solution to the optimization problem, they can only solve straightforward problems. Design problems with complex boundary conditions or multiple components are generally not solvable using analytical techniques. Analytical techniques in structural optimization, with particular emphasis on the creation of "Mitchell Structures," are detailed by Cox (Cox, 1965) and Hemp (Hemp, 1973).

To facilitate the solution of highly-complex problems, numerical structural optimization techniques have been developed. These methods numerically approximate the design and then iteratively modify the design until a near-optimum solution is obtained.

While numerical techniques are unable to obtain exact solutions to an optimization problem, they can address problems of great complexity. All discussion in this article is concerned with numerical structural optimization techniques.

2.2.3 Problem Formulation. Before conducting a structural optimization, the qualitative problem description (*e.g.*, minimize the mass of a cantilevered plate in tension subject to a maximum stress constraint) must first be formulated as a quantitative mathematical statement (Arora, 1989). This begins with the definition of *design variables*, which are parameters controlling the component's design. One design variable is needed for every component attribute allowed to vary during optimization. Because the design variables control the component's design, assigning numerical values to the design variables results in the creation of a particular component design. Note that a particular set of design variable values represents both a particular component design and a particular location, or point, in the search space. A vector of N design variables is shown below:

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{N-1} & x_N \end{bmatrix} \tag{2.1}$$

For example, if optimizing the shape of a geometry containing N/2 vertices (contained in the vector $\mathbf{V}$, shown below), N design variables (contained in the vector $\mathbf{x}$, shown above) would be required to control the vertices' x- and y-coordinates:

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N/2} \end{bmatrix} = \begin{bmatrix} (v_x, v_y)\big|_1 \\ (v_x, v_y)\big|_2 \\ \vdots \\ (v_x, v_y)\big|_{N/2} \end{bmatrix} = \begin{bmatrix} (x_1, x_2) \\ (x_3, x_4) \\ \vdots \\ (x_{N-1}, x_N) \end{bmatrix} \tag{2.2}$$

After defining the design variables, an *objective function* must be developed. Used by the optimization algorithm to make quantitative utility comparisons between candidate component designs, the objective function calculates any given design's utility. Taking as input a particular set of design variable values, the objective function (which is typically non-linear in nature) converts the variable values into their corresponding component design, analyzes the design to determine its structural behavior, and then calculates a scalar quantity describing the utility, or merit, of the design. Hence, the objective function calculates the utility of any specified set of design variable values:

$$\mathit{Utility} = \mathrm{U}(\mathbf{x}) \equiv \mathrm{U}(x_1, x_2, \ldots, x_N) \tag{2.3}$$

Note that many structural optimizations attempt to *minimize* a component's *cost* instead of maximizing its utility. In fact, most optimization algorithms (mathematical programming, optimality criteria, and simulated annealing—see Section 2.2.4) are cost minimization algorithms. However, the optimization technique used in this research, the genetic algorithm, is a maximization algorithm. Hence, the mathematical problem formulation is stated as a utility maximization.

A set of *design constraints* must then be created. Typically used to represent a component's functional requirements and performance constraints, design constraints are (often non-linear) mathematical equalities or inequalities defining constraints on the component's structural behavior. Note that in addition to being functions of the design variables, constraint calculations typically require structural analysis results. An example set of design constraints is depicted below:

$$\mathrm{h}_j(\mathbf{x}) \equiv \mathrm{h}_j(x_1, x_2, \ldots, x_N) = 0; \qquad j = 1 \text{ to } p \tag{2.4}$$

$$\mathrm{g}_i(\mathbf{x}) \equiv \mathrm{g}_i(x_1, x_2, \ldots, x_N) \leq 0; \qquad i = 1 \text{ to } m \tag{2.5}$$

Design constraints establish a component's feasibility—components which do not violate any design constraints are considered *feasible*, while those violating one or more design constraints are considered *infeasible*. For a given set of design variable values $\mathbf{x}^{(k)}$, an inequality constraint is *violated* when positive in value (*i.e.*, $\mathrm{g}_i(\mathbf{x}^{(k)}) > 0$), while an equality constraint is violated when non-zero (*i.e.*, $\mathrm{h}_j(\mathbf{x}^{(k)}) \neq 0$). Also, inequality constraints satisfied as equalities (*i.e.*, $\mathrm{g}_i(\mathbf{x}^{(k)}) = 0$) are considered *active*, while inequality constraints with negative values (*i.e.*, $\mathrm{g}_i(\mathbf{x}^{(k)}) < 0$) are *inactive*. Inequality constraints which are inactive at an optimal solution $\mathbf{x}^*$ can be removed without affecting the optimality of $\mathbf{x}^*$ (*i.e.*, $\mathbf{x}^*$ remains optimal), while removing inequality constraints which are active at $\mathbf{x}^*$ will likely change the optimality of $\mathbf{x}^*$ (*i.e.*, $\mathbf{x}^*$ will likely no longer be optimal). Likewise, removing *any* equality constraint (which must be satisfied at the optimal solution $\mathbf{x}^*$) will likely change the optimality of $\mathbf{x}^*$. Note that removing any equality constraint or inequality (whether active or inactive at the optimal solution $\mathbf{x}^*$) constraint from the optimization problem statement and then re-running the optimization would likely result in the generation of a different optimal solution.

Finally, *side constraints* must be developed to define the allowable ranges of design variable values. As design variable values control the component's design, side constraints define the range of possible designs. An example set of side constraints is depicted below:

$$a_k \le x_k \le b_k; \qquad k = 1 \text{ to N} \tag{2.6}$$

Note that side constraints are often incorporated into the set of inequality design constraints, with each side constraint converting into two inequality design constraints:

$$a_k - x_k \le 0 \tag{2.7a}$$

$$x_k - b_k \le 0 \tag{2.7b}$$

After the definition of side constraints, the problem formulation is complete. Hence, the standard design optimization problem statement is given by:

Find the vector $\mathbf{x}$ of design variable values

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}$$

which maximizes a utility function

$$\mathrm{U}(\mathbf{x}) = \mathrm{U}(x_1, x_2, \ldots, x_N)$$

subject to p equality constraints

$$\mathrm{h}_j(\mathbf{x}) \equiv \mathrm{h}_j(x_1, x_2, \ldots, x_N) = 0; \qquad j = 1 \text{ to } p$$

m inequality constraints

$$\mathrm{g}_i(\mathbf{x}) \equiv \mathrm{g}_i(x_1, x_2, \ldots, x_N) \le 0; \qquad i = 1 \text{ to } m$$

and N side constraints

$$a_k \le x_k \le b_k; \qquad k = 1 \text{ to N}$$

Hence, structural optimization procedures attempt to find the set of design variable values which maximizes an objective function subject to a set of equality and inequality constraints. The set of optimum design variable values represents the optimum component design, while the objective function measures the component's structural performance and the set of equality and inequality constraints represents limitations imposed on the component's structural behavior.

2.2.4 Structural Optimization Subroutines. After creating a mathematical formulation of the structural optimization problem, optimization is performed. The optimization is controlled by an *optimization algorithm*, which first chooses a set of initial design variable values. These variable values are then sent to the objective function, which uses a *modeler* to convert the variable values into a component design. The design is then subjected to *structural analysis* to determine its structural performance. Utilizing results of the structural analysis, the objective function calculates the design's utility and returns the value to the optimization algorithm. The optimization algorithm then modifies the design variable values in an attempt to increase the component's utility while satisfying all constraints. The process then iterates until the optimal component design is found (Figure 2.1).

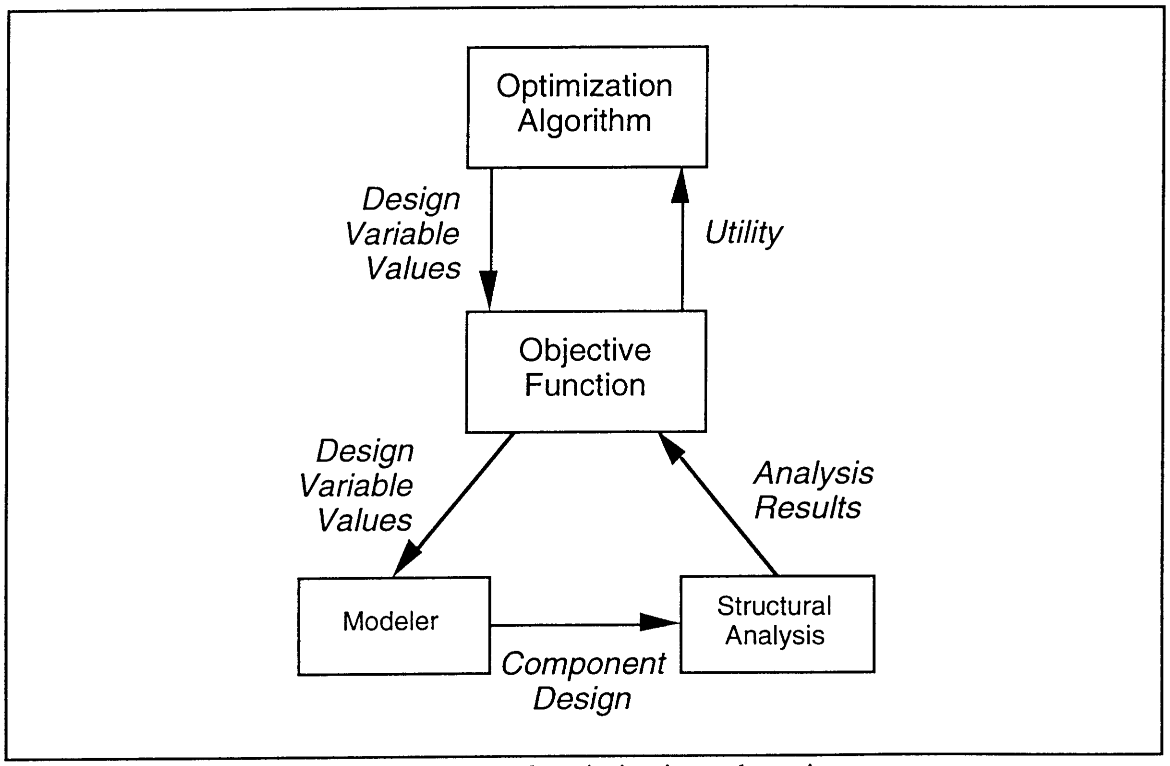


Figure 2.1: Interaction between structural optimization subroutines.

Optimization Algorithm. The main subroutine in a structural optimization procedure is the optimization algorithm. Having overall control of the structural optimization process, the optimization algorithm is responsible for guiding the search to the set of design variable values (corresponding to a particular component design and a particular location in the search space) which maximizes the objective function while satisfying all constraints. Optimization algorithms used in structural optimization procedures generally fall into one of four categories:

- *Mathematical Programming Techniques.* Often referred to as direct methods, mathematical programming techniques perform optimization by deterministically "climbing" to the top of the nearest locally-optimum peak in the search space. This peak corresponds to a set of locally-optimum design variable values and a locally-optimum component design. Search begins with the selection of a set of design variable values corresponding to an initial component design which does not necessarily satisfy any constraints or provide maximum utility. In an iterative process, the design variable values are modified so that the corresponding design's utility is increased and the constraints are satisfied. When all constraints are satisfied and the design can no longer be improved, the design and the design variable values are considered optimal.

  Specifically, any iteration k begins when the values and gradients of the objective function and constraint functions are calculated using the current set of design variable values, $\mathbf{x}^{(k)}$. This information is then used to compute a search direction $\mathbf{d}^{(k)}$ and step size $\alpha_{(k)}$, which indicate where in the search space the optimization algorithm should "move" to obtain maximum improvement in the design's quality. Finally, new design variable values, given by $\mathbf{x}^{(k+1)}$, are obtained using the equation:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha_{(k)}\mathbf{d}^{(k)} \tag{2.8}$$

  Mathematical programming algorithms, by utilizing gradient information, can obtain optimal component designs with relatively little computational expense. However, because these algorithms climb to the top of the nearest locally-optimum peak in the search space, they may avoid globally-optimum peaks in other areas of the search space. Hence, "optimum" component designs generated with mathematical programming algorithms are typically *local* optima. Therefore, mathematical programming-based structural optimizations should be executed with a variety of initial conditions to increase the likelihood of finding globally-optimum component designs.

Many mathematical programming algorithms, each using a different technique to calculate search direction and step size, have been developed for applications in a variety of fields (*e.g.*, engineering, science, and management). Because these algorithms are, in general, specially-tailored to particular problem domains, there is unfortunately no single mathematical programming technique which performs well in all domains. However, techniques such as Quadratic Programming, Sequential Linear Programming, and the Constrained Quasi-Newton method provide satisfactory structural optimization performance (Arora, 1989). Vanderplaats (1993, 1992a, 1992b) details several other mathematical programming algorithms often used to perform structural optimization, as well as state-of-the-art approximation methods which enhance the efficiency of mathematical programming-based structural optimizations.

For a comprehensive introduction to mathematical programming-based structural optimization, please refer to books by Kirsch (Kirsch, 1993), Haftka and Gürdal (Haftka and Gürdal, 1992), and Arora (Arora, 1989).

- *Optimality Criteria Methods.* Developed specifically for applications in structural optimization, optimality criteria methods (often referred to as indirect methods) use an iterative redesign algorithm to generate the component design which satisfies a set of optimality criteria defining the structural behavior required for optimality. Prior to optimization, these criteria are derived and then used to develop the iterative redesign procedure, which defines how any given design should be modified so that it better satisfies the optimality criteria. After the definition of the optimality criteria and the redesign procedure, optimization begins with the selection of an initial component design. Using the design variable values corresponding to the design, the optimality criteria are then evaluated to determine if the current design is optimal. If the criteria are satisfied by the design, optimization is complete. Otherwise, the design variable values are modified according to the redesign procedure and the process is repeated. Over many iterations, the design is gradually modified until the optimality criteria are satisfied.

There are two main components of the optimality criteria method, the first being a set of optimality criteria. Typically differential equations which must be satisfied by the optimal design, optimality criteria define the necessary, and in some cases sufficient, conditions for the optimality of a structure. Note that optimality criteria serve as necessary *and* sufficient conditions only in a limited number of practical structural optimization problems. These criteria, which must be derived prior to optimization, are based on either *intuitive* physical considerations, such as the requirement that each member in an optimal structure be subjected to its maximum allowable stress, or on *rigorous* mathematical considerations, such as the Kuhn-Tucker conditions (Arora, 1989).

The second required component of the optimality criteria method is an iterative redesign procedure. This procedure is a mathematical expression which defines how any given set of design variable values should be modified so that the corresponding design better satisfies the optimality criteria. Specifically, at any iteration k, the redesign procedure relates the current set of design variable values $\left(\mathbf{x}^{(k)}\right)$ to a new set of design variable values $\left(\mathbf{x}^{(k+1)}\right)$ which correspond to a design of higher quality:

$$\mathbf{x}^{(k+1)} = \mathbf{f}\left(\mathbf{x}^{(k)}\right) \tag{2.9}$$

As in the derivation of optimality criteria, the iterative redesign procedure can be based on either *intuitive* heuristics, such as the stipulation that material should be removed from members in a structure which are not fully stressed, or on *rigorous* mathematical considerations such as the Kuhn-Tucker conditions.

Hence, depending upon the techniques used to derive the optimality criteria and the redesign procedure, optimality criteria methods are classified as either intuitive or rigorous. Intuitive methods, which motivated much of the initial interest in optimality criteria techniques and led to the development of rigorous optimality criteria methods, are typically more efficient than rigorous methods. However, while rigorous methods always converge to either locally- or globally-optimal solutions, intuitive methods can converge to non-optimal solutions. Also, intuitive methods are less general than rigorous methods.

The optimality criteria and redesign procedures used for a given structural optimization problem depend upon the structure's boundary and loading conditions, functional requirements, performance constraints, and anticipated structural behavior, as well as the particular design criteria used in utility evaluations. Consequently, a large number of optimality criteria and redesign procedures have been developed for various structural optimization problems (Rozvany, 1989). An early, intuitive optimality criteria method is the Fully Stressed Design (FSD) technique (Rozvany (1989), Kirsch (1993), Haftka and Gürdal (1992)), where material is removed from structural members which are not subjected to their maximum allowable stress. Other early, intuitive optimality criteria include Simultaneous Failure Mode (SFM) and Uniform Energy Density (UED) (Rozvany (1989), Haftka and Gürdal (1992)). Modern optimality criteria methods typically use a rigorous optimality criterion based on the Kuhn-Tucker conditions and a redesign procedure based on heuristics (Haftka and Gürdal (1992)).

Because optimality criteria methods were designed specifically to perform structural optimization, they offer performance and efficiency advantages over mathematical programming techniques. Additionally, while mathematical programming techniques have a maximum problem size of approximately 10^2 design variables (Rozvany *et al.* (1993), Rozvany and Zhou (1991a, 1991b), Zhou and Rozvany (1991)), optimality criteria methods can be applied to problems with *several million* design variables (Rozvany and Zhou, 1991b). However, the performance of optimality criteria methods is limited by the number of displacement (and in some cases stress) constraints in the problem (Rozvany *et al.*, 1993). Also, while the specialized nature of optimality criteria methods enhances their structural optimization performance, it makes them unsuitable for use in other optimization domains (*e.g.*, science and management applications) and necessitates lengthy analytical derivations of optimality criteria for each type of structure and design condition (Rozvany and Zhou, 1991a).

Optimality criteria methods have received only limited acceptance in the structural optimization community because, unlike mathematical programming techniques, they are not necessarily mathematically rigorous. Hence, depending upon the particular optimality criteria selected for a given optimization problem, convergence to a local optimum is not always guaranteed with optimality criteria methods. As in mathematical programming-based structural optimizations, optimizations using optimality criteria techniques should be initiated with a variety of initial conditions to increase the likelihood of generating a globally-optimum component design. While mathematical programming techniques and optimality criteria methods are generally considered to be two distinct types of optimization algorithms, Fleury and Geradin (1978) have demonstrated similarities between the formulations of particular mathematical programming techniques and optimality criteria methods.

Rozvany (1989), Haftka and Gürdal (1992), Kirsch (1993), and Kirsch (1981) provide comprehensive introductions to optimality criteria techniques.

- *Simulated Annealing.* Simulated annealing is a global optimization procedure based on statistical mechanics (Kirkpatrick *et al.*, 1983). In a simulated annealing optimization, a set of initial design variable values is first brought to an elevated energy state by a high "control temperature." At this high energy state, the design variable values may change (*i.e.*, the component design may be modified) with ease, much like atoms at an elevated energy state move throughout their domain with ease. The optimization then progresses by slowly reducing the control temperature to a minimum value. As in an actual annealing process, where atoms tend to place themselves in ground states (*i.e.*, minimal energy configurations) as the temperature reaches its final value, it is hoped that gradually reducing the control temperature of a simulated annealing optimization will enable the set of design variable values to place itself at a search space location corresponding to a globally-optimum component design.

Simulated annealing optimization begins with the selection of a set of design variable values corresponding to an initial component design. Using these design variable values, objective function and constraint calculations are performed, and the design's "energy" (E), or cost, is determined. Note that a particular design's energy is a measure of how well the design minimizes or maximizes the objective function without violating any constraints. Because simulated annealing is a minimization process, a set of design variable values corresponding to a high-quality component design should receive a low energy value, while design variable values corresponding to a low-quality design should receive a high energy value. After evaluating the energy of the current component design, a new design in the vicinity of the current design is chosen (*i.e.*, the current design is *perturbed* to create a new design), and the new design's energy (E') is calculated. If the new design's energy is less than or equal to that of the current design, the new design is automatically accepted to replace the current design. Otherwise, if the new design's energy is greater than that of the current design, the new design is accepted with a probability (P) of:

$$P = e^{-\left(\frac{E'-E}{T}\right)}, \tag{2.10}$$

where T represents the artificial control temperature. After the new design is either accepted or rejected, the process (*i.e.*, perturbing the current design to create a new design, calculating the new design's energy, comparing the new design's energy to that of the current design, and either accepting or rejecting the new design) is repeated for a pre-determined number of iterations at the current control temperature. The control temperature is then decreased, and the entire process then iterates until the temperature reaches a minimum value.

As shown in Equation 2.10, the probability of accepting an inferior design decreases as the control temperature decreases. Hence, during the initial stages of a simulated annealing optimization, the algorithm will often accept new, inferior component designs with the hope that the new design might lead to a globally-optimum design elsewhere in the search space. However, as the search progresses and the control temperature decreases, the algorithm becomes less inclined to explore new, inferior regions of the search space. During the final stages of optimization, the algorithm is heavily biased towards fine-tuning the current design instead of exploring new areas of the search space.

As in actual annealing processes, the rate at which the control temperature is decreased has a large effect on the optimization's outcome. Quickly reducing the temperature minimizes the number of iterations needed to obtain a solution, but the solution will likely be sub-optimal. Conversely, solutions obtained with a slow, gradual temperature reduction are typically locally- or globally-optimum. However, many iterations will be required. The particular frequency and magnitude of temperature reductions, as well as the initial and final control temperatures, are dictated by an *annealing schedule* (sometimes referred to as a *cooling schedule*). Initial control temperatures should be chosen so that virtually all new component designs are accepted, while final control temperatures should result in the acceptance of only those designs which improve on the current design. Aarts and van Laarhoven (1987) and Delyon (1988) detail simulated annealing convergence theory and cooling schedules.

By not using gradient information and by probabilistically accepting inferior component designs with the hope that the inferior designs may eventually lead to globally-optimal designs, simulated annealing optimizations are not usually trapped by local optima—convergence to globally-optimum designs may occur. Hence, unlike mathematical programming techniques and optimality criteria methods, there is no need to start the optimization with a variety of initial conditions in an attempt to obtain a globally-optimum design. Unfortunately, the slow, gradual control temperature decreases required to obtain global optima result in high computational expense.

Simulated annealing is a robust optimization technique applicable to a variety of domains. Unlike optimality criteria methods, no major modifications must be made to the search algorithm when applying simulated annealing to a new domain—only the design variable representation, design perturbation technique (*i.e.*, the technique used to find a new design in the vicinity of the current design), and the energy evaluation function require modification. Because simulated annealing algorithms do not use gradient information, they are ideally suited to multi-modal search spaces and domains using discrete design variables.

For a detailed overview of simulated annealing, please refer to the textbooks by van Laarhoven and Aarts (1987) and van Laarhoven (1988).

- *Genetic Algorithm.* The genetic algorithm is a global optimization procedure based on the theory of natural selection. Optimization occurs through an evolutionary process—populations of chromosomes, where each chromosome represents a possibly-optimal set of design variable values (*i.e.*, a possibly-optimal component design), are created in generations, with child populations arising from parent populations. One "generation" of evolution, or search, begins when a merit function individually evaluates the "fitness" of each chromosome (*i.e.*, component design) in a parent population. The most highly-fit component designs in the parent population are then selected to serve as parents. These parent designs pair and mate via genetic crossover, with each pair of parent designs creating two child designs which each possess traits from both parents. Component designs in the resulting child population are then subjected to infrequent, random mutation, and the child population then replaces the parent population. The process then iterates. After many generations of evolution, the overall quality of component designs should increase because better design characteristics are more likely to propagate into child generations.

This investigation uses the genetic algorithm to perform structural optimization. For a comprehensive introduction to and description of the genetic algorithm, please refer to Section 2.3.

Modeler. When an objective function receives a set of design variable values for utility evaluation, a modeler is used to convert the variable values into their corresponding component design. Conversion begins when the value of each design variable is assigned to the particular component attribute which it controls. Note that relationships between design variables and component attributes are stipulated during problem formulation (Section 2.2.3), and that design variables control various attributes of a component's design (Section 2.2.5). This assignment of component attribute values results in the generation of the component design corresponding to the current set of design variable values. So that the design's structural performance can be evaluated, the modeler then creates a mathematical description of the design. This description, commonly referred to as a geometric model (Mortenson, 1985), is a mathematical representation of the component's geometric characteristics. In most cases, the mathematical representation is a finite element mesh which defines the component's size, shape, and topology. After creating the component's geometric model, the conversion process is complete and the geometric model is then sent to structural analysis routines for evaluation.

Hence, the modeler converts a set of design variable values into a mathematical representation of the component design corresponding to the design variable values. Because this conversion process is specialized, in that it depends upon the geometric characteristics of the component undergoing optimization, the attributes of the component which are allowed to vary, and the design representation requirements of the structural analysis package, there is no single modeler which works for all optimization problems. In fact, most structural optimization problems will necessitate the development of a modeler which is specially tailored to the particular problem.

Structural Analysis. After the current set of design variable values is converted into a mathematical description of the design (*i.e.*, a geometric model), structural analysis is performed. As mentioned previously, most mathematical descriptions are created in the form of a finite element mesh, and most structural analyses are performed using the finite element method (Bathe (1982), Zienkiewicz (1989)). During optimization, when providing finite element routines with a finite element mesh defining the component's design, material properties (*e.g.*, Young's Modulus, Poisson's Ratio) and boundary conditions (*e.g.*, applied loads, support points) must also be supplied. In addition to the finite element method, the boundary element method (Brebbia (1978), Banerjee and Butterfield (1981)) has demonstrated satisfactory structural analysis performance when used in structural optimizations (Sandgren and Wu, 1988). Note that while analytical structural analysis techniques may suffice when optimizing simple structures, nearly all practical problems require numerical analysis techniques such as the finite element or boundary element methods.

2.2.5 Structural Optimization Categories. As detailed in Section 2.2.3, design variables control different attributes of a component's design, and assigning numerical values to the design variables results in the generation of a component design corresponding to the design variable values. Depending upon the type of component attributes controlled by the design variables in a particular structural optimization, the optimization is considered a *sizing*, *shape*, or *topology* optimization. Hence, structural optimization routines iteratively modify either a design's size, shape, or topology until the design exhibits maximum utility subject to performance constraints.

Sizing Optimization. Sizing optimization, the least-complex of the three structural optimization categories, performs optimization by holding a design's shape and topology constant while modifying specific dimensions of the design. Hence, the design variables control particular dimensions of the design, and the values of the design variables define the values of the dimensions. Optimization therefore occurs through the determination of the design variable values which correspond to component dimensions providing optimum structural behavior.

Examples of sizing optimization include the calculation of an optimum cylinder wall thickness, truss member cross-sectional area, or column diameter. Figure 2.2 depicts an example sizing optimization of a beam cross-section. Prior to optimization, the engineer must first define the component's material properties (*e.g.*, Young's Modulus, Poisson's Ratio) and boundary conditions (*e.g.*, simply supported beam under pure bending). The engineer must then specify the structure's shape and topology and indicate which dimensions shall be optimized. In this example, the engineer specified that the beam would have an I-beam type of shape, that no interior holes would exist, and that the web height, flange width, and flange thickness should be optimized. Using the objective function and constraints provided by the engineer (*e.g.*, minimize mass subject to a maximum stress constraint), sizing optimization then determines the optimal dimension values. Note that while the "optimal" answer is likely either locally- or globally-optimal in the search space established by the problem formulation, changes to the beam's shape and topology could possibly provide even higher structural performance.

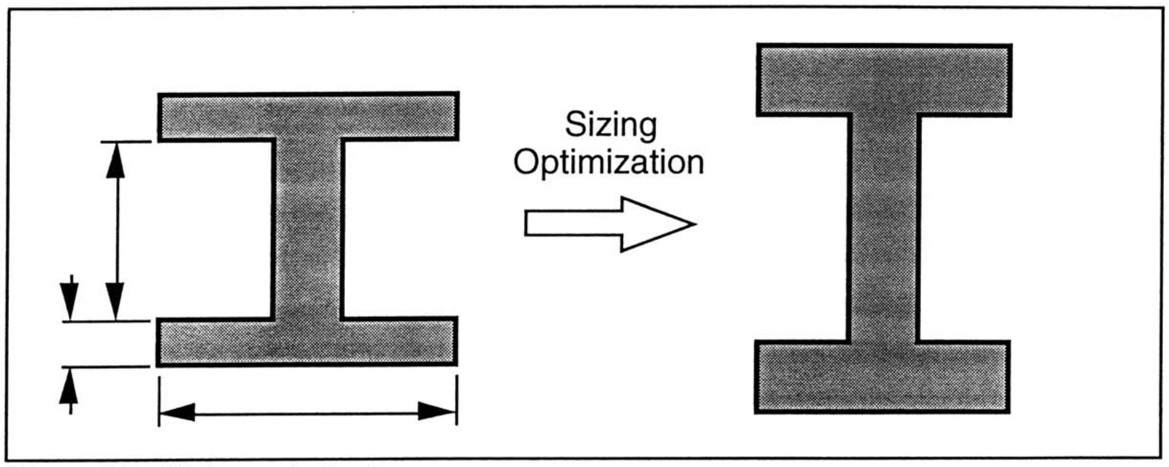


Figure 2.2: Sizing optimization.

Shape Optimization. Shape optimization, which is of intermediate difficulty, performs optimization by holding a design's topology constant while modifying the design's shape. Hence, the design variables control the design's shape, and the values of the design variables define the particular shape of the design. Optimization therefore occurs through the determination of the design variable values which correspond to the component shape providing optimal structural behavior. Note that sizing optimization typically occurs as an incidental byproduct of the shape optimization process.

In nearly all shape optimization problems, the design variables control the coordinates of control points which define the component's size and shape. Therefore, a particular set of design variable values corresponds to a particular set of control point coordinates. For example, if optimizing the shape of a discrete-member truss, each pair of design variables will control the x- and y-coordinates of a particular node in the structure. When working with more general continuum structures such as rods, brackets, or plates, Braibant and Fleury (1984) demonstrated that a component's size and shape can be represented by a collection of B-Spline or Bézier curves (Farin, 1993), where the curve control point locations are controlled by the design variables.

Examples of shape optimization include the determination of the optimum node locations in a 10-bar truss, the optimum fillet radius in a bracket, or the optimum shape of a rod in tension. Figure 2.3 depicts an example shape optimization of a beam cross-section. Prior to optimization, the engineer must first define the component's material properties (*e.g.*, Young's Modulus, Poisson's Ratio) and boundary conditions (*e.g.*, simply supported beam under pure bending). The engineer must then specify the structure's topology and indicate which portions of the component's shape shall be optimized. In this example, the engineer specified that the beam would have no interior holes and selected the entire boundary for shape optimization. Using the objective function and constraints provided by the engineer, shape optimization determines the optimum cross-section size and shape. Note that while the "optimal" answer is likely either locally- or globally-optimal in the search space established by the problem formulation, changes to the beam's topology could possibly provide even higher structural performance.

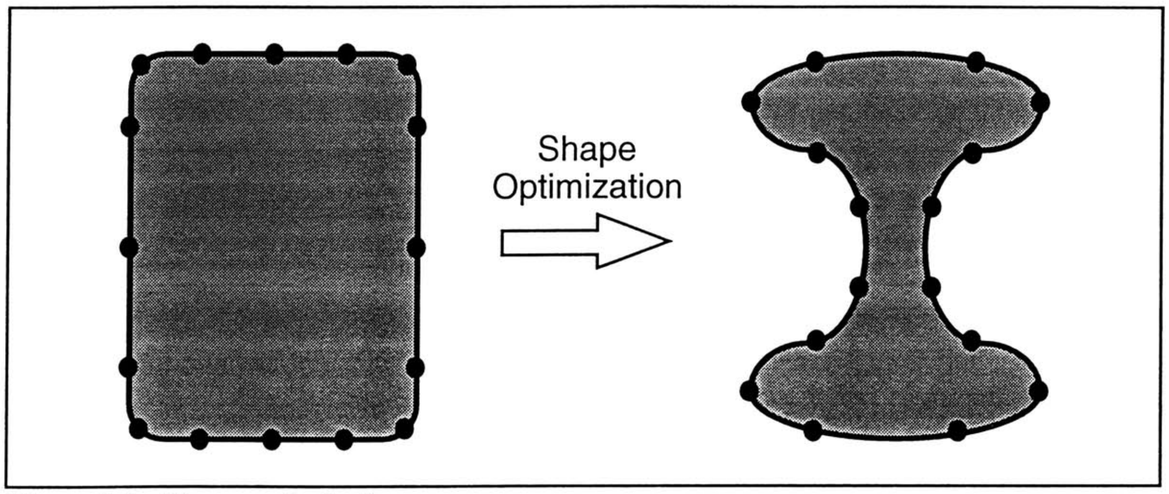


Figure 2.3: Shape optimization.

Topology Optimization. Topology optimization, perhaps the most difficult of the three structural optimization categories, performs optimization by modifying the topology of a design. Hence, the design variables control the design's topology, and the values of the design variables define the particular topology of the design. Optimization therefore occurs through the determination of the design variable values which correspond to the component topology providing optimal structural behavior. Note that sizing and shape optimization typically occur as incidental byproducts of the topology optimization process.

While a set of design variables can easily control a design's size or shape (the design variable values are simply assigned to their corresponding dimension sizes or control point locations), controlling a design's topology is considerably more difficult. In addition to controlling the design's outer boundary, the design variables must create and remove, as well as define the size and shape of, any number of holes in the design's interior. To allow for flexibility in creating, reshaping, and removing internal holes during the course of optimization, the optimization algorithm could use a varying number of design variables, where new design variables are added to control the location and shape of newly-created internal holes and existing design variables are deleted if their corresponding hole is removed. Unfortunately, the computational complexity of this design variable representation renders it unsuitable for topological optimization.

A more general, less complex design representation used by nearly all topological optimization algorithms treats the problem as a *configuration design* problem, where the overall, complex design is created by assembling a large number of basic elements, or "building blocks." By beginning with a set of building blocks representing the structure's maximum allowable "design domain" (*i.e.*, region in space which the structure may occupy) and then allowing each block to either exist or vanish, a unique design is created. Hence, a design is modified by simply changing the states of individual building blocks making up the design. During topological optimization, a design's building blocks are controlled by the design variables, where the value of each design variable determines the existence and characteristics of its corresponding building block. For example, in the topological optimization of a cantilevered plate, the plate is typically discretized into small, rectangular elements, where each element is controlled by a design variable which can vary continuously between 0 and 1. When a particular design variable has a value of 0, the corresponding element is assumed to be a hole. Likewise, when a design variable is equal to 1, its corresponding element contains fully-dense material. Lastly, design variables with intermediate values correspond to elements containing material of intermediate density. So, to create a hole at a particular location in a design, the design variable corresponding to the element at that location is simply set equal to zero. To make the hole larger, the design variables corresponding to elements immediately surrounding the hole are also set equal to zero. Similarly, holes are removed from a design by assigning non-zero values to the design variables corresponding to the elements at the hole location. Hence, because no addition or deletion of design variables is required during optimization, this building block design representation is suitable for topology optimization problems. Chapter 3, which discusses previous work in structural topology optimization, further details the design representations typically used in topological optimization problems.

Figure 2.4 depicts an example topology optimization of a beam cross-section. Prior to optimization, the engineer must first define the component's material properties (*e.g.*, Young's Modulus, Poisson's Ratio) and boundary conditions (*e.g.*, simply supported beam under pure bending). The engineer must then specify the structure's design domain, representing the region which the structure may occupy. In this example, the engineer specified that the beam cross-section must be contained within a rectangular design domain. Using the objective function and constraints provided by the engineer, topology optimization determines the optimum size, shape, and topology of the beam cross-section. Note that the "optimal" answer is in some sense a global optimum, because every aspect of the design (size, shape, topology) was allowed to vary during optimization.

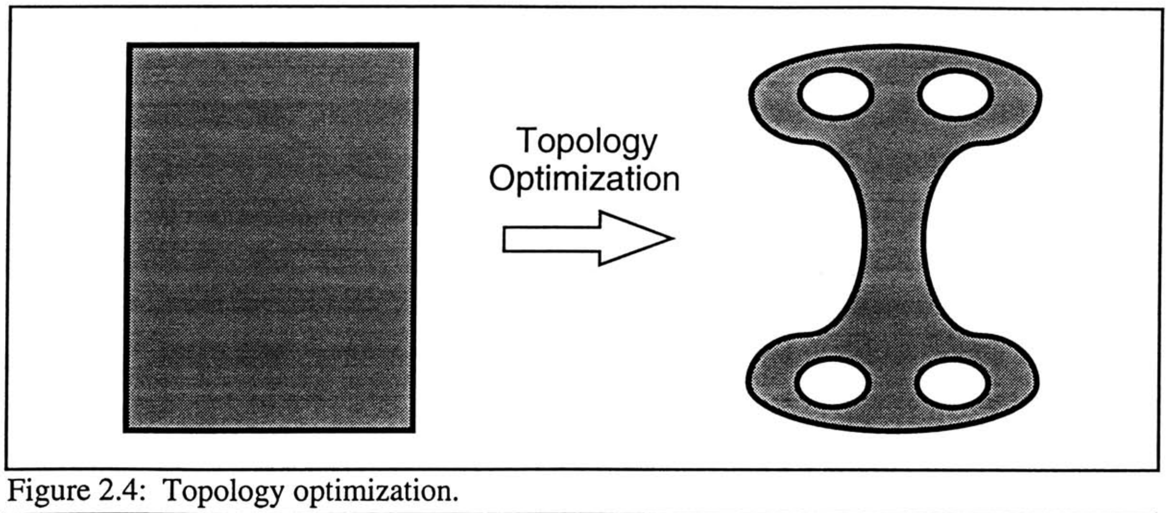


Figure 2.4: Topology optimization.

2.3 The Genetic Algorithm

2.3.1 Introduction. The genetic algorithm (GA) is an optimization strategy based on the theory of natural selection (Holland, 1975). Modeled after biological systems, where populations of organisms evolve (over many generations) so as to best survive in their environment, the genetic algorithm performs optimization through the "evolution" of a population of artificial organisms. These organisms, which each represent a possibly-optimal solution to the optimization problem, compete against one another to best solve the problem. Those organisms which are most highly-fit are allowed to serve as parents, mating with each other to create child organisms. After undergoing random mutation, the child organisms replace the parents, and the evolutionary process iterates. Optimization occurs, therefore, when many generations of evolution improve the quality of the artificial organisms.

2.3.2 Similarities with Biological Systems. In addition to its evolutionary, "survival-of-the-fittest" optimization mechanism, the genetic algorithm shares other similarities with biological systems. For example, each organism in a genetic algorithm population is typically represented by a character string analogous to a chromosome, with each character position being analogous to a gene and each character value being analogous to an allele. As in biological systems, genes in a genetic algorithm chromosome correspond to particular organism traits (*e.g.*, a gene corresponding to hair color), while allele values control trait characteristics (*e.g.*, an allele value corresponding to brown hair).

The genetic algorithm also uses reproductive and mutation operators which are modeled after the mechanics of chromosome recombination and mutation found in biological genetics.

2.3.3 Design Variables as Chromosomes. As detailed above, the genetic algorithm performs optimization through the evolution of a population of chromosomes. Each chromosome in a population is a coded representation of the optimization problem's design variables, with the chromosome's genes corresponding to the design variables and the genes' allele values controlling the design variable values. Note that there isn't necessarily a 1-to-1 correspondence between genes and design variables—as detailed below, genes in a chromosome are typically grouped into multi-gene sub-strings, or "parameters," where each parameter corresponds to, and controls the value of, a particular design variable. Consequently, the number of genes in a chromosome is typically much greater than the number of design variables controlled by the chromosome, while the number of parameters in a chromosome is exactly equal to the number of design variables controlled by the chromosome.

Hence, a chromosome with a particular set of allele values corresponds to a particular set of design variable values, and each chromosome in a genetic algorithm population therefore corresponds to a possibly-optimal set of design variable values. In other words, each chromosome in a genetic algorithm population represents a particular point in the search space, and a population of chromosomes represents a population of points in the search space. By evolving a population of chromosomes, where each chromosome represents a set of possibly-optimal design variable values, the genetic algorithm is able to locate the set of optimal design variable values which minimizes or maximizes the given objective function.

Because every optimization problem has different numbers and types of design variables, there is no single chromosome structure or chromosome-to-design-variable decoding algorithm. In fact, one of the most difficult aspects of configuring a genetic algorithm system is developing a chromosome structure and chromosome-to-design-variable decoding which work well.

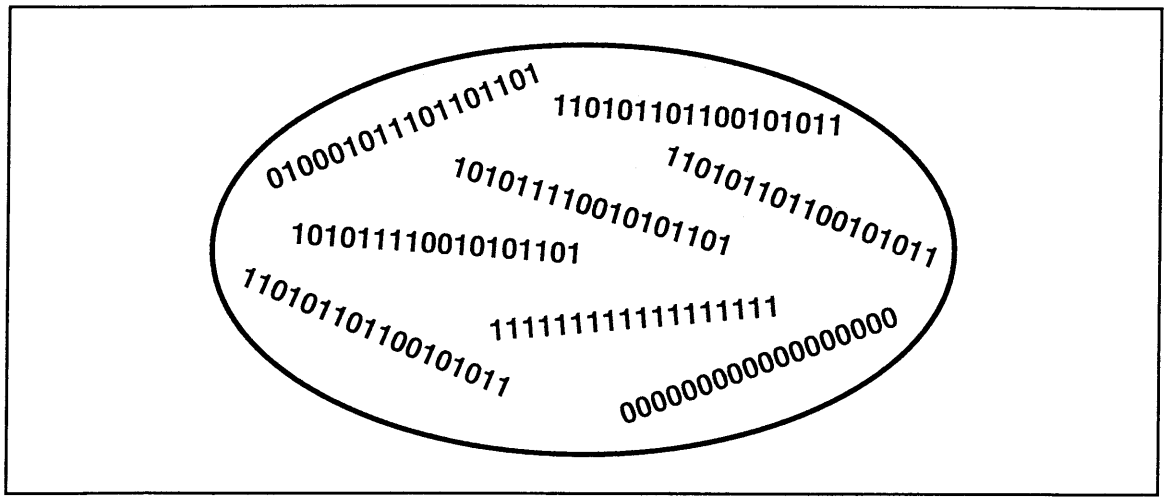


Figure 2.5: A typical genetic algorithm population.

Fortunately, there is a general chromosome structure which is applicable to most optimization problems. As shown in Figure 2.5 (which depicts a genetic algorithm population with eight 18-bit chromosomes), genetic algorithm chromosomes are typically one-dimensional strings of genes, where the allele values (*i.e.*, the values assigned to each gene location) are binary in nature—they can be equal to either 0 or 1. Because chromosomes must each define the value of all design variables in the optimization, they are divided into parameters, where the number of parameters in each chromosome equals the number of design variables in the optimization. So, while appearing to be large, multi-bit binary numbers, chromosomes are actually concatenations of many smaller multi-bit binary numbers (parameters), each of which corresponds to, and controls the value of, a particular design variable.

For example, the 18-bit chromosomes of Figure 2.5 could be comprised of one 18-bit parameter (controlling 1 design variable), two 9-bit parameters (controlling 2 design variables), three 6-bit parameters (controlling 3 design variables), one 6-bit parameter and one 12-bit parameter (controlling 2 design variables), or any other combination of parameters containing a total of 18 bits. The length (in bits) of any given parameter depends upon the range, and possibly resolution, of its corresponding design variable's allowable values. For example, the length of a parameter controlling an integer-valued design variable depends upon the range of the design variable's allowable values: a 2-bit parameter can control a design variable which varies between 0 and 3 (or any other range containing 4 values), while a 6-bit parameter can control a design variable which varies between 1 and 64 (or any other range containing 64 values). Likewise, the length of a parameter controlling a floating-point design variable depends upon the range and desired resolution of the design variable's allowable values: a 4-bit parameter can control a design variable which varies between 0 and 3 in increments of 0.2, while a 5-bit parameter can control a design variable which varies between 0 and 3 in increments of 0.09677 or a design variable which varies between 0 and 6 in increments of 0.1935. In general, the required length (in bits) of a parameter controlling an integer-valued design variable (X_i) with an allowable range of $(V_{MIN} \le X_i \le V_{MAX})$ is given by:

$$l = \frac{1}{\ln 2} \cdot \ln\left[\left(V_{MAX} - V_{MIN}\right) + 1\right], \tag{2.11}$$

while the required length (in bits) of a parameter controlling a floating-point design variable (X_i) with an allowable range of $(V_{MIN} \le X_i \le V_{MAX})$ and a desired resolution of (Resolution = *Increment*$_i$) is given by:

$$l = \frac{1}{\ln 2} \cdot \ln\left[\left(\frac{V_{MAX} - V_{MIN}}{Increment_i}\right) + 1\right]. \tag{2.12}$$

Note that the programmer must specify, *a priori*, the number of parameters in each chromosome and the length (in bits) of each parameter.

When using binary allele values, a chromosome of length l (in bits) corresponds to a search space containing 2^l distinct locations. Hence, when developing a chromosome structure for a particular optimization, care must be taken to minimize chromosome length. The number of design variables should be kept to a minimum, as should design variable ranges and resolution.

To decode a chromosome (with the structure defined above) into its corresponding design variable values, the chromosome must first be divided into its multi-bit parameters. The binary-valued parameters are then converted into either integer or floating-point numbers, depending upon the nature of the corresponding design variables. Note that chromosomes can simultaneously represent both integer and floating-point design variables. While converting a binary-valued parameter into an integer-valued design variable value is relatively straightforward, conversions to floating-point design variable values require slightly more effort. When decoding a parameter of length l into a floating-point design variable $(X_i\text{:}\ \ V_{MIN} \le X_i \le V_{MAX})$, the design variable's value is given by:

$$X_i = \left(ParameterValue \cdot \frac{V_{MAX} - V_{MIN}}{2^l - 1} \right) + V_{MIN}, \tag{2.13}$$

where *ParameterValue* represents the value of the (binary-valued) parameter after conversion into decimal.

As an example, Figure 2.6 details the decoding process for a chromosome representing 3 design variables. Parameter #1, controlling the value of an integer design variable $(X_1\text{:}\ \ 0 \le X_1 \le 7)$, is 3 bits in length and easily converts from binary into the corresponding integer design variable value. Likewise, Parameter #3 controls an integer design variable $(X_3\text{:}\ \ 0 \le X_3 \le 31)$ and is 5 bits in length. Parameter #2, controlling the value of a floating-point design variable $(X_2\text{:}\ \ 1.0 \le X_2 \le 6.0)$, is 10 bits long and converts from a binary-valued parameter into a floating-point design variable value according to Equation 2.13:

$$X_2 = 485 \cdot \frac{6.0 - 1.0}{2^{10} - 1} + 1.0 = 3.37 \tag{2.14}$$

Once a chromosome's parameters have been converted into their corresponding design variable values, the chromosome decoding process is complete.

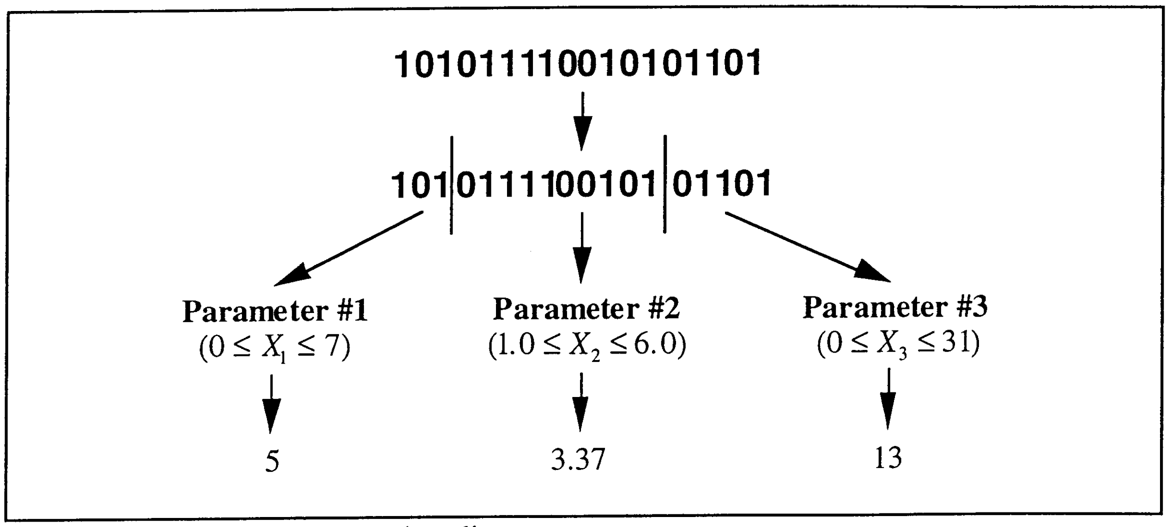


Figure 2.6: The chromosome decoding process.

