An Investigation of Efficient Receiver Structures for CDMA Communication

by

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Abstract

Efficient allocation of the communication resources such as frequency bandwidth is an important problem in multiple access technique. In this thesis, we explore the potential of code-division multiple-access (CDMA) as an efficient resource-sharing technique. We explore recently proposed receivers and develop a unified vector space interpretation. Bit error rates of each receiver in the simple two user case are developed and compared.

The decorrelating detector is subject to a capacity decrease when the number of active interfering users is small compared to the number of potential users. We propose an algorithm to modify the decorrelating detector so that a capacity increase is achieved when the signal-to-noise ratios of interfering users are very high. We analyze the algorithm in a simple case and provide results of Monte-Carlo simulation to demonstrate the viability of the algorithm.

Thesis Supervisor: Gregory W. Wornell
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Chapter 0

Notations

In this thesis, many equations are written in terms of matrices and vectors. We will exclusively use boldface letters for these matrices and vectors.

- A boldface capital letter denotes a matrix. $\mathbf{P}, \mathbf{\Theta}, \cdots$

- A boldface lowercase letter denoted a column vector. $\mathbf{b}, \mathbf{p}, \cdots$

In addition, many subscripts are used in the thesis. To avoid any confusion which may be caused by multiple subscripts, we will use braces for elements of matrices and vectors.

- The (i, j) element of a matrix $\mathbf{A}$ will be denoted as $A\{i, j\}$.

- The i th element of a vector $\mathbf{b}$ will be denoted as $b\{i\}$.

Chapter 1

Introduction

As demand for a wide range communication services is growing rapidly, efficient allocation of limited communication resources such as frequency bandwidth and power emerges as an important problem. The classical resource allocating techniques include FDMA and TDMA. CDMA, a potentially more efficient technique, has been suggested. It has attracted considerable attention in the contest of a new standard for digital mobile communication. For example, R.O. Korea has adopted the CDMA technique as its national standard for the commercial radio telecommunication network. In this thesis, we explain how CDMA works in wireless communication network. We introduce some different types of CDMA receivers, and adopt a vector space interpretation of each receiver in order to explain their differences and similarities. We also suggest an algorithm for improving the interference rejection characteristic of these receivers.

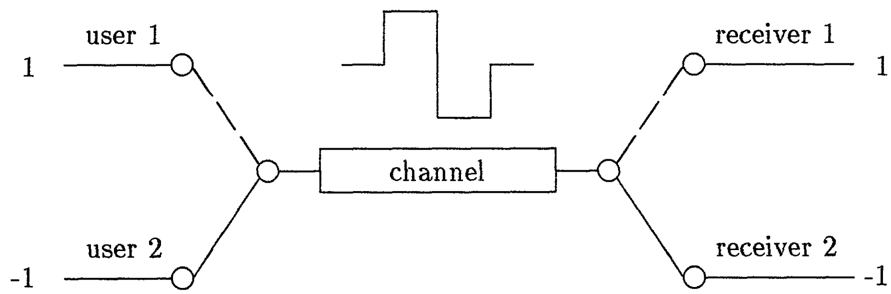
1.1 What is CDMA ?

Spread Spectrum is a transmission methodology in which the signal occupies bandwidth in excess of the minimum necessary to send information. The spreading of the band is accomplished by means of a code which is independent of the data. Synchronized reception with the code at the receiver is used for despreading and subsequent data recovery [1]. The initial application of the spread spectrum system was in military anti-jamming tactical communications and to anti-multipath systems.