2.3.4 Optimization Through Evolution. Again, the genetic algorithm uses an evolutionary, "survival-of-the-fittest" optimization mechanism. A population of chromosomes is allowed to evolve, with generations of child chromosomes arising from, and replacing, parent generations. Chromosomes in a parent generation are first *evaluated* for fitness, or quality. Highly-fit chromosomes are then *selected* to serve as parents of the subsequent child generation. *Reproduction* occurs when parent chromosomes then pair and mate to create a new generation of child chromosomes. *Mutation* is then performed on the child chromosomes. Finally, the child generation replaces the parent generation, and the process iterates. After many generations of evolution, the overall quality of chromosomes should increase because characteristics of better chromosomes are more likely to propagate into child generations.

Hence, the genetic algorithm utilizes genetics-based operators such as evaluation, selection, reproduction, and mutation to evolve a population of artificial chromosomes (Figure 2.7). Because each chromosome in the population corresponds to particular set of design variable values (and therefore a possibly-optimal solution to the optimization problem), evolution of the chromosomes results in optimization of the design variables. As in the evolution of actual biological systems, genetic algorithm-based optimization takes place over hundreds, if not thousands, of "generations."

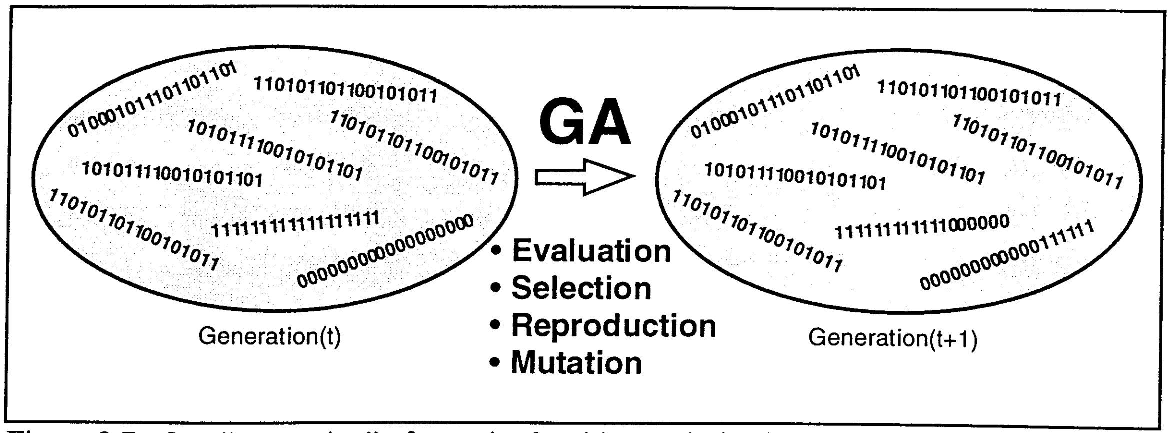


Figure 2.7: One "generation" of genetic algorithm optimization.

Evaluation. One "generation" of optimization begins with *evaluation*, where the "fitness," or quality, of each chromosome in the parent generation is determined. As in biological systems, where organisms are implicitly "graded" according to their prey-catching and predator-avoidance abilities, the genetic algorithm assigns fitness values to chromosomes according to their objective function minimization or maximization abilities.

Fitness evaluation is performed by a "fitness function," which takes an individual chromosome as input, decodes the chromosome into its corresponding design variable values (as detailed in Section 2.3.3), evaluates the objective function using the design variable values, and then returns a fitness value quantitatively describing the extent to which the design variable values minimized or maximized the objective function without violating any constraints (Figure 2.8).

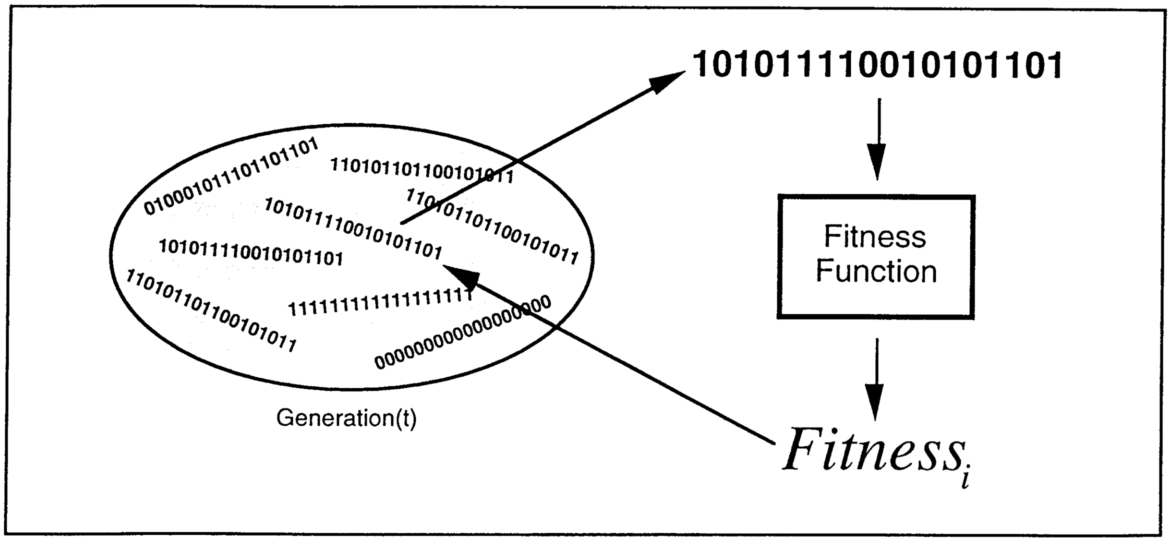


Figure 2.8: Fitness evaluation.

As with the chromosome decoding process, every optimization problem has a unique objective function, and therefore no single algorithm exists for converting the results of an objective function evaluation (using a particular chromosome's design variable values) into a fitness value (for the chromosome). In general, however, calculations must be performed so that high fitness is assigned to chromosomes corresponding to design variable values which do an excellent job of minimizing or maximizing the objective function while satisfying all constraints. Likewise, low fitness should be assigned to chromosomes corresponding to design variable values which violate constraints and/or do a poor job of minimizing or maximizing the objective function. Note that the genetic algorithm is naturally a *maximization* algorithm. Hence, if you wish to minimize an objective function, the fitness function value must increase as the objective function value decreases and must decrease as the objective function value increases. This is typically accomplished by setting the fitness value equal to the reciprocal of the objective function value or equal to the objective function value subtracted from a large constant.

After the fitness function returns the chromosome's fitness to the genetic algorithm routines, the fitness value is assigned to the chromosome. The evaluation process is then repeated for all chromosomes in the population.

Note that, particularly in early generations, large differences in fitness values will exist between chromosomes in any given generation. These differences, if unaddressed, would allow the highest-performance chromosomes to quickly dominate the parenting process, filling subsequent generations with a particular "breed" of possibly-suboptimal design variable values. At the same time, low-performance chromosomes, which likely contain some desirable genetic material, would quickly die off. To prevent this, "fitness scaling" (Goldberg, 1989a) is performed to attenuate fitness value differences. In the procedure, the fitness of high-performance chromosomes is slightly reduced while the fitness of low-performance chromosomes is increased.

Selection. After fitness scaling has been completed, *selection* is performed to determine which chromosomes in the parent generation will serve as parents of the child generation (Figure 2.9). As in actual biological systems, where organisms highly adept at catching prey and avoiding predators are more likely to survive and create offspring, highly-fit genetic algorithm chromosomes are more likely to serve as parents of child generations.

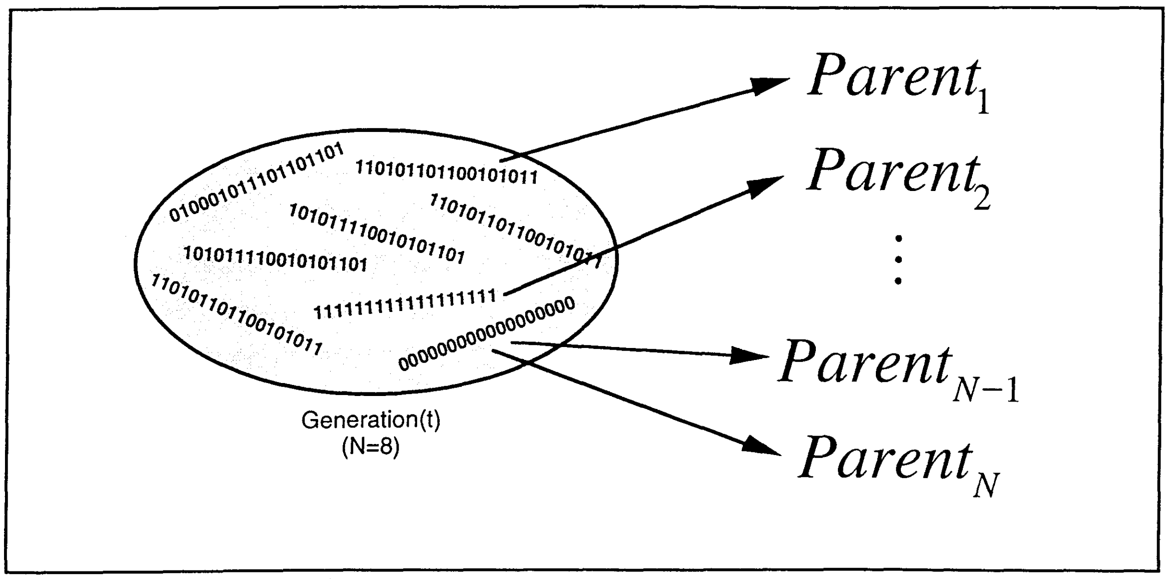


Figure 2.9: Parent selection.

During selection, the number of chromosomes selected to serve as parents must equal the number of chromosomes in the population. However, because the genetic algorithm attempts to obtain increasingly-fit generations of chromosomes, highly-fit chromosomes are likely to serve as a parent multiple times in a single generation, while low-fitness chromosomes are likely to "die off" and not serve as a parent. Additionally, while the selection algorithm must choose a proper number of highly-fit parent chromosomes to insure that child chromosomes will be highly-fit, it must also choose a number of average-to-poor-quality chromosomes in the hope that they contain *some* desirable genetic material.

The number of times a chromosome serves as a parent in one generation is based on its fitness performance in relation to the performance of other chromosomes in the parent generation. As detailed in Baker (1987), the selection process must first determine the "expected value" of each chromosome in the parent generation, and it must then convert the expected values into discrete numbers of parentings. For example, if a chromosome has an expected value of 1.8, it should, on average, serve as a parent 1.8 times in one generation. Note that a chromosome's expected value is equal to its fitness divided by the average fitness of the entire population:

$$ExpectedValue_i = \frac{N \cdot Fitness_i}{\sum_{j=0}^{N} Fitness_j} \tag{2.15}$$

The most straightforward selection algorithm is "Stochastic Sampling with Replacement" (Baker, 1987), otherwise known as "Roulette Wheel" selection. In the algorithm, a "roulette wheel" with an area equal to the number of chromosomes in the population (N) is created. Each chromosome in the population is allocated a slice on the wheel, where the area of the slice is exactly equal to the chromosome's expected value. To choose a parent, a random location on the wheel is chosen (*i.e.*, the wheel is "spun"), and the chromosome located at that position is selected to serve as a parent. To select the N required parent chromosomes, the wheel is simply "spun" N times.

Reproduction. After a group of parents have been selected, *reproduction* must take place to create a generation of child chromosomes. The genetic algorithm's reproduction operator is modeled after biological systems, where "evolution" occurs when highly-fit parent organisms mate to create child organisms (containing genetic material from both parents) which are of higher fitness (*i.e.*, better adapted to their environment) than the parents. Similarly, the genetic algorithm allows pairs of parent chromosomes to mate, creating pairs of child chromosomes containing genetic material from both parents. It is hoped that when two highly-fit chromosomes combine their genetic material to create two children, at least one of the child chromosomes will contain the best "attributes" of both parents, resulting in a chromosome of exceptionally high fitness.

A great amount of previous and ongoing research (Goldberg (1989a), Holland (1975)) has proven that the genetic algorithm's reproduction operator does indeed propagate the best genetic material from generation to generation. Specifically, the *Schema Theorem* (Goldberg, 1989a) states that the genetic algorithm "allocates exponentially increasing (decreasing) numbers of trials to above- (below-) average schemata." Schemata, or genetic "building blocks," are "similarity templates" describing subsets of chromosomes with similar allele values at particular gene locations. In other words, if the genetic algorithm finds that highly-fit chromosomes generally have similar allele values at a particular set of gene locations, it will propagate this high-fitness group of genes to exponentially increasing numbers of child chromosomes. Likewise, if the genetic algorithm finds that low-fitness chromosomes generally have similar allele values at a particular set of gene locations, it will propagate this low-fitness group of genes to exponentially decreasing numbers of child chromosomes. The beauty of the genetic algorithm is that the propagation of highly-fit genetic building blocks is performed in parallel (*i.e.*, many highly-fit building blocks are propagated at once), and it is performed *automatically*—no explicit correlations between building block configuration and chromosome quality need to be made (Goldberg, 1989a).

The genetic algorithm's reproduction operator is inspired by the mechanics of chromosome recombination found in biological systems. Commonly referred to as "genetic crossover" in the genetic algorithm research community, chromosome recombination is the swapping of genetic material between parent chromosomes to create child chromosomes containing genetic material from both parents. As in biological systems, genetic algorithm-based reproduction occurs only between pairs of chromosomes, with each pair of parents creating a pair of children.

The most straightforward reproduction operator, "single point" crossover, is depicted in Figure 2.10. In the technique, two of the chromosomes selected (by the *selection* algorithm) to serve as parents are randomly paired, and a location along the length of the pair is selected at random. "Crossover" then takes place when genetic information beyond the location is swapped, creating two child chromosomes which each possess genetic material from both parents. The child chromosomes are then inserted into the child generation, and the two parent chromosomes are removed from the parent "pool." The process is then repeated until all of the parent chromosomes have mated and generated children.

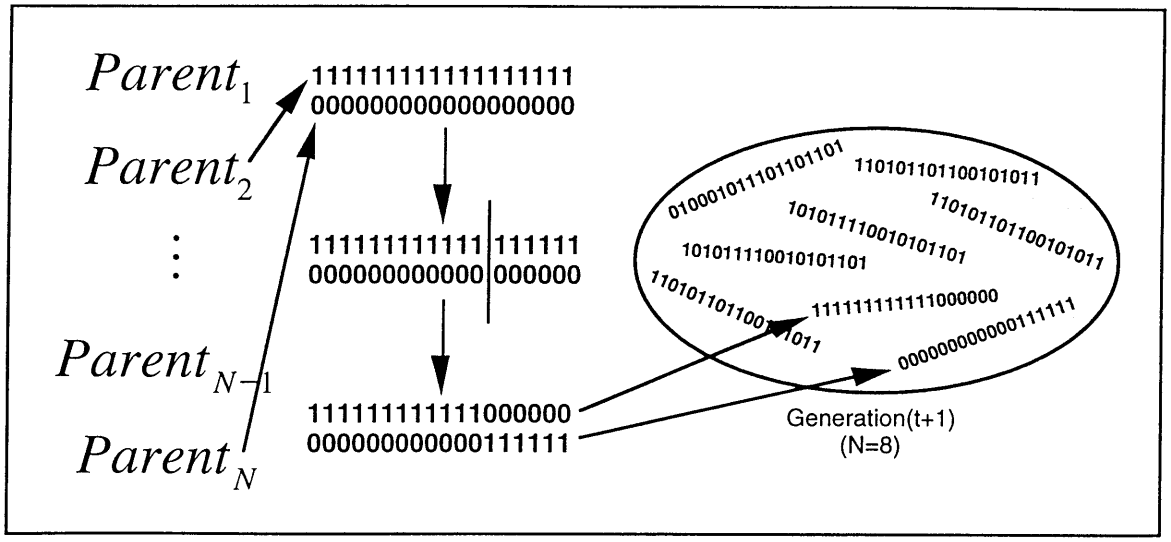


Figure 2.10: Single point crossover.

Note that crossover is *not* performed on every pair of parent chromosomes. Rather, with every pair of parent chromosomes, a "weighted coin toss" determines whether crossover will take place. The probability that crossover will be performed (*i.e.*, the probability that the coin will land on "perform crossover") must be specified *a priori* by the programmer. If crossover is not performed, the two parent chromosomes are inserted into the child generation without any recombination of genetic material.

During the course of this investigation, the performance of single-point crossover was compared with that of uniform crossover (Syswerda, 1989), a reproductive technique shown to outperform single-point crossover in certain problem domains. However, *in this investigation* (*i.e.*, in the examples presented in Chapter 5), single-point crossover demonstrated superior optimization performance.

Mutation. After the child generation has been created, *mutation* is performed on the child chromosomes. As with the genetic algorithm's reproduction operator, the mutation operator is modeled after biological systems, where random, infrequent mutation introduces new, possibly-desirable characteristics into the population. Used in the genetic algorithm, mutation modifies the genetic material in child chromosomes, allowing the genetic algorithm to explore new areas of the search space. Whereas reproduction swaps portions of genetic material between chromosomes, creating new combinations of the same genetic material, mutation slightly changes the genetic material. Hence, if a particular chromosome corresponds to a solution close to the globally-optimum solution, it is hoped that mutation will make the slight changes needed to obtain the optimum solution.

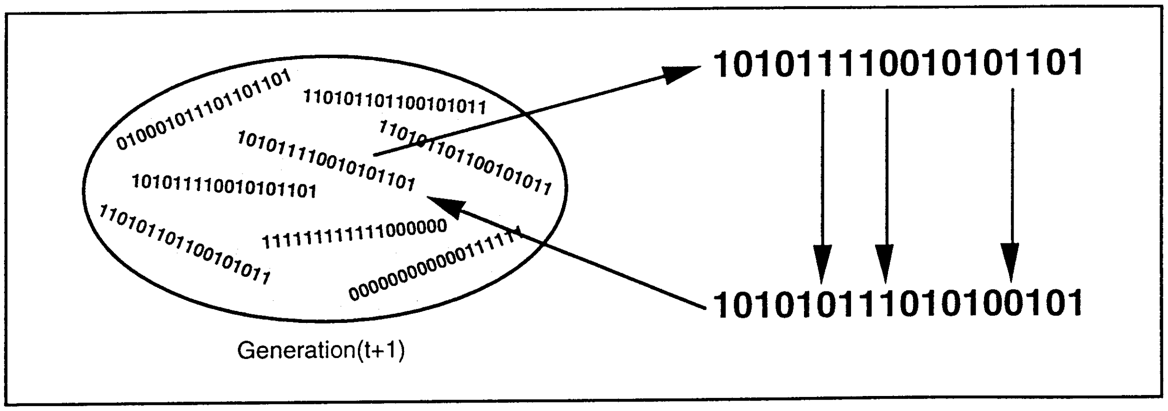


Figure 2.11: Mutation.

Specifically, the mutation algorithm (Figure 2.11) examines every gene of every child chromosome and, using a "weighted coin toss," determines whether the gene's allele value should be toggled. As with the crossover probability discussed in previous paragraphs, the probability that any given gene will mutate (*i.e.*, the probability that the coin will land on "perform mutation") must be specified *a priori* by the programmer. If mutation is performed on a particular gene, the gene's allele value is toggled from 0 to 1 or from 1 to 0. If mutation is not performed on a particular gene, the gene's allele value is not modified.

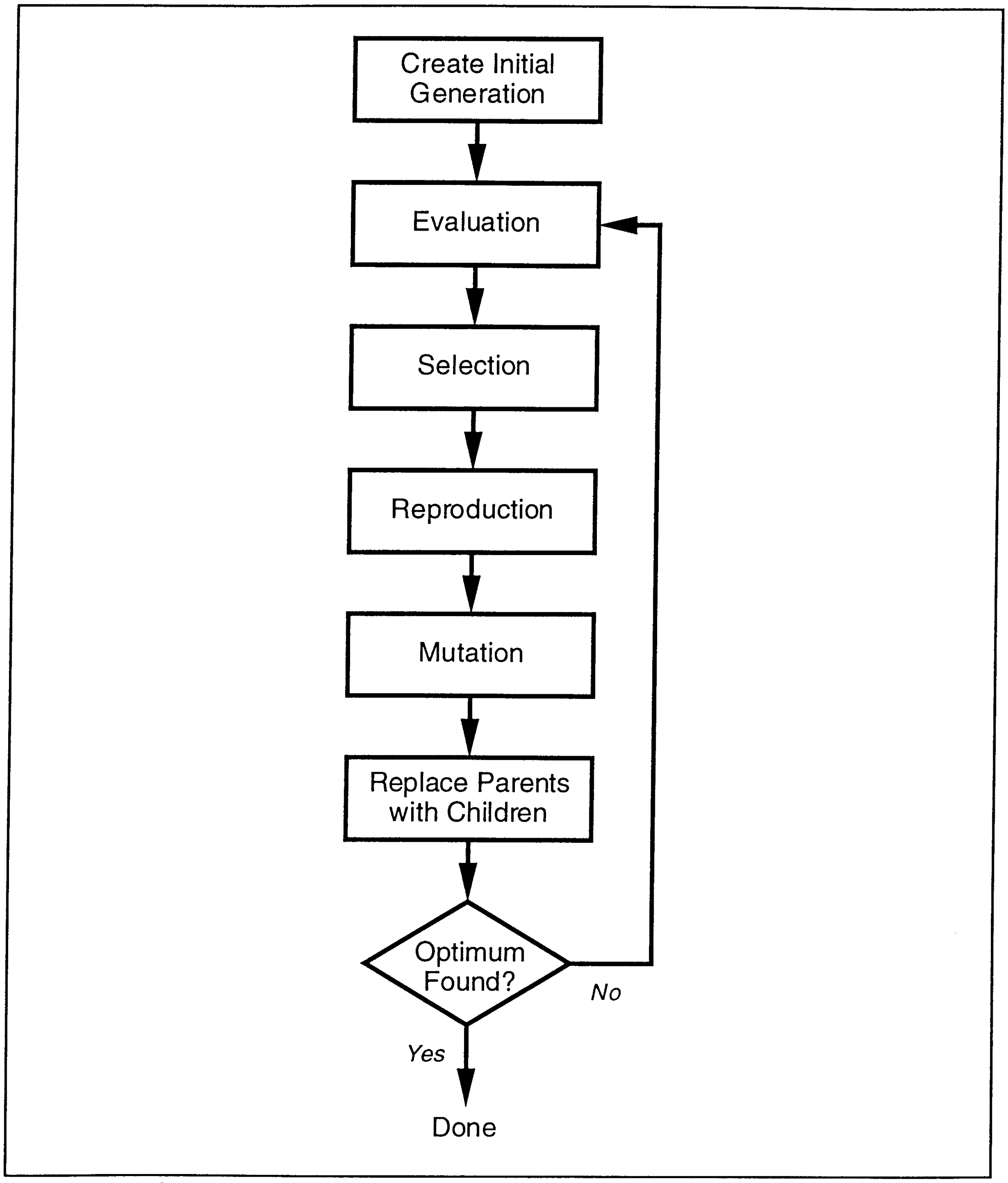


Figure 2.12: Genetic algorithm flowchart.

At the conclusion of mutation, the child generation is complete and is allowed to replace the parent generation. The entire evolution process then iterates (Figure 2.12) until the population has converged to an optimum or has "evolved" for a pre-determined number of generations. Note that Section 2.3.6 provides a brief discussion on determining when a genetic algorithm-based search should end. After many generations of evolution, the overall quality of chromosomes (*i.e.*, the quality of the solutions to the optimization problem) should increase because better chromosomes (*i.e.*, those chromosomes which represent the best solutions to the optimization problem) are more likely to propagate into child generations.

2.3.5 Optimization Parameters.

When using the genetic algorithm, several parameters must typically be specified:

• Probability of Crossover ($P_{CROSSOVER}$)	The probability that crossover will be performed between a particular pair of parent chromosomes.
• Probability of Mutation ($P_{MUTATION}$)	The probability that any given gene on any given chromosome will mutate.
• Fitness Scaling Coefficient (F_{MULT})	A measure of the magnitude of fitness value attenuation. Typically represents the desired ratio of the maximum fitness in a population to the average fitness of the population.
• Population Size	Number of chromosomes in each genetic algorithm generation.
• Selection Algorithm	The technique used to determine which chromosomes in a population will serve as parents for the next generation.
• Crossover Operator	The method used to mate, or combine, two parent chromosomes to create two child chromosomes containing attributes from both parents.

These parameters have a great effect on the performance and efficiency of the search. For example, high crossover and mutation probabilities favor exploration of the search space, while low crossover and mutation probabilities favor exploitation of current genetic material. Likewise, high fitness scaling coefficients bias the search towards immediately-promising areas of the search space at the risk of prematurely converging to a local optimum, while low fitness scaling coefficients avoid premature convergence at the risk of not fully utilizing highly-fit genetic material. Population size also affects search performance, in that large populations are inefficient (*i.e.*, large populations exhibit slow convergence and require more function analyses per generation) while small populations do not provide the variety of genetic material needed to fully explore the search space. Finally, different selection schemes choose parents with varying degrees of accuracy and consistency (Baker, 1987), while different crossover operators offer varying levels of positional and distributional bias (Eshelman *et al.*, 1989) toward genetic "building blocks."

Unfortunately, it is difficult to determine proper parameter values *a priori*. The first comprehensive study of the effects of genetic algorithm parameters was conducted by De Jong (1975). Recently, empirical studies attempting to determine optimum crossover and mutation probabilities as well as population sizes have been conducted by Schaffer *et al.* (1989) and Grefenstette (1986). Goldberg *et al.* (1992) and Goldberg (1989b) conducted analytical studies of optimum population sizes, while Kreinovich *et al.* (1993), Goldberg (1989a), and Grefenstette (1986) provide details on selecting appropriate fitness scaling coefficients. de la Maza and Tidor (1993), Bäck and Hoffmeister (1991), Goldberg and Deb (1991), Grefenstette and Baker (1989), and Baker (1989, 1987, 1985) have investigated parent selection schemes which improve on the standard roulette-wheel selection algorithm. The relative performance of different reproduction operators has been investigated by Spears and De Jong (1991), Eshelman *et al.* (1989), Syswerda (1989), Schaffer and Morishima (1987), and Goldberg and Lingle (1985).

Using the suggestions of Goldberg (1989a), Schaffer *et al.* (1989), Baker (1987), and Grefenstette (1986), the following parameter values were used during the course of this investigation:

$P_{\mathrm{CROSSOVER}}$	=	0.95
P_{MUTATION}	=	0.01
F_{MULT}	=	1.4
Population Size	=	30
Selection Algorithm	=	"Elitist" Stochastic Universal Sampling (Baker, 1987)
Crossover Operator	=	Single-Point Crossover

In addition, this investigation's genetic algorithm routines utilized binary-coded chromosomes (*i.e.*, allowable allele values of either 0 or 1) and randomly-generated initial populations (*i.e.*, chromosome allele values in the initial generation were determined at random). The routines were based on the Simple Genetic Algorithm detailed in Goldberg (1989a).

Note that "elitist" selection strategies differ from normal selection algorithms in that immediately before a parent generation is replaced by its corresponding child generation, a randomly-chosen child chromosome is eliminated and replaced with the parent generation's most-highly-fit chromosome. Because a parent chromosome's highly-fit building blocks may be "destroyed" by crossover (*i.e.*, split between two child chromosomes) or mutation (*i.e.*, erased by allele value changes), highly-fit parent chromosomes do not always produce highly-fit child chromosomes. Hence, "elitist" re-insertion is performed to insure that the child generation's best chromosome is not of lower quality than the parent generation's most-highly-fit chromosome. Elitist selection strategies are particularly useful when using relatively high crossover and mutation probabilities, which tend to destroy highly-fit building blocks in parent chromosomes.

In addition to the above parameters, which define the operation of the genetic algorithm, chromosome fitness value calculations also affect search performance. Specifically, when representing constraints as penalty terms in a fitness function, care must be taken to normalize, or balance, the penalty terms with the actual "quality" terms of the function. Excessive constraint violation penalties typically result in sub-optimal solutions which satisfy all constraints but do not minimize or maximize the objective function. Likewise, insufficient penalties typically result in infeasible solutions which minimize or maximize the objective function but are unable to satisfy the constraints.

While several constraint-representation techniques have been developed (Goldberg, 1989a), most are tailored to a particular problem domain. During the course of this research, a linear penalty term which penalized the objective function value P% for every 10% constraint violation was found to work well. Unfortunately, a great deal of experimentation is generally required to develop a fitness function which correctly balances constraint violation terms (P in this investigation) with objective function "quality" terms.

2.3.6 GA's vs. Traditional Optimization Methods. Several fundamental differences exist between the genetic algorithm and more traditional optimization methods such as mathematical programming techniques, optimality criteria methods, and exhaustive searches.

The most dramatic difference between the genetic algorithm and other techniques is that the GA conducts optimization through the use of a population of points distributed (initially at random) throughout the search space. By gradually combining and modifying the best attributes of the points through probabilistic pairing, crossover, and mutation, the genetic algorithm is able to simultaneously evaluate different areas of the search space, avoiding local optima as it converges on a globally-optimum solution. This differs from mathematical programming and optimality criteria techniques, which start from a single point and then use deterministic "hillclimbing" or heuristic redesign procedures to ascend the nearest peak in the search space. Unfortunately, particularly in multi-modal search spaces, the nearest peak is most likely locally-optimum, with globally-optimum peaks located elsewhere. Hence, with highly multi-modal search spaces, the genetic algorithm will avoid local optima and converge on the globally-optimum solution, while mathematical programming and optimality criteria techniques will likely converge to a local optimum.

Unlike mathematical programming techniques, which utilize gradient information to move from one point in the search space to another, the genetic algorithm uses only objective function evaluations and probabilistic operators such as pairing, crossover, and mutation to move through the search space. Hence, the genetic algorithm is ideally suited to discrete, discontinuous search spaces where derivative information is not available.

Another advantage of the genetic algorithm over other techniques is its ability to work with a variety of design variable types. While the discrete allele values used in genetic algorithm chromosomes are ideally-suited to integer design variables, they can readily represent floating-point design variables with fairly high resolution by using multi-bit, binary encodings. Mathematical programming techniques, on the other hand, are designed specifically for either integer or continuous variables—changes in variable type necessitate the use of a different optimization algorithm.

Unlike mathematical programming and optimality criteria techniques, which obtain only one solution to any given optimization problem, the genetic algorithm obtains a population of solutions. Hence, one optimization run provides the user with a "family" of solutions, where each solution offers varying levels of objective function minimization-maximization and constraint satisfaction abilities. The user can then evaluate the solutions to determine which best satisfies several alternate performance criteria, much like a pareto optimization study. In contrast, when using mathematical programming or optimality criteria techniques, all alternate performance criteria must be explicitly incorporated into the objective and gradient functions.

One difficulty with the genetic algorithm is determining when a search should end. Because the genetic algorithm is probabilistic in nature and does not utilize function gradient information, it is able to search in discrete, discontinuous, multi-modal search spaces. However, these non-deterministic attributes prevent the genetic algorithm from determining whether an optimum has been reached. So, unlike mathematical programming and optimality criteria techniques, which use gradient information or optimality criteria values to explicitly determine if an optimum has been found, the genetic algorithm must rely on "convergence criteria." Typically, the criterion for ending the search is based on the percentage of chromosomes in the population which have converged to similar points in the search space. There is little value in continuing the search when most chromosomes have converged, because the fitness of further generations will increase only through the inefficient method of random gene mutation. Once the convergence criterion is met, it is unknown whether or not the population's highest-fitness chromosome represents an actual optimum. Hence, genetic algorithm searches produce a "pseudo-optimum." The advantage of genetic algorithm search is that it does produce these pseudo-optima in discrete, discontinuous, multi-modal search spaces which would be troublesome to mathematical programming and optimality criteria techniques using gradient or other sensitivity information.

Another disadvantage of the genetic algorithm is computation cost. In general, the genetic algorithm will perform thousands of function evaluations before converging on a solution. Mathematical programming techniques, and particularly optimality criteria methods, typically require many fewer evaluations. Hence, the genetic algorithm is most practical in domains where function evaluations are relatively inexpensive.

2.3.7 Summary. In summary, the genetic algorithm is a robust, general search and optimization technique applicable to a variety of problem domains and design variable types. In many ways, it is a useful compromise between *strong* and *weak* search methods. *Strong* search methods, such as mathematical programming techniques and expert system-based approaches, perform search in an informed manner—there is a rationale for moving from one point in the design space to another. In contrast, *weak* methods, such as exhaustive or random searches, perform search in an uninformed manner—they exhaustively enumerate, or at least extensively sample, the design space in order to find an optimum. Weak methods are computationally expensive, but are more likely to find global optima; strong methods are computationally inexpensive, but are more likely to settle for local suboptima. The genetic algorithm, in contrast to both, uses the weak operations of probabilistic pairing, crossover, and mutation to obtain a strong progression toward globally-optimum solutions.

Interested readers should refer to the Genetic Algorithms chapter in (Levy, 1992) for a discussion of the history of the approach and descriptions of some interesting applications. For a comprehensive introduction to genetic algorithms, including sample computer code, (Goldberg, 1989a) is recommended.

Previous Work 3

3.1 Overview

Sizing and shape optimization based upon structural considerations (*i.e.*, structural sizing and shape optimization) have been active research areas for some time (Schmit (1981), Vanderplaats (1982), Haftka and Grandhi (1986)). However, because of limitations in computer technology, optimization algorithm performance, and structural analysis efficiency, topology optimization based upon structural considerations has only recently been investigated. Nevertheless, structural topology optimization has quickly become an active research area, and several innovative approaches have been developed and applied to a variety of continuum and discrete-membered (*i.e.*, truss) structures.

As detailed in Chapter 1, this investigation addresses the genetic algorithm-based structural topology optimization of continuum structures, where the optimal distribution of material within a discretized design domain is found. So that the genetic algorithm's performance in this domain can be compared with that of other optimization algorithms, this chapter begins by detailing non-genetic-algorithm-based approaches to the topological optimization of continuum structures. Note that non-genetic-algorithm-based topological optimizations of discrete-membered structures are not reviewed. Then, to demonstrate the genetic algorithm's general structural optimization capabilities, genetic algorithm-based approaches to sizing and shape optimization of continuum and discrete-membered structures, as well as topological optimization of discrete-membered structures, are described. Finally, to facilitate the evaluation of this investigation's contributions, previous research in genetic algorithm-based topological optimization of continuum structures is detailed.

For a comprehensive introduction to the current state-of-the-art in structural topology optimization research, please refer to the proceedings of several recent conferences focused on the subject: Bendsøe and Soares (1993), Rozvany (1993), and Eschenauer *et al.* (1991).

3.2 Non-Genetic-Algorithm-Based Topology Optimization

3.2.1 Homogenization Method. Perhaps the most prominent technique used to perform structural topology optimization is the variable density approach based upon material homogenization methods (Strang and Kohn (1986), Kohn and Strang (1986)). Bendsøe and Kikuchi (1988) and associated researchers have applied the method, which minimizes a component's compliance subject to a maximum volume constraint, to a variety of structural topology optimization problems (Bendsøe *et al.* (1993), Bendsøe *et al.* (1991), Díaz and Belding (1993), Fukushima *et al.* (1993), Suzuki and Kikuchi (1993, 1991, 1990)). Current research issues in homogenization-based structural topology optimization are summarized by Kohn (1993).

In the procedure, a design domain is discretized into small, rectangular elements, where each element is assumed to contain composite material of continuously-variable density and orientation. Each element's structural properties are a function of the element's material density and orientation, and are calculated using material homogenization methods. Hence, assigning density and orientation values to each element in the design domain results in the generation of a particular component design with particular structural characteristics. Likewise, modifying the material density or orientation in any element results in the modification of the component's design and structural characteristics. Consequently, because the component's design and structural properties are controlled by the material density and orientation in each design domain element, each element's material density and orientation serve as design variables during optimization. Optimization begins when an initial component design is created by assigning initial material density and orientation values to each design domain element. An optimality criteria method then determines how the material density and orientation in each element should change so that the compliance of the component design is minimized subject to a maximum volume constraint. In an iterative process, an optimal design is generated.

Several different microstructure models have been suggested for use with material homogenization methods when determining the composite material's structural properties. All of the models allow full rotation of the microstructure and enable the material's density to vary continuously between 0 (void) and 1 (solid). Bendsøe and Kikuchi (1988) introduce a "non-optimal" material microstructure comprised of either square or rectangular microscale holes (Figure 3.1), where the size of the holes defines the density of the composite. Other research (Allaire and Francfort (1993), Allaire and Kohn (1993), Bendsøe *et al.* (1993), Jog *et al.* (1993)) studies an "optimal" microstructure created by "Rank-2" layering, which creates a two-scale, orthogonal laminate (Figure 3.2). The volume fractions of strong and weak material define the composite's density. After the microstructure model is defined, material homogenization techniques are then used to determine the relationship between the material's 'density' (*i.e.*, the composite's hole sizes or volume fractions) and orientation and its structural properties. Note that Bendsøe *et al.* (1993) propose an "artificial composite," which eliminates the need for material homogenization methods by using approximate, analytical relationships between material density and orientation and the corresponding structural properties.

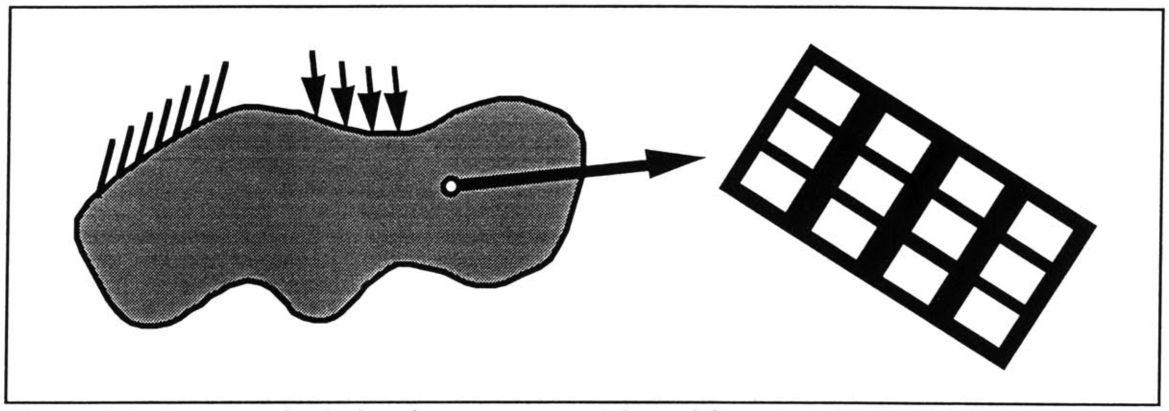


Figure 3.1: Rectangular hole microstructure. Adapted from Bendsøe *et al.* (1993).

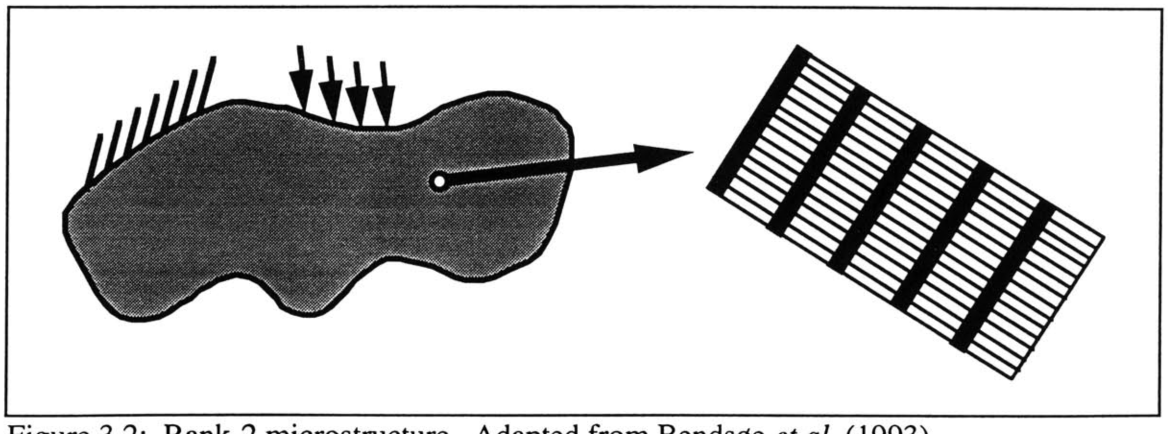


Figure 3.2: Rank-2 microstructure. Adapted from Bendsøe *et al.* (1993).

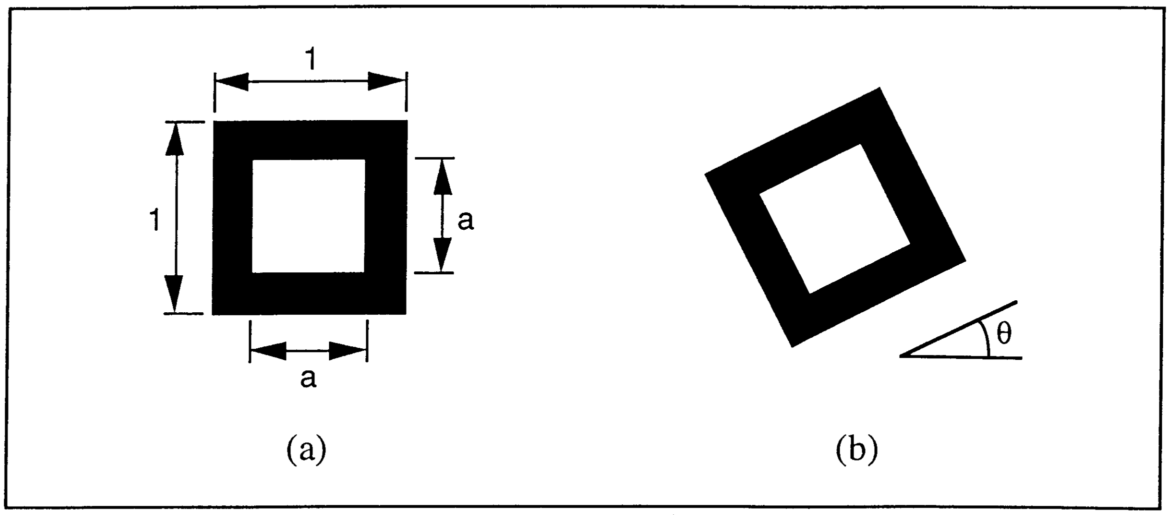


Figure 3.3: Square hole design variables. Adapted from Suzuki and Kikuchi (1991).

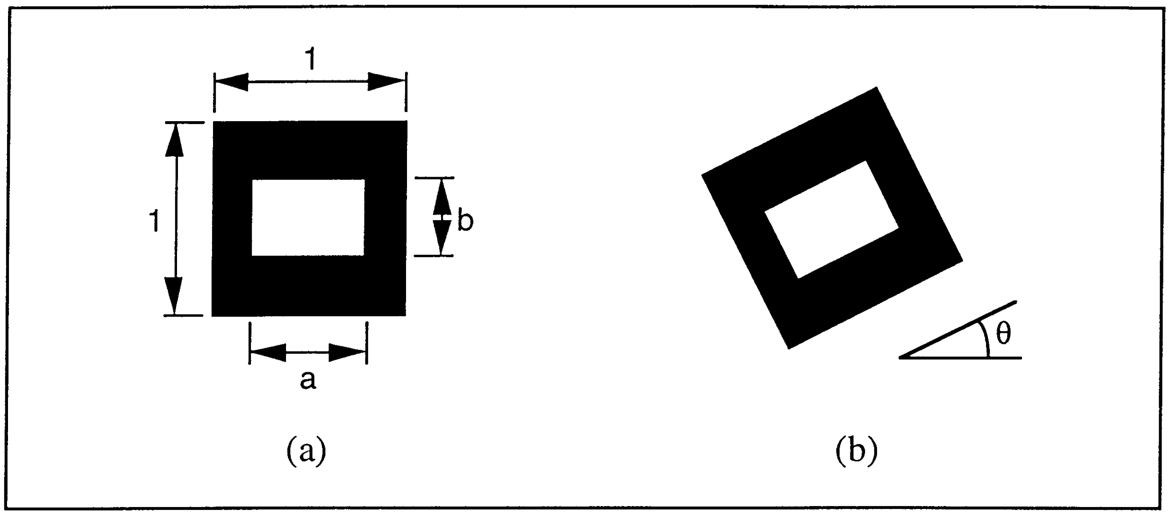


Figure 3.4: Rectangular hole design variables. Adapted from Suzuki and Kikuchi (1991).

Hence, during optimization, the number of design variables required to control the density and structural characteristics of *any single* design domain element depends upon the microstructure used to model the composite material in the design domain. When using a square or rectangular hole microstructure, either two (square size and orientation—Figures 3.3a-b) or three (rectangle height, width, and orientation—Figures 3.4a-b) design variables are needed for each design domain element. Likewise, Rank-2 microstructure models require the use of three (strong material volume fraction, weak material volume fraction, and material orientation) design variables for each design domain element. Artificial composite microstructure models require two (material density and orientation) design variables. So, the *total* number of design variables required in an optimization is equal to the number of elements in the design domain multiplied by the number of design variables required by each element. Note that Figures 3.3 and 3.4 depict an individual *unit cell*, which defines the elementary form of the material microstructure—the composite material in each design domain element is assumed to contain many of these unit cells.

Strang and Kohn (1986) recommend the use of composites in structural optimization problems, stating that a "0-1 dichotomy between holes and material" results in an "ill-posed" minimization problem, where the optimal solution is difficult, if not impossible, to obtain. They suggest that a "relaxation" of the problem, where material in the design domain is modeled as a composite with continuously-varying density, is needed to transform the original problem into one which has a solution. Hence, by modeling the material as a composite and then using material homogenization techniques to determine the composite's structural properties, a minimization problem is created which can be solved by common optimization algorithms.

Optimality criteria methods are typically used to solve the minimization problem created by this "relaxation." Specifically, an iterative redesign procedure gradually modifies an initial component design (*i.e.*, an initial set of material density and orientation values) until the design satisfies a set of optimality criteria. While an optimum design has been generated when the problem's optimality criteria are satisfied, there is no guarantee that the design is *globally* optimum. Bendsøe *et al.* (1993) demonstrate that an optimal component design's shape depends upon both the initial material density values and the material microstructure model when using homogenization-based techniques.

Another problem with homogenization-based approaches is that elements of intermediate density may exist in the optimal component design—boundary definition using a density threshold may be necessary to create a design containing only solid material. Papalambros and Chirehdast (1990) and Chirehdast *et al.* (1992) use binary image analysis approaches to extract precise topological boundaries, while Olhoff *et al.* (1991) rely on manual boundary definition by the user. The resulting shape can then be modeled with a set of spline curves and subjected to classical, mathematical programming-based shape optimization.

3.2.2 Simulated Annealing. Another approach to the topological optimization of structural components is the work by Anagnostou *et al.* (1992) and Ghaddar *et al.*

(1993), who use simulated annealing to perform topology optimization based on strength, manufacturability, and heat transfer considerations.