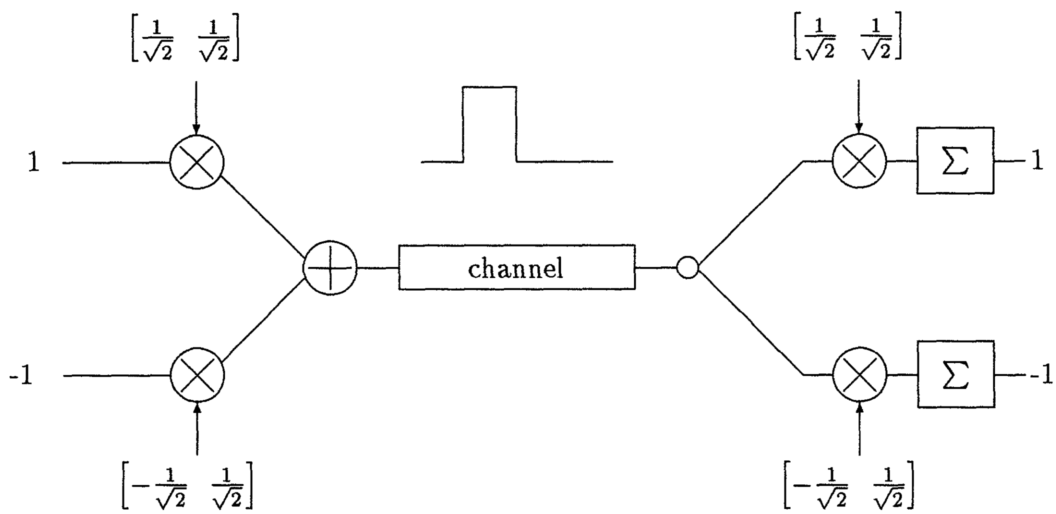
Multiple access refers to techniques with which two or more independent information sources (transmitters) transmit information through one physical channel to their counterparts (receivers). Interference among the signals requires a receiver which extracts desired information out of the received signal by suppressing the interference.

Spread spectrum techniques have been proposed for code division multiple access (CDMA) in order to support simultaneous digital communication among a large number of users. With the Direct-Sequence Code-Division Multiple-Access (DS CDMA), each user is given its own pseudo-random code or signature form, which is quasi-orthogonal (*i.e.*, has low correlation) with the codes of the other users. Receivers use the same codes to suppress the interference. While all users use disjoint frequency bands in Frequency-Division Multiple-Access (FDMA) or transmit sequentially in time in Time-Division Multiple-Access (TDMA), users of the CDMA system are allowed to transmit simultaneously in time and to occupy the same frequency band as well.

In order to explain briefly how the CDMA system achieves the multiple access task, we will compare the CDMA system with the heuristic TDMA system in a noise-free environment. Suppose that there are two transmitters and two receivers. The first transmitter (user 1) tries to send one bit to the first receiver (receiver 1), and the second transmitter (user 2) tries to send one bit to the second receiver (receiver 2). Bits sent by transmitters are either 1 or -1. Assume that the bit that user 1 wants to send is 1, and the bit that user 2 wants to send is -1. The TDMA system, whose model is given in figure 1.1(a), works as follows. For the first time slot, switches at both ends of the channel connect the user 1 to the receiver 1 through the channel. Then user 1 sends the desired bit to receiver 1 through the channel, while user 2 and receiver 2 are disconnected from the channel. During the second time slot, the switches connect user 2 to receiver 2. It is a very simple way to share a common



(a) A TDMA system



(b) A CDMA system

Figure 1-1: System models for multiple access.

channel between two or more users. It is also very easy to see that synchronization between users is crucial for TDMA systems.

The CDMA system adopts a more complicated modulation/demodulation method than the TDMA system. A simplified CDMA system model is depicted in figure 1.1(b). User 1, who wants to send 1, spreads the bit into the two time slots by multiplying 1 with its signature sequence, say $[\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}]$. User 2, who wants to send -1, also spreads the bit into two time slots by multiplying -1 with its signature sequence, say $[-\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}]$. Those two spreaded sequences now add up and get transmitted through the common channel to their destinations. After the transmitting waveform reaches its receivers, it gets multiplied by the same signature sequence as those at the transmitters, *i.e.* $[\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}]$ for receiver 1 and $[-\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}]$ for receiver 2. These multipliers, which operate as matched filters, will produce 1 for receiver 1 and -1 for receiver 2.

1.2 Why CDMA ?

While the FDMA and TDMA techniques also provide good multiple access flexibility, CDMA offers some unique advantages which are listed below.

- When the code for a particular user group is only distributed among authorized users, the CDMA process can provide some degree of communication privacy, since the transmissions are less easily intercepted by unauthorized users without the code.