In the technique, a design domain is discretized into small, rectangular elements, where each element can contain either material or void—no intermediate densities are allowed. Assigning material or void to each element in the design domain (*i.e.*, designating the *state* of each design domain element) defines the distribution of material and void within the design domain, and therefore establishes the component's topology (Figure 3.5). Likewise, modifying the state of any element (*i.e.*, toggling a material element to void or a void element to material) results in the modification of the component's topology. Hence, the material-void states of the design domain elements serve as design variables during optimization. Optimization begins when an initial component design is generated by assigning initial states to the elements in the design domain. Simulated annealing is then used to determine the optimal configuration of material and void within the design domain (*i.e.*, the optimal set of design domain element states) such that the component's structural performance is maximized and all constraints are satisfied.

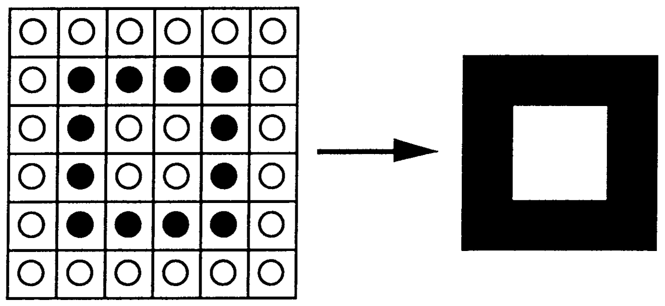


Figure 3.5: Design domain to topology mapping. Adapted from Ghaddar *et al.* (1993).

Anagnostou *et al.* (1992) first investigate the topological optimization of a beam, where the beam's volume, surface area, and number of edges are minimized while the beam's strength is maximized. Another example determines the topology of a thermal fin which provides minimum volume, surface area, and number of edges while exhibiting maximum heat dissipation abilities. Ghaddar *et al.* (1993) extend the work of Anagnostou *et al.* by investigating the previous beam example at a finer discretization (Figure 3.6 provides optimization results using three different discretizations) and optimizing the topology of a cantilevered plate subject to a downward vertical load (Figure 3.7). Similar to optimizations performed using homogenization-based techniques, the cantilevered plate's compliance (as well as its perimeter and number of corners) is minimized subject to a maximum volume constraint. While satisfactory cantilevered plate topologies are obtained, slight differences in objective function and volume constraint formulation prevent direct comparison with structures obtained using homogenization-based techniques.

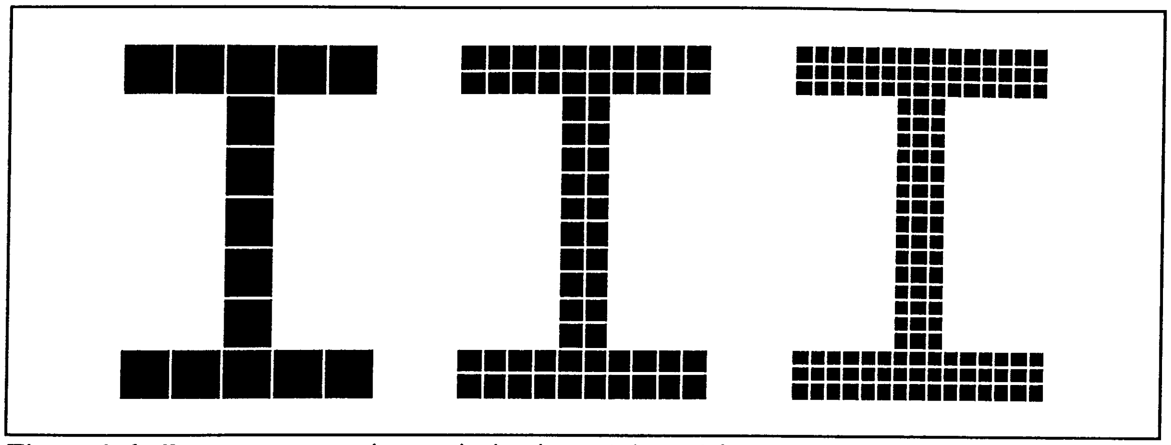


Figure 3.6: Beam cross-section optimization. Adapted from Ghaddar *et al.* (1993).

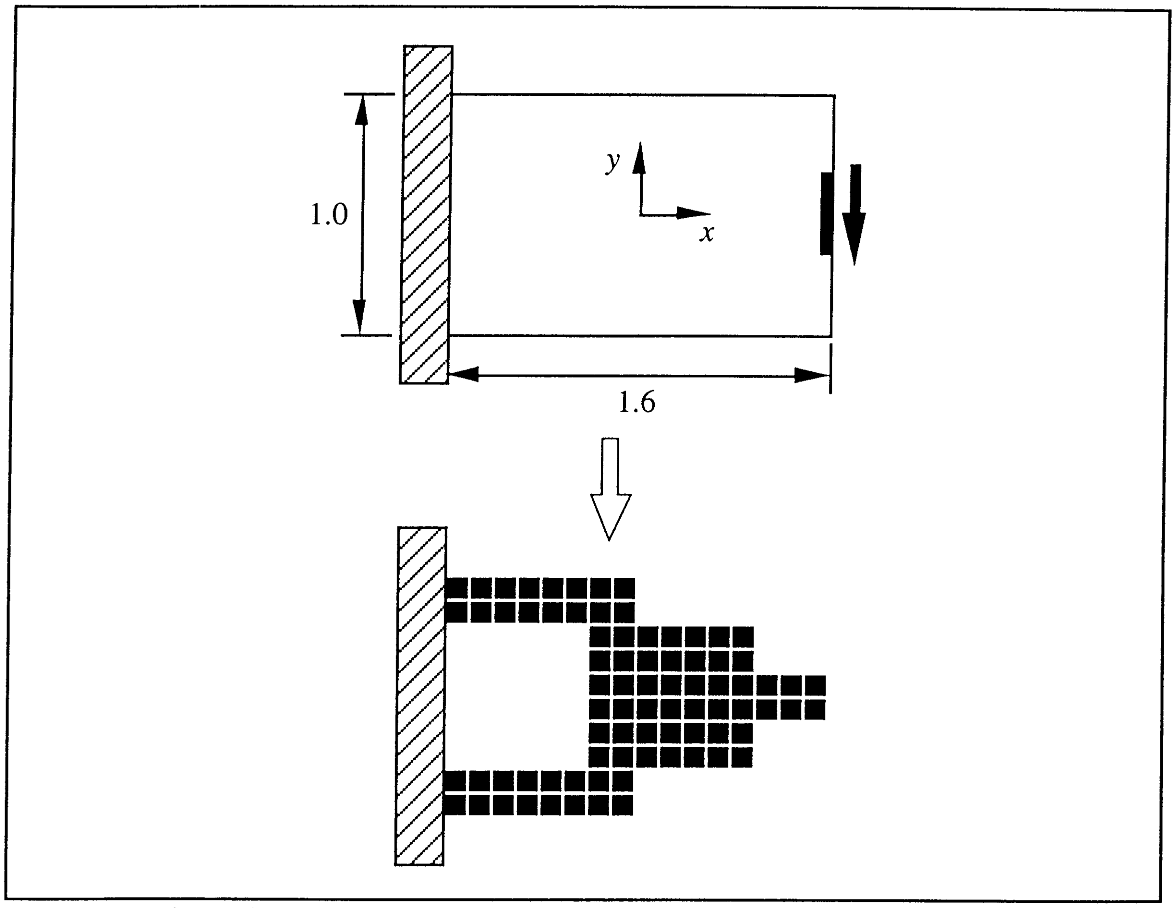


Figure 3.7: Compliance minimization. Adapted from Ghaddar *et al.* (1993).

3.3 Genetic Algorithm-Based Structural Optimization

3.3.1 Introduction. A third approach, which has been applied by a variety of researchers and serves as the focus of this investigation, is genetic algorithm-based structural topology optimization. This section details previous efforts in genetic algorithm-based structural topology optimization, and, in an attempt to demonstrate the genetic algorithm's general structural optimization capabilities, describes previous investigations of genetic algorithm-based structural sizing and shape optimization.

As detailed in Section 2.2.4, the genetic algorithm performs structural optimization by "evolving" a population of artificial chromosomes, where each chromosome represents a possibly-optimal component design and the population of chromosomes therefore represents a population of component designs. Optimization begins when an initial population of designs is generated. After first evaluating the structural performance and constraint satisfaction abilities of each design in the population, the highest-quality designs are then selected to serve as parents. Parent designs then pair and mate via genetic crossover, creating a population of child designs. These child designs are then subjected to random, infrequent mutation, after which they replace the parent designs. The process then iterates until an optimal component design is found.

Because each chromosome in a genetic algorithm population is a coded representation of the optimization problem's design variables (Section 2.3.3), the particular design characteristics controlled by a chromosome depend upon the type of structural optimization which is being performed (Section 2.2.5). For example, each chromosome in a genetic algorithm-based sizing optimization represents a particular set of component dimensions. Likewise, chromosomes in a genetic algorithm-based shape optimization each represent a particular component shape, and chromosomes in a genetic algorithm-based topology optimization each represent a particular component topology.

3.3.2 Sizing Optimization. The first investigation (to the investigator's knowledge) of genetic algorithm-based structural optimization is that of Goldberg and Samtani (1986), who examine the sizing optimization of a 10-member truss structure (Figure 3.8). In the investigation, the truss structure's geometry and topology are held constant, and the optimal cross-sectional area of each truss member is determined. Hence, the truss member cross-sectional areas, which may vary continuously between given minimum and maximum values, serve as design variables. Each genetic algorithm chromosome in the optimization is therefore divided into 10 parameters, where each parameter specifies the floating-point cross-sectional area of a particular truss member. (Refer to Section 2.3.3 for details on how genetic algorithm chromosomes are concatenations of parameters, where each parameter controls the value of a particular integer or floating-point design variable.) The optimization, which attempts to minimize the structure's weight subject to minimum and maximum stress constraints in each member, produces satisfactory results.

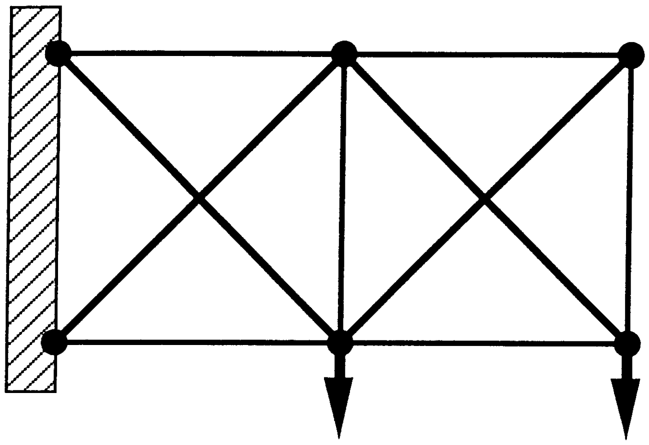


Figure 3.8: 10-member truss structure with applied loads. Adapted from Goldberg and Samtani (1986).

Hajela (1990) also investigates the sizing optimization of a 10-member truss. However, instead of the time-invariant loading used by Goldberg and Samtani, the truss examined by Hajela is subjected to harmonic excitation. In the example, the structure's weight is minimized subject to a maximum displacement constraint at a particular node. A second example in the investigation describes the sizing optimization of a two-element, thin-walled, cantilever torsional rod. Using the rod's thicknesses as design variables, the rod's weight is minimized subject to maximum stress constraints. Finally, the cross-sectional areas of a two-beam grillage structure are optimized in an attempt to minimize the structure's weight subject to maximum stress constraints.

Hajela (1992) provides additional examples of genetic algorithm-based sizing optimization of discrete-membered truss structures. The investigation first details the sizing optimization of a single bay, single story portal frame, where the frame's cross-sectional areas serve as design variables. In the example, the frame's weight is minimized subject to several plastic collapse constraints. Three other examples in the investigation detail the sizing optimization of 3-, 6-, and 25-member truss structures. All three examples attempt to minimize weight subject to maximum stress and displacement constraints. As in other sizing optimization examples, the cross-sectional areas of the truss members serve as design variables during optimization.

Rajeev and Krishnamoorthy (1992) extend the work of Goldberg and Samtani by optimizing the member cross-sectional areas of 10-, 25-, and 160-member truss structures. All three examples attempt to minimize weight subject to maximum stress and displacement constraints. As opposed to the truss structure sizing optimizations detailed above, which allow truss member cross-sectional areas to vary continuously between given minimum and maximum values, Rajeev and Krishnamoorthy specify that truss member cross-sectional areas must be selected from a list of standardized cross-sectional areas. Hence, each truss member's cross-sectional area serves as a discrete design variable, where the variable's value specifies a particular entry, or cross-sectional area, from the list of allowable cross-sectional areas. So, just as genetic algorithm chromosomes controlled floating-point design variables (*i.e.*, continuously-variable cross-sectional areas) in other investigations, Rajeev and Krishnamoorthy use genetic algorithm chromosomes to control integer-valued design variables (*i.e.*, discrete cross-sectional areas).

Lin and Hajela (1993) investigate an innovative technique for optimizing the member cross-sectional areas of discrete-member truss structures. As in the sizing optimization studies detailed above (with the exception of Rajeev and Krishnamoorthy), Lin and Hajela use genetic algorithm chromosomes divided into parameters, where each parameter controls, with a particular resolution, the floating-point cross-sectional area of an individual truss member. However, Lin and Hajela modify this standard, generation-invariant chromosome-to-design-variable representation by introducing a "multistage," variable-granularity genetic search strategy. The procedure, which enables the genetic algorithm to quickly evaluate large regions of the search space in early generations and then "fine-tune" a high-quality design in final generations, periodically increases the resolution with which floating-point truss member cross-sectional areas are represented. This increase in design variable resolution is achieved by increasing the length (in bits) of the parameters in each chromosome. For example, optimization begins with a coarse design variable representation—each truss member's cross-sectional area may vary between A_{MIN} and A_{MAX} in increments of ΔA_0 (where ΔA_0 depends upon the length of the parameters controlling the truss member cross-sectional areas and the range of allowable area values, $A_{MAX} - A_{MIN}$). Optimization is then performed for a given number of generations, after which the design is assumed to have converged to a "coarse optimum." The lengths of the chromosome parameters (and therefore the lengths of the chromosomes) are then increased so that each truss member's cross-sectional area may vary between A_{MIN} and A_{MAX} in increments of ΔA_1, where $\Delta A_1 < \Delta A_0$. Optimization is again performed for a certain number of generations, and the entire resolution enhancement procedure is then repeated in an iterative process. Lin and Hajela use this multistage technique to optimize the cross-sectional areas of 10-, 25-, and 72-member truss structures. The 10-member truss structure is optimized for minimum weight subject to maximum tensile and compressive stress constraints, while the 25- and 72-member truss structures are optimized for minimum weight subject to stress and displacement constraints.

Finally, Wang *et al.* (1993) investigate the genetic algorithm-based sizing optimization of several three-dimensional structures. In one example, a structure comprised of truss, plate, and brick elements is optimized. Specifically, the truss cross-sectional areas and the plate thicknesses serve as design variables, and the structure's weight is minimized subject to stress and displacement constraints. A second example, which optimizes a similar structure comprised of plate elements, determines the plate thicknesses which minimize the structure's weight subject to stress and displacement constraints.

3.3.3 Shape Optimization. The sizing optimization studies detailed above optimized only cross-sectional dimensions of structures—the shape and topology of the designs remained constant. Consequently, while the "optimal" designs which were obtained are likely either locally- or globally-optimal in the search spaces established by the problem formulations, modifications to the shape or topology of the designs could possibly lead to higher structural performance. The following investigations, which perform genetic algorithm-based shape optimization, modify the shape of designs in an attempt to obtain structural performance greater than that possible through sizing optimization alone. Note that these investigations typically perform sizing optimization during the shape optimization process.

Jenkins (1991a, 1991b), in addition to performing sizing optimizations of a 3-member truss structure and a thin-walled cross-section, studies the shape optimization of several discrete-member truss structures. In one example, the geometry of a trussed-beam roof structure is optimized in an attempt to minimize the structure's mass subject to stress and deflection constraints. A second example, which optimizes the shape of an 18-member truss structure, uses the structure's nodal point coordinates as design variables. The example obtains an optimal truss geometry with a volume only 5% greater than the corresponding Mitchell structure (see Section 2.2.2 for a list of textbooks detailing Mitchell structures). In another study, Jenkins (1992) details the shape optimization of a cable-stayed bridge, where the locations of the cable attachment points on the girder and tower, as well as the cable cross-sectional areas, serve as design variables.

Richards and Sheppard (1992) investigate the use of a genetic algorithm-based classifier system in the shape optimization of a two-dimensional structural component under tensile loading. Much like an expert system, a classifier system (Goldberg, 1989a) uses a set of rules to govern its behavior in a particular environment. However, unlike expert systems, which commonly employ pre-coded rule sets, classifier systems use the genetic algorithm to learn rules. Hence, in the example detailed by Richards and Sheppard, a classifier system uses the genetic algorithm to learn which component shape modifications result in a shape of maximum structural performance. Specifically, the classifier system-based shape optimization attempts to find the component shape providing minimum mass subject to a maximum stress constraint. During optimization, cubic spline curves represent the component's shape, and the curves' control point locations serve as design variables. Note that the control points are evenly distributed along the x-direction length of the component, and that the y-coordinates of the control points serve as the actual design variables (Figure 3.9).

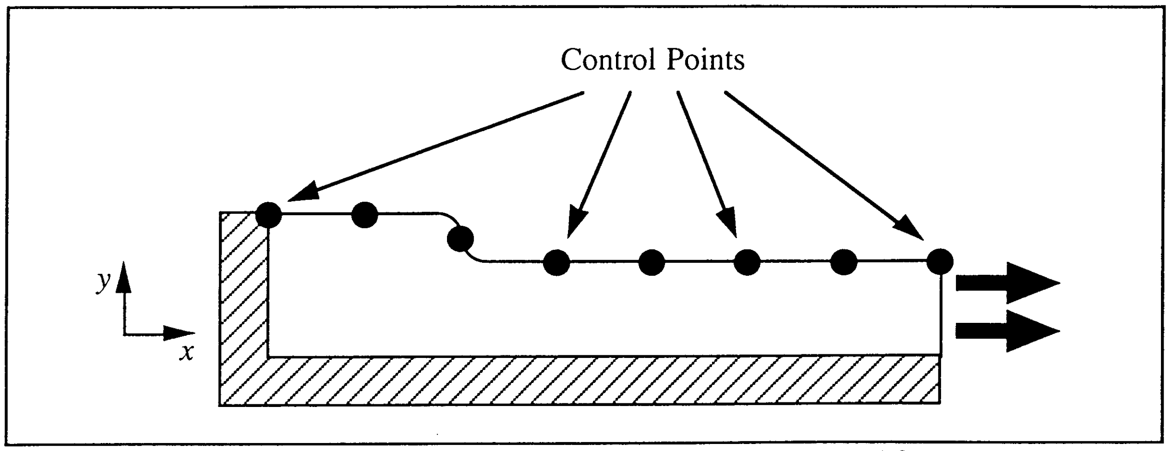


Figure 3.9: Control points defining the component's shape. Adapted from Richards and Sheppard (1992).

Finally, Watabe and Okino (1993) study the shape optimization of a 20-member truss structure. Specifically, they use the genetic algorithm to generate the truss structure shape which provides minimum mass subject to a maximum stress constraint. While previous truss structure shape optimizations (*e.g.*, Jenkins (1991a, 1991b)) performed optimization by directly modifying truss node locations, Watabe and Okino use a unique design representation in which the structure's nodal coordinates *do not* serve as design variables. Instead, using a technique called Free-Form Deformation (FFD), the genetic algorithm chromosomes control the locations of a set of control points which, in turn, define the locations of the structure's nodes. Hence, by specifying the locations of the control points (which are initially configured in a grid pattern), the genetic algorithm is able to specify the locations of the structure's nodes and therefore optimize the truss structure's shape (Figure 3.10).

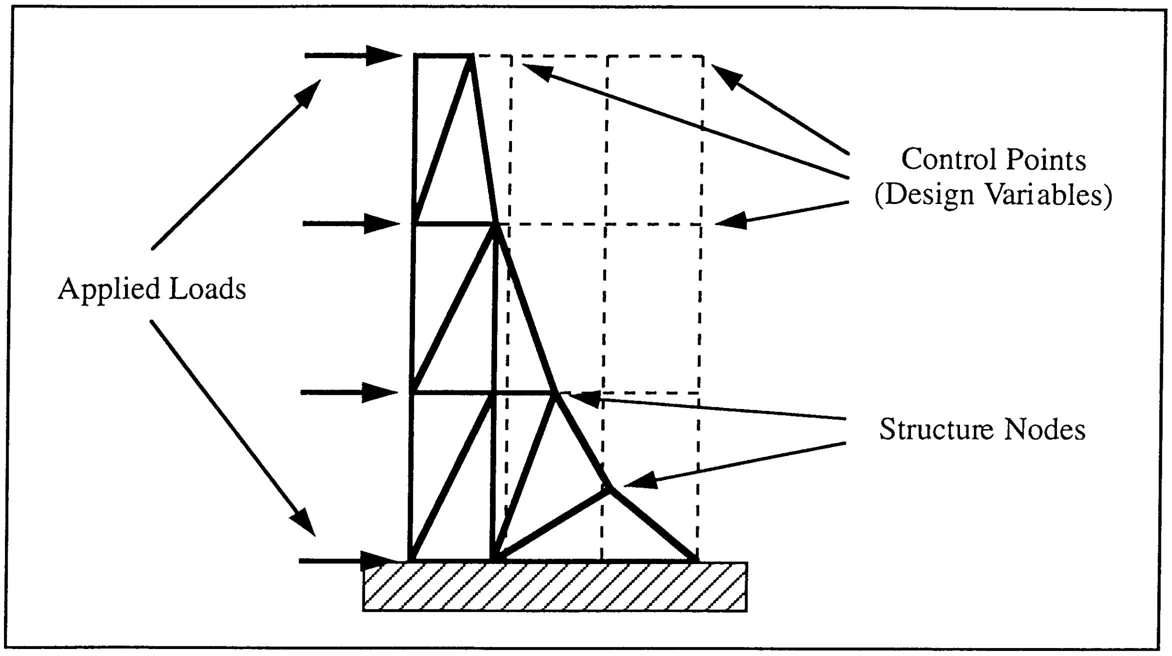


Figure 3.10: Shape representation using FFD. Adapted from Watabe and Okino (1993).

3.3.4 Topology Optimization. The shape optimization studies detailed above optimized only the shapes of structures—the topology of the designs remained constant. Consequently, while the "optimal" designs which were obtained are likely either locally- or globally-optimal in the search spaces established by the problem formulations, modifications to the topology of the designs could possibly lead to higher structural performance. The following investigations, which perform genetic algorithm-based topology optimization of *discrete-member* and *continuum* structures, modify the topology of structures in an attempt to obtain structural performance greater than that possible through sizing and shape optimization alone. Note that these investigations typically perform sizing and shape optimization during the topology optimization process.

Discrete-Member Truss Structures. The first investigation of genetic algorithm-based topology optimization of discrete-member truss structures is that of Shankar and Hajela (1991), who attempt to find a truss topology with minimum weight subject to displacement, stress, and natural frequency constraints. Given a set of required load application points and candidate support locations, a heuristics-based, "stagewise decomposition approach" is first used to create an initial population of stable truss topologies (*i.e.*, topologies which can support the applied loads). These topologies, containing truss members of nominal cross-sectional dimensions, then serve as the initial population for a genetic algorithm-based topological optimization which generates an optimal truss topology. This optimum truss topology, again containing members of nominal cross-sectional dimensions, then serves as the initial seed of a second genetic algorithm-based optimization which determines the optimal cross-sectional areas of the truss structure's members.

Hajela *et al.* (1993) perform a similar two-level-GA topological optimization of truss structures, where an initial genetic algorithm optimization generates stable topologies and a second genetic algorithm optimization then performs sizing optimization on the stable topologies. Specifically, optimization begins with the definition of candidate support points, required load application points, and allowable truss member connection points (Figure 3.11). Using this information, an initial genetic algorithm search generates a population of stable truss topologies, where each topology's members are of nominal cross-sectional area. The least-weight topologies generated by this first optimization then serve as the initial population of a second genetic algorithm-based search which performs sizing optimization on the topologies. Hence, this second genetic algorithm-based search generates the truss topology and corresponding cross-sectional dimensions which provide minimum weight and satisfy maximum stress and displacement constraints.

Grierson and Pak (1993) investigate the topological optimization of skeletal building structures. Specifically, they attempt to generate the cross-sectional dimensions, member lengths, and structure topologies which provide minimum weight subject to a maximum displacement constraint. However, unlike the topological optimizations detailed above (Shankar and Hajela (1991), Hajela *et al.* (1993)), which utilize large search spaces (*i.e.*, the total number of possible topologies is large), the example provided by Grierson and Pak chooses between only *two* candidate topologies. Hence, the optimization focuses on determining the structure's optimal cross-sectional areas and column lengths. An approximate fitness evaluation method, utilizing design sensitivity analysis, is used to reduce the number of structural analyses required during optimization.

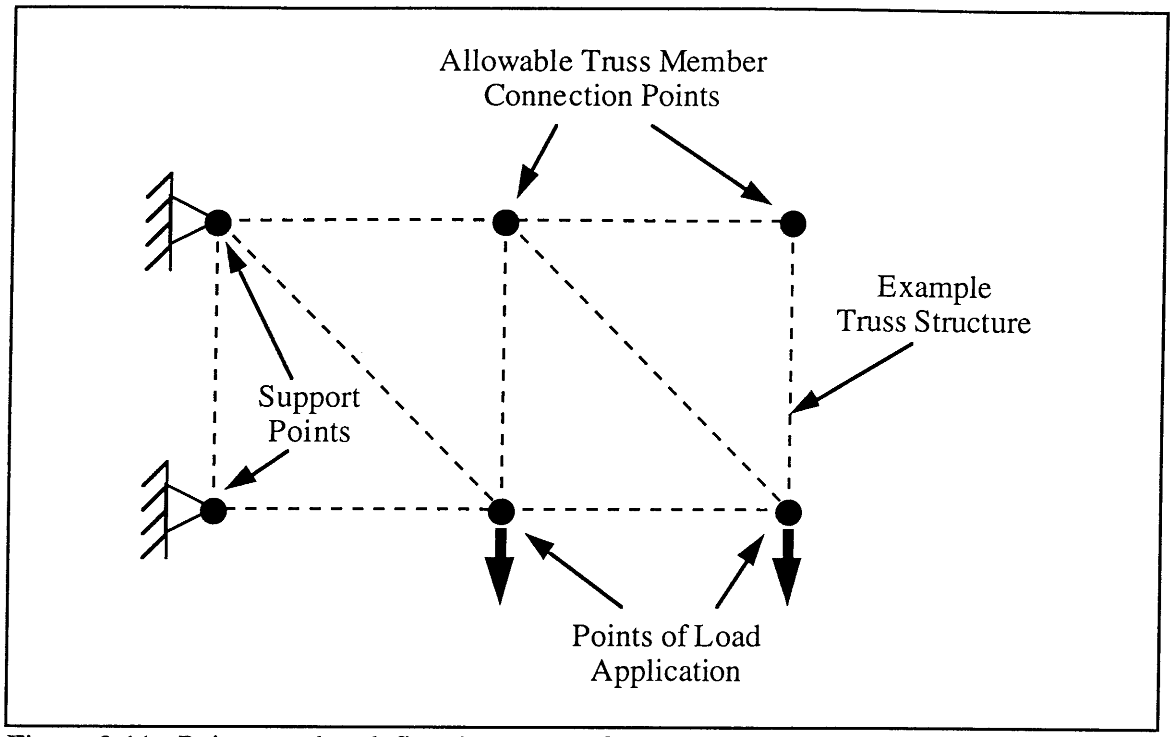

Allowable Truss Member Connection Points
Example Truss Structure
Support Points
Points of Load Application

Figure 3.11: Points used to define the nature of a truss structure, along with an example truss structure using the points. Adapted from Hajela *et al.* (1993).

Finally, Sakamoto and Oda (1993) couple the genetic algorithm with an optimality criteria technique to optimize the topology and cross-sectional dimensions of discrete truss structures. The procedure begins with the establishment of a *ground structure*, which represents the structure's maximum possible connectivity—given the structure's pre-defined node locations (*i.e.*, points of support, points of load application, and allowable truss member connection points), each node is linked to all other nodes via truss members (Figure 3.12). After creating the ground structure, a population of genetic algorithm chromosomes is configured so that each chromosome controls the existence of every truss member in the ground structure. Specifically, each gene in any given chromosome controls a particular truss member—an allele value of 1 specifies that the corresponding truss member exists, while an allele value of 0 specifies that the corresponding truss member does not exist. Hence, the population of chromosomes represents a population of possibly-optimal truss topologies. Genetic algorithm-based optimization then generates the truss topology with minimum weight subject to a maximum displacement constraint. Note that the genetic algorithm is only responsible for the topological optimization of the truss—all sizing optimization is performed by an optimality criteria-based algorithm.

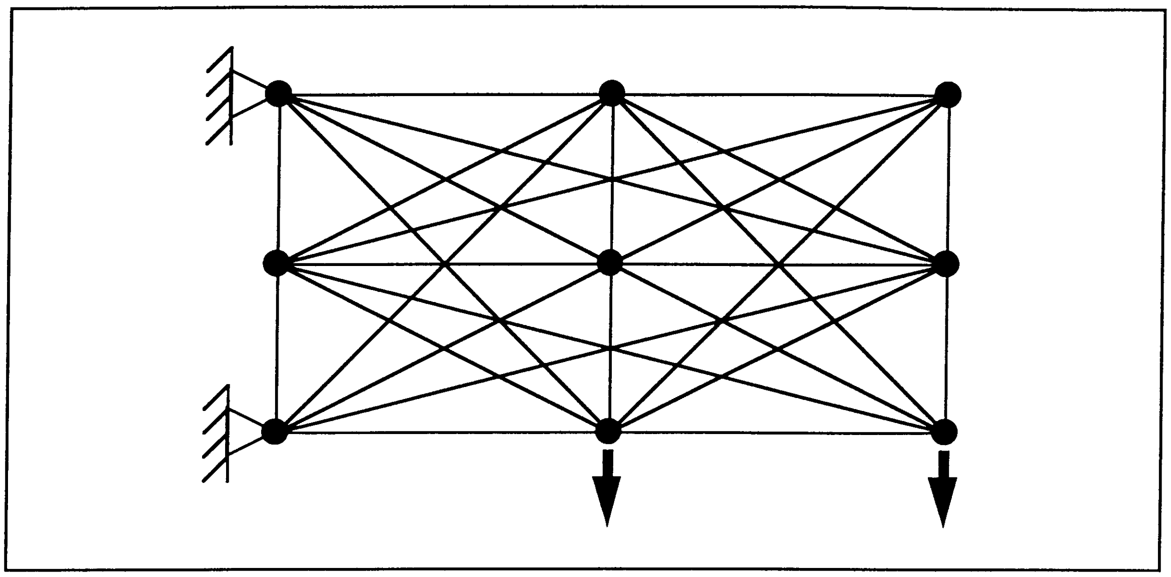


Figure 3.12: An example ground structure. Adapted from Sakamoto and Oda (1993).

Continuum Structures. In addition to the genetic algorithm-based topological optimizations detailed above, which generate the optimal topologies of *discrete*-membered truss structures, the genetic algorithm has also been applied to the topological optimization of *continuum* structures. Similar to the homogenization- and simulated annealing-based techniques detailed in Section 3.2, these genetic algorithm-based approaches attempt to find the optimal distribution of material and void within a discretized design domain.

While genetic algorithm-based topological optimization of continuum structures is the focus of this investigation, the first efforts in this field are those of Jensen and associated researchers (Sandgren *et al.* (1990), Sandgren and Jensen (1992), Jensen (1992)), who apply the genetic algorithm to the topological optimization of a variety of continuum structures. Specifically, Jensen and associated researchers use the genetic algorithm to generate component topologies with minimum weight subject to displacement and (in some cases) stress constraints.

Jensen utilizes a variety of design domain representations to convert genetic algorithm chromosomes into component topologies during optimization. In most examples, Jensen uses a representation similar to that of the previously-detailed simulated annealing-based investigations, where the design domain is discretized into small, rectangular elements and each element can contain either material or void—no intermediate densities are allowed. With this representation, each chromosome in a genetic algorithm population controls the material-void states of the elements in the design domain, and therefore corresponds to a particular distribution of material and void within the domain. Hence, evolving a population of chromosomes using this binary, material-void representation generates the optimal distribution of material and void within the design domain. Other examples detailed by Jensen use design domain elements of variable thickness, where each element can assume one of three allowable material thickness values. In this representation, each chromosome in a genetic algorithm population controls the thicknesses of the design domain elements, and therefore corresponds to a particular material thickness distribution within the domain. Hence, evolving a population of chromosomes using this three-thickness representation generates an optimal distribution of material thickness within the design domain. Finally, one of Jensen's examples models the design domain as an assemblage of beam elements, where each beam element may either exist or vanish. With this representation, each chromosome in a genetic algorithm population controls the existence of the beam elements, and therefore corresponds to a particular configuration of existing and non-existing beam elements within the design domain. Hence, evolving a population of chromosomes using this beam-element representation generates an optimal configuration of existing and non-existing beam elements within the design domain.

Jensen's initial efforts in genetic algorithm-based structural optimization, detailed by Sandgren *et al.* (1990), optimize the topology of an automotive bumper beam's cross-section as well as the topology of an automotive body panel. The automotive bumper beam example, which uses a binary, material-void design domain to represent the beam's cross-section, attempts to generate the beam cross-section topology which provides minimum weight subject to a maximum stress constraint. In the automotive body panel example, the topology of an automobile trunk lid is optimized so that it provides minimum weight and satisfies maximum stress and displacement constraints. During optimization, the trunk lid's topology is modeled using a three-thickness design domain representation.

Sandgren and Jensen (1992) also examine the topological optimization of bumper beam cross-sections and trunk lids. However, they extend the original work of Sandgren *et al.* by providing new design representations and obtaining improved results. The automotive trunk lid example, instead of using the previous investigation's three-thickness design domain representation, models the structure as an assemblage of 110 beam elements. Note that the example attempts to generate the trunk lid topology with minimum weight subject to a maximum displacement constraint. In the automotive bumper beam example, the bumper is modeled as a multi-segmented, simply-supported beam subject to bending and shear (Figure 3.13). Using a binary, material-void design domain representation, the beam's three individual cross-section topologies are optimized for minimum weight subject to a maximum displacement constraint.

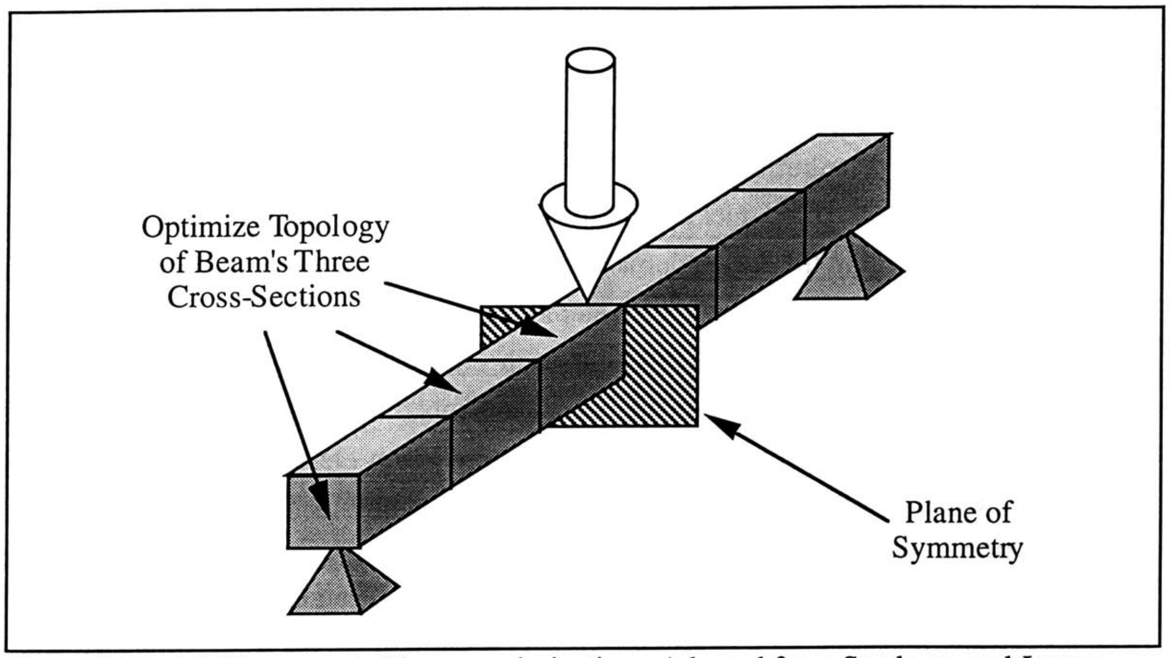


Figure 3.13: Multi-segmented beam optimization. Adapted from Sandgren and Jensen (1992).

A comprehensive overview of the investigation is provided by Jensen (1992). First, to obtain values of genetic algorithm control parameters (*i.e.*, population size, mutation rate, etc.) which are appropriate for structural topology optimization, Jensen conducts experiments with a "similarly formed" pattern matching problem. Actual structural topology optimization examples are then detailed, the first being an automotive bumper beam optimization. In addition to containing the multi-segmented bumper beam example detailed by Sandgren and Jensen (1992), the example details the topological optimization of a simpler, single-segment beam (Figure 3.14). As in the more complicated, multi-segmented example, the beam's cross-section is modeled with a binary, material-void design domain, and the optimization attempts to generate the beam cross-section providing minimum weight subject to a maximum displacement constraint. Results of the optimization are depicted in Figure 3.15.

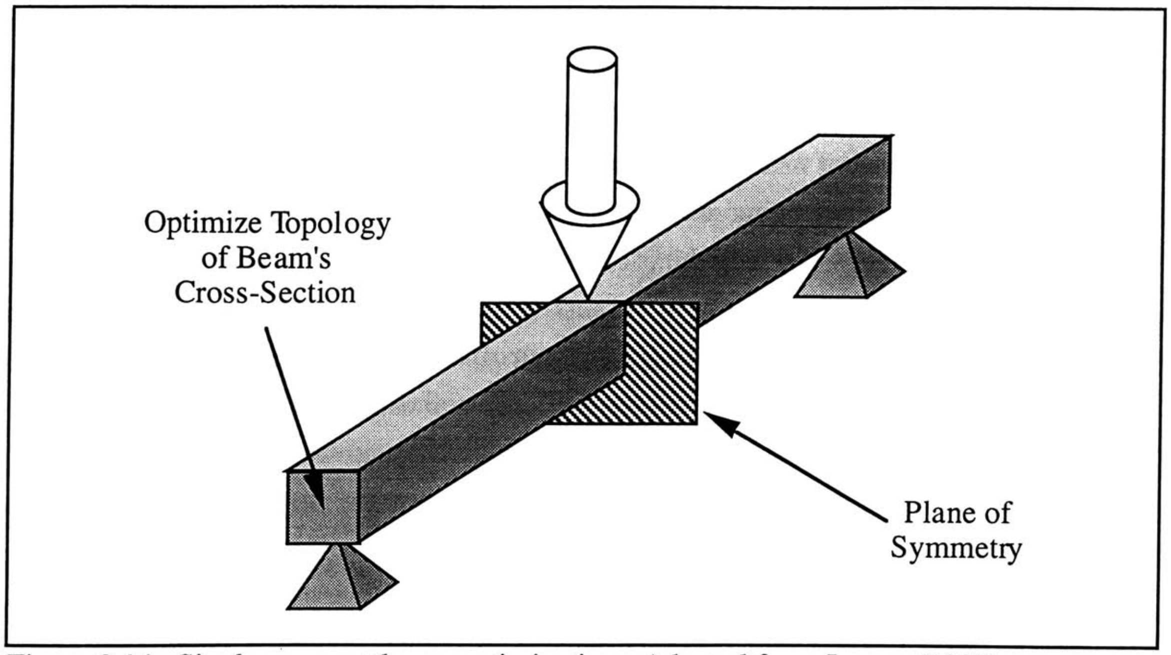


Figure 3.14: Single-segment beam optimization. Adapted from Jensen (1992).

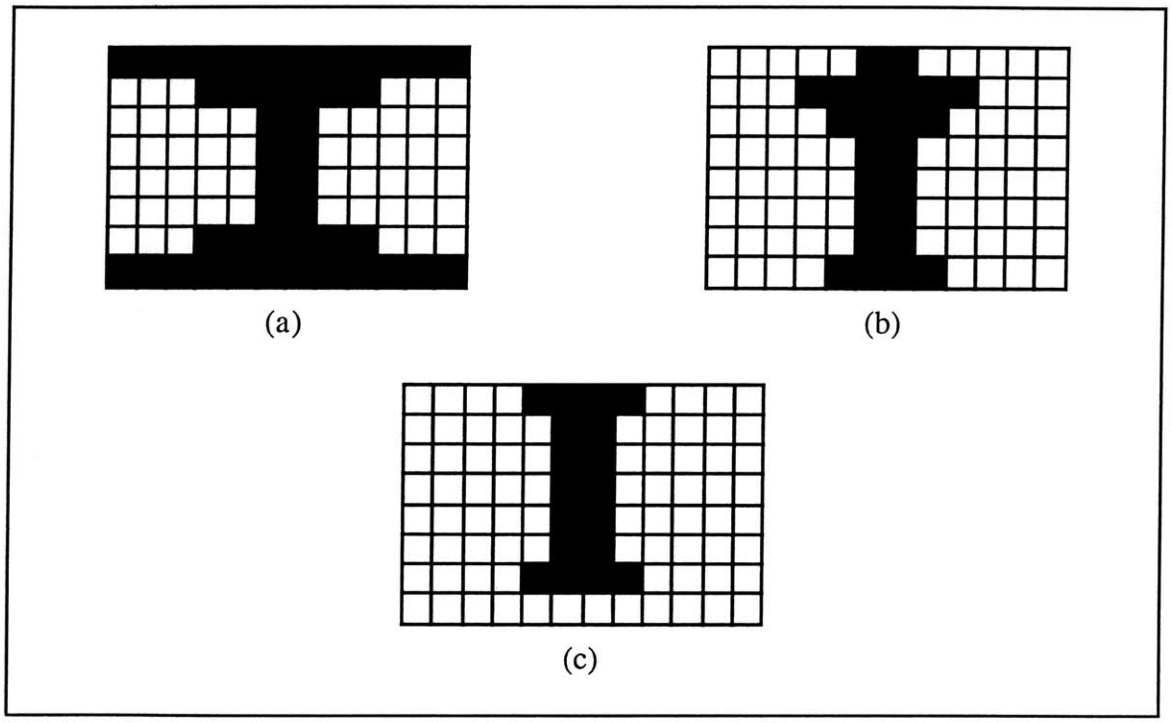


Figure 3.15: Optimal beam cross-sections for (a) plastic, (b) aluminum, and (c) steel. Adapted from Jensen (1992).

Another example detailed by Jensen (1992) attempts to find the optimal topology of a tall, thin cantilevered plate subject to a downward vertical load. Using a binary, material-void design domain, the optimization attempts to minimize the structure's weight subject to a maximum displacement constraint. Jensen's cantilevered plate example is similar to the examples detailed in this investigation, in that the topology of a cantilevered plate is optimized using a binary, material-void design domain discretization. However, whereas this investigation's use of Connectivity Analysis (Section 4.4.4) prevents the generation of structure topologies containing material elements connected only at corners, Jensen's "optimal" plate topology is a very thin, "chain-link" topology comprised almost entirely of material elements connected only at corners (Figure 3.16). Note that planar material elements connected only at corners (*i.e.*, planar material elements which do not share edges) cannot support torques or compressive loads and can therefore lead to an unstable structure. Hence, Jensen's topology should *collapse* when subjected to the given loading condition. Section 5.5.4 provides further details of the plate topology generated by Jensen and also demonstrates how the topology differs from topologies generated using this investigation's technique.

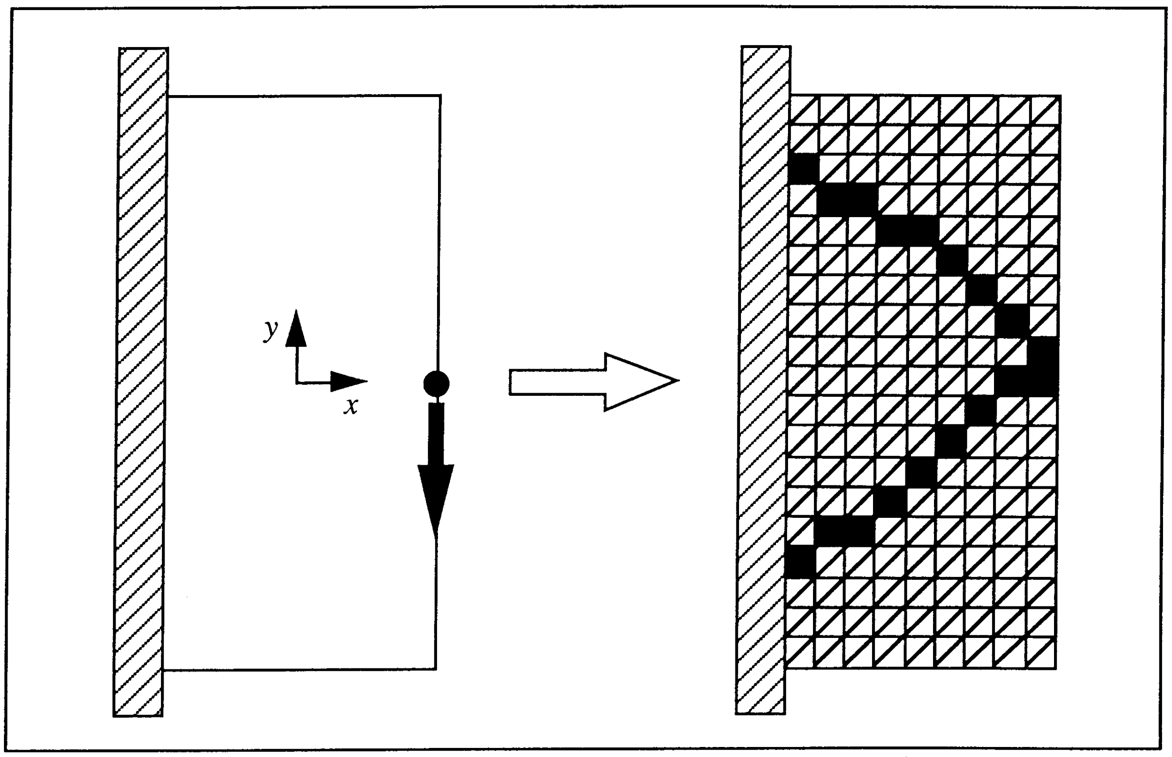


Figure 3.16: Cantilevered plate optimization. Adapted from Jensen (1992).

Finally, Jensen (1992) provides two automotive body panel examples. The first example, which attempts to generate the trunk lid topology providing minimum weight subject to a maximum displacement constraint, models the trunk lid with three-thickness design domain elements and is nearly identical to the automotive body panel optimization detailed by Sandgren *et al.* (1990). The second body panel optimization, which also uses a three-thickness design domain to model a trunk lid, attempts to generate the trunk lid topology which provides minimum weight subject to maximum stress and displacement constraints. In addition to using a slightly different objective function, this second trunk lid optimization uses six different loading conditions and uses a slightly finer design domain discretization.

The only other investigation (to the investigator's knowledge) of genetic algorithm-based structural topology optimization of continuum structures is that described in this article. Detailed in subsequent chapters, this investigation extends Jensen's work in the topological optimization of mechanical components modeled with binary, material-void design domains. Specifically, this investigation examines, in great detail, problems similar to Jensen's beam cross-section and cantilevered plate optimizations. For further details regarding this investigation, please refer to articles by Chapman and Jakiela (1994) and Chapman *et al.* (1993a, 1993b).

This Investigation 4

4.1 Overview

This chapter first introduces the genetic algorithm-based structural topology optimization technique developed in this investigation and then describes the areas in which this investigation extends previous work in genetic algorithm-based structural topology optimization. Finally, details regarding the implementation of this investigation's technique are provided.

4.2 The Technique

4.2.1 Introduction. This investigation uses the genetic algorithm to perform structural topology optimization of continuum structures, where the optimal distribution of material within a discretized design domain is found. The technique developed in this investigation can be applied to a variety of structures, which may be optimized based on strength, stiffness, compliance, or manufacturability considerations.

4.2.2 Performing Optimization. In the technique, a design domain is first discretized into small, square elements, where each element can contain either material or void—no intermediate densities are allowed. Hence, defining the material-void state of each element in the design domain establishes a particular structure topology, and modifying the material-void state of any design domain element results in a modification of the structure's topology. Prior to optimization, a population of genetic algorithm chromosomes is configured so that each chromosome controls the material-void state of every element in the design domain. Consequently, each chromosome in the genetic algorithm population represents a possibly-optimal structure topology, and the population of chromosomes therefore represents a population of structure topologies.

Optimization begins when an initial population of chromosomes (*i.e.*, an initial population of structure topologies) is generated at random. After first evaluating the structural performance and constraint satisfaction abilities of each structure topology in the population, the highest-quality topologies are selected to serve as parents. Parent topologies then pair and mate via genetic crossover, creating a population of child topologies. These child topologies are then subjected to random, infrequent mutation, after which they replace the parent topologies. The process then iterates until an optimal structure topology is found.

4.3 Extensions of Previous Work

4.3.1 Introduction. This investigation focuses and extends the work of Jensen and associated researchers (Section 3.3.4) in genetic algorithm-based structural topology optimization of continuum structures. Specifically, whereas Jensen's research examines the genetic algorithm-based topological optimization of a variety of structure types (beam cross-sections, cantilevered plates, automotive body panels) using a variety of design domain representations (material-void, three-thickness, beam assemblage), this investigation focuses on, and examines in great detail, genetic algorithm-based topological optimization of cantilevered plates using a binary, material-void design domain representation.

4.3.2 Specific Extensions of Previous Work. While Jensen does use his genetic algorithm-based structural topology optimization approach to generate the optimal topology of a cantilevered plate using a binary, material-void design domain discretization (please refer to Section 3.3.4 for details), only one example is provided, and that example only presents two optimal plate topologies (one with symmetry assumptions, one without symmetry) and then compares the optimal plates' general shape to that of an analytical solution—no attention is paid to alternate finite element meshing techniques, finer design domain discretizations, different fitness function formulations, etc.

Conversely, after first establishing the viability of its genetic algorithm-based structural topology optimization approach by examining the straightforward topological optimization of a beam cross-section, this investigation focuses entirely on the topological optimization of cantilevered plates. Using genetic algorithm-based topological optimization of cantilevered plates as its specific application domain, this investigation demonstrates and enhances the abilities and efficiency of its approach by examining the following areas in detail:

- Varied loading conditions and design domain geometries, as well as a variety of increasingly-fine design domain discretizations.

- Advantages and disadvantages of deterministically removing disconnected material elements from cantilevered plate topologies.

- The efficiency and performance of several finite element meshing techniques.

- Effective and efficient techniques for obtaining finely-discretized cantilevered plate topologies.

- The genetic algorithm's ability to generate families of highly-fit cantilevered plate topologies.

- Methods for obtaining cantilevered plate topologies which combine high strength with high manufacturability.

- The genetic algorithm's structural topology optimization abilities, compared with those of homogenization-based techniques.

So, while this investigation's technique is similar to that of Jensen and associated researchers, in that it performs genetic algorithm-based structural topology optimization of continuum structures, the examples detailed in this investigation are considerably more focused and comprehensive than those examined by Jensen.

4.3.3 Fundamental Differences Between the Techniques. In addition to the differences in content provided by the investigations' examples, fundamental differences in *chromosome structure* and *fitness function formulation* exist between this investigation's approach to genetic algorithm-based structural topology optimization and that of Jensen and associated researchers.

Chromosome Structure. To provide an intuitive correspondence between genetic algorithm chromosomes and the two-dimensional structure topologies which they represent, Jensen uses two-dimensional chromosomes and a two-dimensional crossover operator. So, instead of representing each topology with a string of genes and swapping sub-strings during reproduction (*i.e.*, instead of using standard, one-dimensional genetic algorithm chromosomes and a one-dimensional crossover operator), Jensen represents each topology with an array of genes and swaps rectangular gene regions during reproduction (shown schematically in Figure 4.1). A similar two-dimensional chromosome structure and reproduction operator is detailed by Cartwright and Harris (1993). Note, however, that the Schema Theorem (Section 2.3.4) is not applicable to these two-dimensional chromosomes.

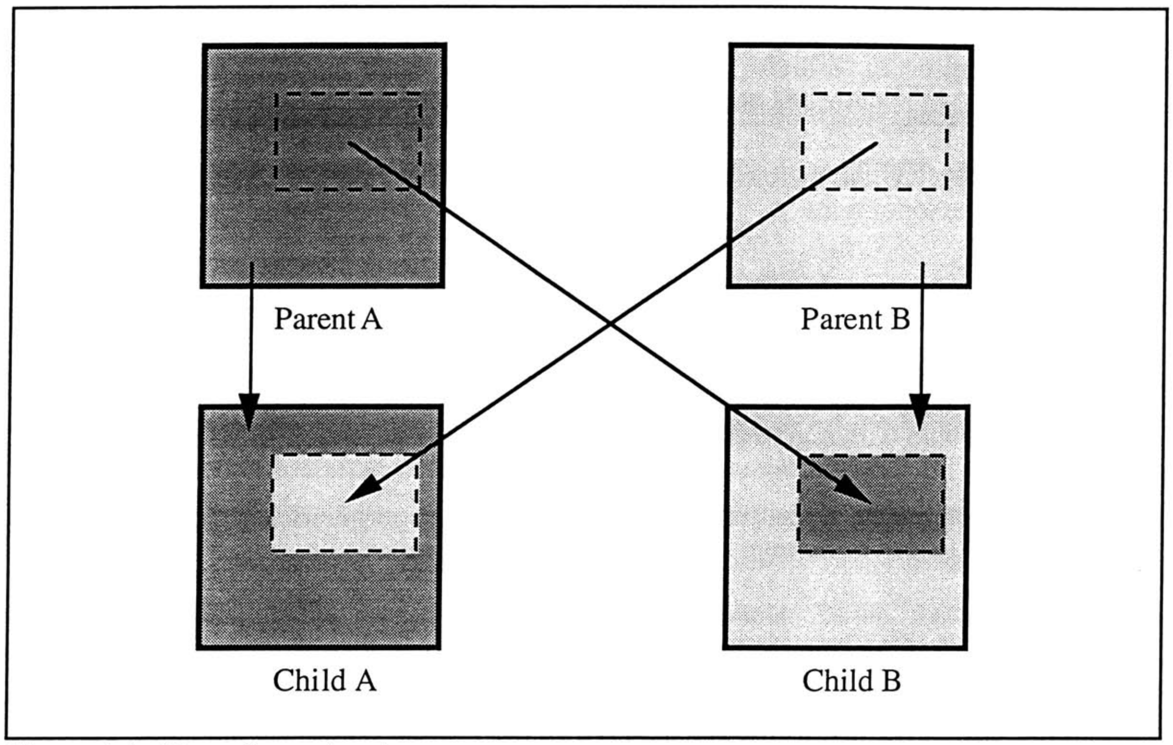


Figure 4.1: Two-dimensional crossover.

This investigation utilizes one-dimensional chromosomes and a one-dimensional crossover operator to represent and mate two-dimensional structure topologies. Hence, each structure topology is represented by a string of genes, and reproduction occurs by swapping sub-strings between parents. While this chromosome representation and crossover operator are the simplest possible and do not provide the intuitive 2D-chromosome-to-2D-design-domain correspondence of Jensen's chromosome representation, they have provided satisfactory results during the course of this investigation and are applicable to the Schema Theorem. Note that no experimentation with two-dimensional chromosomes or crossover was undertaken in this investigation.

Fitness Function Formulation. The topological optimizations examined by Jensen and associated researchers all attempt to minimize a structure's weight subject to maximum displacement and (in some cases) maximum stress constraints. Hence, the fitness functions (to be maximized) used by Jensen during chromosome fitness calculations are generally of the form:

$$Fitness = \mathrm{f}(Weight) - \mathrm{g}(Displacement, Stress), \tag{4.1}$$

where $\mathrm{f}(Weight)$ represents a function which increases in value as the structure's weight decreases and $\mathrm{g}(Displacement, Stress)$ represents a function which increases in value as the structure's constraint satisfaction abilities diminish and decreases in value as the structure's constraint satisfaction abilities improve. Note that a great amount of experimentation is required to use these fitness functions effectively, for the magnitude of the penalty terms must be properly balanced with the quality terms. In other words, $\mathrm{g}(Displacement, Stress)$ must be properly balanced with $\mathrm{f}(Weight)$. If the penalty terms are given too much emphasis, the optimization process will typically result in a structure with low stress and deflection levels, but which contains an excessive amount of material. Likewise, if the penalty terms are given too little emphasis, the optimization will likely result in a light structure with high levels of stress and deflection. Jensen examines a situation where these fitness functions can lead to "optimal" structures containing no material (termed *null* structures), and details techniques for avoiding such situations. Note that because these fitness functions attempt to minimize weight subject to maximum displacement and stress constraints, the optimal topology generated by the technique depends upon the structure's material properties and the magnitude of the applied load. Hence, optimization simultaneously addresses the structure's size, shape, and topology.

In an attempt to avoid the penalty term scaling problems inherent with constrained optimization fitness functions, this investigation introduces an unconstrained fitness function (to be maximized) which bases a structure's fitness on its Strength-to-Weight or Stiffness-to-Weight ratio:

$$Fitness = \frac{Strength}{Weight} \tag{4.2a}$$

or

$$Fitness = \frac{Stiffness}{Weight}. \tag{4.2b}$$

Note that these fitness functions *automatically* assign the highest fitness value to the structure which offers the best combination of light weight and load-carrying ability. Hence, there is no need to carefully balance magnitudes of quality terms with those of penalty terms—these unconstrained fitness functions eliminate all penalty coefficients and penalty term scaling. This normalization allows the genetic algorithm to focus on optimizing the structure's shape and topology, independent of the magnitude of the applied load or the structure's material properties. Hence, the proper size of the structure is left for later sizing optimization.

While the majority of the examples provided in this investigation utilize the unconstrained fitness functions detailed above, these fitness functions can easily be modified to account for various constraints. A linear penalty term, penalizing the strength-to-weight or stiffness-to-weight ratio by P% for every 10% constraint violation, provides satisfactory performance in many of the examples detailed in Chapter 5. However, as with Jensen's fitness functions, great care is required when choosing a penalty parameter magnitude (in this case, P) which is well balanced with the quality terms of the fitness function. This investigation uses penalty-based fitness functions to generate structures combining high strength-to-weight ratio with low stress levels, structures combining high stiffness-to-weight ratio with high manufacturability, and structures with minimum mean compliance subject to a maximum volume constraint.

4.4 Implementation

4.4.1 Introduction. When applying the genetic algorithm to a particular application domain, several points are worthy of note. First, the general structure of the genetic algorithm requires no modification—optimization always occurs through an "evolutionary" process, where a population of artificial chromosomes is subjected to genetics-based operators such as evaluation, selection, reproduction, mutation, and replacement (detailed in Section 2.3.4). Second, the specific operation of the selection, reproduction, mutation, and replacement operators also requires no modification—these operators perform their duties without regard to the particular application domain (again, detailed in Section 2.3.4). These operators, however, *do* require modification if the general one-dimensional, string-of-binary-digits chromosome structure is modified. Finally, because each chromosome in the population is a coded representation of the optimization problem's design variables, the chromosome structure must be specially-tailored to the design variables of the optimization problem to which the genetic algorithm is being applied (detailed in Section 2.3.3). Likewise, because the fitness function must take an individual chromosome as input, decode the chromosome into its corresponding design variable values, and then quantitatively grade the chromosome according to the objective function minimization (or maximization) and constraint satisfaction abilities of its corresponding design variable values, the fitness function must also be specially-tailored to the particular optimization problem (detailed in Section 2.3.4).

Hence, in this work, where the genetic algorithm is used to optimize the topology of structural components, each chromosome must be configured so that it represents a structure topology. Likewise, the fitness function must be formulated so that it can take an individual chromosome as input, convert the chromosome into its corresponding structure topology, evaluate the topology's structural performance, and then assign a quantitative fitness value to the chromosome according to the topology's structural performance and constraint satisfaction abilities.

The following sections describe the technique used to represent structure topologies with chromosomes (*i.e.*, the design representation), the specific procedures used to convert any given chromosome into a structure topology, the technique used to remove disconnected material elements from a structure topology, the methods used to analyze a topology's structural performance, and the approaches taken to convert a topology's structural performance into a quantitative fitness value for the corresponding chromosome.

4.4.2 Design Representation. During optimization, this investigation uses a rectangular, two-dimensional design domain to establish the maximum allowable size of the component being optimized. This design domain is discretized into small, square elements, where each element can contain either material or void—no intermediate densities are allowed. Figure 4.2 depicts an example design domain discretization for a square, cantilevered plate subject to a downward vertical load. Note that the design domain shape and the discretization resolution depend upon the particular problem and the available computational resources. Once the discretized design domain has been created, assigning material or void to each element in the domain (*i.e.*, designating the *state* of each design domain element) defines a distribution of material and void within the design domain, therefore establishing a structure topology. Likewise, modifying the state of any element (*i.e.*, toggling a material element to void or a void element to material) results in a modification of the structure's topology. Hence, the material-void states of the design domain elements serve as design variables during optimization. Figure 4.3 depicts an example distribution of material and void within the example design domain, along with the structure topology corresponding to this example material-void distribution.

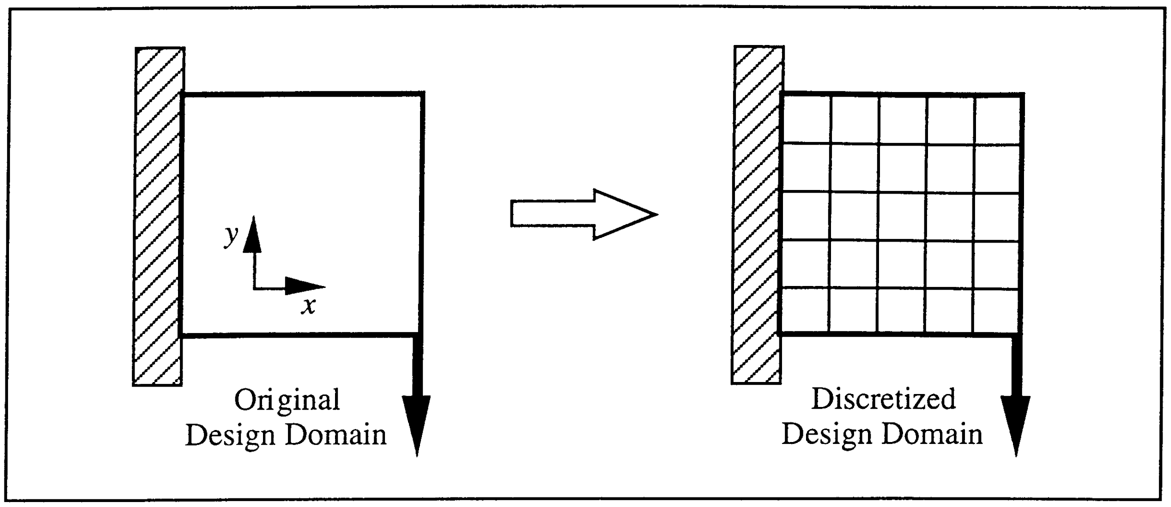


Figure 4.2: Design domain discretization.

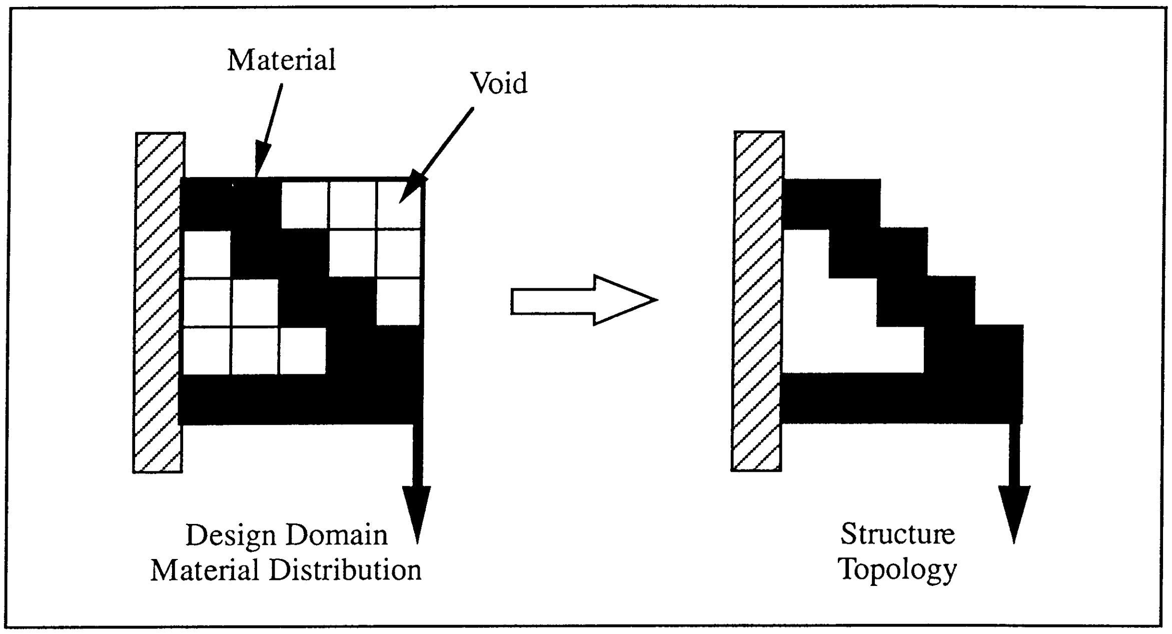


Figure 4.3: Design domain material distribution to structure topology correspondence.

The chromosomes in the genetic algorithm population are configured so that each chromosome controls the material-void state of every element in the design domain. Specifically, each gene in any given chromosome controls the state of a particular design domain element—an allele value of 1 specifies that the corresponding element contains material, while an allele value of 0 specifies that the corresponding element contains void. Hence, each chromosome represents a structure topology, and the population of chromosomes therefore represents a population of structure topologies (Figure 4.4).

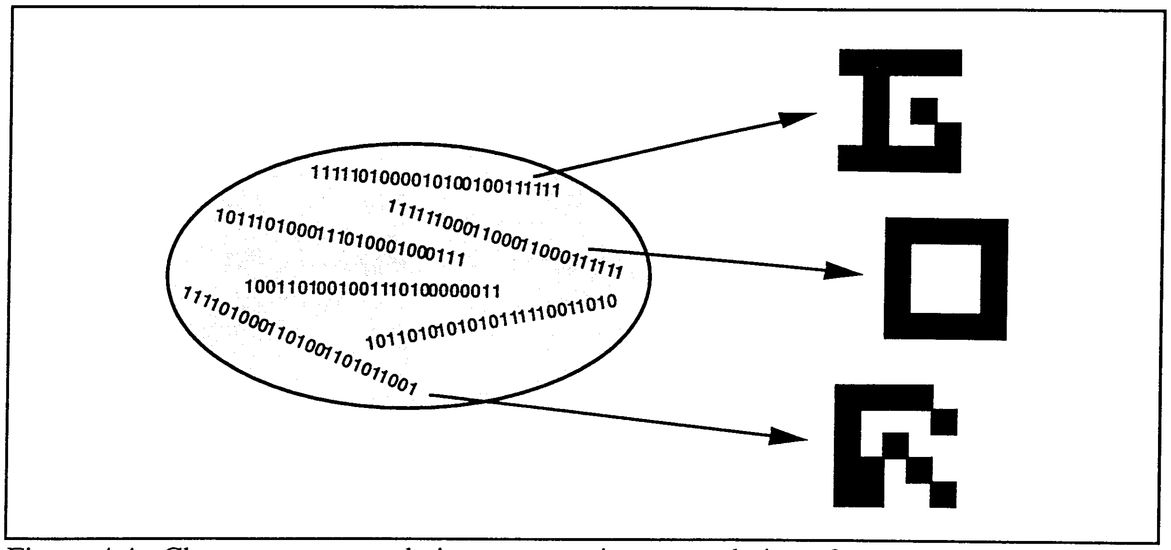


Figure 4.4: Chromosome population representing a population of structure topologies.

While this binary, material-void design domain representation is used in other structural topology optimization investigations based on simulated annealing (Section 3.2.2) and the genetic algorithm (Section 3.3.4), it results in a discrete, typically nonconvex search space (Anagnostou *et al.* (1992), Ghaddar *et al.* (1993)) and is not recommended for shape optimization (Strang and Kohn, 1986). However, this binary, material-void design domain representation does serve as an excellent test of the genetic algorithm's ability to find optima in discrete, nonconvex search spaces, and it allows for a natural conversion between chromosomes (strings of 0's and 1's) and the corresponding topologies (distributions of void and material). Also, the discrete nature of the domain allows for a precise, although discretized, topology boundary. Note that homogenization-based methods (Section 3.2.1) attempt to avoid discretization and nonconvexity of the search space by using a design domain representation which allows the density of any design domain element to vary continuously between 0 (void) and 1 (solid material).

4.4.3 Converting a Chromosome into a Topology. When the fitness function receives a genetic algorithm chromosome for fitness evaluation, it must first convert the chromosome into a structure topology. As detailed above, each chromosome in the genetic algorithm population contains one gene for every element in the design domain, and each gene corresponds to, and controls the material-void state of, a particular design domain element. Hence, to convert a chromosome into a topology, the chromosome is simply mapped into the design domain, and each element in the domain then assumes the material-void state specified by the allele value of the chromosome gene controlling the element—elements controlled by genes with allele values of 1 become material, while elements controlled by genes with allele values of 0 are set to void (Figure 4.5). Once the distribution of material and void within the design domain is defined, the structure topology corresponding to the current chromosome is established. Because the genetic algorithm chromosomes in this research are one-dimensional strings of genes, the one-dimensional chromosomes are mapped into the two-dimensional design domain from left to right, top to bottom.

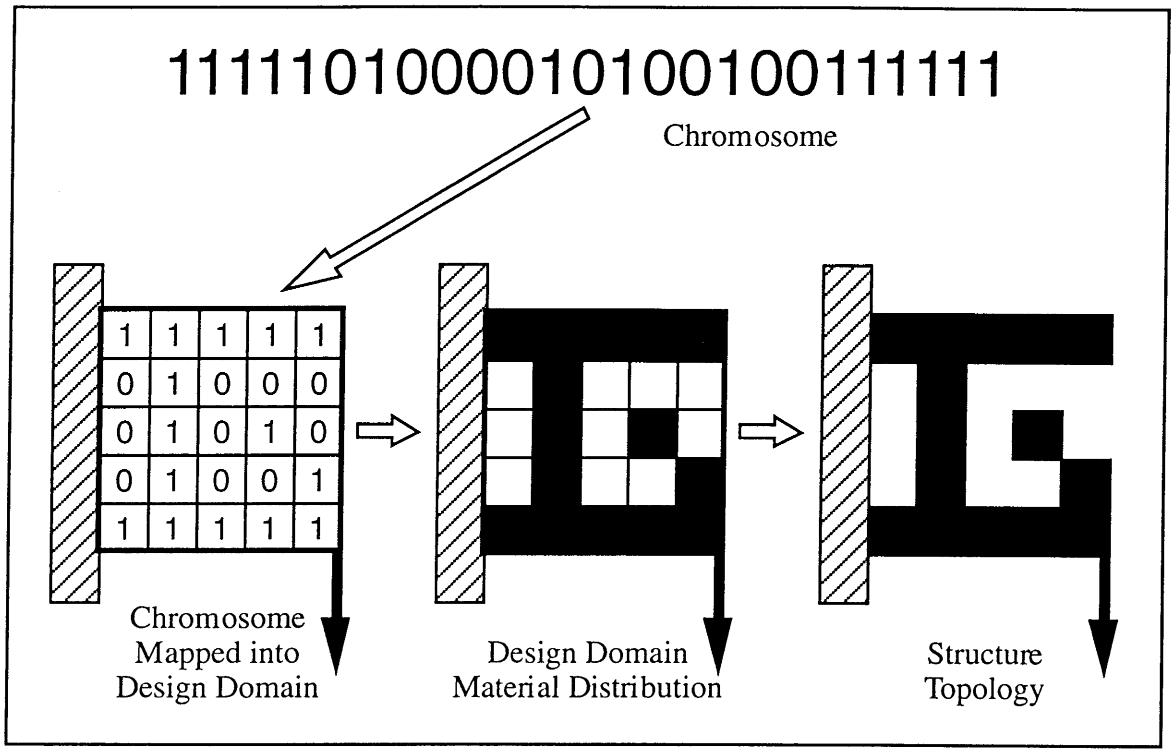


Figure 4.5: Mapping a genetic algorithm chromosome into the design domain to create a structure topology.

4.4.4 Connectivity Analysis. After the current chromosome has been converted into a structure topology, the topology is analyzed for connectedness. This "connectivity analysis" sets to void all material elements in the design domain which are not connected (whether directly or indirectly via other material elements) to one or more *seed* elements. *Seed* elements are those elements in the design domain which are required to contain material so that they may serve as a support boundary condition or point of load application. Figure 4.6 depicts the connectivity analysis of an example structure topology with an example set of seed elements, and the resulting structure. Note that for any two material elements to be considered connected, they must share an edge; material elements sharing only a corner are considered disconnected (Figure 4.7).

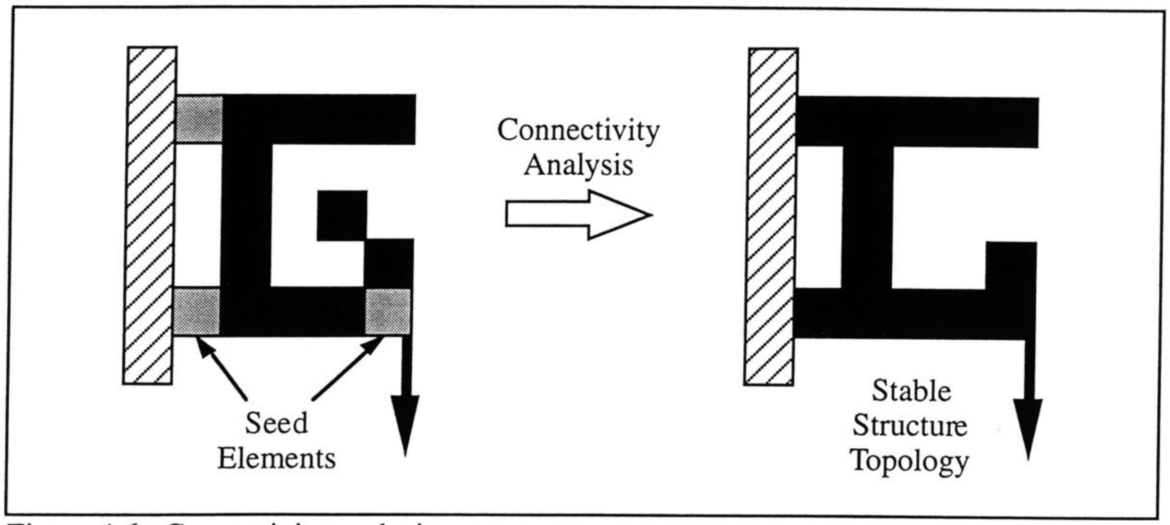


Figure 4.6: Connectivity analysis.

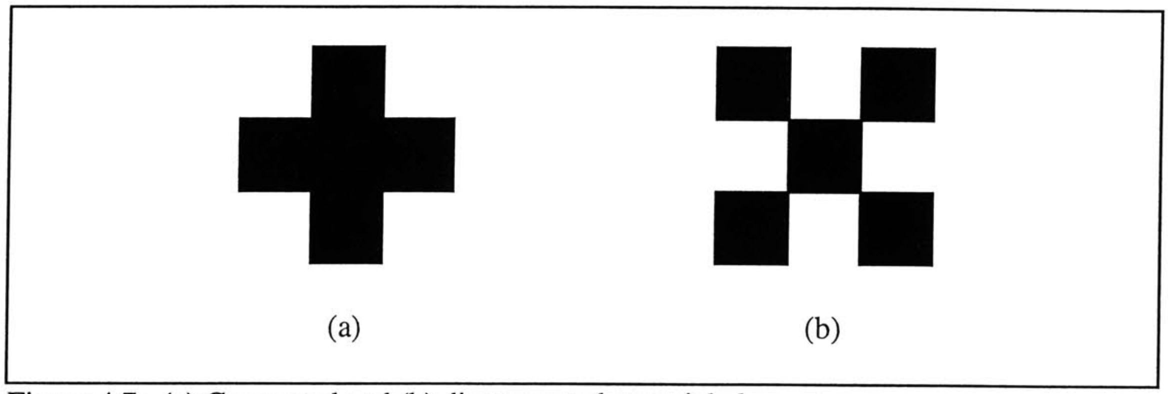


Figure 4.7: (a) Connected and (b) disconnected material elements.

When connectivity analysis is activated, disconnected material elements are treated as if they are void—they are assigned zero weight and volume and are considered to be void when performing structural analysis. Conversely, when connectivity analysis is deactivated, disconnected material elements are treated as if they are material—they are counted in weight and volume calculations and are considered to be material when performing structural analysis. Note that when connectivity analysis is performed, the corresponding chromosome allele value is not modified when a disconnected material element is switched to void or a seed element is switched to material—only the state of the design domain element is changed. In other words, connectivity analysis does not alter a chromosome's *genotype* (*i.e.*, the chromosome's genetic code) and instead alters the chromosome's *phenotype* (*i.e.*, the design variable values corresponding to the chromosome genotype).

Connectivity analysis is performed so that all structure topologies are stable. Planar material elements connected only at a corner cannot withstand applied torques about the corner, and can therefore lead to a structure which cannot support various loads. An "optimal" structure comprised almost entirely of material elements connected corner-to-corner is generated by Jensen (Figure 3.16) and detailed in Sections 3.3.4 and 5.5.4. The structure, which supports tensile and *compressive* loads, is likely assisted by the soft-material void elements surrounding the structure. Note that because this investigation performs connectivity analysis, optimal structure topologies generated in this investigation do not contain material elements connected corner-to-corner as in Jensen's example. In fact, Example 3 in Chapter 5 demonstrates that the quality of the structure topologies generated by this investigation's approach *decreases* when connectivity analysis is deactivated.

Connectivity analysis does not guarantee that a structure topology is stable in a classical sense (*i.e.*, it does not guarantee that any given structure topology will resist buckling), for a structure topology containing disconnected material elements could indeed resist buckling while a structure topology containing only connected material elements could fail due to buckling. So, it may be more appropriate to state that connectivity analysis is performed because disconnected material elements will *likely* lead to structural instability. Hence, connectivity analysis is not equivalent to, nor does this investigation perform, classical stability or buckling analysis. However, connectivity analysis is sufficient for the genetic algorithm-based generation of optimal structure topologies, which are generated without stress, displacement, or buckling constraints, while classical stability and buckling analyses are best suited to subsequent sizing optimization, in which the size of the optimal structure topology could be optimized to account for a given applied load magnitude and stress, displacement, and buckling constraints.

While disconnected material elements may lead to unstable structures and are therefore removed from structure topologies via connectivity analysis, genetic algorithm chromosomes corresponding to structures containing disconnected material elements are not penalized and allele values of chromosome genes creating disconnected material elements are not altered. Hence, the genetic algorithm search is not driven towards structure topologies which combine, without the aid of connectivity analysis, high structural performance and few disconnected material elements. Instead, the genetic algorithm search is driven towards structure topologies with maximum structural performance, regardless of the number of disconnected material elements in the structure (which are, because of connectivity analysis, ignored during fitness calculations). So, because disconnected material elements are ignored during optimization and the genetic algorithm search is not driven towards structure topologies containing few disconnected material elements, groups of disconnected material elements are allowed to propagate into subsequent generations with the possibility that the elements can, as a result of mating or mutation, become connected via edges to create a structure topology of exceptionally high fitness.

4.4.5 Structural Analysis. At the conclusion of connectivity analysis, the current genetic algorithm chromosome has been converted into a stable, possibly-optimal structure topology. The topology's structural performance must then be determined so that the chromosome can be assigned a quantitative fitness value for use in genetic algorithm parent selection.

This investigation performs structural analyses using either analytical or finite element techniques. However, only Example 1 in Chapter 5 uses analytical structural analysis—all other examples in this investigation utilize finite element analysis to determine a topology's structural performance. Hence, details of this investigation's analytical structural analysis approach are provided in Example 1, while this section details this investigation's use of the finite element method.

After the current genetic algorithm chromosome has been converted into a stable structure topology, a finite element mesh representing the entire discretized design domain is created. Note that either two or four triangular finite elements are used to represent every square design domain element, and that triangular finite elements are used for compatibility with an existing finite element software package. Using two triangular finite elements per design domain element yields a finite element node at each corner of every design domain element, while using four triangular finite elements per design domain element yields a finite element node at each corner, and in the center, of every design domain element. Once the mesh has been created, the finite element nodes corresponding to points of support are defined to have zero displacement in the finite element analysis, and a concentrated load is applied to the appropriate finite element node. Figure 4.8 depicts the creation of finite element meshes (containing either two or four finite elements per design domain element) corresponding to an example design domain discretization and an example set of seed elements. Note that the only finite element nodes constrained to have zero displacement are those which both correspond to a seed element and are adjacent to the support—finite element nodes adjacent to the support which correspond to non-seed design domain elements are not constrained to have zero displacement (except in Example 8 of Chapter 5, where all finite element nodes adjacent to the support, regardless of whether or not they correspond to a seed element, are constrained to have zero displacement).

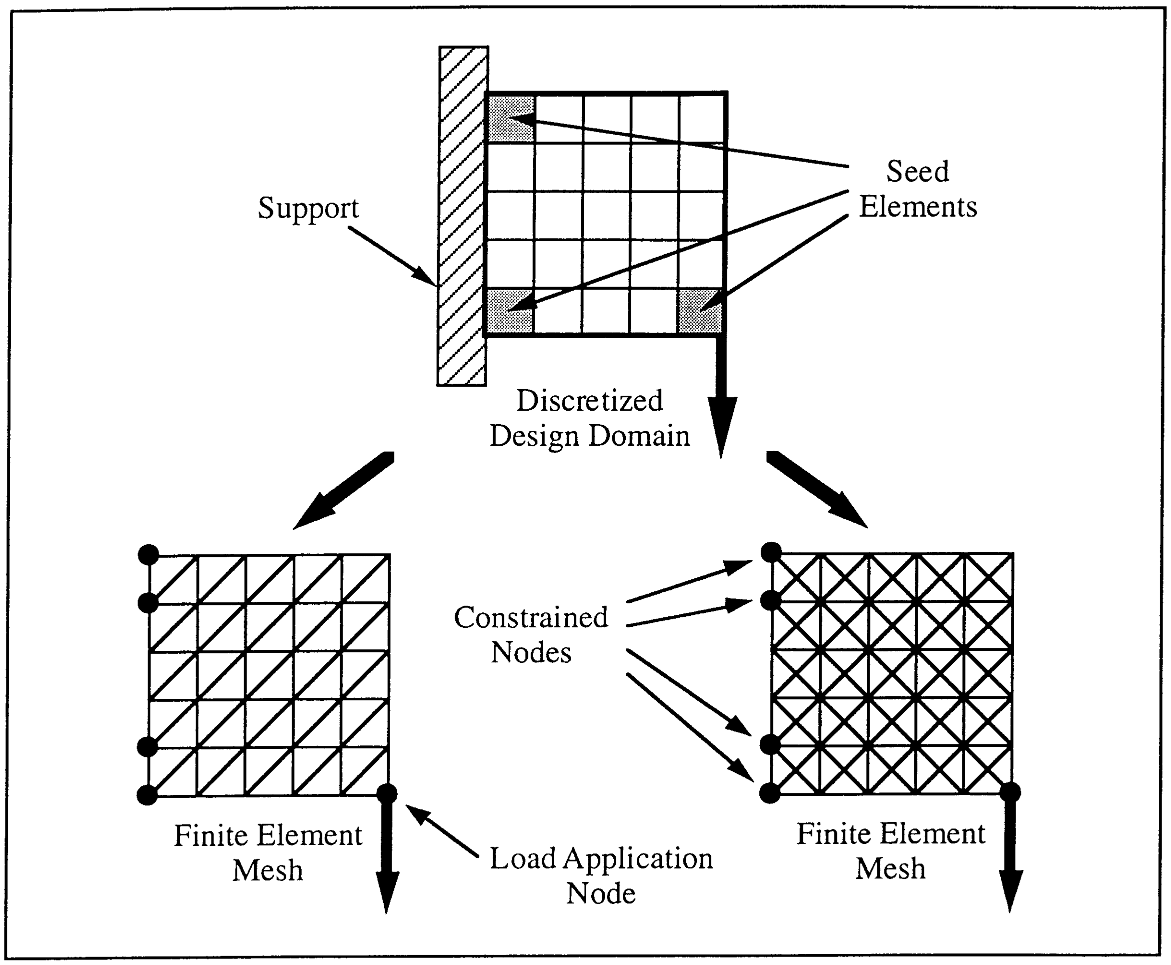


Figure 4.8: Finite element mesh generation.

After a finite element mesh corresponding to the discretized design domain has been created, the mesh must be modified so that it represents the current structure topology (and therefore the current genetic algorithm chromosome). This modification is performed using either an *adaptive* or a *constant* meshing technique. Once the mesh has been modified, *finite element analysis* takes place.

Adaptive Meshing Technique. To create a finite element mesh corresponding to the current structure topology, the adaptive meshing technique begins with the finite element mesh corresponding to the entire discretized design domain and then removes all finite elements from the mesh which correspond to void elements in the current structure. Hence, for every design domain element containing void, either two or four finite elements are removed from the finite element mesh. At the conclusion of the procedure, the only finite elements which remain are those corresponding to connected material elements, and the finite element mesh therefore exactly corresponds to the current structure topology.

While this technique offers the advantage of creating a finite element mesh which exactly corresponds to the current structure topology, it suffers from the disadvantage that a new finite element mesh must be generated for every topology (*i.e.*, every chromosome) in every genetic algorithm generation. Consequently, this technique exhibits considerable computational expense. Figure 4.9 depicts the results of this adaptive meshing technique using an example design domain discretization and an example structure topology.

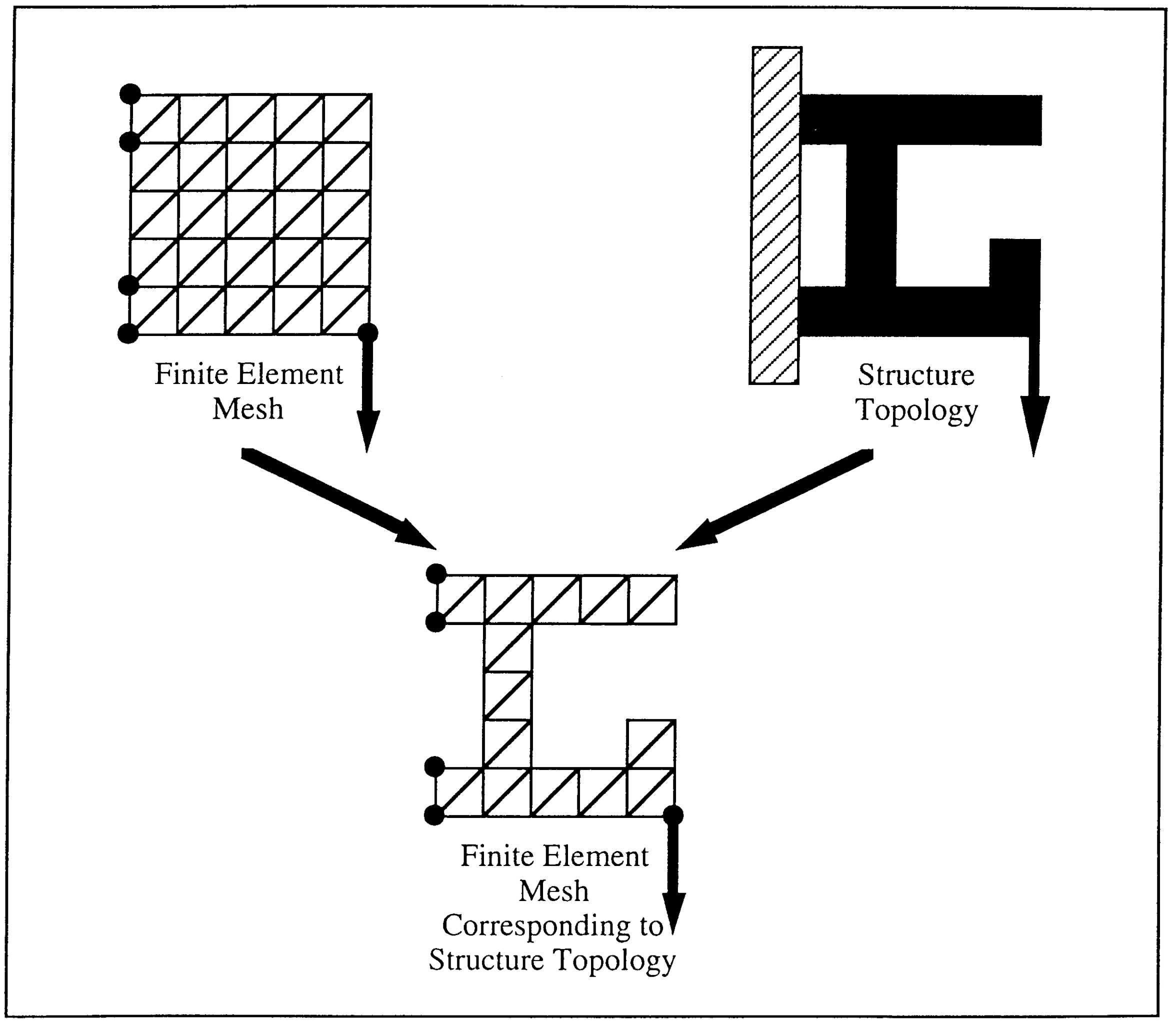


Figure 4.9: Adaptive meshing technique.

Constant Meshing Technique. To create a finite element mesh corresponding to the current structure topology, the constant meshing technique begins with the finite element mesh corresponding to the entire discretized design domain and then assigns a small Young's Modulus value to all finite elements in the mesh which correspond to void elements in the current structure topology. Likewise, finite elements corresponding to material elements in the current structure topology are assigned a standard Young's Modulus value. Hence, finite elements corresponding to void elements in the structure topology are not removed from the finite element mesh—instead, they are simply "weakened." So, at the conclusion of the procedure, the original finite element mesh corresponding to the entire discretized design domain is intact and contains a distribution of large and small Young's Modulus values corresponding to the current structure topology.

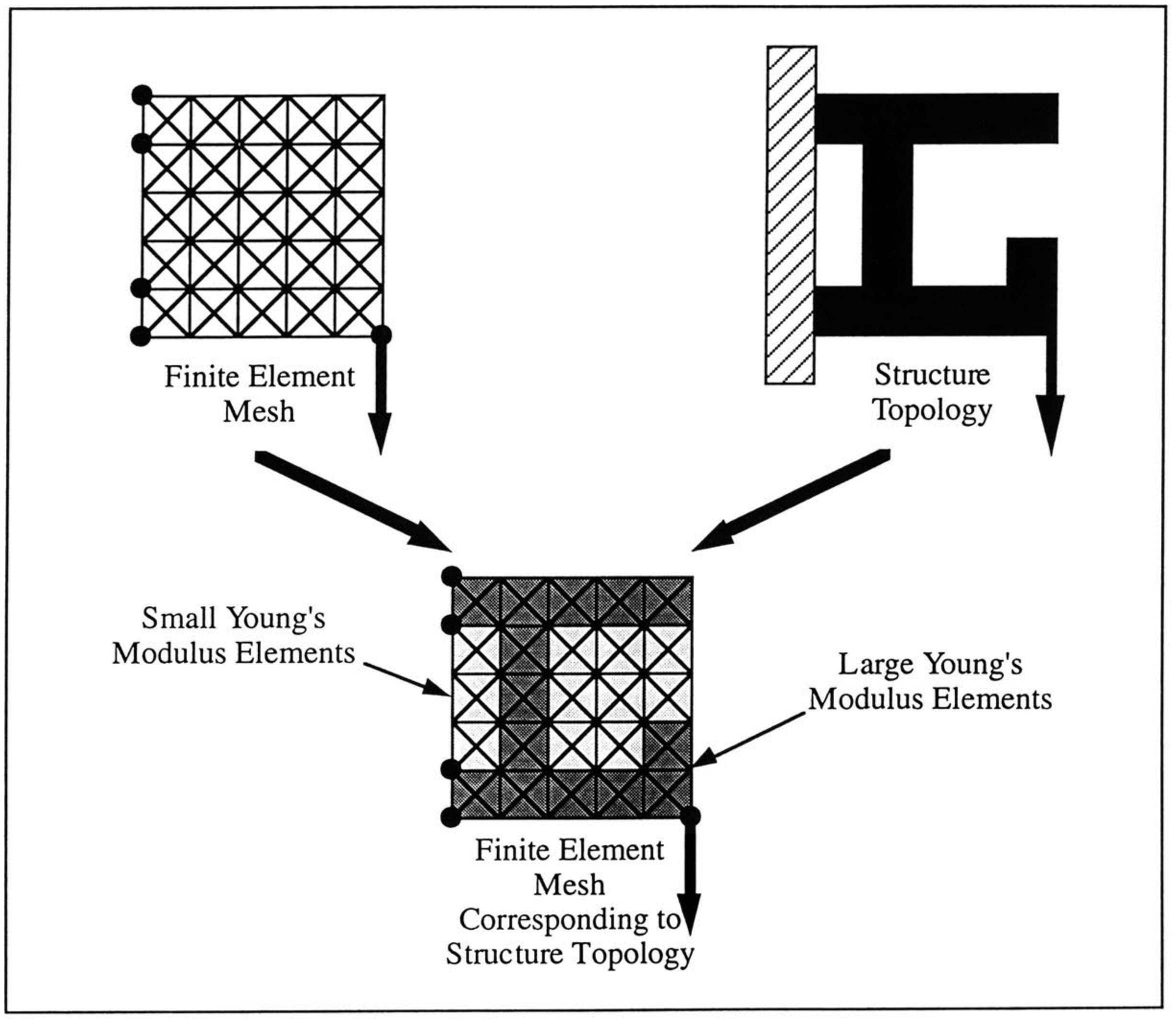

Finite Element Mesh
Structure Topology
Small Young's Modulus Elements
Large Young's Modulus Elements
Finite Element Mesh Corresponding to Structure Topology

Figure 4.10: Constant meshing technique.

While the finite element mesh generated by this technique does not exactly correspond to the structure topology (*i.e.*, finite elements, albeit of low strength, exist in areas where no material is present), this technique does not require the generation of a new finite element mesh for every structure topology in every genetic algorithm generation. Consequently, this technique is relatively inexpensive computationally. In fact, as detailed by Example 3 in Chapter 5, the constant meshing technique provides, at much less computational expense, optimization and finite-element analysis performance comparable to that of the adaptive technique. Figure 4.10 depicts the results of this constant meshing technique using an example design domain discretization and an example structure topology.