- If a particular portion of the spectrum is characterized by fading, signals in that frequency range are attenuated. In an FDMA scheme, a user who was unfortunate enough to be assigned to the fading portion of the spectrum may experience highly degraded communications for as long as the fading persists. Time-selective fading can cause the same problem in TDMA. In a CDMA scheme, however, all users occupy the frequency band and time equally. Therefore, degradation caused by fading is shared among all users.

- There is no need for precise time coordination among the various simultaneous transmitters with CDMA schemes. Of course, time coordination information increases capacity when available, but it is not critically important. On the contrary, the TDMA system largely depends on a common clock for coordinating the transmission because only one transmitter can send a symbol at a given time. However, requiring two spatially isolated transmitters to share a common clock is, although not impossible, non-trivial.

- In a multiple access environment with bursty users, such as voice communication, it is typical that at a given time, only few of a large number of potential users are actively transmitting. FDMA (TDMA, respectively) allocates statically nonoverlapping frequency bands (time slots) to all potential users. Hence, as the ratio of simultaneously active users to the number of potential users decreases, FDMA (TDMA) becomes increasingly inefficient due to the increase in the number of unused frequency bands (time slots). On the contrary, the signature waveforms of the CDMA users overlap in both the frequency and the time domains. Therefore, as the ratio of active users to potential users decreases, CDMA becomes more efficient due to the decrease in the inter-user interference. In essence, CDMA is a dynamic channel sharing strategies which perform better in bursty user environments.

1.3 Thesis Outline

The traditional CDMA receiver, or the matched filter receiver, suffers from the *near-far problem* caused by the different distances of spatially distributed transmitters to the receiver. Recently, a lot of receivers which overcome the near-far problem have been proposed [5]-[9].

In Chapter 2, we introduce a model of a CDMA system. We describe three synchronous linear CDMA receivers and explore their characteristics. We also consider the differences between these three receivers and show that one receiver converges to another in some special cases. We present the bit error rates of each receiver and include a graph that shows the bit error rates of detectors in various situations.

In Chapter 3, we address the dynamic multi-access aspect of the CDMA system. Two of the three CDMA receivers introduced in Chapter 2 experience some capacity increases as the interference decreases while the other one does not have this property. We investigate this behavior of the receiver and suggest an algorithm to modifies the detector so that it overcomes this shortcoming. Moreover, by analyzing a simple case and using the Monte-Carlo method, we show that the algorithm results in a capacity increase in high interference situations.

Chapter 4 concludes the thesis by summarizing the results and presenting some possible directions for further work.

Chapter 2

CDMA Receivers

CDMA is increasingly of interest for use in wireless communication environments. Unlike wired communication where signals travel through the wire without substantial attenuation, wireless communication involves major signal attenuation. In general, the power at the receiver is inversely proportional to the cube of the distance between the transmitter and the receiver, referred to as path loss. This path loss, as well as the fact that wireless communication is mostly used in mobile communication systems, makes it hard to achieve reliable wireless communication. This leads to the *near-far problem* [5], which we develop in the next section, and has been the major obstacle in building high performance CDMA receivers.

2.1 Model for a Wireless CDMA Communication System

In a typical multiple access wireless communication scenario, there are many spatially distributed transmitters and a central base station. The goal of each transmitter is to send symbols to the base station. Each transmitter radiates symbols modulated by a waveform. The base station receives the transmitted waveforms and detects all or some of the transmissions. The signal from each transmitter, scattered spatially in an uncoordinated manner, reaches the base station with various energies depending on the distances between the transmitter and the base station.

Let assume that there are M users transmitting symbols. The received signal at the base station is the sum of the M transmissions plus additive noise. Throughout the thesis, we will assume that the noise is a stationary white Gaussian process. The received signal due to the user j is given by

$$r_j(t) = \sqrt{P_j} \sum_{n=-L_1}^{L_2} b_j[n] s_j(t - nT - \nu_j) \qquad 1 \leq j \leq M \tag{2.1a}$$

where P_j is the power of the user j, $b[n]$ is the symbol to be transmitted, $s_j(t)$ is the signature waveform of the user j, T is the symbol interval, and $\nu_j \in [0, T)$ is the delay of the user j. L_1 and L_2 in (2.1a) denote the points where transmissions start and stop, respectively. We will restrict $b_j[n]$ to be independent and equally likely to be 1 or -1. Without loss of generality, we assume that $\int_0^T s_j^2(t)\, dt = 1$ for all j. The received signal due to all M users is then given by

$$r(t) = \sum_{j=1}^{M} r_j(t) + n(t) \tag{2.1b}$$

where $n(t)$ is white Gaussian noise with power spectral density of $N_0/2$.