The use of "soft" material elements to represent void is suggested by Bendsøe and Kikuchi (1988), who state that soft material can be considered a void or hole if its Young's Modulus is 10^{-2} to 10^{-3} times that of a hard material. Jensen (Section 3.3.4) uses a similar approach to create finite element meshes, although he reduces the thicknesses of finite elements corresponding to void elements instead of reducing their Young's Modulus values. Because reducing a finite element's thickness is equivalent to reducing its Young's Modulus value, Jensen's meshing technique is identical to this investigation's constant meshing technique. However, while Jensen specifies that the thickness of void elements should be 10^{-2} times that of material elements, this investigation stipulates that void elements receive a Young's Modulus 10^{-5} times that of a material element.

Finite Element Analysis. After the finite element mesh corresponding to the current structure topology has been created, finite element analysis is performed. This finite element analysis determines the displacement, in the x- and y-directions, of every node in the finite element mesh. Note that no stress calculations are performed by the finite element routines.

4.4.6 Fitness Calculations. At the conclusion of finite element analysis, results of the analysis are used to calculate a fitness value which quantitatively describes the quality of the genetic algorithm chromosome which created the current structure topology. Again, while fitness calculations performed by Example 1 in Chapter 5 are based on the results of analytical structural analyses, all other examples in this investigation utilize finite element analysis results to calculate a topology's fitness. Hence, details of fitness calculations based on analytical analyses are provided in Example 1, while this section details fitness calculations based on finite element analyses.

Because all finite element analysis-based examples in this investigation attempt to optimize structures which are subject to a single applied load, all finite element analyses in this investigation use a single concentrated load applied to a single finite element node. Consequently, these finite element analysis-based optimizations utilize the current structure's displacement at the point of load application ($\mathbf{u}$) as the basis for all fitness calculations. So, after performing finite element analysis on the current structure topology, $\mathbf{u}$ is set equal to:

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \tag{4.3}$$

where u_x and u_y represent the displacement, in the x- and y-directions, of the finite element node where the load was applied. After $\mathbf{u}$ has been determined, fitness calculations may begin.

The fitness calculations performed in this investigation attempt to either maximize a structure's *stiffness-to-weight ratio* or minimize a structure's *compliance* subject to a maximum volume constraint. Again, the fitness calculations performed in Example 1, which attempt to drive the genetic algorithm search towards structure topologies with maximum strength-to-weight ratio, are detailed in the text of Example 1.

Stiffness-To-Weight Ratio Maximization. As detailed in Section 4.3.3, most examples provided in this investigation attempt to maximize a structure's stiffness-to-weight ratio. Hence, the fitness value which these examples assign to any given genetic algorithm chromosome is equal to the stiffness-to-weight ratio of the structure topology corresponding to the chromosome. Stiffness-to-weight ratio calculations begin when a structure's "stiffness" is assumed to be inversely proportional to the magnitude of the structure's displacement at the point of load application:

$$\mathit{Stiffness} \approx \frac{1}{|\mathbf{u}|} = \frac{1}{\sqrt{(u_x)^2 + (u_y)^2}}. \tag{4.4}$$

Using the area of connected material in the structure as a qualitative measure of the structure's weight, the structure's stiffness-to-weight ratio (and therefore the fitness, to be maximized, of the chromosome corresponding to the structure) is given by:

$$Fitness = \frac{Stiffness}{Weight} = \frac{\left(\frac{1}{|\mathbf{u}|}\right)}{Area} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}. \tag{4.5}$$

Note that disconnected material elements which were switched to void by connectivity analysis are not counted in the above "weight" calculations.

Compliance Minimization. Example 8, detailed in Chapter 5, does not attempt to maximize a structure's stiffness-to-weight ratio. Rather, it attempts to minimize a structure's mean compliance subject to a maximum volume constraint. Hence, any given chromosome's fitness is determined by the compliance minimization and volume constraint satisfaction abilities of the structure topology corresponding to the chromosome. Specifically, fitness calculations are performed so that high fitness is assigned to chromosomes corresponding to structures exhibiting low mean compliance without violating the maximum volume constraint. Likewise, low fitness is assigned to chromosomes corresponding to structures exhibiting large mean compliance and/or violating the maximum volume constraint.

Instead of using a structure's displacement at the point of load application $(\mathbf{u})$ to calculate stiffness, these fitness calculations use $\mathbf{u}$ to determine the structure's mean compliance. Note that a structure's mean compliance represents twice the strain energy of the structure as it supports the applied load(s). In Example 8, where a single concentrated load is applied to a single finite element node, a structure's mean compliance is equal to the applied load vector $(\mathbf{F}_{APPLIED})$ multiplied by the structure's displacement at the point of load application $(\mathbf{u}^{\mathrm{T}})$:

$$Compliance = \mathbf{u}^{\mathrm{T}} \cdot \mathbf{F}_{APPLIED}, \tag{4.6}$$

where

$$\mathbf{u}^{\mathrm{T}} = \begin{bmatrix} u_x & u_y \end{bmatrix} \tag{4.7}$$

and

$$\mathbf{F}_{APPLIED} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \tag{4.8}$$

Note that F_x and F_y represent the magnitude, in the x- and y-directions, of the single concentrated load acting on the structure.

In addition to the structure's compliance, these fitness calculations also require the structure's volume. So, just as the stiffness-to-weight ratio fitness calculations use a structure's area to represent weight, these calculations use the area of connected material in a structure as a qualitative measure of the structure's volume. Note that disconnected material elements which were switched to void by connectivity analysis are not counted in "volume" calculations.

After determining a structure's volume and mean compliance, the structure's fitness (to be maximized) is initially set equal to:

$$Fitness = \frac{1}{\ln(Compliance)}. \tag{4.9}$$

Two clarifications are needed regarding the above fitness value. First, as opposed to the unconstrained stiffness-to-weight ratio fitness calculations detailed above, fitness calculations for this constrained optimization problem require the use of a linear penalty term to penalize a structure's mean compliance based on the extent of volume constraint violation. However, mean compliance values typically decrease by several orders of magnitude during optimization, creating difficulty in selecting volume constraint violation penalty coefficients which work well for both early and mature genetic algorithm populations. Hence, to reduce this variation, the natural log of mean compliance is used in fitness calculations. Second, the reciprocal of the natural log of mean compliance is used because the genetic algorithm is naturally a maximization algorithm and this optimization attempts to minimize the structure's mean compliance. By maximizing the above fitness function, the genetic algorithm generates structures with minimum mean compliance.

After the above fitness value is determined, the structure's volume is compared to the maximum allowable volume. If there is no constraint violation, the chromosome corresponding to the structure is assigned the fitness value above. If there is a violation, the structure's fitness is penalized by 6% for every 10% volume constraint violation. Note that the fitness penalty is linearly attenuated according to the current generation number, with no penalty at generation 0 and full (6%) penalty at generation 175. Hence, prior to generation 175, the fitness of a chromosome corresponding to a structure topology violating the maximum volume constraint is given by:

$$Fitness = \left\{ 1.0 - \left[\frac{generation}{175} \cdot \frac{0.06}{0.10} \cdot \left(\frac{V - V_{MAX}}{V_{MAX}} \right) \right] \right\} \frac{1}{\ln(Compliance)}, \qquad (4.10)$$

where *generation* represents the current generation of genetic algorithm evolution, V represents the volume (*i.e.*, the area) of the current structure topology, and V_{MAX} represents the maximum allowable volume of the structure. After generation 175, the fitness of a chromosome corresponding to a structure topology violating the maximum volume constraint is given by:

$$Fitness = \left\{ 1.0 - \left[\frac{0.06}{0.10} \cdot \left(\frac{V - V_{MAX}}{V_{MAX}} \right) \right] \right\} \frac{1}{\ln(Compliance)}. \qquad (4.11)$$

4.4.7 Summary. After calculating the fitness value of the current structure topology and then assigning the value to the chromosome corresponding to the topology, the entire fitness evaluation process is repeated for every chromosome in the current genetic algorithm generation. This fitness evaluation process is performed during each generation of genetic algorithm-based structural topology optimization.

Examples | 5

5.1 Overview

This chapter details the application of this investigation's genetic algorithm-based structural topology optimization approach to a variety of example problems.

5.2 Example 1: Beam Cross-Section.

5.2.1 Introduction. This example attempts to establish the viability of this investigation's genetic algorithm-based structural topology optimization approach by examining the relatively straightforward topological optimization of a beam cross-section. Specifically, this example first attempts to generate the cross-section topology of a simply-supported beam in pure bending which provides maximum strength-to-weight ratio, independent of the magnitude of bending stresses in the beam. A second portion of this example then attempts to generate the cross-section topology of a simply-supported beam in pure bending which provides maximum strength-to-weight ratio subject to a maximum stress constraint.

5.2.2 Design Domain. The beam's cross-section is modeled using a square design domain discretized into a 15×15 grid of binary, material-void elements (Figure 5.1). Because each genetic algorithm chromosome must contain one gene for every element in the design domain, this 15×15 design domain discretization results in a chromosome length of 225 genes. Hence, the search space contains 2^{225} distinct locations. During optimization, no symmetry constraints are imposed on the beam cross-section, and the cross-section topology is assumed to be constant throughout the length of the beam. Also, the middle design domain element along the cross-section's top surface serves as a seed element. During structural analysis, no particular assumption is made as to how the moments are physically applied to the beam—analysis routines simply use a stress distribution which would result from pure bending.

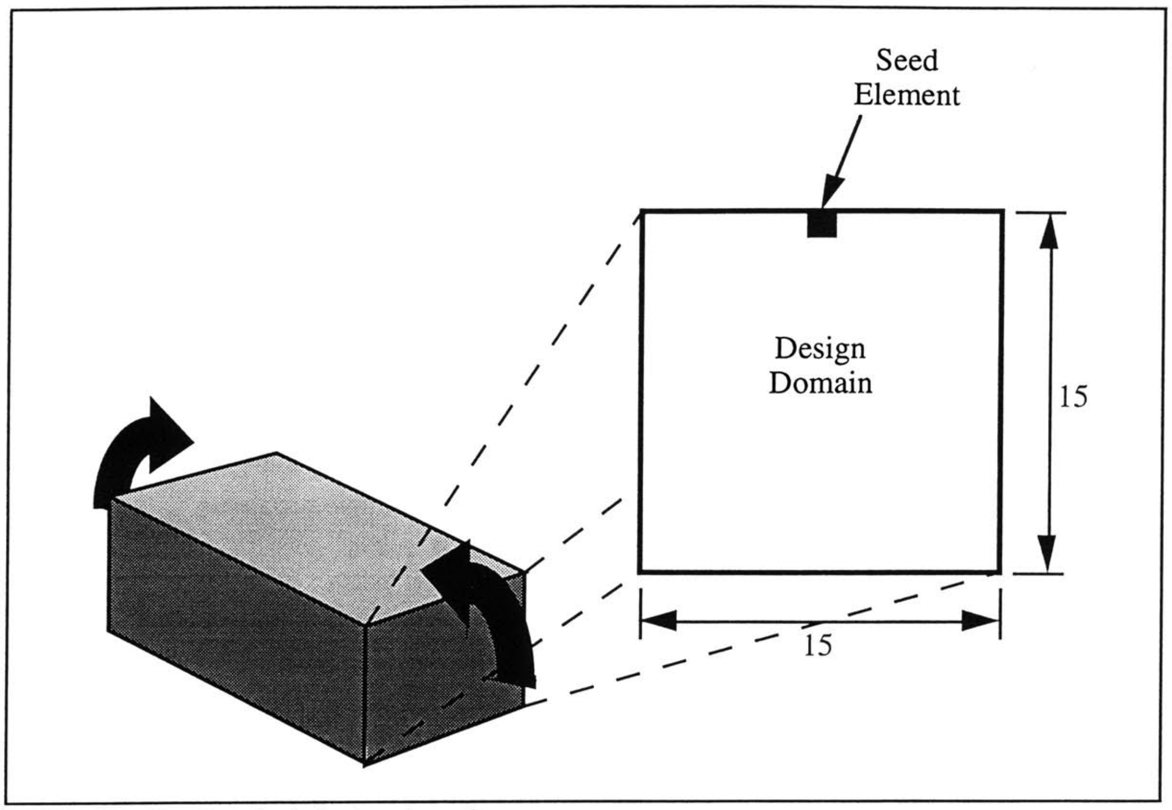


Figure 5.1: Example 1 design domain.

5.2.3 Fitness Calculations—Part A. This first portion of Example 1 attempts to generate the beam cross-section topology which exhibits maximum strength-to-weight ratio. As detailed in Section 4.4, the fitness of any given genetic algorithm chromosome is determined by converting the chromosome into its corresponding structure topology, analyzing the structure topology for connectedness, evaluating the topology's structural performance, and then converting the topology's structural performance into a quantitative fitness value for the corresponding chromosome. Hence, fitness calculations in this example convert a chromosome into its corresponding beam cross-section topology, perform connectivity analysis on the cross-section topology, calculate the cross-section's strength-to-weight ratio, and then set the chromosome's fitness value equal to the cross-section's strength-to-weight ratio:

$$Fitness = \frac{Strength}{Weight}. \tag{5.1}$$

Strength-to-weight ratio calculations begin when a cross-section's moment of inertia is determined using analytical techniques and then used to represent the cross-section's strength. Likewise, as in the stiffness-to-weight ratio fitness calculations detailed in Section 4.4.6, the cross-section's "weight" is assumed to be equal to the area of connected material elements in the cross-section topology. Hence, the strength-to-weight ratio of any given cross-section topology (and therefore the fitness, to be maximized, of the chromosome corresponding to the cross-section topology) is given by:

$$Fitness = \frac{Moment\ of\ Inertia}{Area}. \tag{5.2}$$

As detailed in Sections 4.4.5 and 4.4.6, this example's fitness calculations differ from all other examples in this investigation in that they are based on the results of analytical structural analysis techniques instead of finite element analysis.

5.2.4 Results—Part A. Results of the optimization are shown in Figure 5.2, which depicts best-of-generation cross-section topologies at 50-generation intervals, and in Figure 5.3, which plots the genetic algorithm population's maximum, average, and

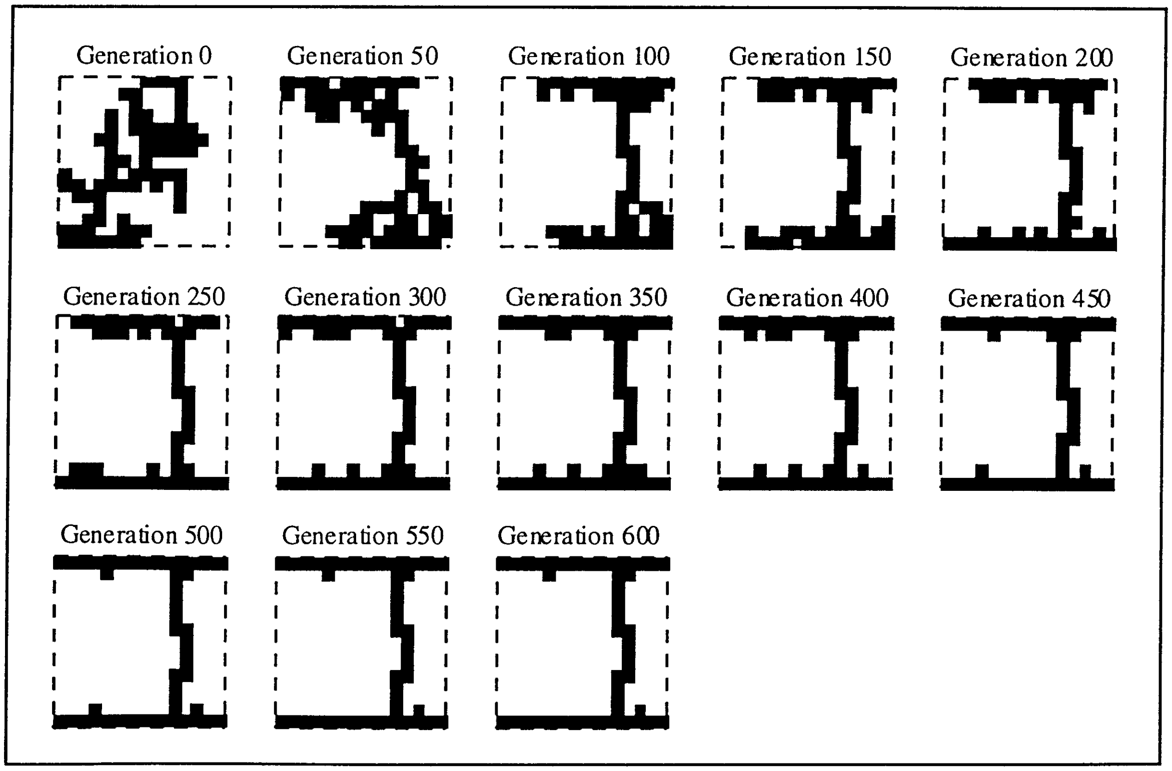


Figure 5.2: Best-of-generation beam cross-sections.

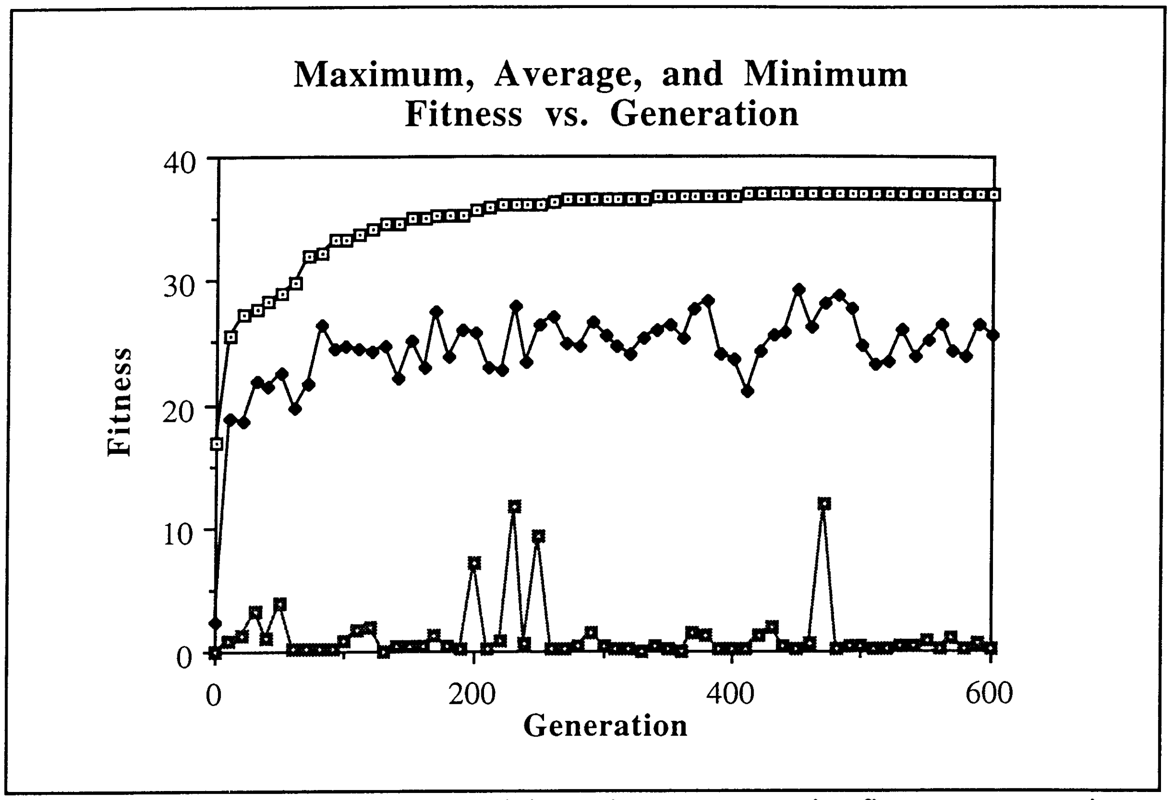


Figure 5.3: Maximum, average, and minimum beam cross-section fitness vs. generation.

minimum chromosome fitness (*i.e.*, cross-section strength-to-weight ratio) values at 10-generation intervals. Starting at 16.96, the cross-section's stiffness-to-weight ratio increased during optimization to a value of 36.95, and the resulting optimal cross-section topology closely resembles an I-beam cross-section, which is the expected result. During the 600 generations of "evolution," 18,000 fitness evaluations were performed:

$$(600\ Generations)\cdot\left(30\ \frac{Chromosomes}{Generation}\right) = 18{,}000\ Chromosomes. \tag{5.3}$$

Note that improvements in the cross section's fitness were so small and infrequent by generation 600 that little, if any, improvement would be obtained if the search were allowed to continue (Figure 5.3 graphically demonstrates the asymptotic behavior of the maximum cross section fitness). Hence, the optimization was stopped after 600 generations of search.

5.2.5 Fitness Calculations—Part B. This second portion of Example 1 attempts to generate the beam cross-section topology which exhibits maximum strength-to-weight ratio subject to a maximum stress constraint. Hence, this example's fitness calculations convert a chromosome into its corresponding beam cross-section topology, perform connectivity analysis on the cross-section topology, calculate the cross-section's strength-to-weight ratio, evaluate the beam's stress levels, and then assign a fitness value to the chromosome according to the cross-section's strength-to-weight ratio maximization and stress constraint satisfaction abilities. Specifically, high fitness is assigned to chromosomes corresponding to beam cross-sections which exhibit large strength-to-weight ratio while satisfying the maximum stress constraint, and low fitness is assigned to chromosomes corresponding to beam cross-sections which exhibit low strength-to-weight ratio and/or violate the maximum stress constraint.

Fitness calculations begin when a beam cross-section's strength-to-weight ratio is calculated as in Part A of this example (Section 5.2.3). Then, using a specified applied moment, the maximum stress in the beam is calculated analytically using the equation (Popov, 1978):

$$\sigma_{MAX} = \frac{M \cdot c}{Moment\ Of\ Inertia}, \tag{5.4}$$

where σ_{MAX} is the maximum stress in the beam, M is the magnitude of the applied moment, c is the distance from the beam's neutral axis to either the top or bottom surface of the beam (whichever is farther), and *Moment of Inertia* is the cross-section's moment of inertia (which is determined during strength-to-weight ratio calculations).

The maximum stress in the beam (σ_{MAX}) is then compared to the beam's specified yield stress (σ_{YIELD}). If the maximum stress in the beam is less than or equal to the beam's yield stress (*i.e.*, $\sigma_{MAX} \leq \sigma_{YIELD}$), the chromosome corresponding to the current beam cross-section topology is assigned a fitness value equal to the cross-section's strength-to-weight ratio:

$$Fitness = \frac{Moment\ of\ Inertia}{Area}. \tag{5.5}$$

If the stress in the beam exceeds the beam's yield stress (*i.e.*, $\sigma_{MAX} > \sigma_{YIELD}$), the beam cross-section's strength-to-weight ratio is penalized using a linear penalty term. As detailed in Section 4.3.3, the magnitude of the penalty term has a great effect on the optimal cross-section topology generated by the genetic algorithm—if the penalty term is too small, the optimal cross-section topology will have an excellent strength-to-weight ratio, but the beam's maximum stress will exceed the yield stress. Likewise, if the penalty term is too large, the optimal cross-section topology will have a maximum stress far below the yield stress, but the topology will have a poor strength-to-weight ratio. In this example, a linear penalty term which penalizes the cross-section's strength-to-weight ratio by 6% for every 10% stress constraint violation is used. Hence, a chromosome corresponding to a beam cross-section topology which violates the maximum stress constraint is assigned a fitness value equal to:

$$Fitness = \left[1.0 - \left(\frac{0.06}{0.10} \cdot \frac{\sigma_{MAX} - \sigma_{YIELD}}{\sigma_{YIELD}}\right)\right] \frac{Moment\ Of\ Inertia}{Area}. \tag{5.6}$$

During fitness calculations, each element in the design domain is assumed to have dimensions of $1\text{ cm} \times 1\text{ cm}$, and a yield stress (σ_{YIELD}) of 480 MPa is used. Also, optimization is performed using applied moments of either 150,000 N·m or 200,000 N·m.

5.2.6 Results—Part B. Results of the optimization are shown in Figure 5.4, which displays the optimal cross-section topologies generated after 600 generations of search (again, improvements in the cross section's fitness were so small and infrequent by generation 600 that continued search was deemed unnecessary). The optimization of each cross-section topology, as in the optimization detailed by Part A of this example, required

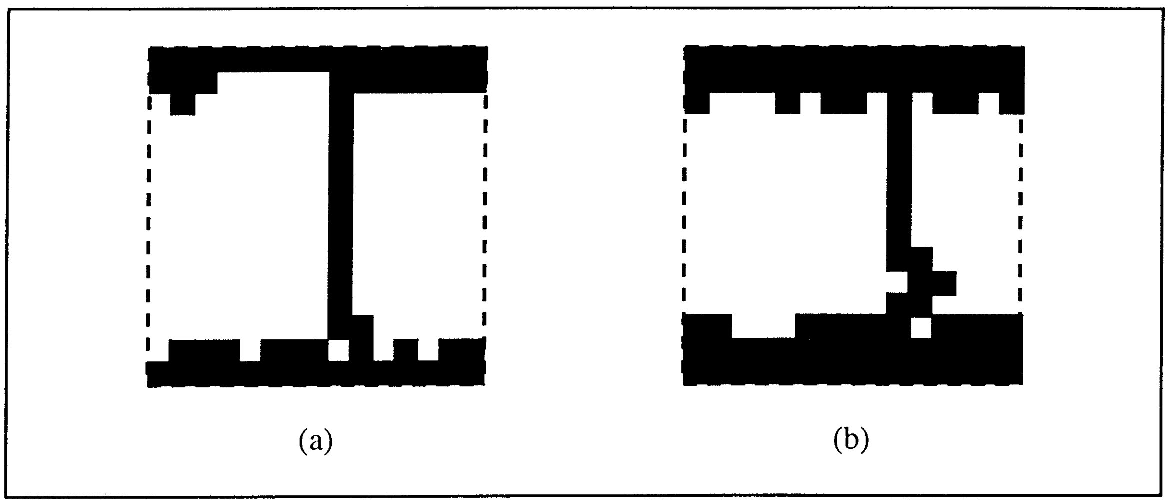


Figure 5.4: Optimal cross-section topologies for applied moments of (a) 150,000 N·m and (b) 200,000 N·m.

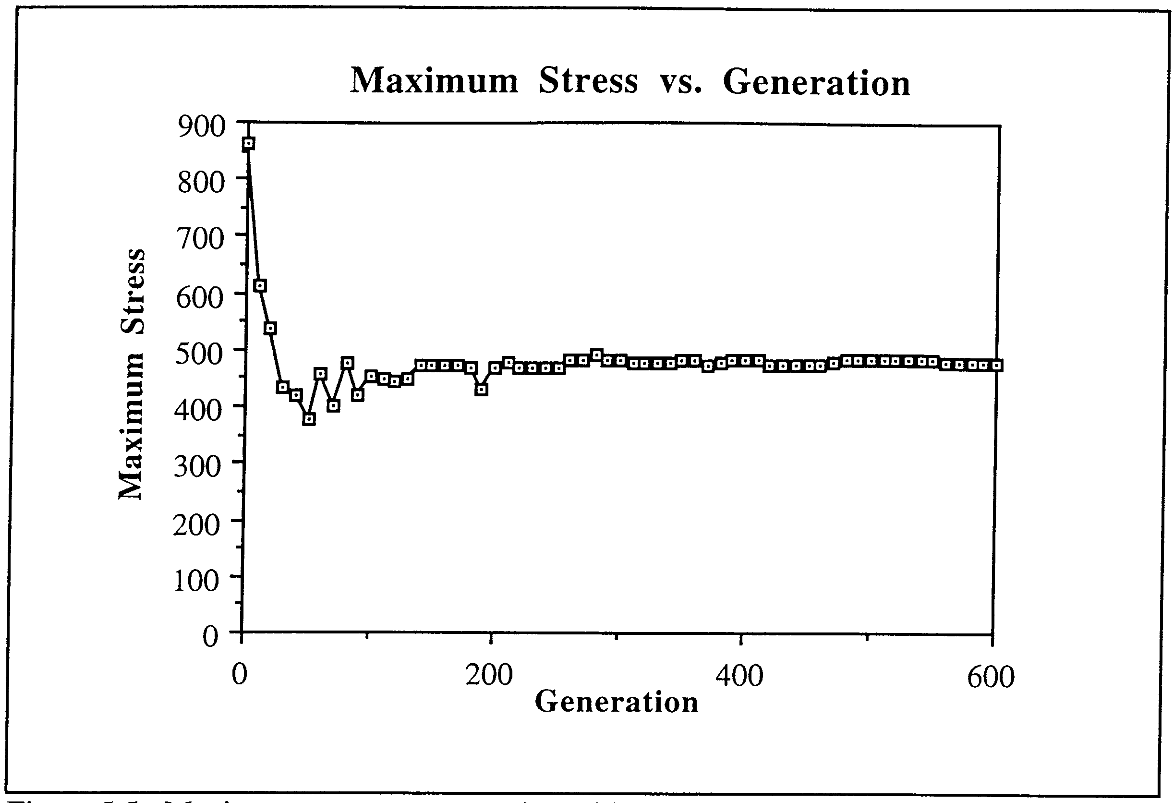


Figure 5.5: Maximum stress vs. generation with applied moment of 150,000 N·m.

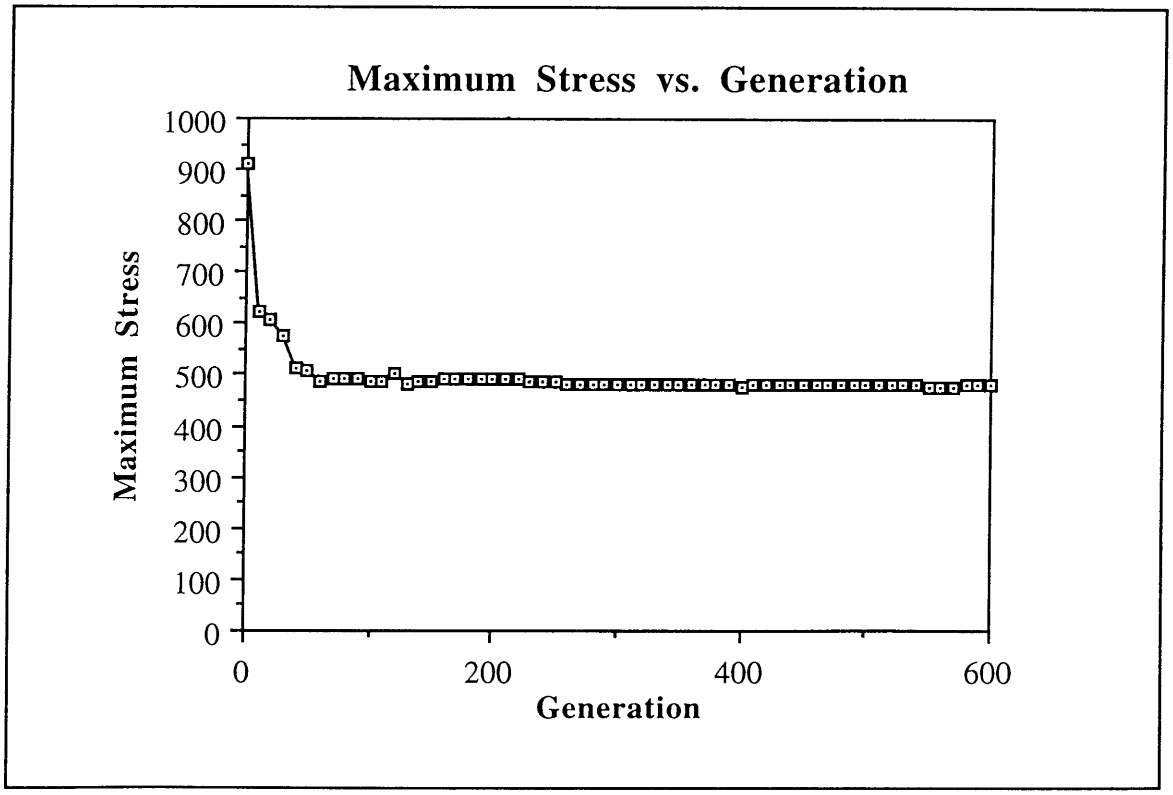


Figure 5.6: Maximum stress vs. generation with applied moment of 200,000 N·m.

18,000 fitness evaluations. Note that Figure 5.4a depicts the optimal cross-section topology for the applied moment of 150,000 N·m, while Figure 5.4b depicts the optimal cross-section topology for the applied moment of 200,000 N·m. As expected, the optimal cross-section topologies closely resemble I-beam cross-sections, and the outer flanges become thicker and the web remains thin as the applied moment is increased. Figure 5.5 depicts, in 10-generation intervals, the maximum stress in the beam subject to the applied moment of 150,000 N·m. During optimization, the maximum stress in the beam decreased from 860.95 MPa to a final value of 477.65 MPa. Likewise, Figure 5.6 shows the maximum stress in the beam subject to the applied moment of 200,000 N·m. In this second beam, the maximum stress decreased from an initial value of 909.69 MPa to a final value of 481.5 MPa.

When examining these results, it is interesting to make comparisons between the genetic algorithm and other basic search techniques which are also able to search in discrete, discontinuous, multi-modal search spaces (*e.g.*, exhaustive and random searches). In this example, a near-optimum beam cross-section is found after 18,000 function evaluations, *i.e.*, after examining 18,000 locations in the search space. As the search space contains 2^{225} distinct locations, the genetic algorithm searches only a tiny fraction of the space before finding a near-optimum location. This is compared to an exhaustive search, which would examine all 2^{225} locations. A random search would likely find the optimum location after searching one-half of the 2^{225} locations (note that the random search would not know that it had found the optimum because it does not use gradient information). Hence, when attempting to locate near-optimum topologies, the genetic algorithm is much more efficient than other basic techniques which are also able to search ill-conditioned search spaces.

As detailed in Section 4.4.4, this investigation uses connectivity analysis to insure that material elements connected only at corners are removed from topologies. While these disconnected material elements cannot withstand any applied torque and could therefore lead to an unstable structure, they *can* withstand the bending stresses encountered in this example. Hence, utilizing disconnected material elements in this example's moment of inertia and area calculations (*i.e.*, deactivating connectivity analysis) might have resulted in higher-fitness beam cross-section topologies. However, any cross-section topology containing such elements would be unrealistic and nearly impossible to manufacture. Hence, connectivity analysis is used to eliminate these disconnected material elements.

Note that placing a seed element on the beam cross-section's upper surface could possibly bias the genetic algorithm search towards topologies with material along the upper surface. However, the results given in Figures 5.2 and 5.4 show that while material congregates along the upper surface of the design domain, material also congregates along the lower surface. Because the stress distribution resulting from a pure bending loading configuration is symmetric about the neutral axis, the configuration of material should also be symmetric about the neutral axis. Hence, the fact that material places itself along the lower surface of the design domain without the "assistance" of a material constraint suggests that the material along the upper design domain surface is an unbiased optimal configuration of material.

5.3 Example 2: Small Cantilevered Plate.

5.3.1 Introduction. This example details this investigation's first attempt at using the genetic algorithm in conjunction with finite element analysis to perform structural topology optimization. This example's use of finite element analysis techniques is a significant enhancement over the previous example's analytical techniques, as nearly all practical structural topology optimization problems require the use of numerical analysis methods such as the finite element method. In the example, the genetic algorithm is used to generate the topology of a square cantilevered plate in tension which provides maximum stiffness-to-weight ratio.

5.3.2 Design Domain. The plate is modeled using a square design domain discretized into a 12×12 grid of binary, material-void elements (Figure 5.7). During optimization, the plate is constrained to be symmetric about the x-axis, and genetic algorithm chromosomes therefore control the material distribution in one-half of the design domain. Hence, fitness calculations must map each chromosome into one-half of the design domain and then "mirror" the resulting distribution of material and void elements about the line of symmetry. This design domain discretization consequently requires a chromosome length of 72 genes $(6 \cdot 12 = 72)$ and corresponds to a search space containing 2^{72} distinct locations. To allow for the application of the horizontal concentrated load, the two design domain elements on the plate's right-hand surface immediately adjacent to the point of load application (which is in the middle of the plate's right-hand surface) serve as seed elements. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface.

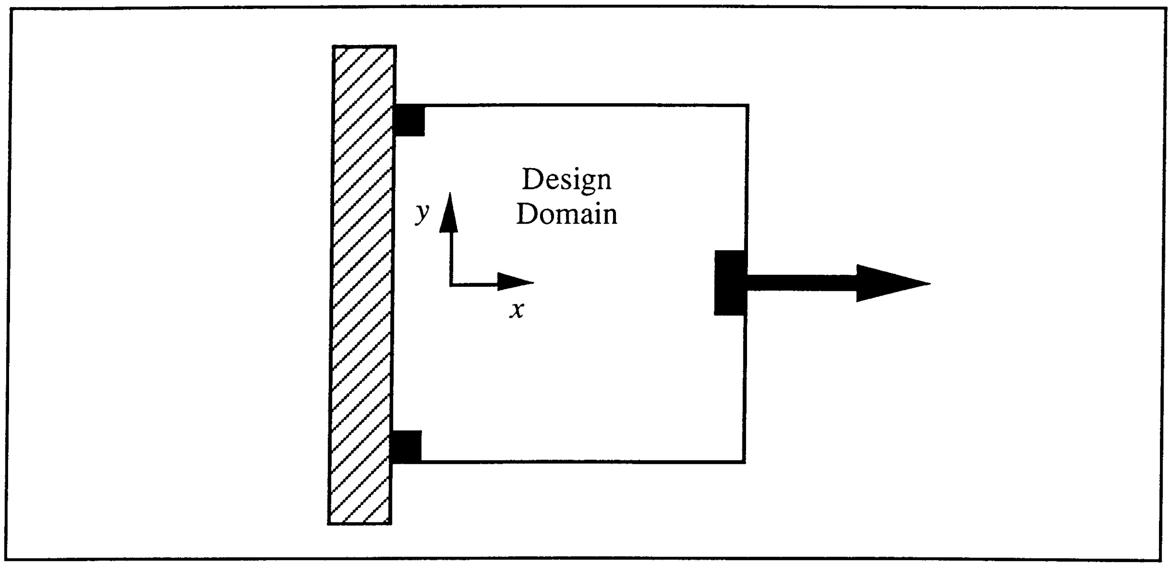


Figure 5.7: Example 2 design domain.

5.3.3 Fitness Calculations. As this example attempts to maximize the plate's stiffness-to-weight ratio, all fitness calculations are performed as detailed in Section 4.4. Hence, to evaluate the fitness of a particular chromosome, the chromosome is first converted into its corresponding plate topology (*i.e.*, mapped into one-half of the design domain and then mirrored about the line of symmetry) and connectivity analysis is then performed. A finite element mesh containing two triangular finite elements for every material design domain element is then generated using the adaptive meshing technique (Section 4.4.5), and structural analysis is performed using the finite element method. Finally, the plate topology's stiffness-to-weight ratio, and therefore the fitness (to be maximized) of the chromosome corresponding to the plate topology, is given by:

$$Fitness = \frac{Stiffness}{Weight} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}, \qquad (5.7)$$

where *Area* is equal to the area of connected material elements in the plate topology and u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

Note that while stress or displacement constraints could be used with this stiffness-to-weight ratio fitness function (just as stress constraints were used in Example 1, Part B), a finer design domain discretization would be needed so that the genetic algorithm could satisfy the constraint(s) without adding excessive amounts of material.

The above fitness calculations assume that the plate topology contains material elements connecting the point of load application to the points of support (again, all of the material elements connecting the point of load application to the points of support must be connected along *edges*). If the plate topology does not contain material connecting the point of load application to the points of support, the topology cannot withstand any tensile load, and the corresponding finite element mesh cannot connect the point of load application to the points of support via finite elements. Hence, any finite element analysis is meaningless in this situation. Unfortunately, particularly in early generations, many genetic algorithm chromosomes correspond to plate topologies which *do not* connect the point of load application to the points of support with material elements.

Consequently, before generating a finite element mesh and performing fitness calculations for the current chromosome and its corresponding plate topology, this example's fitness function first evaluates the plate topology to determine if the point of load application is indeed connected to the points of support with material elements. Specifically, a graph search (Nilsson, 1971) is used to determine if the plate topology provides a path of connected material elements between the seed elements corresponding to the points of support and the seed elements corresponding to the point of load application. If the plate topology does connect the point of load application to the points of support (*i.e.*, if the plate topology is *connected*), fitness calculations proceed as detailed above. If the plate topology is unable to connect the point of load application to the points of support (*i.e.*, if the plate topology is *disconnected*), the chromosome corresponding to the topology is assigned a small, non-zero fitness value. While the particular fitness value assigned to these disconnected plate topologies has no physical significance, it was chosen so that disconnected plate topologies always have lower fitness than any connected plate topology—the genetic algorithm is therefore driven towards connected plate topologies. Note that fitness values of zero are not assigned to chromosomes corresponding to these disconnected plate topologies, as most of these chromosomes contain some high-quality genetic material—assigning them a fitness value of zero would automatically prevent them from mating with other chromosomes in an attempt to generate high-quality child chromosomes.

A brief explanation of the terminology used in this article: a plate topology is considered *connected* if the point of load application is connected to the points of support via material elements, while a plate topology is considered *disconnected* if it does not connect the point of load application to the points of support via material elements.

Conversely, individual material elements in the design domain are considered to be either *connected* or *disconnected* during connectivity analysis (Section 4.4.4). So, during fitness evaluation, a plate topology is first subjected to connectivity analysis, which removes all *disconnected* material elements from the topology. Hence, after connectivity analysis, the plate topology is comprised entirely of *connected* material elements. If these remaining material elements connect the point of load application to the points of support, the plate topology, as well as the chromosome corresponding to the plate topology, are considered *connected.* Likewise, if these remaining material elements do not connect the point of load application to the points of support, the plate topology and the chromosome corresponding to the plate topology are considered *disconnected.*

5.3.4 Results. Results of the optimization are shown in Figure 5.8, which displays the best-of-generation plate topologies at 12-generation intervals, and in Figure 5.9, which plots the genetic algorithm population's maximum, average, and minimum chromosome fitness (*i.e.*, plate topology stiffness-to-weight ratio) values at 4-generation intervals. During optimization, the plate topology's stiffness-to-weight ratio increased from an initial value of 2.0 (corresponding to an unconnected plate topology) to a final value of 20.47. Because the optimization "evolved" for 160 generations, a total of 4,800 fitness evaluations were performed:

$$(160\ \mathit{Generations})\cdot\left(30\ \frac{\mathit{Chromosomes}}{\mathit{Generation}}\right)=4{,}800\ \mathit{Chromosomes}. \tag{5.8}$$

However, because many chromosomes, particularly in early generations, created disconnected plate topologies which did not require finite-element analysis (*e.g.*, the Generation 0 plate topology in Figure 5.8), less than 4,800 structural analyses were performed during the optimization.

As shown in Figure 5.8, the optimization provides reasonable results. Note that the symmetric 12×12 design domain discretization, corresponding to a 72-gene chromosome, is rather small and the genetic algorithm therefore has little difficulty finding a near-optimal plate topology. However, even with short chromosome lengths, the adaptive meshing technique used to create a unique finite element mesh for *each connected chromosome in each generation* is very computationally expensive. Also, in the early stages of optimization, few chromosomes correspond to plate topologies which connect the point of load application to the points of support. Hence, many chromosomes cannot be subjected

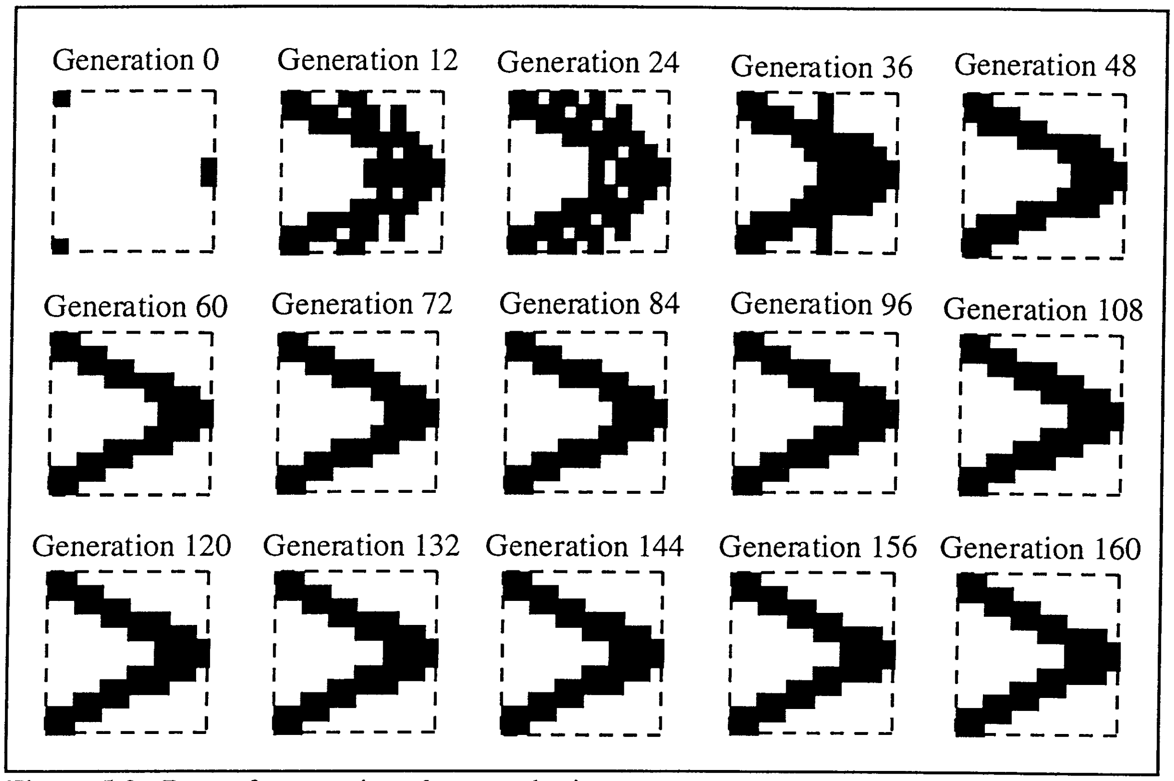


Figure 5.8: Best-of-generation plate topologies.

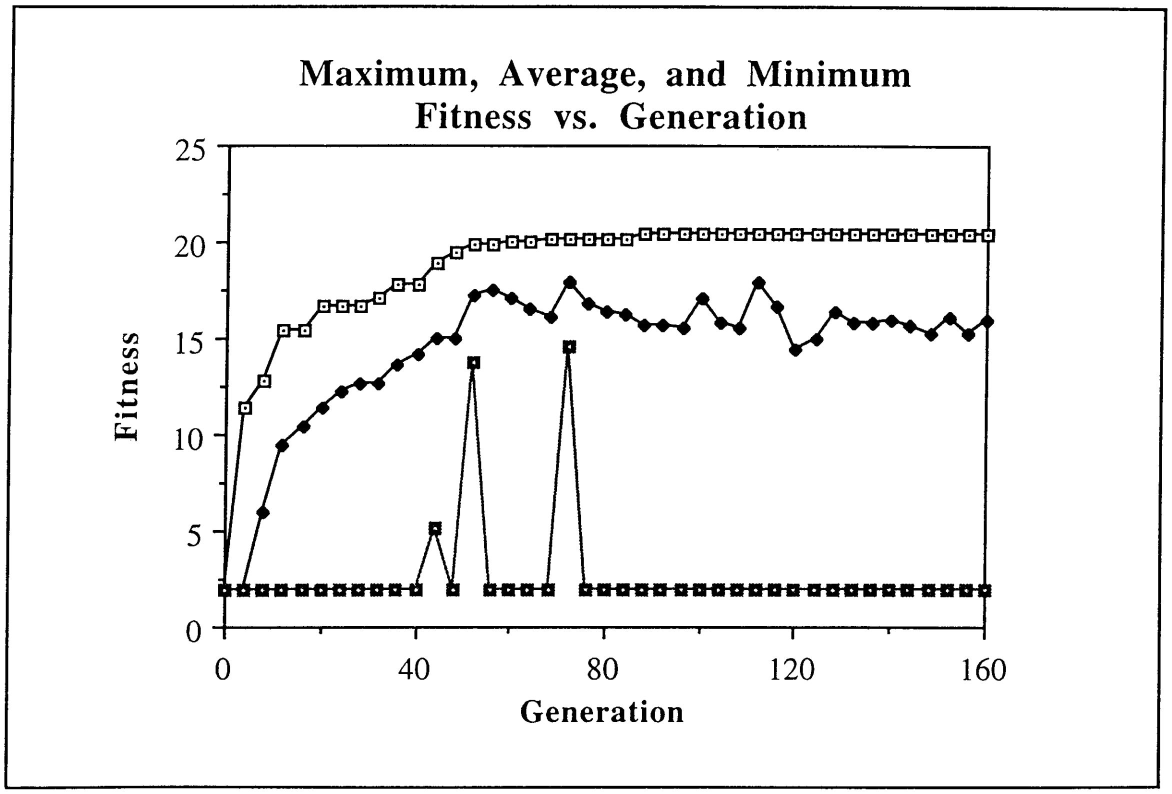


Figure 5.9: Maximum, average, and minimum plate topology fitness vs. generation.

to finite element analysis and instead must be assigned a small, non-zero fitness value. Unfortunately, it is then impossible for the genetic algorithm to give preference to any of these chromosomes, as they all have the same fitness value. For example, one chromosome might correspond to a disconnected plate topology which would require major modification to become connected (*i.e.*, the chromosome is truly of poor quality), while another chromosome might correspond to a disconnected plate topology which requires only slight modification to become connected (*i.e.*, the chromosome is of reasonably-high quality, and mating it with another chromosome may generate a highly-fit child chromosome). While preference should be given to the second chromosome, this fitness formulation would assign the same small, non-zero fitness value to both of these example chromosomes. In another example situation, if the genetic algorithm population contains only one chromosome which creates a connected plate topology, this fitness formulation will assign to the chromosome a fitness value which is much higher than that assigned to the other chromosomes in the population. Consequently, this highly-fit chromosome will quickly dominate the genetic algorithm population, and high-quality genetic material in chromosomes corresponding to disconnected topologies will be ignored.

Because the adaptive meshing technique used in this example's fitness calculations results in high computational expense and difficulties in finding connected plate topologies, using a finer design domain discretization would only increase the computational expense and enhance the difficulty of finding high-quality topologies. Hence, the following examples describe this investigation's examination of other techniques which allow for better computational speed and finely-discretized design domains.

5.4 Example 3: Comparison of Finite Element Meshing Techniques.

5.4.1 Introduction. This example attempts to establish the viability of the constant finite element meshing technique (detailed in Section 4.4.5) as well as compare the constant meshing technique's optimization capabilities and computational expense to those of the adaptive meshing technique (also detailed in Section 4.4.5) used in Example 2. Also examined are the advantages and disadvantages of performing connectivity analysis (detailed in Section 4.4.4) during optimization.

5.4.2 Adaptive Finite Element Meshing. As detailed in Example 2, the adaptive finite element meshing technique results in considerable computational expense, for a unique finite element mesh must be generated for each chromosome in every genetic algorithm generation which corresponds to a connected plate topology. Another problem with the adaptive meshing technique is that connected topologies are difficult to obtain in early generations, leading to the undesireable situation where only a few plate topologies are submitted to finite element analysis while the vast majority of topologies are disconnected and must be assigned small, non-zero fitness values.

While the optimization detailed in Example 2 avoided the above shortcomings of adaptive meshing by using a relatively coarse design domain discretization, the adaptive meshing technique typically cannot be used with finely-discretized design domains such as those common to homogenization-based structural topology optimization examples (Section 3.2.1). In addition to increasing the computational expense to prohibitive levels, using finer discretizations only enhances the difficulty of finding connected topologies in early generations. During the course of this investigation, the adaptive meshing technique was applied to a variety of finely-discretized design domains, and the technique demonstrated extreme computational expense and was unable, after many generations of search, to find *any* connected plate topologies.

5.4.3 Constant Finite Element Meshing. In an effort to increase the efficiency of finite element meshing and allow for fine design domain discretizations, the constant finite element meshing technique was developed. Unlike the adaptive meshing technique, which generates a new finite element mesh for each chromosome in every genetic algorithm generation, the constant meshing technique uses a single finite element mesh throughout the duration of an optimization and simply modifies the Young's Modulus values of the mesh's finite elements according to each chromosome. Because changing the Young's Modulus values of a mesh's finite elements is much easier than generating a new finite element mesh, the constant meshing technique is considerably less expensive computationally than the adaptive meshing technique.

Also, whereas the adaptive meshing technique cannot create finite element meshes for disconnected plate topologies, the constant meshing technique can generate a valid finite element mesh for any plate topology, regardless of whether or not the topology connects the point of load application to the points of support via material elements. Specifically, because the constant meshing technique begins with a finite element mesh representing the entire design domain and then assigns small Young's Modulus values to finite elements corresponding to void elements in a plate topology, the point of load application is *always* connected to the points of support via finite elements. So, while some of these finite elements may be of small Young's Modulus (*i.e.*, the plate topology may be disconnected), the finite elements always connect the point of load application to the points of support, and finite element analysis can therefore be performed on any plate topology.

Hence, when using the constant finite element meshing technique, the fitness evaluation process does not examine plate topologies to determine if they are connected or disconnected, and disconnected plate topologies are therefore never simply assigned small, non-zero fitness values. Instead, all plate topologies, whether or not they connect the point of load application to the points of support with material, are evaluated using finite element analysis, and all fitness calculations are based on a plate's displacement at the point of load application. So, plate topologies which connect the point of load application to the points of support will exhibit little displacement and will therefore receive high fitness, while plate topologies which do not connect the point of load application to the points of support with material elements will exhibit large displacements and will therefore receive low fitness. Also, unlike the adaptive meshing technique, the constant meshing technique allows fitness calculations to distinguish between different qualities of disconnected plate topologies. For example, a disconnected plate topology which would require major modification to become connected will exhibit greater displacement than a disconnected plate topology which requires only slight modification to become connected. Hence, the chromosome corresponding to the disconnected plate topology requiring only slight modification will receive a higher fitness value than the chromosome corresponding to the disconnected plate topology requiring major modification, and both of these chromosomes will receive a fitness lower than that assigned to a chromosome corresponding to a plate topology which connects the point of load application to the points of support with material elements.

5.4.4 Comparison of Adaptive and Constant Meshing. In an attempt to determine if the constant meshing technique can provide optimization results similar to those obtained using the adaptive meshing technique, the topological optimization of a square, cantilevered plate detailed in Example 2 is now re-examined using the constant meshing technique. Hence, the results of Example 2, which used the adaptive meshing technique, serve as the benchmark for evaluating the performance of the constant meshing technique. In addition to establishing the viability of the constant meshing technique, this example also hopes to determine the proper relationship between the Young's Modulus value assigned to finite elements representing void design domain elements and that assigned to finite elements representing material design domain elements (otherwise referred to as the "Young's Modulus Ratio"). Finally, this example provides insight regarding the effectiveness of connectivity analysis.

Design Domain. This investigation attempts to generate the optimal topology of a square, cantilevered plate in tension (Figure 5.10). As in Example 2, where this problem was originally solved, this example uses a square design domain discretized into a 12×12 grid of binary, material-void elements. During optimization, the plate is constrained to be symmetric about the x-axis, and genetic algorithm chromosomes therefore control the material distribution in one-half of the design domain. Hence, during fitness calculations, chromosomes are mapped into one-half of the design domain and the resulting distribution of material and void elements is then "mirrored" about the line of symmetry. This design domain discretization consequently requires a chromosome length of 72 genes $(6 \cdot 12 = 72)$ and corresponds to a search space containing 2^{72} distinct locations. To allow for the application of the horizontal concentrated load, the two design domain elements on the plate's right-hand surface immediately adjacent to the point of load application (which is in the middle of the plate's right-hand surface) serve as seed elements. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface.

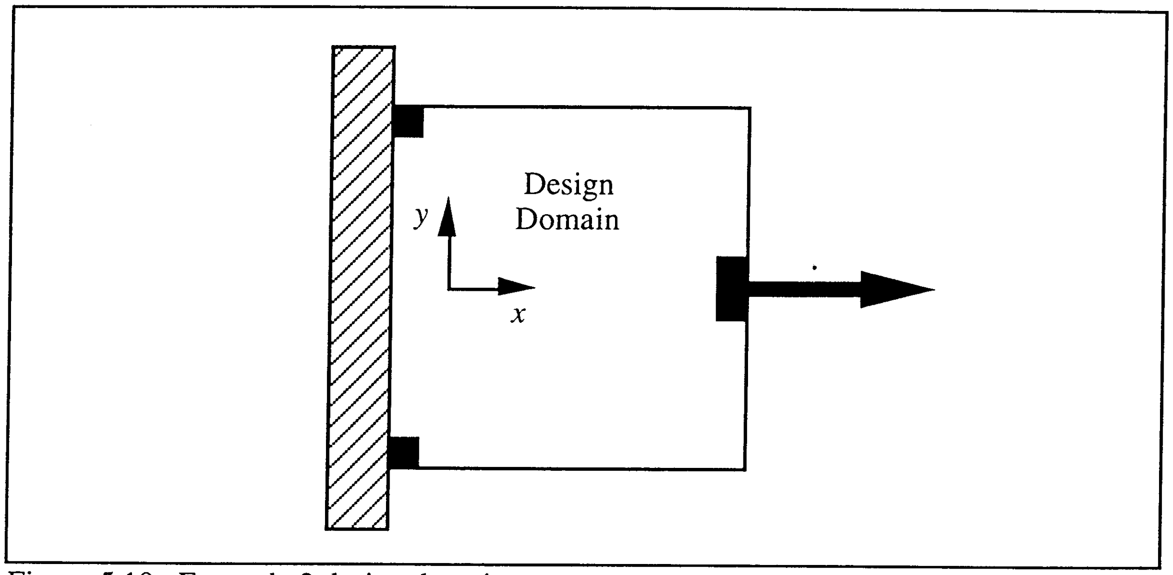


Figure 5.10: Example 3 design domain.

Fitness Calculations. As in Example 2, this example attempts to maximize the plate's stiffness-to-weight ratio, and all fitness calculations are performed as detailed in Section 4.4. Hence, to evaluate the fitness of a particular chromosome, the chromosome is first converted into its corresponding plate topology (*i.e.*, mapped into one-half of the design domain and then mirrored about the line of symmetry) and connectivity analysis is then performed. Then, whereas Example 2 used the adaptive meshing technique to generate a finite element mesh representing the plate topology, this example uses the constant meshing technique to create a mesh containing two triangular finite elements for every design domain element. After a finite element mesh has been created, structural analysis is performed using the finite element method. Note that during finite element analysis, the four nodes along the finite element mesh's left-hand surface which correspond to the two seed elements are constrained to have zero displacement, while the concentrated load is applied to the middle node along the finite element mesh's right-hand surface. Finally, the plate topology's stiffness-to-weight ratio, and therefore the fitness (to be maximized) of the chromosome corresponding to the plate topology, is given by:

$$Fitness = \frac{Stiffness}{Weight} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}, \tag{5.7}$$

where *Area* is equal to the area of connected material elements in the plate topology and u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

Unlike Example 2, where disconnected plate topologies are given small, non-zero fitness values, this example's use of the constant meshing technique enables all plate topologies, regardless of whether or not they connect the point of load application to the points of support with material elements, to be subjected to finite element analysis to determine u_x and u_y.

Experiment Design. In an effort to establish the viability of the constant meshing technique, as well as determine the proper Young's Modulus Ratio and evaluate the effectiveness of connectivity analysis, optimization is performed using four different values of Young's Modulus Ratio and with Connectivity Analysis activated and deactivated. Specifically, topological optimizations are run using the following parameter settings:

- Young's Modulus Ratio
  10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}

- Connectivity Analysis
  Activated and Deactivated

Eight unique combinations of Young's Modulus Ratio and Connectivity Analysis activation are represented in the above list of parameter settings. In the experiment, optimization is performed twice using each of the eight combinations (for a total of 16 optimization runs). At the conclusion of each optimization, the optimum plate topology is recorded, as is the plate's stiffness-to-weight ratio. Note that each genetic algorithm-based optimization is evolved for 200 generations, and that random initial seeds (*i.e.*, different, randomly-generated initial populations) are used in each optimization. Hence, each optimization performs 6,000 structural analyses:

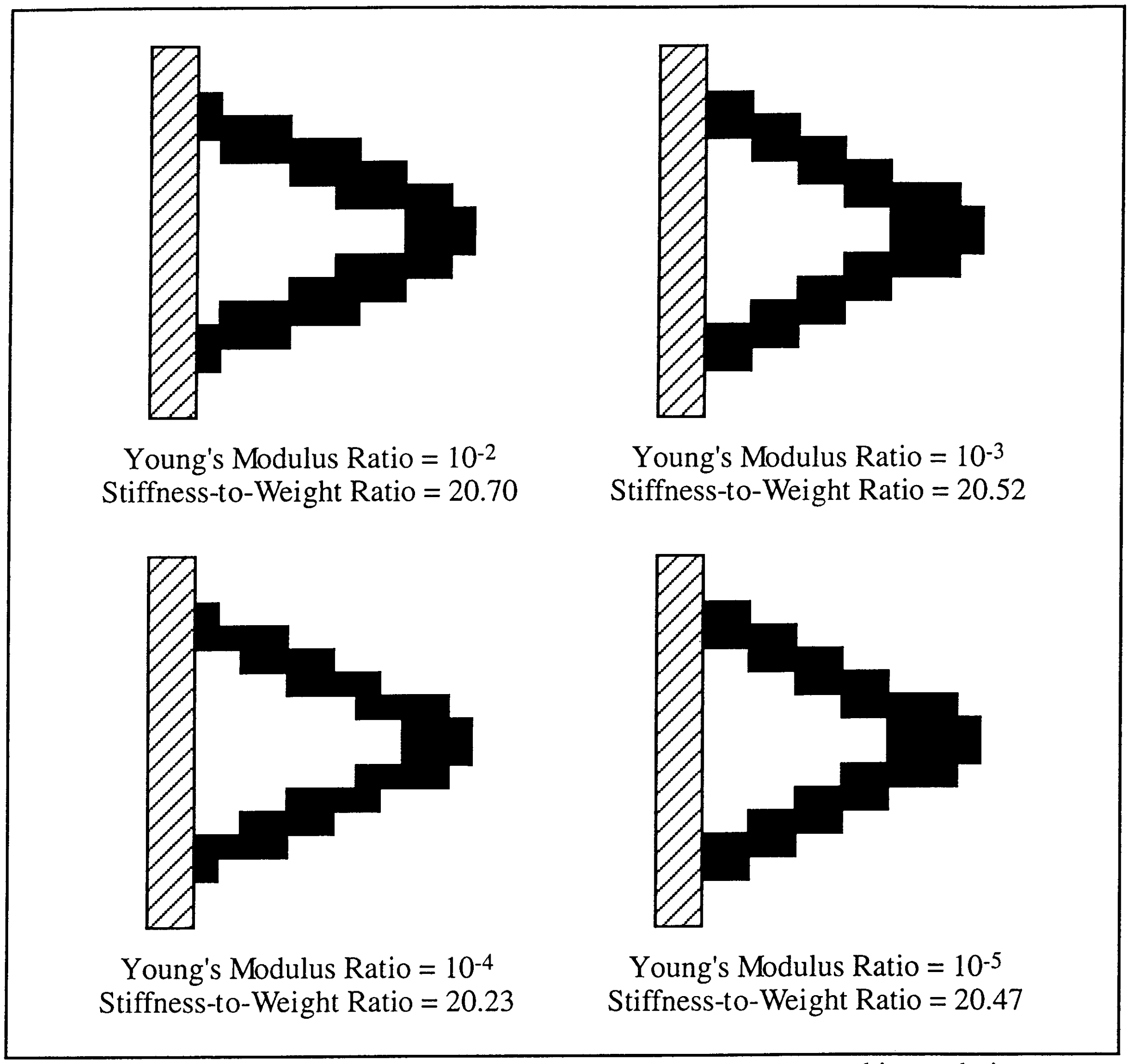
$$(200\ Generations)\cdot\left(30\ \frac{Chromosomes}{Generation}\right) = 6{,}000\ Chromosomes. \tag{5.9}$$

Results. Table 5.1 details the results of the experiment. Note that each of the eight fitness values listed in the table represents the average of the two optimal stiffness-to-weight ratios obtained for a particular combination of Young's Modulus Ratio and Connectivity Analysis activation. Also, of the 8 genetic algorithm optimizations conducted using connectivity analysis, Figure 5.11 depicts the plate topologies with highest stiffness-to-weight ratio for each of the four different Young's Modulus Ratio values.

Young's Modulus Ratio $\frac{E_{VOID}}{E_{MATERIAL}}$	Maximum Stiffness-to-Weight Ratio with Connectivity Analysis	Maximum Stiffness-to-Weight Ratio without Connectivity Analysis
10^{-2}	20.46	19.26
10^{-3}	20.44	18.07
10^{-4}	20.12	17.77
10^{-5}	20.47	18.24

Table 5.1: Optimization performance.

From these results, it is evident that the constant finite element meshing technique, when used with connectivity analysis, generates plate topologies and plate topology stiffness-to-weight ratios comparable to those found using the adaptive meshing technique. In fact, the constant meshing technique, when used with connectivity analysis and a Young's Modulus Ratio of 10^{-5}, obtains the *exact* plate topology found using adaptive meshing. Furthermore, when using connectivity analysis and a Young's Modulus Ratio of 10^{-2}, the constant meshing technique is able to generate a slightly better plate topology than that obtained using adaptive meshing. Note that while the topologies resulting from the two meshing techniques are similar, the computational expense of the constant meshing technique is much less than that of the adaptive meshing technique. Consequently, using the constant meshing technique with connectivity analysis provides, at considerably less computational expense, optimization performance equal to that of the adaptive meshing technique. Hence, the constant meshing technique (with a Young's Modulus ratio of 10^{-5}) and connectivity analysis are used in all subsequent examples in this investigation.


Young's Modulus Ratio = 10^{-2}
Stiffness-to-Weight Ratio = 20.70
Young's Modulus Ratio = 10^{-3}
Stiffness-to-Weight Ratio = 20.52
Young's Modulus Ratio = 10^{-4}
Stiffness-to-Weight Ratio = 20.23
Young's Modulus Ratio = 10^{-5}
Stiffness-to-Weight Ratio = 20.47
Figure 5.11: Optimal plate topologies obtained using the constant meshing technique.

As depicted in Table 5.1, the optimal plate topologies generated with connectivity analysis deactivated have considerably lower stiffness-to-weight ratios than topologies generated with connectivity analysis activated. This decrease in performance is most likely caused by the fact that when connectivity analysis is deactivated, disconnected material elements are included in weight calculations but cannot significantly assist in supporting the applied load. Hence, these disconnected material elements increase the plate's weight but do little to increase the plate's stiffness. However, it is possible that disconnected material elements could arrange themselves in a "chain-link" (*i.e.*, a group of material elements connected corner-to-corner, creating a chain-like structure), which, if situated in a suitable location within the structure, could assist in withstanding a *tensile* load. In that situation, if connectivity analysis is deactivated, the disconnected material elements might increase the topology's structural performance. This, however, was rarely found to occur.

5.5 Example 4: Large Cantilevered Plate.

5.5.1 Introduction. This example details the topological optimization of a large cantilevered plate. While similar to the topological optimizations detailed in Examples 2 and 3, in that it uses the genetic algorithm in conjunction with finite element analysis to optimize the topology of a cantilevered plate, this example extends the work of Examples 2 and 3 by using a much finer design domain discretization, unsymmetric loading, no symmetry constraints, and the constant meshing technique. Specifically, this example attempts to generate the topology of a rectangular cantilevered plate subject to a downward vertical load which provides maximum stiffness-to-weight ratio. This example is a first attempt to determine if this investigation's genetic algorithm-based topology optimization approach can perform optimization using design domain discretizations approaching those used by homogenization-based methods.

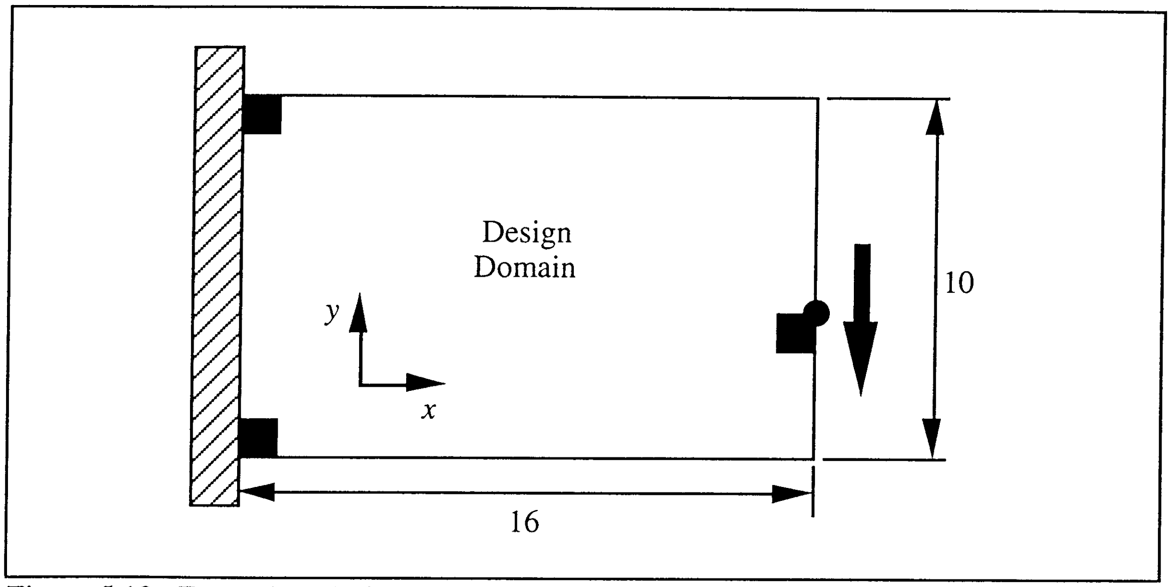


Figure 5.12: Example 4 design domain.

5.5.2 Design Domain. The cantilevered plate is modeled using a rectangular design domain discretized into 10×16 (*i.e.*, 10 elements tall by 16 elements wide),

15×24, or 20×32 grids of binary, material-void elements (Figure 5.12). During optimization, no symmetry constraints are imposed on the plate topology, and the genetic algorithm chromosomes control the distribution of material and void throughout the entire design domain. As each genetic algorithm chromosome must therefore contain one gene for every design domain element, these design domain discretizations result in chromosome lengths of 160, 360, and 640 genes. Hence, the search space contains either 2^{160}, 2^{360}, or 2^{640} distinct locations. To allow for the application of the vertical concentrated load, the design domain element on the plate's right-hand surface immediately below the point of load application (which is $\frac{2}{5}$ of the distance from the bottom) serves as a seed element. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface.

5.5.3 Fitness Calculations. As this example attempts to maximize the plate's stiffness-to-weight ratio, all fitness calculations are performed as detailed in Section 4.4. Hence, to evaluate the fitness of a particular chromosome, the chromosome is first converted into its corresponding plate topology (because no symmetry constraints are imposed, the chromosome is mapped into the entire discretized design domain to create a plate topology) and connectivity analysis is then performed. A finite element mesh containing four triangular finite elements for every design domain element is then generated using the constant meshing technique (Section 4.4.5), and structural analysis is performed using the finite element method. Note that during finite element analysis, the four nodes along the finite element mesh's left-hand surface which correspond to the two seed elements are constrained to have zero displacement, while the concentrated load is applied to the node on the finite element mesh's right-hand surface which is $\frac{2}{5}$ of the distance from the bottom. Finally, the plate topology's stiffness-to-weight ratio, and therefore the fitness (to be maximized) of the chromosome corresponding to the plate topology, is given by:

$$Fitness = \frac{Stiffness}{Weight} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}, \tag{5.7}$$

where *Area* is equal to the area of connected material elements in the plate topology and u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

5.5.4 Results. Results of the optimization are shown in Figure 5.13, which displays the plates generated at the three different discretizations with maximum stiffness-to-weight ratio. Figure 5.13a depicts the optimal topology generated using the 10×16 design domain discretization, obtained after 225 generations of genetic algorithm search. Hence, 6,750 fitness evaluations were performed during the generation of the 10×16 plate topology:

$$(225\ Generations) \cdot \left(30\ \frac{Chromosomes}{Generation}\right) = 6,750\ Chromosomes. \qquad (5.10)$$

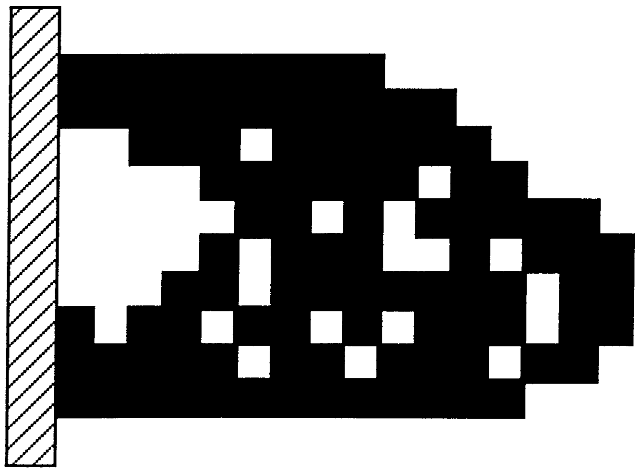
Note that because the constant finite element meshing technique was used, structural analysis was performed in all of the above 6,750 fitness evaluations.

Figures 5.13b and 5.13c display the optimal topologies generated using the 15×24 and 20×32 design domain discretizations. As these two finely-discretized design domains correspond to exceptionally long chromosome lengths and therefore create optimization problems of considerable difficulty, 600 generations of search were required to generate the plate topologies. Hence, 18,000 fitness evaluations (and therefore 18,000 structural analyses) were performed during each of the two optimizations:

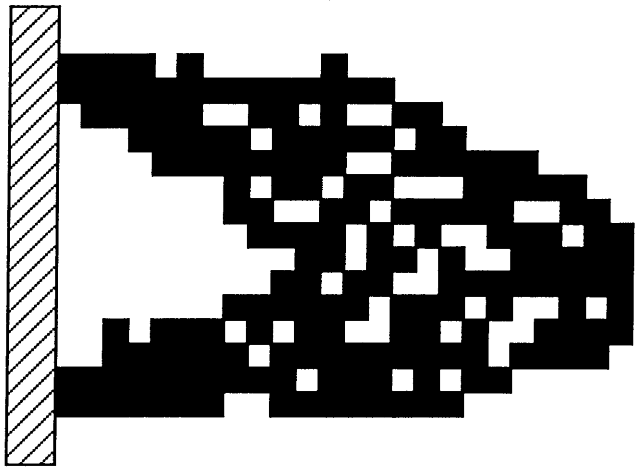
$$(600\ Generations) \cdot \left(30\ \frac{Chromosomes}{Generation}\right) = 18,000\ Chromosomes. \qquad (5.11)$$

Note that whereas improvements in the plate topology's fitness became small and infrequent by generation 225 when using the 10×16 design domain discretization, 600 generations of search were performed using the 15×24 and 20×32 design domain discretizations before improvements in the structure's fitness became small and infrequent.

Examining the results in Figure 5.13, it is interesting to note that the topologies have well-defined solid-material outer boundaries, while the interior regions generally have a composite-like internal structure comprised of equally-distributed material and void. The 20×32 topological optimization was able to "hollow out" several large holes in the interior region, producing truss-like members. While homogenization-based methods minimize mean compliance subject to a maximum volume constraint, and this genetic algorithm-



(a)



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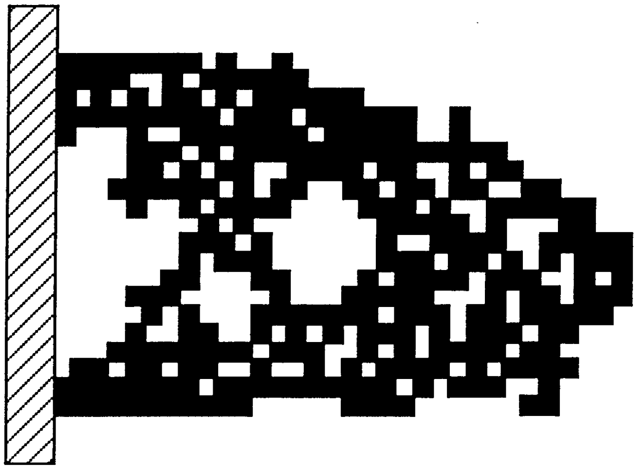


(c)

Figure 5.13: Optimal plate topologies using (a) 10×16, (b) 15×24, and (c) 20×32 design domain discretizations.

based method maximizes stiffness-to-weight ratio, results of the two methods share some *general* similarities. As in this example's 20×32 optimization, homogenization-based methods result in a topology with a well-defined, solid-material outer boundary and a large internal section of voids traversed by small truss members (see Papalambros and Chirehdast (1990) for the homogenization-based solution to a problem using boundary conditions, loading conditions, and a design domain geometry similar to those used in this example). Also, the outer boundaries of the structures found using the two methods are somewhat similar in shape.

Jensen (1992) also investigates the topological optimization of a finely-discretized cantilevered plate subject to a downward concentrated load. The example attempts to generate the plate topology which provides minimum weight subject to a maximum displacement constraint. Similar to the examples provided in this investigation, Jensen's example uses a binary, material-void design domain discretization and a constant finite element meshing technique. However, as opposed to the design domains used in this investigation, the design domain examined by Jensen is more than twice as tall as it is wide (Figure 3.16), and is discretized into a 19×9 grid. Hence, Jensen uses 171-gene chromosomes in his example. Note that the constant meshing technique used by Jensen varies the thickness of finite elements by a factor of 100 to represent material and void design domain elements, which is exactly equivalent to using a 10^{-2} Young's Modulus Ratio with this investigation's constant meshing technique.

Using a population size of 140 chromosomes, Jensen's example generates an optimal plate topology after 150 generations of search. However, the optimal plate topology is a very thin, "chain-link" topology almost entirely comprised of material elements connected at corners. As detailed in Section 4.4.4, groups of material elements connected only at corners (*i.e.*, elements which do not share edges) cannot support torques or compressive loads. Hence, the plate topology generated by Jensen should *collapse* when subjected to the given loading condition. Jensen's plate topology most likely does not collapse because it is "supported" by the void finite elements surrounding it, which have a thickness 0.01 times that of the material finite elements comprising the topology. Consequently, the plate topology generated in Jensen's example must be thought of as a "topology schematic" which the designer may use, along with his or her knowledge of the application domain, to create a physically-realizable topology.

This investigation's use of connectivity analysis guarantees that an unstable plate topology such as that obtained by Jensen cannot be generated—referring to the plate topologies depicted in Figure 5.13, please note that every material element in the design domain shares an edge with at least one other material element.

5.6 Example 5: Hierarchical Design Domain Subdivision.

5.6.1 Introduction. While the resolutions of the plate topologies generated in Example 4 are satisfactory for determining the general shape of the load-bearing structure, they are much coarser than the finely-discretized domains used in homogenization-based methods. Unfortunately, attempts to increase the design domain discretization beyond the maximum 20×32 resolution in Example 4 were unsuccessful, as the system was unable to create plate topologies where the point of load application was connected to the points of support with high-Young's-Modulus material. Specifically, whereas the genetic algorithm was able to locate connected plate topologies within the first few generations of search when using the 10×16, 15×24, and 20×32 design domain discretizations, the genetic algorithm was unable to find connected topologies for finer discretizations, even after hundreds of generations. This poor performance with fine design domain discretizations is most likely caused by the exceptionally large search spaces resulting from the long chromosome lengths required for the fine discretization. When searching in these large search spaces, the genetic algorithm is simply unable to locate, with its population of 30 chromosomes, regions of the search space which correspond to connected plate topologies.

5.6.2 Overview of the Technique. In an attempt to obtain finely-discretized plate topologies, this example introduces a hierarchical design domain subdivision technique. The technique, which enables the genetic algorithm to determine the general shape of the optimal plate topology in early generations and then "fine-tune" the high-quality topology in final generations, periodically increases both the resolution of a design domain's discretization and the number of genetic algorithm populations controlling the distribution of material and void throughout the domain.

5.6.3 Details of the Technique. The hierarchical subdivision technique begins genetic algorithm-based structural topology optimization with a coarse design domain discretization, resulting in short chromosomes and a correspondingly small search space. (Figure 5.14 depicts, for an example optimization problem, an example coarse design domain discretization.) As demonstrated in Examples 2 and 3, short chromosomes and small search spaces enable the genetic algorithm to easily generate connected plate topologies in early populations. Using this coarse design domain discretization, genetic algorithm-based structural topology optimization is performed for a specified number of generations, and an optimal coarse plate topology is generated. Note that the resolution of this coarse plate topology provides only a general shape of the structure, and is insufficient for the creation of truss-like members or holes in the structure's inner region. (Figure 5.15 depicts an example optimal coarse plate topology using the example coarse design domain discretization.) To refine this coarse plate topology, the resolution of the design domain discretization is then quadrupled by subdividing each design domain element into four smaller elements, where each small element is exactly $\frac{1}{4}$ the size of an original design domain element. The optimal coarse plate topology is then mapped into the new, finely-discretized design domain, with each element in the coarse plate topology converting into four elements in the new topology. (Figure 5.16 depicts the example optimal coarse plate topology being mapped into an example finely-discretized design domain.) During conversion, every material element in the coarse topology corresponds to four material elements in the new topology, while every void element in the coarse topology corresponds to four void elements in the new topology.

Because this new, finely-discretized design domain contains four times as many elements as the coarse design domain, the genetic algorithm chromosomes required by this new discretization would contain four times as many genes as the chromosomes optimizing the material-void distribution within the original, coarse design domain. However, it is

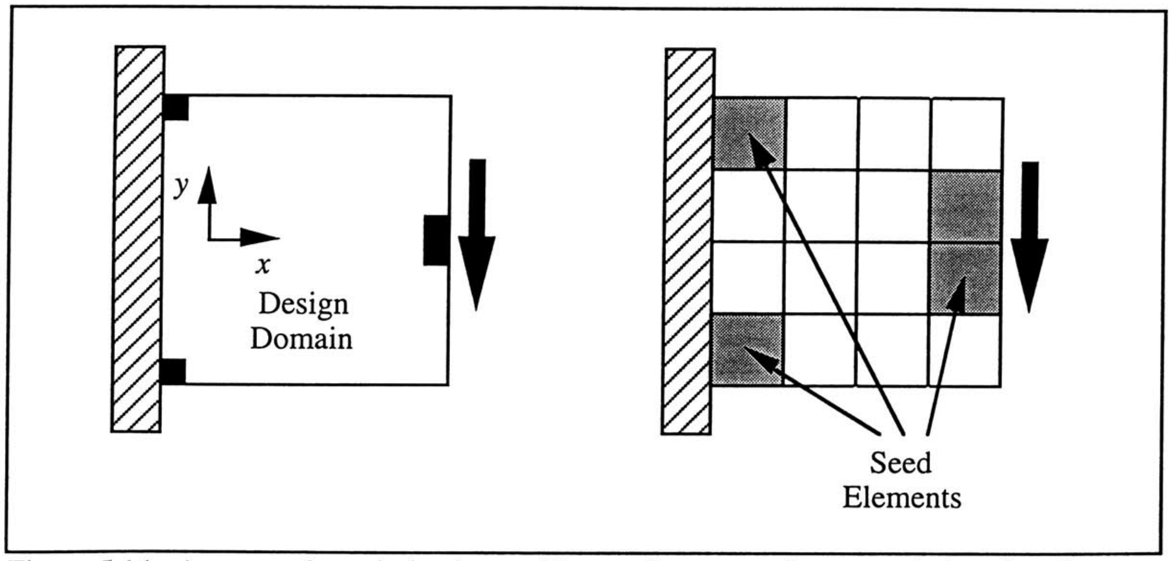


Figure 5.14: An example optimization problem and an example coarse design domain discretization corresponding to the problem.

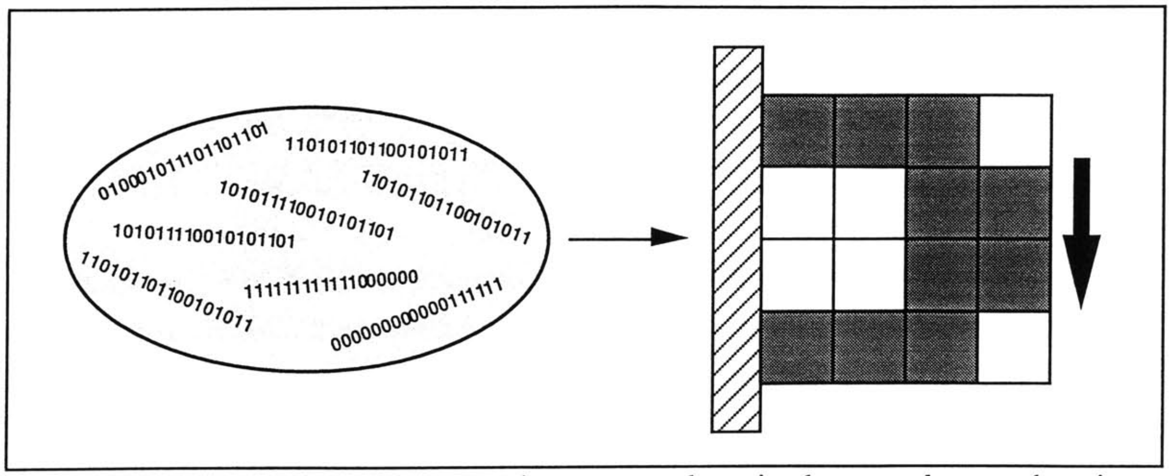


Figure 5.15: The genetic algorithm creating an example optimal coarse plate topology in the example coarse design domain.

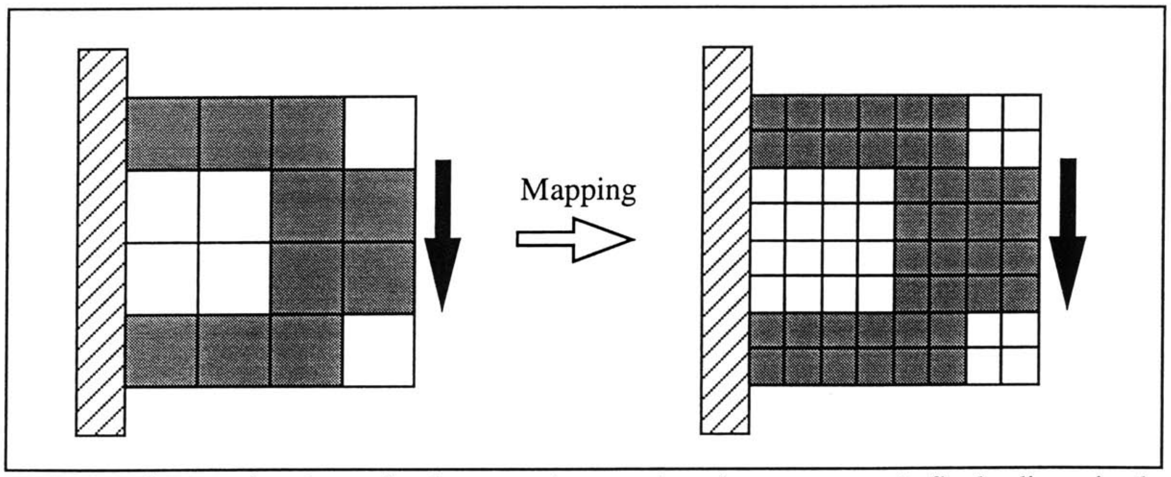


Figure 5.16: Mapping the optimal coarse plate topology into an example finely-discretized design domain.

beneficial to keep chromosome length constant, for genetic algorithm performance typically diminishes as chromosome length (and therefore search space size) increases. Hence, instead of increasing the lengths of the chromosomes in the genetic algorithm population and then using the population to optimize the material-void distribution within the finely-discretized design domain, hierarchical subdivision divides the new, finely-discretized design domain into four "quadrants" and uses four genetic algorithm populations to optimize the distribution of material and void within the quadrants (Figure 5.17). So, whereas a single genetic algorithm population was used to optimize the distribution of material and void within the original coarse design domain, *four* genetic algorithm populations are used to optimize the distribution of material and void within the new, finely-discretized design domain. Specifically, each of the four populations optimizes the distribution of material and void within a particular design domain quadrant, where each quadrant contains the same number of design domain elements as the original, coarse design domain. Note that the populations each contain the same number of chromosomes as the original, single population used with the coarse design domain, and the chromosomes each contain the same number of genes as those in the original population.

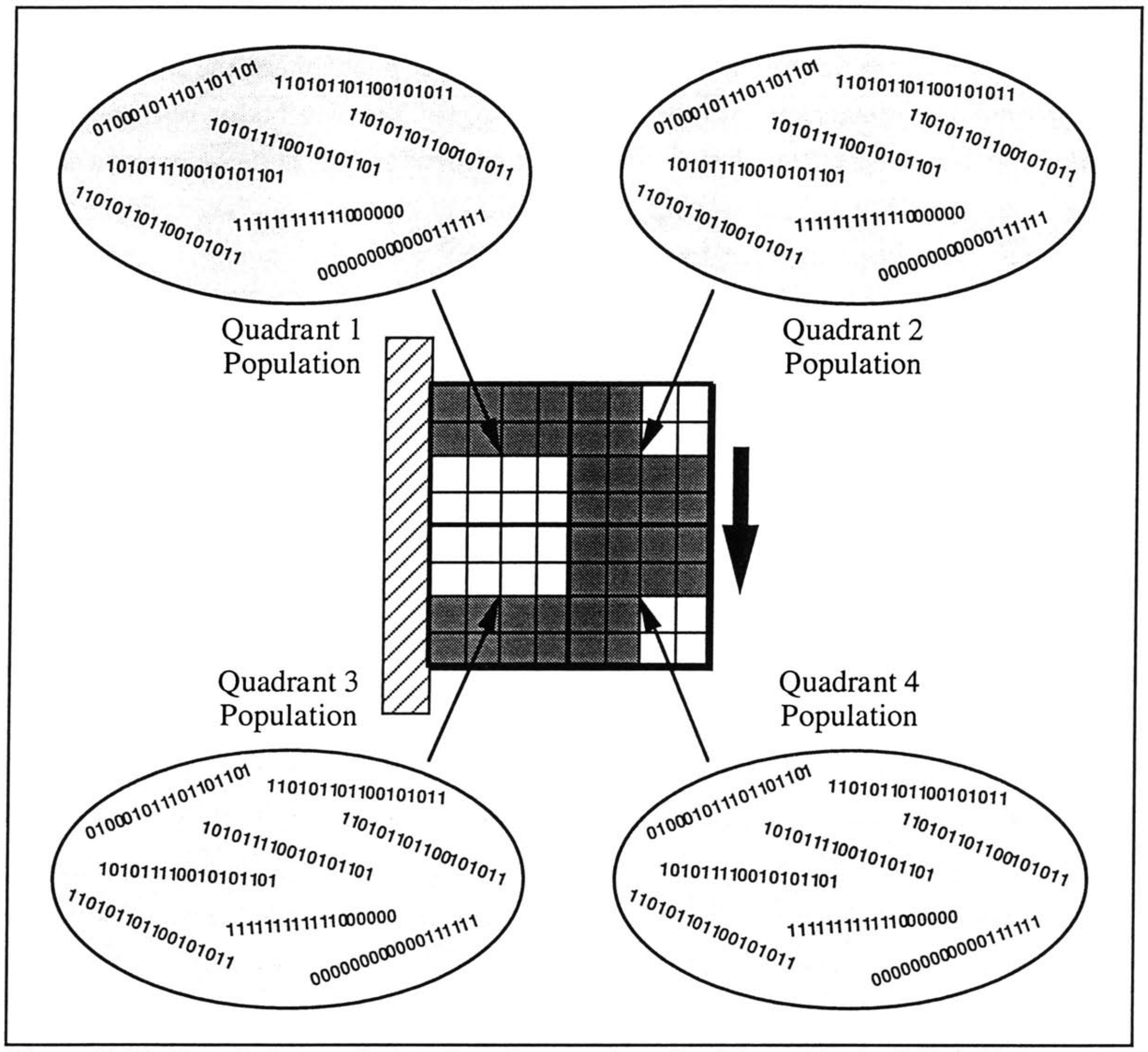


Figure 5.17: Population-to-design-domain mapping after hierarchical subdivision.

To initialize the four populations, the material-void distribution in each quadrant of the finely-discretized design domain (again, the finely-discretized design domain currently contains the optimal coarse plate topology which was converted into the finer discretization) is mapped onto the chromosomes in the population controlling the quadrant—material elements in the design domain set corresponding chromosome genes to 1, while void elements in the design domain set corresponding chromosome genes to 0. Hence, after initialization, all chromosomes in each population correspond to the distribution of material and void within the design domain quadrant controlled by the population. To create diversity in each population, the chromosomes are then subjected to high-probability mutation (performed on every gene of every chromosome with a 0.15 probability) similar to a "nuking" procedure used by Jensen (1992).

Before any optimization is performed in the four populations, the fitness of each recently-nuked chromosome must be evaluated. Again, the distribution of material and void in the finely-discretized design domain currently corresponds to the optimal coarse plate topology which was previously mapped into the domain. So, beginning with the population controlling Quadrant 1 of the finely-discretized design domain, a chromosome in the population is mapped into the quadrant to create a distribution of material and void within the quadrant. This distribution of material and void, when used with the distributions of material and void within the three other quadrants, creates a material distribution throughout the *entire* design domain and consequently generates a finely-discretized plate topology. This finely-discretized plate topology is then subjected to connectivity analysis, and the constant meshing technique is used to generate a finite element mesh corresponding to the plate topology. Finite element analysis is then performed on the plate topology, and fitness calculations are conducted as in previous examples. The resulting fitness value is then assigned to the chromosome which created the current material-void distribution in Quadrant 1. This process (*i.e.*, mapping a chromosome into the quadrant, generating a plate topology from the material-void distribution throughout the entire domain, performing connectivity analysis on the plate topology, creating a finite element mesh corresponding to the plate topology, performing finite element analysis on the plate topology, and then assigning a fitness value to the chromosome) is then repeated for each chromosome in the population controlling Quadrant 1. After all of the chromosomes in the population have been evaluated, the most-highly-fit chromosome in the population is mapped back into Quadrant 1 so that it may be used in the fitness calculations of the other quadrants. The populations controlling Quadrants 2, 3, and 4 are then evaluated (in that order) using the same technique.

Optimization proceeds by performing standard genetic algorithm optimizations in each of the four quadrants. First, the population controlling Quadrant 1 is evolved a single generation: parents are selected and mated to create a generation of child topologies, each of which is subjected to mutation and then evaluated for fitness (fitness evaluations are performed as detailed in the previous paragraph). The child population then replaces the parent population, and the most-highly-fit child chromosome is mapped into Quadrant 1. The populations controlling Quadrants 2, 3, and 4 (in that order) are then evolved a single generation. After each quadrant has evolved one generation (with the resulting optimal chromosome from each population being mapped into its corresponding design domain quadrant), the "cycle" is repeated. Figure 5.18 depicts an example finely-discretized plate topology after several cycles of optimization.