When the base station receives the waveform (2.1), it extracts all or some of the $b_j[n]$ from the received waveform. In this thesis, the ultimate goal of the receiver is to detect the transmission of user 1, $b_1[n]$, which will be referred to as the desired transmission. To achieve this goal, the base station has some or all of the following *a priori* knowledge.

- The signature waveform of the desired user, $s_1(t)$.
- The signature waveforms of the interfering users, $s_2(t), \cdots, s_M(t)$.
- The timing of the desired user, ν_1.

- The timing of each of the interfering users, $\nu_2, \cdots, \nu_M$.
- The received power of each user, $P_1, \cdots, P_M$.

2.2 Synchronous Linear CDMA Receiver

In this section, we will introduce three CDMA receiver structures. We assume that the transmission is symbol synchronous, *i.e,* $\nu_1 = \cdots = \nu_M = 0$. Though synchronous CDMA systems are not typical, the extension of this analysis to the asynchronous case can be done without any conceptual difficulty. When the users are synchronous, it is sufficient to consider the one-shot version of (2.1),

$$r(t) = \sum_{j=1}^{M} \sqrt{P_j} b_j s_j(t) + n(t) \qquad 0 \le t \le T. \tag{2.2}$$

The receivers of this section are linear CDMA receivers, *i.e.,* they have the form

$$\begin{aligned} y_1 &= \int_0^T r(t) d_1(t)\, dt && \text{(2.3a)} \\ &= \sqrt{P_1} b_1 \int_0^T s_1(t) d_1(t)\, dt + \int_0^T n(t) d_1(t) + \sum_{j=2}^{M} \sqrt{P_j} b_j \int_0^T s_j(t) d_1(t)\, dt. && \text{(2.3b)} \end{aligned}$$

Let's denote $\hat{b}_1$ as the estimate of the bit b_1. Then

$$\hat{b}_1 = \operatorname{sgn}(y_1) = \begin{cases} 1 & \text{if } y_1 > 0 \\ -1 & \text{if } y_1 < 0 \\ 0 & \text{if } y_1 = 0 \end{cases} \tag{2.3c}$$

where $d_1(t)$ is a linear combination of the signature waveforms. Note that the decision (2.3c) is invariant to positive scaling of $d_1(t)$. The splitter (2.3c) appears in each detector that we present in the section. The difference among the three receivers is the correlating waveform $d_1(t)$. We sometimes consider the detector waveform $d_1(t)$ as the detector itself.

2.2.1 Matched Filter Detector

The matched filter detector is the linear CDMA detector with the detector waveform of $d_{1,\mathrm{mtch}} = s_1(t)$. Note that the matched filter detector does not exploit any knowledge of the signature waveforms of the interfering users when detecting the desired transmission. The matched filter detector is the maximum-likelihood detector when the signal is corrupted only by white Gaussian noise and no multiple access interference [13]. Due to its simplicity, the matched filter detector is often used even when the noise is not white Gaussian. In the multiple access system of the previous section, the overall interference is not white Gaussian because of the interference from undesired users. The output of the matched filter detector is

$$\begin{aligned} y_{1,\mathrm{mtch}} &= \int_0^T r(t)s_1(t)\,dt \\ &= \sqrt{P_1}b_1 + \int_0^T n(t)s_1(t) + \sum_{j=2}^{M} \sqrt{P_j}b_j \int_0^T s_j(t)s_1(t)\,dt. \end{aligned} \tag{2.4}$$

The third term in (2.4) represents the effect of the interfering users. In theory, the effect of the multiple access interference component can be eliminated by choosing the signature waveform to be orthogonal, *i.e.*, $\int_0^T s_i(t)s_j(t)\,dt = 0$. In practice, however, orthogonal signal constellations are more the exception than the rule because of limited bandwidth. The asynchronism between users also makes this orthogonality difficult to impose. In the asynchronous case, the correlations have to remain small for all possible relative delays. There has been extensive research looking for sets of $s_j(t)$'s which have minimal correlation among $s_j(t)$'s for all possible delays [15]. For the thesis, it is sufficient to mention making the signatures waveforms orthogonal is not typical.