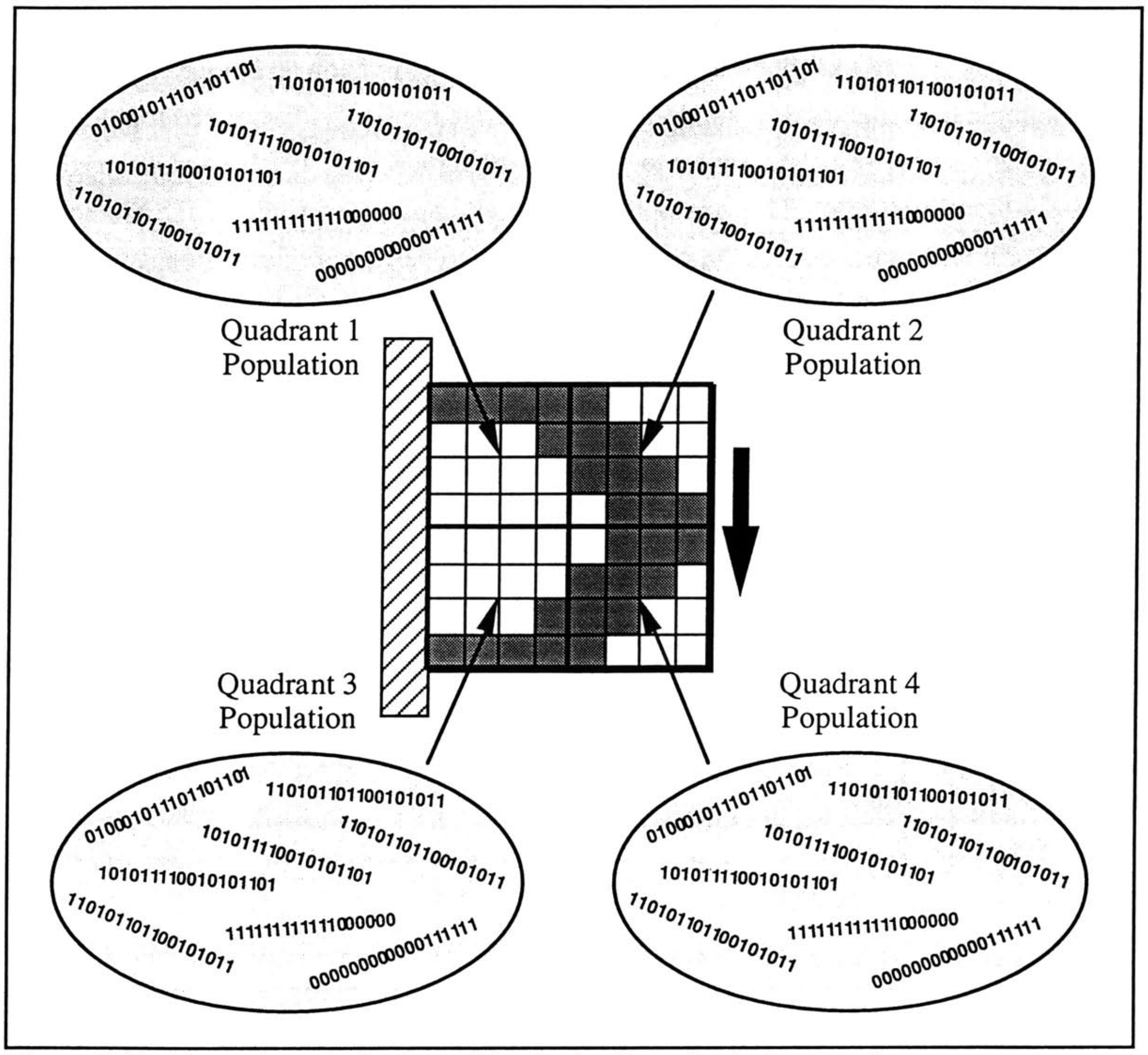


Figure 5.18: Example optimization of the finely-discretized plate topology.

After a pre-determined number of cycles, the entire *subdivision* process is repeated. Each quadrant of the design domain is divided into four sub-quadrants, where each sub-quadrant has the same resolution as the original quadrant. The resulting design domain has

16 times the resolution of the original, coarse design domain discretization and is controlled by 16 genetic algorithm populations, where each population's chromosomes are equal in length to the original chromosomes which controlled the coarse design domain. The optimization then proceeds by cycling through the 16 sub-quadrants (from left to right, top to bottom), allowing each population to evolve one generation during each cycle. This process could be repeated indefinitely.

5.6.4 Example. In an attempt to establish the viability of the hierarchical subdivision technique, the topological optimization of a cantilevered plate detailed in Example 4 is now re-examined using the hierarchical subdivision technique. Specifically, this example uses hierarchical subdivision to generate the finely-discretized topology of a rectangular cantilevered plate subject to a downward vertical load which provides maximum stiffness-to-weight ratio. This example is a first attempt to determine if the hierarchical subdivision technique enables the genetic algorithm to perform optimization using design domain discretizations higher than those investigated in previous examples.

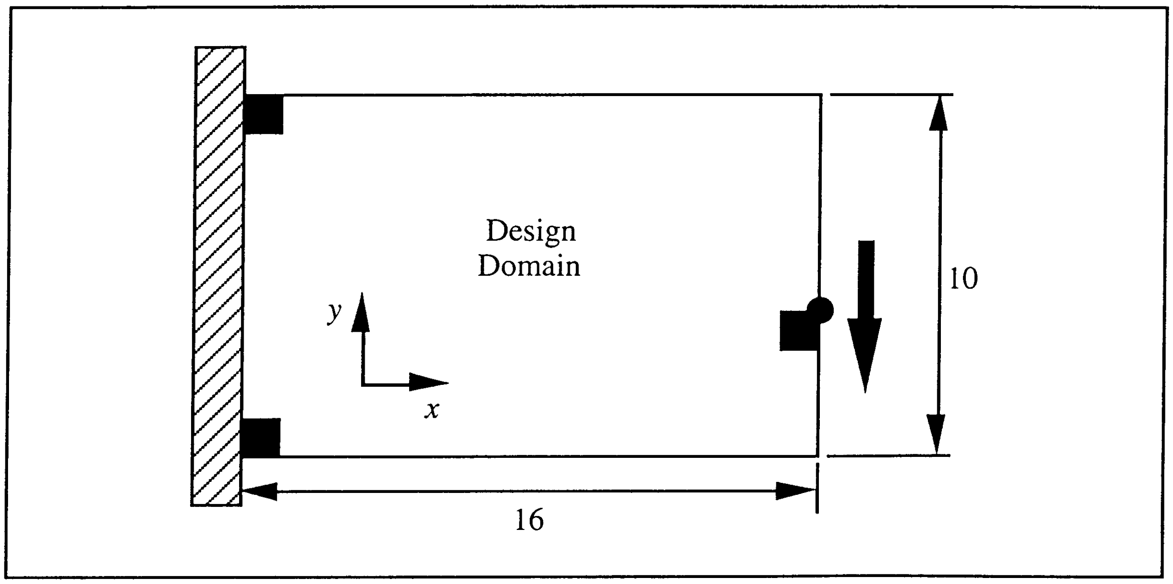


Figure 5.19: Example 5 design domain.

Design Domain. As in Example 4, the cantilevered plate is modeled using a rectangular design domain 10 units tall by 16 units wide (Figure 5.19), and no symmetry constraints are imposed on the plate topology. The design domain is initially discretized into a 10×16 grid of binary, material-void elements, and subsequent hierarchical subdivision-based resolution increases result in design domain discretizations of 20×32 and finally 40×64. Hence, as the initial design domain discretization (10×16) establishes the genetic algorithm chromosome length, all chromosomes used in the optimization (regardless of the design domain discretization or number of genetic algorithm populations controlling the plate's topology) contain 160 genes. To allow for the application of the vertical concentrated load, the design domain element on the plate's right-hand surface immediately below the point of load application (which is $\frac{2}{5}$ of the distance from the bottom) serves as a seed element. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface.

Fitness Calculations. As this example uses hierarchical subdivision to maximize the plate's stiffness-to-weight ratio, fitness calculations begin when a chromosome is converted into its corresponding plate topology as detailed in Section 5.6.3. Then, once the plate topology has been generated, connectivity analysis, constant finite element meshing (using four triangular finite elements for every design domain element), and finite element analysis are performed. Note that during finite element analysis, the four nodes along the finite element mesh's left-hand surface which correspond to the two seed elements are constrained to have zero displacement, while the concentrated load is applied to the node on the finite element mesh's right-hand surface which is $\frac{2}{5}$ of the distance from the bottom. Finally, the plate topology's stiffness-to-weight ratio, and therefore the fitness (to be maximized) of the chromosome corresponding to the plate topology, is given by:

$$Fitness = \frac{Stiffness}{Weight} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}, \tag{5.7}$$

where *Area* is equal to the area of connected material elements in the plate topology and u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

Hierarchical Subdivision Parameters. Optimization begins with a single genetic algorithm population optimizing the plate topology with a coarse, 10×16 design domain discretization. Note that this population contains 30 160-gene chromosomes. After performing 250 generations of evolution at this initial discretization, hierarchical subdivision is performed to quadruple the resolution of the design domain. Hence, following hierarchical subdivision, four genetic algorithm populations optimize the plate's topology using a 20×32 design domain discretization. At this discretization, each design domain element is $\frac{1}{4}$ the size of the original, coarse design domain elements, and each of the four populations contains 30 160-gene chromosomes. Then, after performing optimization for 50 cycles (*i.e.*, 50 generations of evolution in each design domain quadrant) at the 20×32 discretization, the resolution of the design domain is again quadrupled. After this second hierarchical subdivision, sixteen genetic algorithm populations optimize the plate's topology using a 40×64 design domain discretization. At this discretization, each design domain element is $\frac{1}{16}$ the size of the original, coarse design domain elements, and each of the sixteen populations contains 30 160-gene chromosomes. Optimization concludes after 10 cycles at this discretization.

Results. Results of the optimization are shown in Figure 5.20, which displays the optimal plate topologies generated at the three different discretizations. 7,500 structural analyses were performed during the generation of the coarse, 10×16 plate topology:

$$250\ \mathit{Generations} \cdot 30 \frac{\mathit{Chromosomes}}{\mathit{Generation}} = 7{,}500\ \mathit{Chromosomes}, \tag{5.12}$$

while an additional 6,000 structural analyses were performed to obtain the 20×32 plate topology:

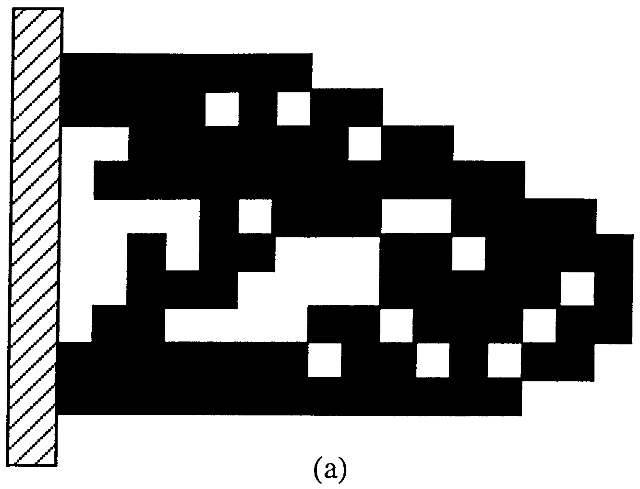
$$50\ \mathit{Cycles} \cdot 4 \frac{\mathit{Generations}}{\mathit{Cycle}} \cdot 30 \frac{\mathit{Chromosomes}}{\mathit{Generation}} = 6{,}000\ \mathit{Chromosomes} \tag{5.13}$$

and an additional 4,800 analyses were performed to obtain the 40×64 plate topology:

$$10\ \mathit{Cycles} \cdot 16 \frac{\mathit{Generations}}{\mathit{Cycle}} \cdot 30 \frac{\mathit{Chromosomes}}{\mathit{Generation}} = 4{,}800\ \mathit{Chromosomes}. \tag{5.14}$$

Hence, a total of 18,300 finite element analyses were required to obtain the 40×64 plate topology.

As shown in Figure 5.20, this investigation's hierarchical subdivision technique provides reasonable results. While the 40×64 plate's outer boundaries are somewhat jagged, the inner region does contain several well-defined, truss-like members and several large areas of void. Comparing the plate topologies generated in this example to those detailed in Example 4 (Figure 5.13), which are generated using a constant design domain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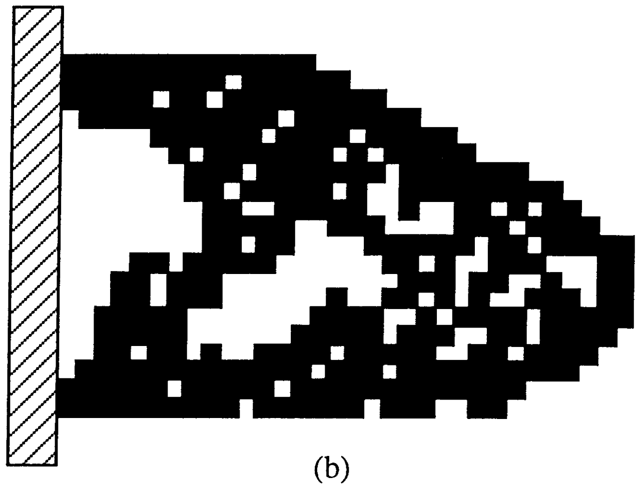
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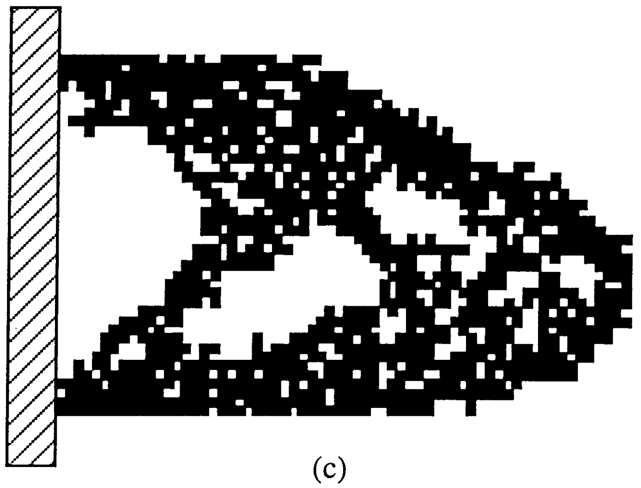
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Figure 5.20: Optimal plate topologies generated using (a) 10×16, (b) 20×32, and (c) 40×64 design domain discretizations.

discretization, it is interesting to note that this example's hierarchical subdivision technique generates a 40×64 plate topology using only slightly more structural analyses (18,300 vs. 18,000) than Example 4's constant design domain discretization requires to generate a 20×32 plate topology. Also, when generating an optimal 20×32 plate topology, the hierarchical subdivision technique requires considerably fewer structural analyses (13,500 vs. 18,000) than Example 4's constant design domain discretization technique. Finally, while Example 4 requires 18,000 finite element analyses with a 20×32 finite element mesh to generate an optimal 20×32 plate topology, this example's hierarchical subdivision technique obtains an optimal 20×32 plate topology by performing 7,500 finite element analyses with a 10×16 finite element mesh and 6,000 analyses with a 20×32 mesh. Hence, in addition to requiring fewer structural analyses, more than half of the analyses performed by the hierarchical subdivision technique use a much coarser finite element mesh.

By beginning optimization with a coarse design domain discretization and a correspondingly small search space, hierarchical subdivision enables the genetic algorithm to easily locate the *general* location of the optimal plate topology in the search space. Then, having found a high-quality region in the search space, hierarchical subdivision increases the resolution of the search space so that a more exact plate topology can be located. In other words, hierarchical subdivision uses a coarse design domain discretization to find the general shape of the optimal plate topology and then increases the design domain's resolution so that the plate topology can be fine-tuned. As this increased design domain resolution necessitates an increase in the number of genetic algorithm genes controlling the material-void distribution in the design domain, hierarchical subdivision increases the number of genetic algorithm populations controlling the design domain.

Note that instead of increasing the number of populations controlling the domain, the length of the chromosomes in the single, original genetic algorithm population could be increased to account for the finer design domain resolution. Lin and Hajela (Section 3.3.2) use such an approach, where chromosome length is periodically increased so that floating-point design variables (truss-member cross-sectional areas in the work of Lin and Hajela) may be represented with greater resolution. Unfortunately, expanding the lengths of chromosomes in a single population results in an exceptional expansion of the search space, and, during the course of this investigation, was found to provide unsatisfactory results in comparison to hierarchical subdivision's multiple population approach. Hence, by using multiple genetic algorithm populations, each responsible for a particular portion of the design domain, hierarchical subdivision increases the resolution of the overall search space while keeping the size of each individual population's search space constant.

Because the constant design domain discretization used in previous examples forces the genetic algorithm to generate both the general plate topology and the finely-tuned, optimal plate topology using a highly-discretized design domain, the genetic algorithm experiences great difficulty in finding *either*. On the other hand, hierarchical subdivision's use of a variable-resolution design domain enables the genetic algorithm to find the general shape of the optimal plate topology using a coarse design domain discretization and then fine-tune the optimal plate topology using a highly-discretized design domain. Consequently, when generating finely-discretized plate topologies, the hierarchical subdivision technique outperforms the constant design domain discretization used in previous examples.

5.7 Example 6: Design Families.

5.7.1 Introduction. Throughout this investigation, the genetic algorithm has been used to evolve a population of plate topologies in an attempt to generate an optimal plate topology. Hence, at the conclusion of any given optimization, a population of highly-fit plate topologies has been created. However, while nearly all of the plate topologies in a final population are highly-fit, previous examples have considered only the most-highly-fit plate topology in a final population. Unfortunately, by examining only the plate topology which maximizes the optimization problem's fitness function (which certainly does not consider all possible design criteria), previous examples have ignored other highly-fit plate topologies in the final population which, when evaluated using alternate design criteria, may outperform the most-highly-fit plate topology.

This example attempts to demonstrate the genetic algorithm's ability to obtain, during a single optimization, a family of plate topologies which a designer can evaluate using alternate, secondary criteria not addressed in the fitness function which generated the designs. Specifically, this example uses the genetic algorithm to obtain a family of plate topologies with maximum stiffness-to-weight-to-perimeter ratio, which the designer can then select from using manufacturing, weight, and/or displacement criteria. Note that this new stiffness-to-weight-to-perimeter ratio fitness function, which will be detailed in Section 5.7.3, is used in an attempt to drive the genetic algorithm search towards plate topologies which combine high stiffness-to-weight ratio with high manufacturability. As in Examples 4 and 5, this example optimizes the topology of a rectangular, cantilevered plate subject to a downward vertical load and uses unsymmetric loading, no symmetry constraints, and the constant meshing technique. Note that this example *does not* use hierarchical subdivision.

5.7.2 Design Domain. As in Examples 4 and 5, the cantilevered plate is modeled using a rectangular design domain discretized into a 10×16 grid of binary, material-void elements (Figure 5.21). During optimization, no symmetry constraints are imposed on the plate topology, and the genetic algorithm chromosomes control the distribution of material and void throughout the entire design domain. As each genetic algorithm chromosome must therefore contain one gene for every design domain element, this 10×16 design domain discretization results in a chromosome length of 160 genes and a search space containing 2^{160} distinct locations. To allow for the application of the vertical concentrated load, the design domain element on the plate's right-hand surface immediately below the point of load application (which is $\frac{2}{5}$ of the distance from the bottom) serves as a seed element. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface.

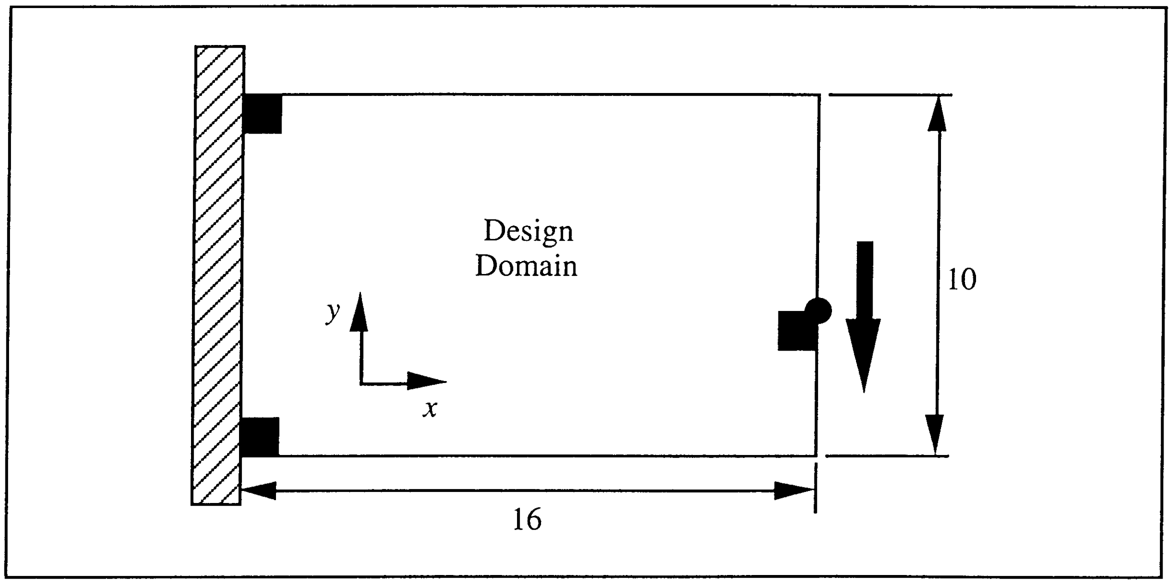

Figure 5.21: Example 6 design domain.

5.7.3 Fitness Calculations. As this example attempts to maximize the plate's stiffness-to-weight-to-perimeter ratio, fitness calculations are based on the stiffness-to-weight ratio calculations detailed in Section 4.4 and used in Example 4. Specifically, to evaluate the fitness of a particular chromosome, the chromosome is first converted into its corresponding plate topology (because no symmetry constraints are imposed and hierarchical subdivision is not used, the chromosome is mapped into the entire discretized design domain to create a plate topology) and connectivity analysis is then performed. A finite element mesh containing four triangular finite elements for every design domain element is then generated using the constant meshing technique, and structural analysis is performed using the finite element method. Note that during finite element analysis, the four nodes along the finite element mesh's left-hand surface which correspond to the two seed elements are constrained to have zero displacement, while the concentrated load is applied to the node on the finite element mesh's right-hand surface which is $\frac{2}{5}$ of the distance from the bottom. As in Examples 4 and 5, the plate topology's stiffness-to-weight ratio is then set equal to:

$$\frac{Stiffness}{Weight} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}, \tag{5.15}$$

where *Area* is equal to the area of connected material elements in the plate topology and u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

Again, this example attempts to drive the genetic algorithm towards plate topologies which combine high stiffness-to-weight ratio with high manufacturability. While manufacturability can have various definitions and any given plate's manufacturability can be based on a variety of different attributes, this example assumes that a plate's manufacturability is based entirely on the plate's porosity—highly-porous plate topologies are considered to have low manufacturability, while plate topologies with low porosity are considered to have high manufacturability. Note that a plate's porosity is assumed to be equal to the number of internal holes in the structure. Hence, fitness calculations are formulated so that plate topologies which combine high stiffness-to-weight ratio with a low number of internal holes receive high fitness, while plate topologies with low stiffness-to-weight ratio and/or a high number of internal holes receive low fitness.

A variety of fitness formulations can be used to simultaneously maximize stiffness-to-weight ratio and minimize internal holes. For example, an optimization could attempt to maximize stiffness-to-weight ratio subject to a constraint on the maximum number of internal holes in the structure; maximize stiffness-to-weight ratio subject to a constraint on the maximum area of internal holes; or maximize the value of the stiffness-to-weight ratio divided by the number of internal holes in the structure. Again, these examples represent only a partial list of the possible fitness formulations. As in any optimization, the particular fitness formulation chosen depends upon the available computational tools as well as the intuition of the designer.

To simultaneously maximize stiffness-to-weight ratio and minimize internal holes, this example attempts to drive the genetic algorithm towards plate topologies with a high stiffness-to-weight ratio and a small perimeter. Hence, this example's fitness formulation maximizes a plate topology's stiffness-to-weight-to-perimeter ratio. Note that in this example, a plate's perimeter is assumed to be equal to the sum of the plate's outer perimeter and the perimeter of the plate's internal holes.

So, after calculating a plate topology's stiffness-to-weight ratio (given by Equation 5.15), the plate's perimeter is calculated. The plate topology's stiffness-to-weight-to-perimeter ratio, and therefore the fitness (to be maximized) of the chromosome corresponding to the plate topology, is then given by:

$$Fitness = \frac{\left(\frac{Stiffness}{Weight}\right)}{Perimeter} = \frac{1}{Area \cdot Perimeter \cdot \sqrt{(u_x)^2 + (u_y)^2}}. \tag{5.16}$$

5.7.4 Results. After 225 generations of genetic algorithm-based evolution, a population, or family, of 30 possibly-optimal plate topologies was generated. During the search, a total of 6,750 structural analyses were performed:

$$225\ Generations \cdot 30 \frac{Chromosomes}{Generation} = 6{,}750\ Chromosomes. \tag{5.17}$$

Figure 5.22 depicts the plate topologies which exhibit, for the six minimum possible levels of porosity, maximum stiffness-to-weight ratio. In other words, of the plate topologies in the family which have either 0, 1, 2, 3, 4, or 5 internal holes, Figure 5.22 depicts the plate topologies with maximum stiffness-to-weight ratio. Also, Table 5.2 quantifies each plate topology's number of internal holes (*i.e.*, manufacturability), stiffness-to-weight ratio, displacement, and weight. Note that while the genetic algorithm-based optimization only addressed the minimization of internal holes and the maximization

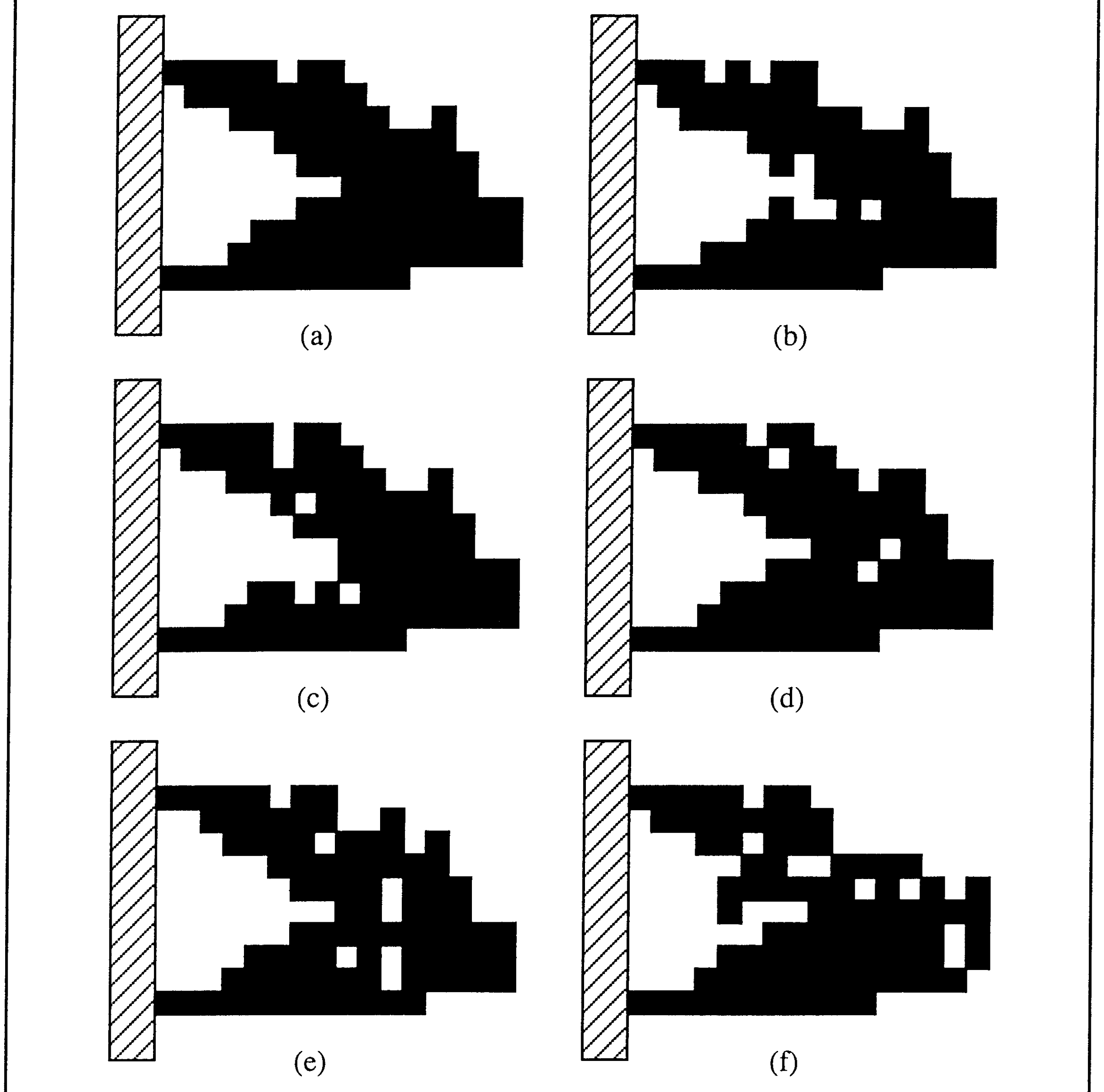

Figure 5.22: Family of plate topologies with (a) 0, (b) 1, (c) 2, (d) 3, (e) 4, and (f) 5 internal holes.

Topology	Number of Internal Holes	Stiffness-to-Weight Ratio	Displacement	Weight
(a)	0	1.19	0.0092	91
(b)	1	1.15	0.0102	85
(c)	2	1.11	0.0106	85
(d)	3	1.20	0.0094	89
(e)	4	1.14	0.0102	86
(f)	5	1.14	0.0101	87

Table 5.2: Plate topology performance data.

of stiffness-to-weight ratio, a designer may consider any or all of these attributes (number of internal holes, stiffness-to-weight ratio, displacement, weight) when selecting an optimal plate topology.

This family of possibly-optimal plate topologies, generated by one genetic algorithm-based optimization, could be used in several different ways. First, as shape optimization does not alter a structure's topology, the six plate topologies provided in Figure 5.22 could serve as highly-fit initial plate topologies for classical, mathematical programming-based shape optimization. Second, using each plate topology's performance data (Table 5.2) in conjunction with knowledge of the particular application domain and the environment in which the plate will serve, a designer could conduct a pareto optimization study to select an optimal topology based on several alternate criteria. For example, if manufacturability (*i.e.*, lack of internal holes) is of utmost importance, the designer would choose topology (a). Conversely, if weight is important, the designer would choose topology (b) or (c).

Note that during this optimization, the genetic algorithm search was not explicitly driven towards a population of designs exhibiting varying levels of stiffness-to-weight ratio, internal holes, displacement, and weight. In other words, chromosomes in the genetic algorithm population were not explicitly driven to distribute themselves throughout the search space. Instead, the genetic algorithm's population-based search paradigm, as well as its randomly-generated initial populations and probabilistic operators, automatically generated different plate topologies. Hence, when using the genetic algorithm to perform structural topology optimization, there is no need for the designer to develop a complicated objective function which specifies the relative importance of secondary criteria—he or she may simply run the genetic algorithm-based optimization with a straightforward objective function, and the genetic algorithm will automatically provide a family of design alternatives in a single optimization. To obtain families of design alternatives using traditional optimization methods such as mathematical programming and optimality criteria methods, optimization must be performed several times using a variety of initial conditions, with the hope that the search will converge to different plate topologies.

5.8 Example 7: Manufacturability Considerations.

5.8.1 Introduction. Examples 4 and 5 detailed genetic algorithm-based topological optimizations of cantilevered plates represented by finely-discretized design domains. The optimizations, which attempted to maximize a plate's stiffness-to-weight ratio, all generated optimal plate topologies containing large numbers of internal holes. These porous topologies, while perhaps exhibiting "optimal" stiffness-to-weight ratio, are poor from a manufacturing perspective and could also lead to difficulties during parameterization for sizing and shape analysis.

This example attempts to determine if the genetic algorithm can generate finely-discretized plate topologies combining high stiffness-to-weight ratio with high manufacturability. As detailed in Example 6, this investigation assumes that a plate's manufacturability depends only upon porosity (*i.e.*, the number of internal holes in the plate topology). Hence, highly-porous plate topologies are assumed to have low manufacturability, while low-porosity plate topologies are assumed to have high manufacturability. In addition to evaluating the genetic algorithm's abilities in generating plate topologies which combine high stiffness-to-weight ratio with high manufacturability, this example also intends to establish what, if any, relationship exists between a structure's porosity and its stiffness-to-weight ratio. Finally, this example provides insight as to why previous genetic algorithm-based topological optimizations (*i.e.*, Examples 4 and 5) have resulted exclusively in highly-porous plate topologies.

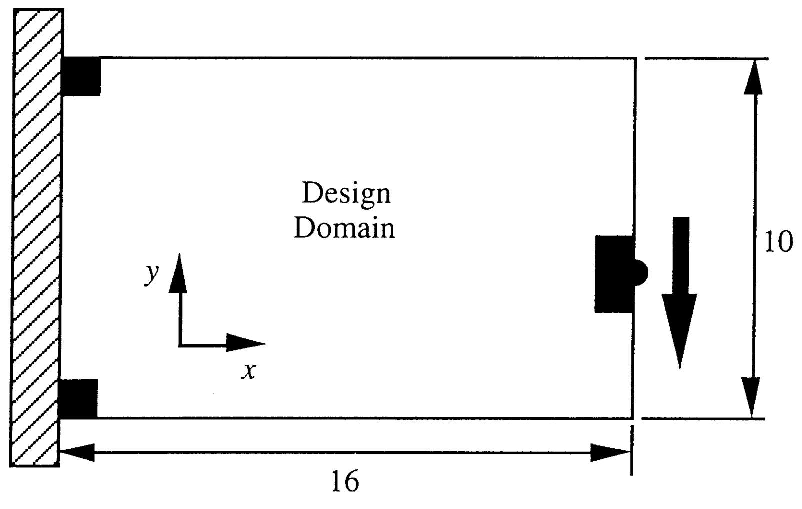

Figure 5.23: Example 7 design domain.

5.8.2 Design Domain. As in Examples 4, 5, and 6, the cantilevered plate is modeled using a rectangular design domain discretized into a 10×16 grid of binary, material-void elements (Figure 5.23). During optimization, no symmetry constraints are imposed on the plate topology, and the genetic algorithm chromosomes control the distribution of material and void throughout the entire design domain. As each genetic algorithm chromosome must therefore contain one gene for every design domain element, this 10×16 design domain discretization results in a chromosome length of 160 genes and a search space containing 2^{160} distinct locations. To allow for the application of the vertical concentrated load, the two design domain elements on the plate's right-hand surface immediately adjacent to the point of load application (which is $\frac{2}{5}$ of the distance from the bottom) serve as seed elements. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface. Note that this example *does not* use hierarchical subdivision.

5.8.3 Fitness Calculations. As this example attempts to simultaneously maximize stiffness-to-weight ratio and minimize porosity, fitness calculations are based on the stiffness-to-weight ratio calculations detailed in Section 4.4 and used in Examples 4, 5, and 6. Specifically, to evaluate the fitness of a particular chromosome, the chromosome is first converted into its corresponding plate topology (because no symmetry constraints are imposed and hierarchical subdivision is not used, the chromosome is mapped into the entire discretized design domain to create a plate topology) and connectivity analysis is then performed. A finite element mesh containing four triangular finite elements for every design domain element is then generated using the constant meshing technique, and structural analysis is performed using the finite element method. Note that during finite element analysis, the four nodes along the finite element mesh's left-hand surface which correspond to the two seed elements are constrained to have zero displacement, while the concentrated load is applied to the node on the finite element mesh's right-hand surface which is $\frac{2}{5}$ of the distance from the bottom. As in previous examples, the plate topology's stiffness-to-weight ratio is then set equal to:

$$\frac{Stiffness}{Weight} = \frac{1}{Area \cdot \sqrt{(u_x)^2 + (u_y)^2}}, \tag{5.15}$$

where *Area* is equal to the area of connected material elements in the plate topology and u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

Once the plate's stiffness-to-weight ratio has been calculated, additional calculations must be performed in order to drive the genetic algorithm towards plate topologies combining high stiffness-to-weight ratio and low porosity. As detailed in Example 6, a variety of different fitness functions can be used to obtain such plate topologies. Hence, because of the availability of fitness functions which would likely provide adequate optimization performance, as well as the inherent *ad hoc* nature of genetic algorithm fitness function design, this example uses nine candidate fitness functions, all based on a plate's stiffness-to-weight ratio (obtained above), to generate plate topologies combining high stiffness-to-weight ratio with low porosity. Besides the stiffness-to-weight-to-perimeter ratio maximization fitness function used in Example 6, this example provides seven newly-developed functions which, in addition to maximizing stiffness-to-weight ratio, use a variety of techniques to reduce porosity. The standard stiffness-to-weight ratio maximization function used in previous articles is also included (Function 1) so that it may serve as the basis for comparison. A list of the nine candidate fitness functions, as well as definitions of parameters used in the fitness functions, are provided below.

<u>Variable Definitions:</u>

• *Area*	Area of connected material elements in the plate topology. Serves as a qualitative measure of the plate's weight. Equivalent to the Area variable used in previous stiffness-to-weight ratio calculations.
• *Displacement*	Magnitude of the plate's displacement (vector sum of x- and y-direction displacements) at the finite element node where the concentrated load is applied. Equivalent to $\|\mathbf{u}\|$ (*i.e.*, $\sqrt{(u_x)^2+(u_y)^2}$) in previous stiffness-to-weight ratio calculations.
• *HoleArea*	Total area of the plate's internal holes. An internal hole is a void element, or group of void elements, which are surrounded in all directions by material elements. Hence, groups of void elements between the plate's outer surface and the design domain boundaries are not considered internal holes.
• *HolePerimeter*	Perimeter of all internal holes in the plate topology.
• *NumberOfHoles*	Number of internal holes in the plate topology.
• *Perimeter*	The plate's total perimeter. Represents the sum of the plate's outer perimeter and the perimeter of its internal holes.

Candidate Fitness Functions:

- *Fitness Function 1:* Maximize Stiffness-to-Weight Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area}$$

- *Fitness Function 2:* Simultaneously Maximize Stiffness-to-Weight Ratio and Weight-to-Perimeter Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area} + c\frac{Area}{Perimeter}$$

$$c = 0.1,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 2.0$$

- *Fitness Function 3:* Maximize Stiffness-to-Weight-to-Perimeter Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area \cdot Perimeter}$$

- *Fitness Function 4:* Maximize Stiffness-to-Weight-to-HoleArea Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area \cdot HoleArea}$$

- *Fitness Function 5:* Maximize Stiffness-to-Weight-to-HolePerimeter Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area \cdot HolePerimeter}$$

- *Fitness Function 6:* Maximize Stiffness-to-Weight-to-NumberOfHoles Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area \cdot (NumberOfHoles + 1)}$$

- *Fitness Function 7:* Simultaneously Maximize Stiffness-to-Weight Ratio and HoleArea-to-HolePerimeter Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area} + c\frac{HoleArea}{HolePerimeter}$$

$$c = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 2.0$$

- *Fitness Function 8:* Simultaneously Maximize Stiffness-to-Weight Ratio and Minimize HolePerimeter-to-HoleArea Ratio.

$$Fitness = \frac{1}{Displacement \cdot Area} - c\frac{HolePerimeter}{HoleArea}$$

$$c = 0.05,\ 0.10,\ 0.15,\ 0.20,\ 0.25,\ 0.30$$

- *Fitness Function 9:* Maximize Stiffness-to-Weight Ratio subject to a maximum porosity constraint. Similar to the compliance minimization fitness function detailed in Section 4.4.6 and used in Example 8, this function penalizes a structure's stiffness-to-weight ratio by a (a = 6, 10, 14, 18, 22, or 26) percent for every internal hole in addition to the b (b = 0, 1, 2, 3, or 4) allowable holes. The penalty is also attenuated according to the current generation number.

$$Fitness = \left\{1.0 - \left[\frac{a}{100} \cdot \frac{generation}{175} \cdot (NumberOfHoles - b)\right]\right\}\frac{1}{Displacement \cdot Area}$$

$$a = 6,\ 10,\ 14,\ 18,\ 22,\ 26$$
$$b = 0,\ 1,\ 2,\ 3,\ 4$$

As in all other examples detailed in this article, after a plate topology's fitness has been calculated the fitness value is assigned to the genetic algorithm chromosome corresponding to the topology.

5.8.4 Experiment Design. Including all possible parameter values (*i.e.*, coefficient values in Functions 2, 7, and 8) and parameter combinations (*i.e.*, combinations of parameters a and b in Function 9), 54 unique fitness functions are represented in the above function suite and used to optimize the plate's topology. So that statistically-significant comparisons can be made between the candidate fitness functions, 10 replications of the experiment are conducted with each function variant. Hence, a total of 540 optimizations are performed, with each optimization evolving for 225 generations. 6,750 structural analyses are therefore performed during each optimization:

$$225\ Generations \cdot 30\frac{Chromosomes}{Generation} = 6{,}750\ Chromosomes. \qquad (5.18)$$

Note that different random initial seeds (*i.e.*, different, randomly-generated initial populations) are used in each optimization. At the conclusion of each optimization, the optimal plate topology is recorded, as are the plate topology's stiffness-to-weight ratio and porosity (*i.e.*, number of internal holes).

5.8.5 Results. Figure 5.24, which provides initial results of the experiment, depicts the plate topologies which exhibit, for the six minimum possible levels of porosity, maximum stiffness-to-weight ratio. In other words, of the plate topologies obtained which contain either 0, 1, 2, 3, 4, or 5 internal holes, Figure 5.24 displays the plate topologies with maximum stiffness-to-weight ratio. While casual inspection of these plate topologies suggests that the genetic algorithm can generate topologies which combine high stiffness-to-weight ratio and low porosity, these topologies give little, if any, insight as to which of the above candidate fitness functions can *consistently* generate plate topologies with the highest stiffness-to-weight ratio and lowest porosity. Also, these six plate topologies do not provide any insight as to whether or not a relationship exists between porosity and stiffness-to-weight ratio.

In an attempt to determine which candidate fitness functions consistently generate plate topologies with the highest stiffness-to-weight ratio and the lowest porosity, an analysis of variance (ANOVA), followed by comparisons using the Bonferroni procedure (Devore, 1987), is performed on the experimental data. The null hypothesis is confidently rejected ($\alpha < 0.05$) in the analysis, indicating statistically-significant stiffness-to-weight ratio and porosity differences between the fitness functions. Results are provided in Table 5.3, which ranks the candidate fitness functions according to their ability to generate plate topologies with high stiffness-to-weight ratio, and in Table 5.4, which ranks the candidate fitness functions according to their ability to generate plate topologies with few internal holes. Note that the "Mean Stiffness-to-Weight Ratio of optimal topologies" column in Table 5.3 details, for each of the nine candidate fitness functions, the mean stiffness-to-weight ratio of the optimal plate topologies generated by that fitness function. Likewise, the "Mean number of internal holes in optimal topologies" column in Table 5.4 details, for each of the nine candidate fitness functions, the mean number of internal holes in the optimal plate topologies generated by that function.

As demonstrated by the ANOVA results in Tables 5.3 and 5.4, best stiffness-to-weight ratio performance is obtained using fitness functions 1, 2, 3, 7, and 9 (which provide statistically-equivalent stiffness-to-weight ratio performance), while structures with lowest porosity are obtained using fitness functions 3, 4, 5, 6, and 9 (which generate

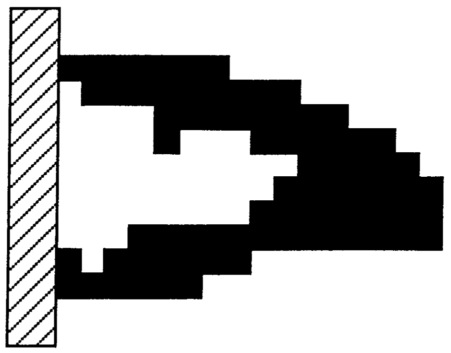
Figure 5.24a
Number of Internal Holes = 0
Stiffness-to-Weight Ratio = 1.290
Generated by Fitness Function 9

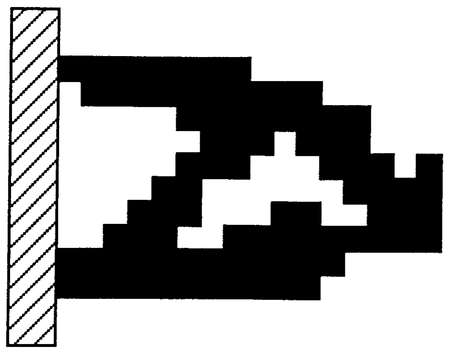
Figure 5.24b
Number of Internal Holes = 1
Stiffness-to-Weight Ratio = 1.355
Generated by Fitness Function 9

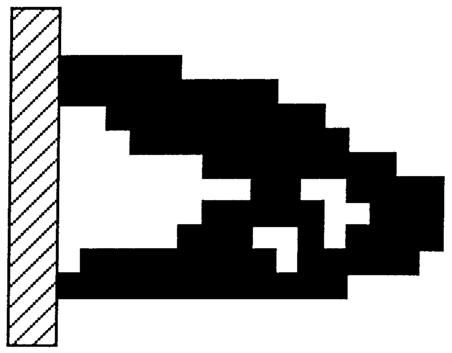
Figure 5.24c
Number of Internal Holes = 2
Stiffness-to-Weight Ratio = 1.436
Generated by Fitness Function 9

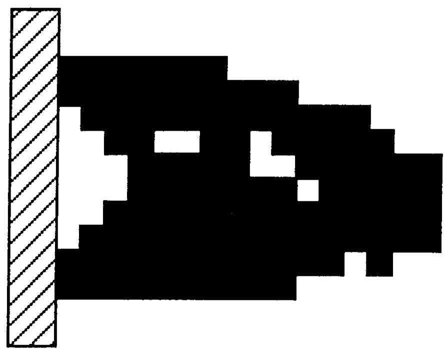
Figure 5.24d
Number of Internal Holes = 3
Stiffness-to-Weight Ratio = 1.418
Generated by Fitness Function 2

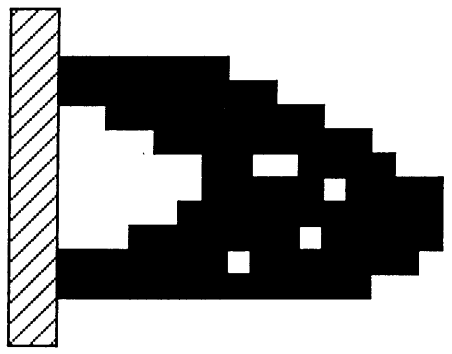
Figure 5.24e
Number of Internal Holes = 4
Stiffness-to-Weight Ratio = 1.479
Generated by Fitness Function 2

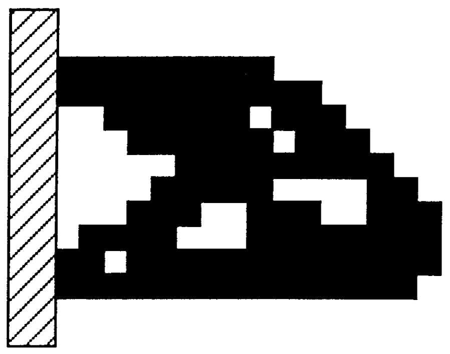
Figure 5.24f
Number of Internal Holes = 5
Stiffness-to-Weight Ratio = 1.483
Generated by Fitness Function 2

Figure 5.24: High stiffness-to-weight ratio, low-porosity plate topologies.

Fitness Function	# of optimizations performed using this fitness function	Mean Stiffness-to-Weight Ratio of optimal topologies	Bonferroni Grouping *Means with the same letter are not significantly different*	
1	10	1.44445	A	
2	70	1.38307	A	
3	10	1.25356	A	
9	300	1.21846	A	B
7	60	1.18592	A	B
6	10	0.95431	C	B
8	60	0.84680	C	
4	10	0.78166	C	D
5	10	0.57193		D

Table 5.3: Ranking of candidate fitness functions according to Stiffness-to-Weight Ratio.

Fitness Function	# of optimizations performed using this fitness function	Mean number of internal holes in optimal topologies	Bonferroni Grouping *Means with the same letter are not significantly different*	
5	10	0.000	A	
6	10	0.200	A	
4	10	0.800	A	
9	300	2.290	A	B
3	10	2.700	A	B
8	60	4.350	C	B
7	60	6.100	C	
2	70	7.043	C	
1	10	12.000	D	

Table 5.4: Ranking of candidate fitness functions according to number of internal holes.

statistically-equivalent numbers of internal holes). Hence, fitness functions 3 and 9 obtain topologies combining highest stiffness-to-weight ratio with lowest porosity. This result is also demonstrated graphically in Figure 5.25, which plots the mean stiffness-to-weight ratio against the mean number of holes for each of the nine candidate fitness functions. Note that in Figure 5.25, fitness functions 3 and 9 are located in the area representing plate topologies combining high stiffness-to-weight ratio with low porosity.

Casual examination of this example's experimental data, provided in Figure 5.26 (which plots stiffness-to-weight ratio vs. number of internal holes data from all 540 optimization runs), suggests that a plate's stiffness-to-weight ratio may increase as its porosity increases. This relationship is also suggested by Bendsøe *et al.* (1993), who, using a homogenization-based technique (see Section 3.2.1 for details of the technique), find that plate topologies containing large areas of intermediate-density (*i.e.*, highly-porous) material (obtained using a Rank-2 microstructure model) are slightly stiffer than

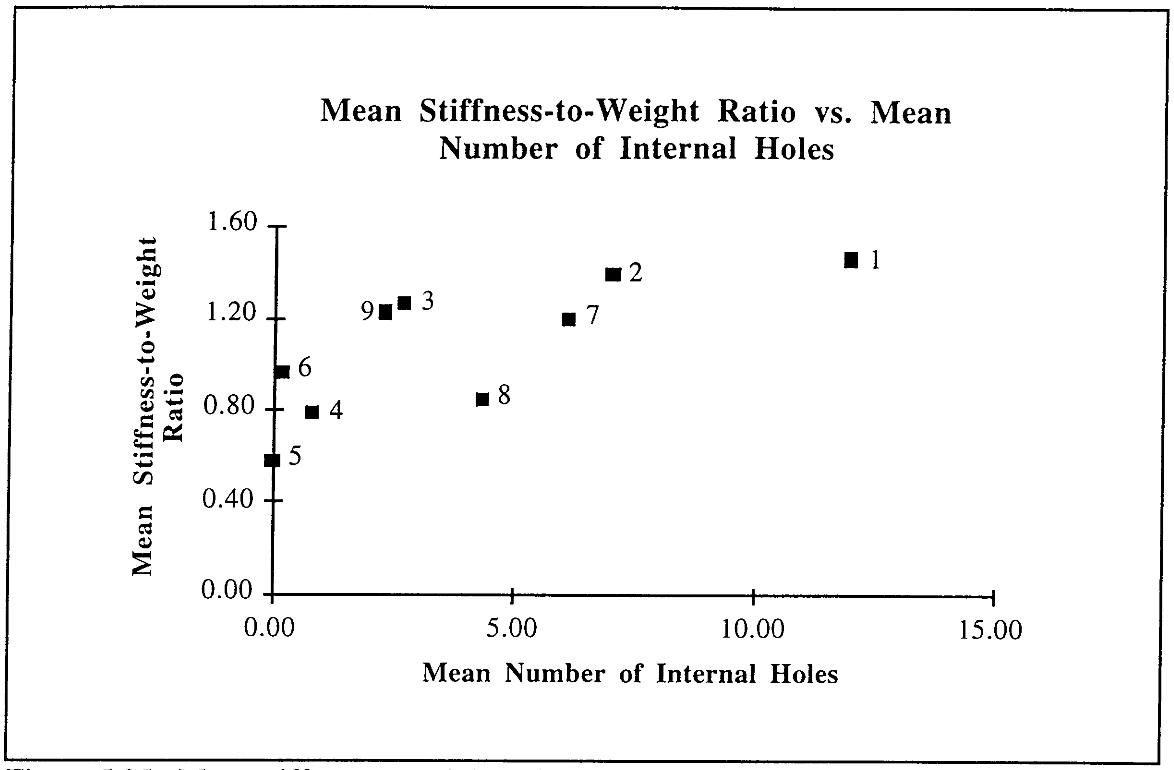

Figure 5.25: Mean stiffness-to-weight ratio vs. mean number of internal holes.

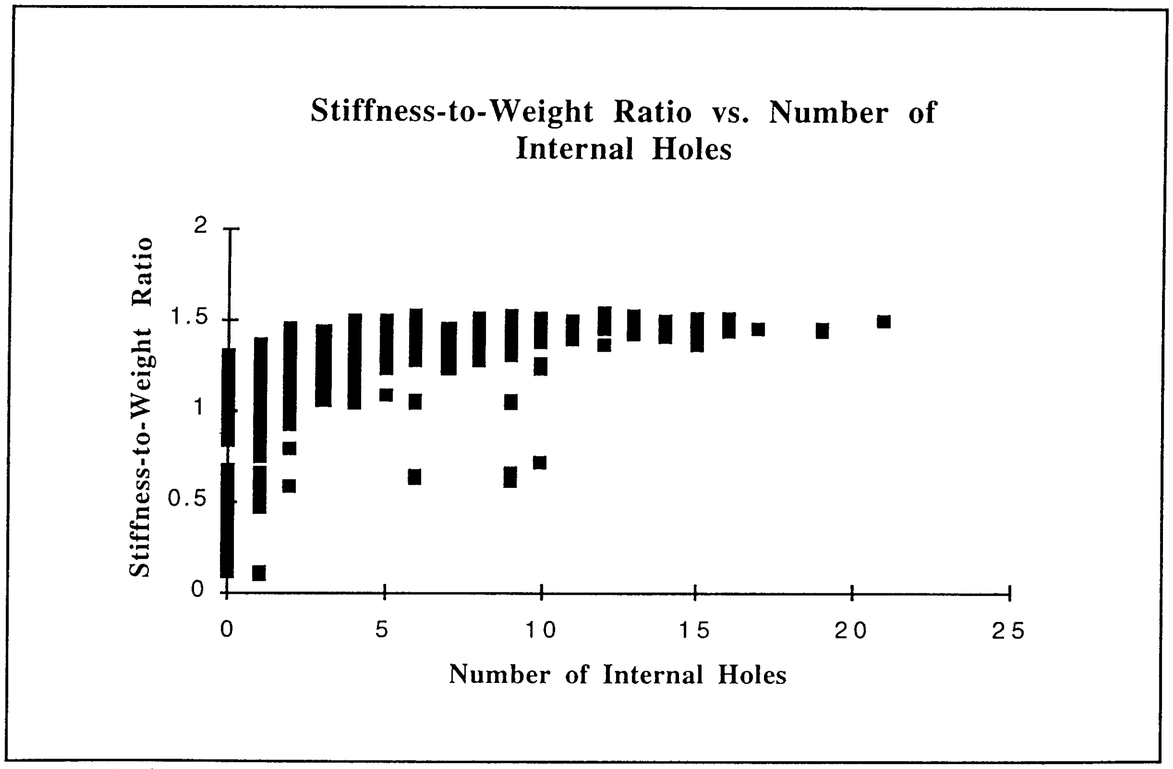

Figure 5.26: Stiffness-to-weight ratio vs. number of internal holes. Note that this graph contains data from all 540 optimization runs.

topologies with well-defined, truss-like members and small numbers of large, internal holes (obtained using a rectangular hole microstructure model). However, an analysis of variance performed on this investigation's experimental data indicates that little correspondence exists. As shown in Table 5.5, the only statistically-significant relationship which can be inferred from the data is that plate topologies with zero internal holes have lower stiffness-to-weight ratios than structures with two or more holes.

Number of Internal Holes	# of topologies containing the number of holes	Mean Stiffness-to-Weight Ratio of optimal topologies	Bonferroni Grouping *Means with the same letter are not significantly different*	
21	1	1.4793	A	
16	2	1.4654	A	
12	4	1.4572	A	
14	5	1.4564	A	
13	6	1.4527	A	
17	1	1.4467	A	
11	10	1.4396	A	
15	8	1.4356	A	
19	1	1.4290	A	
8	15	1.3905	A	
7	9	1.3491	A	
10	13	1.3435	A	
5	18	1.3312	A	
6	16	1.3271	A	
9	16	1.3075	A	
4	67	1.2974	A	
3	77	1.2580	A	
2	103	1.2185	A	
1	101	1.0711	A	B
0	67	0.6763		B

Table 5.5: Correspondence between porosity and stiffness-to-weight ratio.

Examining the results in Tables 5.3 and 5.4, it is interesting to note that fitness function 1, the standard stiffness-to-weight ratio maximization fitness function used in previous examples, provides stiffness-to-weight ratio performance statistically-equal to that of several other candidate fitness functions, while it obtains structures of statistically-higher porosity than any other fitness function. Given that other fitness functions in this study find low-porosity structure topologies with stiffness-to-weight ratios equal to those of the high-porosity topologies generated by fitness function 1, why does the standard stiffness-to-weight ratio maximization fitness function generate only highly-porous structures?

Several factors may be responsible for fitness function 1's natural tendency towards porous plate topologies. One possible cause is that the structure topology with globally-optimal stiffness-to-weight ratio is indeed highly-porous, but optimizations using fitness function 1 have not been allowed to evolve long enough (*i.e.*, for a sufficient number of generations) to converge to the globally-optimal structure topology (and have instead generated highly-porous structure topologies which exhibit stiffness-to-weight ratios no better than those of low-porosity structures obtained by other fitness functions). Another possibility is that fitness function 1 *does* consistently generate plate topologies with higher stiffness-to-weight ratios than other fitness functions, but additional experiment replications are required to establish statistically-significant differences. Finally, the exceptionally large search space resulting from this example's finely-discretized design domain may be responsible for the natural tendency towards porous structures—while fitness function 1 found plate topologies with high stiffness-to-weight ratio and low porosity when using relatively small search spaces in Examples 2 and 3, it may be unable to efficiently search larger search spaces for high stiffness-to-weight ratio, low-porosity plate topologies. Note that simply increasing the genetic algorithm population size and the number of generations performed during search would likely improve the genetic algorithm's abilities in finding high stiffness-to-weight ratio, low-porosity plate topologies.

When examining the results and conclusions provided by this example, it is important to note that unlike homogenization-based techniques, which are based on deterministic principles, the genetic algorithm is based on probabilistic principles. Hence, it is impossible to determine whether a structure generated by the genetic algorithm is indeed locally- or globally-optimal, and it is also impossible to predict, using either mathematical techniques or results from a single genetic algorithm optimization run, the nature of the optimal structure topology. Consequently, this example's conclusions that fitness functions 3 and 9 generate structures combining highest stiffness-to-weight ratio with lowest porosity, and that little relationship exists between a structure's porosity and its stiffness-to-weight ratio, have little rigorous mathematical basis and are based entirely on the experimental results of the 540 optimization runs. However, statistical analyses of this example's experimental results do, with a high level of confidence, substantiate these conclusions.

Note also that the results of this study, as well as the conclusions which this example makes based on the results, could possibly change if different genetic algorithm population sizes, varied crossover and mutation probabilities, new fitness functions, coarser or finer design domain discretizations, or different design domains and boundary conditions were used. In fact, this example clearly demonstrates that the fitness function used has a great effect on the type of structure obtained. However, additional experiment replications would, given the genetic algorithm's probabilistic nature, produce slightly different experimental results even if all of the above parameters were held *constant*.

So, the genetic algorithm's probabilistic nature prevents the development of mathematically-rigorous convergence criteria or error bounds (and investigators must therefore use statistical analysis to model the genetic algorithm's behavior and capabilities), and the particular parameter values (*i.e.*, population size, probabilities of crossover and mutation, fitness function formulation, design representation, etc.) used in a genetic algorithm search have a great effect on the optimization's outcome. However, while this example's findings could indeed change if other parameter values were used, statistical analysis nonetheless shows with a high level of confidence that, given the set of parameter values used in this experiment, little correspondence exists between porosity and stiffness-to-weight ratio. Consequently, this study presents a strong counterexample to the notion that porosity and structural performance (here stiffness-to-weight ratio) are related.

However, it is not this example's intent to make general conclusions regarding the nature of optimal structure topologies. Instead, this example simply attempts to provide insight into the behavior exhibited by the genetic algorithm in Examples 4 and 5. Specifically, given the:

- design domain representation, geometry, and discretization
- boundary conditions
- genetic algorithm parameters

used in Examples 4 and 5 (which generated highly-porous optimal structure topologies), this example attempts to determine:

- if the genetic algorithm can generate structure topologies combining high stiffness-to-weight ratio with low porosity.
- if highly-porous structures indeed have higher stiffness-to-weight ratios than low-porosity structures (*i.e.*, what, if any, relationship exists between stiffness-to-weight ratio and porosity).
- why the genetic algorithm was unable to generate low-porosity, high-stiffness-to-weight-ratio structures in Examples 4 and 5.

This example was also investigated with the hope that it might demonstrate both the varied, probabilistic behavior of the genetic algorithm as well as the *ad hoc* nature of genetic algorithm fitness function design. This example also hoped to demonstrate that vastly different fitness functions can be used in an attempt to achieve a given objective (in this case, maximize stiffness-to-weight ratio while minimizing porosity), and that these different fitness functions typically provide varying levels of optimization performance.

5.9 Example 8: Comparison with Homogenization-Based Techniques.

5.9.1 Introduction. Previous examples in this investigation used the genetic algorithm to optimize the topologies of cantilevered plates with design domain geometries and discretizations, as well as boundary and loading conditions, similar to those examined using homogenization-based techniques (Papalambros and Chirehdast (1990), Suzuki and Kikuchi (1990)). Unfortunately, no quantitative comparisons could be made between the two methods because this investigation's genetic algorithm-based technique maximizes a plate's stiffness-to-weight ratio while homogenization-based techniques typically minimize a structure's mean compliance subject to a maximum volume constraint.

This example, by using the genetic algorithm to minimize a plate's mean compliance subject to a maximum volume constraint, provides a quantitative comparison between this investigation's genetic algorithm-based approach and homogenization-based methods. Specifically, this example evaluates the optimization abilities of the genetic algorithm, a global search technique which typically avoids local optima, in relation to those of the optimality criteria methods used in homogenization-based methods, which can be expected to converge to local extrema (Bendsøe *et al.*, 1993).

Hence, as in Examples 4, 5, 6, and 7, this example optimizes the topology of a rectangular, cantilevered plate subject to a downward vertical load. No symmetry constraints are imposed on the topology, and the constant meshing technique and hierarchical subdivision are used to facilitate the efficient generation of finely-discretized plate topologies. However, unlike previous examples which attempted to maximize a plate's stiffness-to-weight ratio, this example minimizes a plate's mean compliance subject to a maximum volume constraint.

5.9.2 Design Domain. The cantilevered plate is modeled using a rectangular design domain 10 units tall by 16 units wide (Figure 5.27), and no symmetry constraints are imposed on the plate's topology. The design domain is initially discretized into a

10×16 grid of binary, material-void elements, and subsequent hierarchical subdivision increases the design domain resolution to a 20×32 discretization. Hence, as the initial design domain discretization (10×16) establishes the genetic algorithm chromosome length, all chromosomes used in the optimization (regardless of the design domain discretization or number of genetic algorithm populations controlling the material-void distribution within the design domain) contain 160 genes. To allow for the application of the vertical concentrated load, the two design domain elements on the plate's right-hand surface immediately adjacent to the point of load application (which is in the middle of the plate's right-hand surface) serve as seed elements. Likewise, two seed elements corresponding to the points of support are placed at the top and bottom of the plate's left-hand surface.

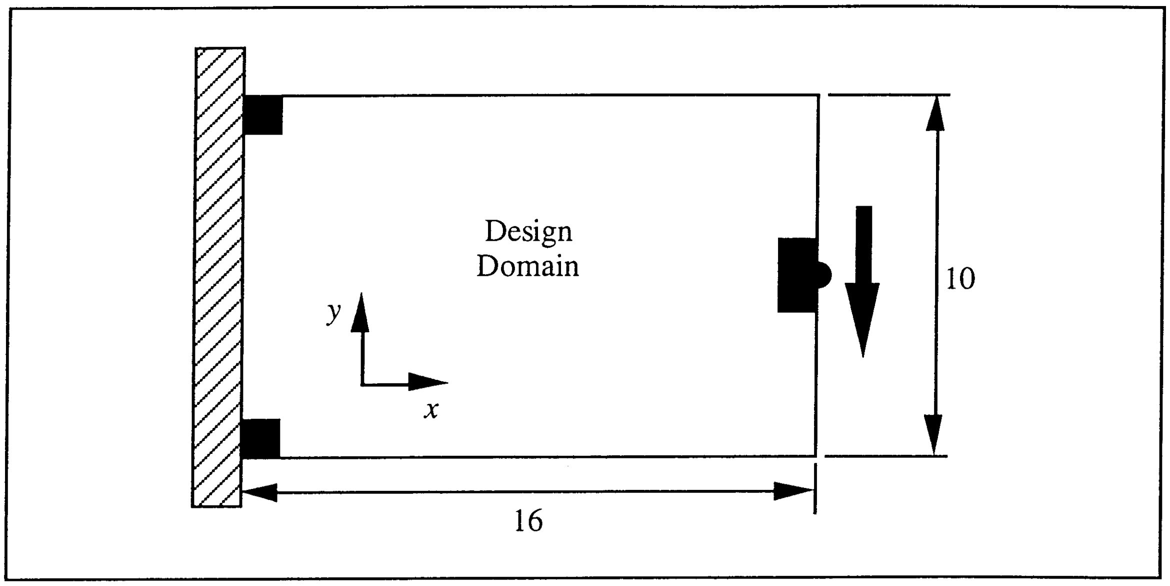

Figure 5.27: Example 8 design domain.

5.9.3 Fitness Calculations. As this example uses the hierarchical subdivision technique to minimize the plate's compliance subject to a maximum volume constraint, fitness calculations begin when a chromosome is converted into its corresponding plate topology as detailed in Section 5.6.3. Then, once the plate topology has been generated, connectivity analysis, constant finite element meshing (using four triangular finite elements for every design domain element), and finite element analysis are performed. Note that during finite element analysis, *all* nodes along the finite element mesh's left-hand surface are constrained to have zero displacement (as opposed to all previous examples in this investigation, which constrained only the four nodes corresponding to the two seed elements), while the concentrated load is applied to the middle node along the finite element mesh's right-hand surface. Finally, fitness calculations are performed as detailed in Section 4.4.6. Again, chromosomes corresponding to plate topologies which exhibit minimum mean compliance and satisfy the maximum volume constraint are assigned high fitness, while chromosomes corresponding to plate topologies which exhibit large mean compliance and/or violate the maximum volume constraint are assigned low fitness. Hence, the fitness of a plate topology (and therefore the fitness, to be maximized, of the chromosome corresponding to the plate topology) which satisfies the maximum volume constraint is given by:

$$Fitness = \frac{1}{\ln(Compliance)}, \tag{4.9}$$

where *Compliance* represents the mean compliance of the plate topology. As detailed in Section 4.4.6, a plate's mean compliance is equal to the applied load vector $(\mathbf{F}_{APPLIED})$ multiplied by the plate's displacement at the point of load application $(\mathbf{u}^{\mathrm{T}})$:

$$Compliance = \mathbf{u}^{\mathrm{T}} \cdot \mathbf{F}_{APPLIED}, \tag{4.6}$$

where

$$\mathbf{u}^{\mathrm{T}} = \begin{bmatrix} u_x & u_y \end{bmatrix} \tag{4.7}$$

and

$$\mathbf{F}_{APPLIED} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \tag{4.8}$$

Again, F_x and F_y represent the magnitude, in the x- and y-directions, of the single concentrated load acting on the plate while u_x and u_y represent the displacement, in the x- and y- directions, of the finite element node where the concentrated load is applied.

Then, if the plate topology violates the maximum volume constraint (as detailed in Section 4.4.6, these fitness calculations use the area of connected material in a plate topology as a qualitative measure of the plate's volume, and the maximum volume constraint is therefore actually a maximum area constraint), the above fitness function is penalized by 6% for every 10% volume constraint violation. Note that this fitness penalty is linearly attenuated according to the current generation number, with no penalty at generation 0 and full (6%) penalty at generation 175. Hence, prior to generation 175, the fitness of a chromosome corresponding to a structure topology violating the maximum volume constraint is given by:

$$Fitness = \left\{1.0 - \left[\frac{generation}{175} \cdot \frac{0.06}{0.10} \cdot \left(\frac{V - V_{MAX}}{V_{MAX}}\right)\right]\right\} \frac{1}{\ln(Compliance)}, \tag{4.10}$$

where *generation* represents the current generation of genetic algorithm evolution, V represents the volume (*i.e.*, the area of connected material elements) of the current plate topology, and V_{MAX} represents the maximum allowable volume of the plate. After generation 175, the fitness of a chromosome corresponding to a plate topology violating the maximum volume constraint is given by:

$$Fitness = \left\{1.0 - \left[\frac{0.06}{0.10} \cdot \left(\frac{V - V_{MAX}}{V_{MAX}}\right)\right]\right\} \frac{1}{\ln(Compliance)}. \tag{4.11}$$

Again, chromosomes corresponding to plate topologies which *do not* violate the maximum volume constraint are assigned the fitness value given by Equation 4.9.

During fitness calculations, a maximum volume constraint of 25 percent is used. Again, because these fitness calculations use the area of connected material in a plate topology as a qualitative measure of the plate's volume, the maximum volume constraint is actually a maximum area constraint. Hence, because the design domain has an area of 160 units^2, the maximum volume constraint stipulates that the maximum allowable area of connected material elements in a plate topology is 40 units^2.

5.9.4 Hierarchical Subdivision Parameters. Optimization begins with a single genetic algorithm population optimizing the plate topology using the coarse, 10×16 design domain discretization. Note that this population contains 30 160-gene chromosomes. After performing 225 generations of evolution at this initial discretization, hierarchical subdivision (with a nuking probability of 0.3) is performed to quadruple the resolution of the design domain. Hence, following hierarchical subdivision, four genetic algorithm populations optimize the plate's topology using a 20×32 design domain discretization. At this discretization, each design domain element has $\frac{1}{4}$ the area of the original, coarse design domain elements, and each of the four populations contains 30 160-gene chromosomes. Optimization then concludes after 150 cycles (*i.e.*, 150 generations of evolution in each design domain quadrant) have been performed at this 20×32 design domain discretization.

Best optimization results were obtained when the maximum volume constraint is relaxed during early stages of the search. Specifically, when using the coarse design domain discretization, the maximum allowable volume fraction is set to 1.5 times the desired 25 percent, or 37.5 percent. Then, after performing hierarchical subdivision and switching to the finely-discretized design domain, the maximum allowable volume fraction is returned to the desired 25 percent.

Hence, as the maximum volume constraint is actually a maximum area constraint, the initial 37.5 percent maximum volume constraint corresponds to a maximum allowable plate area of 60 units^2 $(.375 \cdot 160 = 60)$. Because this initial area constraint is used with the coarse design domain discretization, where the 10 unit tall by 16 unit wide design domain is discretized into a 10×16 grid of elements and each element therefore has an area of 1 unit^2, the maximum allowable plate area of 60 units^2 corresponds to a maximum of 60 design domain elements containing material. Then, when hierarchical subdivision is performed and the maximum allowable volume fraction is returned to its desired value of 25 percent, the maximum allowable plate area becomes 40 units^2 $(0.25 \cdot 160 = 40)$. Note that this maximum area constraint of 40 units^2 corresponds to a maximum of 160 design domain elements containing material, for the 10 unit tall by 16 unit wide design domain is discretized into a 20×32 grid of elements, where each element has an area of $\frac{1}{4}$ units^2.

5.9.5 Results. The result of the genetic algorithm-based topological optimization is shown in Figure 5.28, while the homogenization-based solution to the problem is shown in Figure 5.29 (Rodrigues, 1993). 6,750 structural analyses were required to generate an optimal coarse, 10×16 plate topology:

$$225 \; Generations \cdot 30 \frac{Chromosomes}{Generation} = 6{,}750 \; Chromosomes, \tag{5.19}$$

while an additional 18,000 structural analyses were performed to obtain the final, 20×32 plate topology:

$$150\ Cycles \cdot 4\frac{Generations}{Cycle} \cdot 30\frac{Chromosomes}{Generation} = 18{,}000\ Chromosomes. \qquad (5.20)$$

Hence, generating the final, 20×32 plate topology required a total of 24,750 finite element analyses.

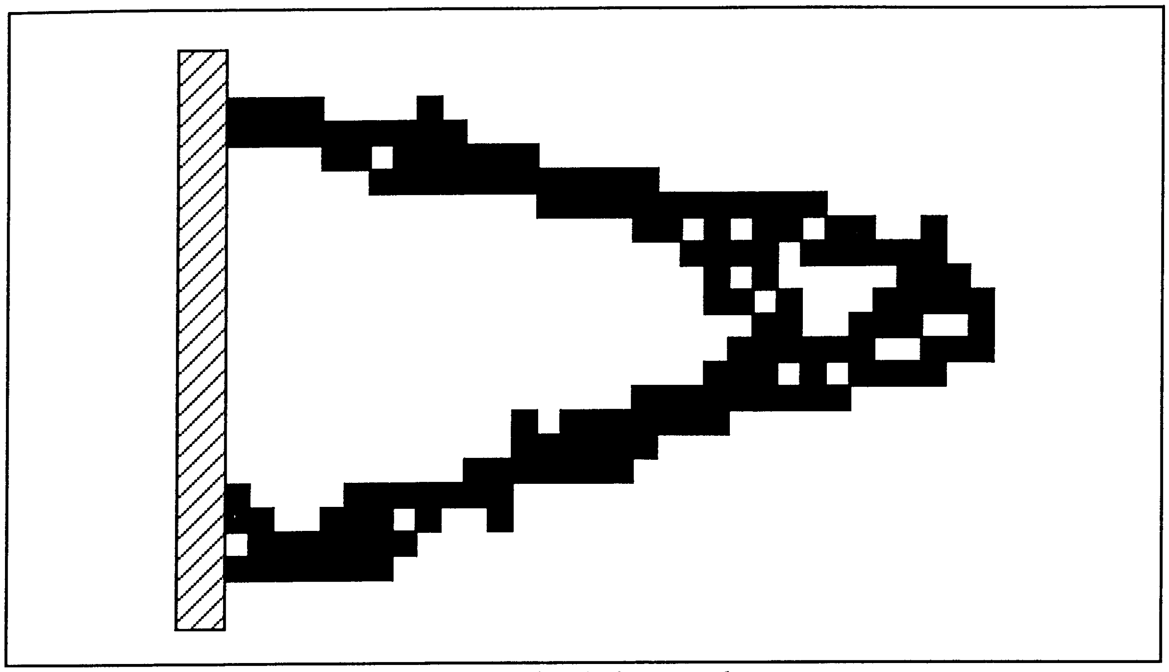
Figure 5.28: Genetic algorithm-based optimal plate topology.

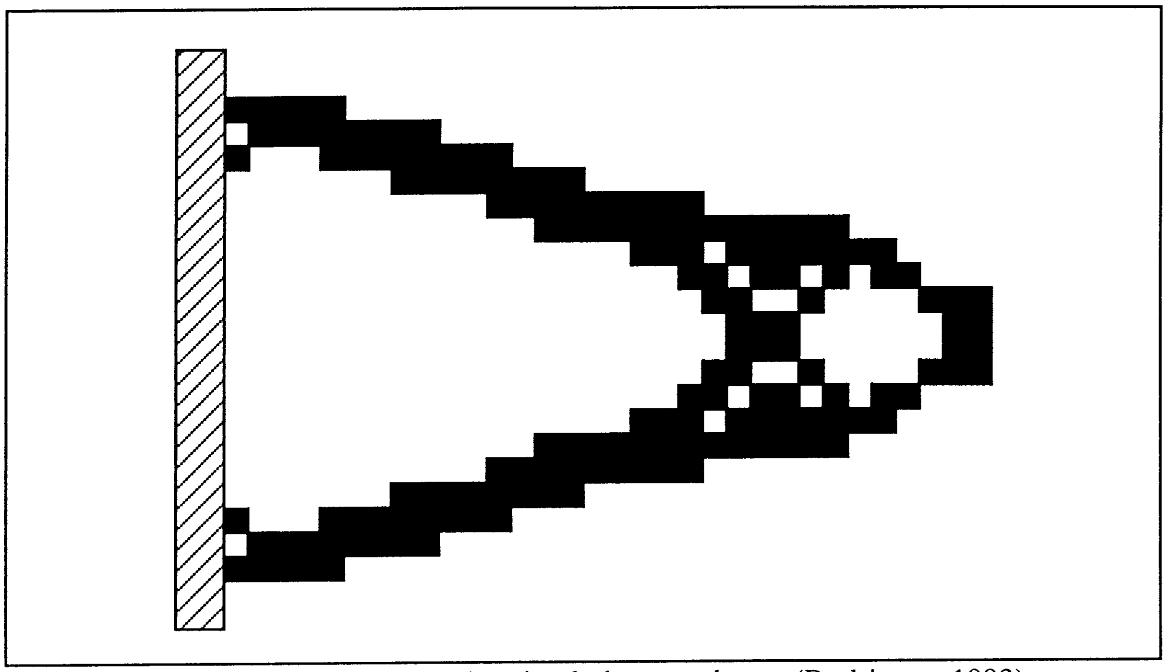
Figure 5.29: Homogenization-based optimal plate topology. (Rodrigues, 1993).

Again, because of the probabilistic nature of the genetic algorithm, every execution of this investigation's genetic algorithm-based structural topology optimization approach results in the generation of a unique plate topology. Hence, this investigation's genetic algorithm-based approach was executed many times in an attempt to generate a structure exhibiting mean compliance lower than that exhibited by the structure obtained through homogenization-based methods (Figure 5.29). Of the many structures which were generated during this investigation's examination of compliance minimization, Figure 5.28 depicts the lowest-compliance plate topology which the genetic algorithm was able to generate.

As shown in Figure 5.28, the genetic algorithm, when used with hierarchical subdivision, provides reasonable results when attempting to minimize a plate's mean compliance subject to a maximum volume constraint. While the optimal plate topology generated using this investigation's genetic algorithm-based topology optimization approach does not exactly correspond to the plate topology obtained using homogenization-based methods, the overall shapes of the two designs are quite similar. Compared with the homogenization-based solution, this investigation's genetic algorithm-based solution contains 3% less material and exhibits 12% greater mean compliance. This small difference in material and large difference in mean compliance demonstrate that mean compliance is highly sensitive to the amount and distribution of material in a structure.

While this investigation's genetic algorithm-based structural topology optimization approach has considerably greater computational expense than homogenization-based techniques, the genetic algorithm's probabilistic nature does offer an advantage over the deterministic techniques used in homogenization-based optimizations. As detailed in Example 6, examining the entire population resulting from a single genetic algorithm optimization or executing a genetic algorithm optimization multiple times provides a designer with a family of possibly-optimal plate topologies. In the context of this example, each design in a family will exhibit approximately the same compliance (Figure 5.30 shows that mean compliance values vary by approximately 15 percent) as the other topologies, but will have a unique geometry and topology. The designer can then evaluate the different designs to determine which best satisfies several secondary performance criteria, much like a pareto optimization study. An example design family is provided in Figure 5.30, which depicts several plate topologies obtained during other executions of this example's genetic algorithm-based compliance minimization routines. Note that percentages given in Figure 5.30 represent each plate topology's variation from the homogenization-based result.

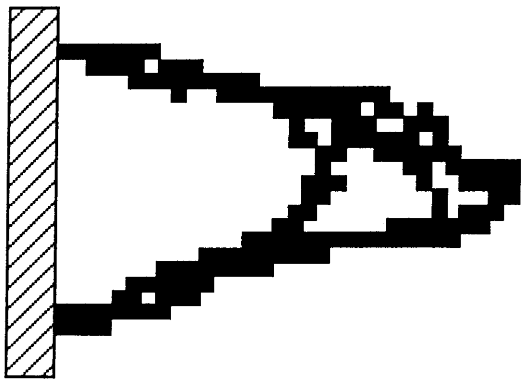
Figure 5.30a.
2.9% less material
14.6% greater compliance

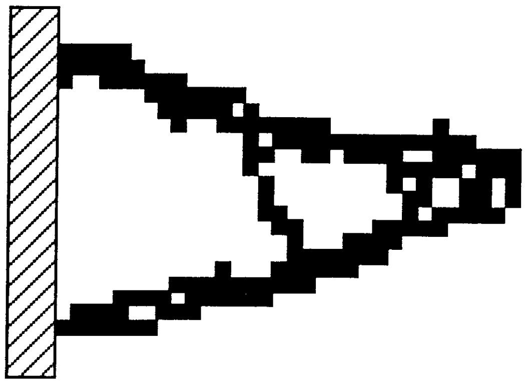
Figure 5.30b.
3.5% less material
18.5% greater compliance

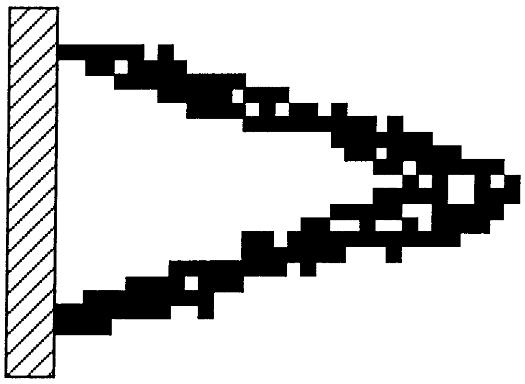
Figure 5.30c
2.9% less material
13.4% greater compliance

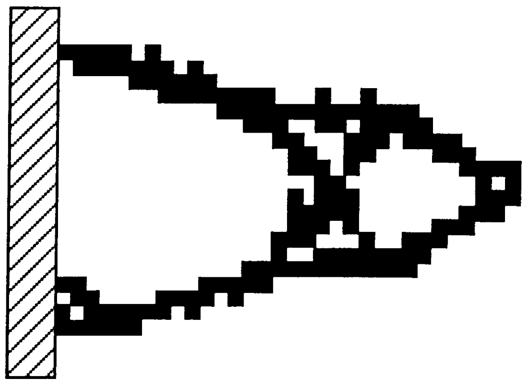
Figure 5.30d
2.9% less material
20.6% greater compliance

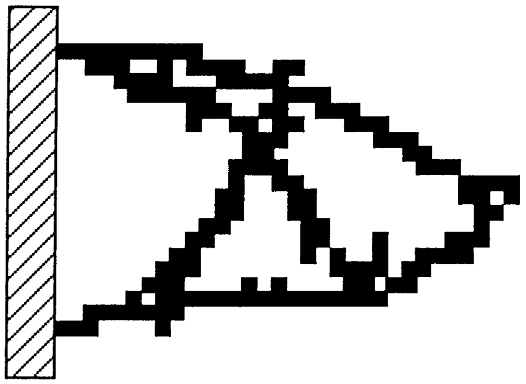
Figure 5.30e
2.9% less material
32.4% greater compliance

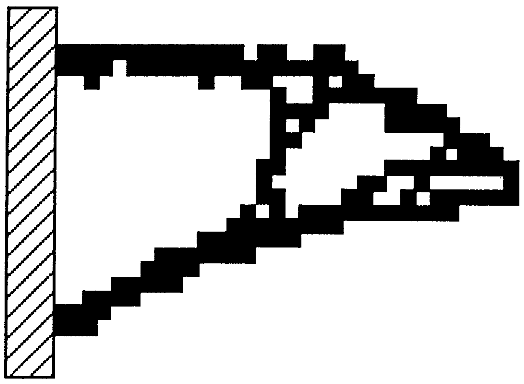
Figure 5.30f
3.5% less material
25.0% greater compliance

Figure 5.30: Family of minimum mean compliance plate topologies.

Conclusions 6

6.1 Overview

This chapter first overviews the contributions of this investigation, and then provides conclusions regarding the abilities and limitations of the genetic algorithm-based structural topology optimization technique developed in this investigation. Finally, potential areas of future work are offered.

6.2 Contributions of This Investigation

This investigation applied the genetic algorithm, a search and optimization technique based on the theory of natural selection, to the structural topology optimization of continuum structures, where the optimal distribution of material within a discretized design domain was found. Significantly extending previous work in this area, this investigation focused on, and examined in great detail, the genetic algorithm-based structural topology optimization of cantilevered plates using a binary, material-void design domain representation. The technique developed in this investigation was utilized to generate the optimal topologies of a variety of cantilevered plates, which were optimized based upon stiffness, mean compliance, weight, and manufacturability considerations.

As a preliminary verification of the abilities of this investigation's technique, the unconstrained topological optimization of a beam cross-section was examined. After many generations of search, the genetic algorithm generated a cross-section topology with material concentrated in locations furthest away from the neutral axis. This was the expected result. Then, to establish the technique's constrained optimization abilities, the beam cross-section optimization was re-examined using a maximum stress constraint.

With the constraint, the genetic algorithm generated optimal cross-section topologies containing additional material in the flanges, which again is the expected result.

The genetic algorithm was then used in conjunction with finite element analysis to optimize the topology of a small cantilevered plate in tension. This also yielded the predicted results.

As the adaptive finite element meshing technique used in the small cantilevered plate optimization proved to be computationally expensive, a constant finite element meshing technique was investigated. When used with connectivity analysis, the constant meshing technique was shown to generate, with less computational expense, plate topologies exhibiting structural performance equal to that of structures obtained using the adaptive finite element meshing technique.

The topological optimization of a finely-discretized, cantilevered plate was then examined to determine if this investigation's genetic algorithm-based structural topology optimization technique could perform optimization using design domain discretizations approaching those used by homogenization-based methods. Satisfactory results were obtained.

In an attempt to enhance the genetic algorithm's optimization abilities and efficiency when using finely-discretized design domains, a hierarchical subdivision technique was introduced. Compared to previous examples which used a constant design domain discretization, hierarchical subdivision was able to produce highly-fit plate topologies of equal discretization while requiring fewer structural analyses. The hierarchical subdivision technique also enabled the genetic algorithm to generate plate topologies of much higher discretization than that possible with a constant design domain discretization.

The genetic algorithm's ability to obtain, during a single optimization, a family of plate topologies which a designer can evaluate using alternate, secondary criteria (*e.g.*, manufacturability, weight, displacement, etc.) was then demonstrated. To obtain families of designs when using traditional optimization methods such as mathematical programming or optimality criteria techniques, optimization must be performed multiple times using a variety of initial conditions, with the hope that the search will converge to different plate topologies.

As many of the plate topologies generated in these examples contained large numbers of internal holes, which are undesireable from a manufacturing standpoint, an experiment was then conducted to determine if the genetic algorithm could generate plate topologies with high stiffness-to-weight ratio and low porosity. A group of high-stiffness-to-weight ratio, low-porosity structures was obtained. Results of the experiment were then used to establish that little statistically-significant correspondence exists between a structure's porosity and its stiffness. Possible reasons as to why previous examples only generated highly-porous plate topologies were also discussed.

To provide a quantitative comparison between the optimization capabilities of this investigation's genetic algorithm-based approach and those of homogenization-based methods, the genetic algorithm was used to minimize a plate's mean compliance subject to a maximum volume constraint. While the optimal plate topology generated using this investigation's approach did not exactly correspond to the homogenization-based solution, the overall shapes of the two designs were quite similar. Compared with the homogenization-based solution, this investigation's genetic algorithm-based solution contained less material but exhibited greater mean compliance.

These examples significantly focused and extended the previous work of Jensen and associated researchers (Section 3.3.4). Whereas Jensen's research examines the genetic algorithm-based topological optimization of a variety of structure types using a variety of design domain representations, this investigation emphasized the topological optimization of cantilevered plates using a binary, material-void design domain representation. Specifically, this investigation extended Jensen's work by examining the following:

- Design domain discretizations much finer than those used by Jensen, as well as hierarchical subdivision and constant finite element meshing techniques which improved the genetic algorithm's optimization abilities and efficiency when using finely-discretized design domains. While Jensen used a meshing technique similar to that used in this investigation, this investigation's introduction of hierarchical subdivision was a significant extension and enhancement of Jensen's constant design domain discretizations.

- A connectivity analysis technique which deterministically removed disconnected material elements from cantilevered plate topologies. This connectivity analysis provided a guarantee of structural stability—no "chain-link" type structures (which are unable to withstand any torque or compressive load) were allowed by the analysis.

- Fitness functions for unconstrained optimization problems which were based on a structure's strength-to-weight or stiffness-to-weight ratio. Unlike the fitness functions used by Jensen, which involve a combination of weight minimization and constraint violation terms, the fitness functions introduced in this investigation do not require carefully-chosen penalty terms and therefore attempt to directly influence the structure's topology above all else. These fitness functions also provided adequate performance when used in constrained optimization problems.

- The genetic algorithm's ability to generate, in a single optimization run, a family of highly-fit cantilevered plate topologies which can then be compared using alternate, secondary criteria. Jensen examines only those chromosomes which maximize a given fitness function.

- Methods for driving the genetic algorithm search towards plate topologies combining high stiffness-to-weight ratio with high manufacturability. In his examples, Jensen considers only weight, stress, and displacement.

- A direct, quantitative comparison between the genetic algorithm's structural topology optimization abilities and those of homogenization-based methods, which are currently the most widely-used approach to structural topology optimization. Jensen provides no such comparison.

6.3 Conclusions

This investigation's genetic algorithm-based structural topology optimization approach was able to generate structure topologies of relatively high quality and discretization. When examining the "optimal" structures generated using this investigation's approach, it is evident that many of the structures would require modification before they could be considered optimal or manufacturable. For example, many structures are unsymmetric when they should be symmetric, and other structures contain a great number of "stray" material elements which, while connected to the structure, do little to support the applied load. Hence, the performance of these structures would likely improve if they were symmetric or did not contain stray material elements (or the stray material was moved to other locations within the structure). Unfortunately, because genetic algorithm search is probabilistic rather than deterministic in nature and does not utilize sensitivity information (such as function gradients), the genetic algorithm cannot determine whether an optimum has been reached and therefore cannot "fine-tune" a structure in an attempt to generate an *exact* optimum. Hence, genetic algorithm search typically produces a "pseudo-optimum," which is similar, but not exactly equal, to the truly-optimal structure topology. Conversely, the optimality criteria optimization methods used in homogenization-based techniques use sensitivity information to determine if an exact optimum has been generated, and search is not concluded until a true optimum is found. Hence, the structures obtained using homogenization-based techniques are symmetric when they should be, and they typically do not contain stray material elements. Note, however, that while the genetic algorithm's probabilistic nature typically prevents the generation of exact optima, it does allow the genetic algorithm to search in discrete, discontinuous, multi-modal search spaces which would be troublesome to mathematical programming and optimality criteria techniques.

Additionally, most of the structures generated using this investigation's genetic algorithm-based structural topology optimization technique were of lower discretization than structures typically obtained using homogenization-based methods. This is a direct result of the genetic algorithm's extreme computational expense. Specifically, because the genetic algorithm must evaluate the fitness of every chromosome in every generation of a search, structural topology optimizations in this investigation required thousands of finite element analyses. Hence, as increases in design domain discretization result in an increase in the computational expense of each finite element analysis, using design domain discretizations approaching those used by homogenization-based methods would make an already-computationally-expensive optimization prohibitively expensive. Consequently, structures subject to genetic algorithm-based structural topology optimization must currently be of limited size and complexity. Because the optimality criteria methods used in homogenization-based structural topology optimization approaches require many fewer analyses and provide good performance independent of the number of design variables, homogenization-based techniques can be applied to structures of exceptionally high discretization. Note that while the genetic algorithm's populations of chromosomes and probabilistic nature do result in great computational expense, they enable the genetic algorithm to search in discrete, discontinuous, multi-modal search spaces (which again would be troublesome to optimality criteria methods) and they allow the genetic algorithm to automatically generate families of structures which the designer can evaluate using secondary criteria.

Finally, many of the plate topologies generated during this investigation contained a large number of internal holes, which could lead to difficulties when parameterizing the topology for sizing and shape analysis. However, as demonstrated in this investigation, simple modifications to the fitness function can eliminate many of these internal holes. Note that most topologies found using homogenization-based techniques will also lead to interpretation difficulties, as they generally contain material of intermediate density. Hence, when parameterizing topologies obtained with homogenization-based techniques (particularly those topologies obtained using Rank-2 microstructure models), the designer must select a density threshold. The density threshold chosen likely has a great effect on the resultant topology. While the topologies obtained using this investigation's approach are generally porous, they contain only material and void—no density threshold is needed during parameterization.

Indeed, the motivation for using the genetic algorithm to perform structural topology optimization is not an enhanced ability to find exact optima or an increase in computational speed, but an increase in simplicity and generality. Unlike most other optimization algorithms, the genetic algorithm can be applied to a wide variety of (possibly ill-behaved) problem domains simply by changing the chromosome-to-design-variable mappings and fitness function procedures. In this investigation, the genetic algorithm was easily applied to the domain of structural topology optimization and was able to perform admirably.

6.4 Future Work

Future investigations of genetic algorithm-based structural topology optimization must focus on decreasing the computational expense of this approach. For example, while increasing the genetic algorithm population size or the number of generations performed in a search would likely allow the genetic algorithm to generate structure topologies of higher quality than those found in this investigation, either of these changes would increase the number of structural analyses required by the genetic algorithm and would therefore make the genetic algorithm search prohibitively expensive computationally. Whereas the computational expense of the actual genetic algorithm routines is relatively low and would be difficult to reduce, structural analysis is the major source of this approach's computational expense. Possible enhancements include a parallel implementation of this genetic algorithm-based structural topology optimization approach, where multiple processors would perform finite element analysis on the population of structure topologies. Approximate fitness calculations could also be used in an attempt to reduce this approach's computational complexity.

While this investigation used one-dimensional, binary-string genetic algorithm chromosomes to represent structure topologies, several other chromosome representations might provide better chromosome-to-design-domain correspondence. For example, when using a constant design domain discretization, two-dimensional chromosomes (*i.e.*, arrays of binary digits) would provide an intuitive 2D-chromosome-to-2D-design-domain correspondence. Likewise, when using hierarchical subdivision, where each element in an initial, coarse design domain is periodically subdivided into smaller and smaller elements, a quadtree chromosome structure would allow the genetic algorithm to adaptively increase design domain discretization in those regions of the domain requiring fine discretization, while regions of the domain which do not require fine discretization would remain at the original, coarse discretization.

Finally, this investigation demonstrated the great effect which fitness calculations have on the results of a genetic algorithm search—slight modifications in penalty parameter value or fitness formulation can drive the genetic algorithm to vastly different "optimal" structure topologies. To eliminate this great dependency on the fitness function designer's intuition, an implicit fitness function could be used. Instead of the technique used in this investigation, where the fitness function is defined *a priori* and the genetic algorithm chromosomes are then evaluated using the time-invariant fitness function, an implicit fitness function would itself evolve during optimization until both an optimal fitness function and structure topology are found. This would minimize the *ad hoc* nature of the fitness function development process and would likely enhance the genetic algorithm's abilities in finding highly-fit structure topologies.

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