For $\hat{b}_{1,\mathrm{mtch}}$ (or $\mathrm{sgn}(y_{1,\mathrm{mtch}})$) to be a reliable estimate of b_1, the sum of the second term and the third term in (2.4) has to be small compared to the first term. Arbitrarily small correlations between users can be achieved by making the signature waveforms longer, implying greater bandwidth. However, the ratio of P_j to P_1 is determined by the ratio of the geometrical distance between the receiver and the user j to the distance between user 1 and the receiver. When CDMA is used for mobile radio communication network, the distances between the users and the base station cannot generally be controlled by the receiver. When the distance between the desired user and the receiver is much bigger than the distance between user j and the receiver, the third term in (2.4) dominates $y_{1,\mathrm{mtch}}$, thereby making the estimate unreliable. This problem, referred to as the near-far problem, makes the matched filter an inadequate receiver unless P_j's are made small.

2.2.2 Power Control

The near-far problem is caused by the big interfering signal power of the undesired users which happen to be much closer to the receiver than the desired user is. Reducing the interfering signal power makes the matched filter receiver perform better. The interfering power, however, cannot be reduced to zero because an interfering user for one receiver is a desired user for another one. In most cases, matched filter receivers for all users are located at the central base station.

With the power control method [2], the central base station measures the geometrical distance between the receiver and each transmitter. The base station then controls the power with which each transmitter sends its signals so that the power of each received signal becomes the same for all users. Using the previous notation, the power control method makes all P_j's equal. This power control method, combined with a low correlation constellation between signature waveforms, can be used with a matched filter receiver to get reasonably good performance. However, although this power control scheme suppresses the near-far problem greatly, it is self-defeating in a sense that the transmitted powers of the strong users should be reduced in order for the weaker users to achieve reliable communication, which decreases the over-all capacity of the resources.

2.2.3 Decorrelating Linear Detector

As we have seen, the traditional matched filter receiver for CDMA communication suffers from the near-far problem. The power control method with low mutual correlations between the user signature waveforms is the only remedy implemented in practice to cure the near-far problem [6]. However, the near-far problem is not an inherent characteristic of CDMA systems. Rather, it is the inability of the conventional matched-filter receiver to exploit the structure of the multiple-access interference. Recently, the decorrelating detector has been proposed [5]. It is based on *a priori* knowledge of the signature waveforms of interfering users which is not required for the conventional matched filter detector, and overcomes the near-far problem.

The decorrelating detector $d_{1,\text{dec}}(t)$ is a function which is orthogonal to all the interfering signature waveform, $s_2(t), \cdots, s_M(t)$. The third term in (2.3b) is made to be zero for the decorrelating detector $d_{1,\text{dec}}(t)$. This ensures that the decorrelating detector avoids the near-far problem.

Let $d_{1,\text{dec}}(t)$ be a linear combination of $s_1(t), s_2(t), \cdots, s_M(t)$, *i.e.*,

$$d_{1,\text{dec}}(t) = \sum_{j=1}^{M} a_j s_j(t). \tag{2.5}$$

From the orthogonality of $d_{1,\text{dec}}(t)$ to the interfering signature waveform, we have following $M-1$ equations,

$$\int_0^T d_{1,\text{dec}}(t) s_j(t)\, dt = 0 \quad \text{for} \quad j = 2, \cdots, M. \tag{2.6a}$$

We will restrict our attention to the case in which $s_1(t)$ is not a linear combination of $s_2(t), \cdots, s_M(t)$. Without loss of generality, we assume

$$\int_0^T d_{1,\text{dec}}(t) s_1(t)\, dt = 1. \tag{2.6b}$$

Define a matrix $\mathbf{R}$ whose (i,j) component is the correlation between user i and user j, *i.e,*

$$R\{i,j\} = \int_0^T s_i(t)s_j(t)\,dt. \tag{2.7}$$

Define a vector $\mathbf{a}$ whose jth component is a_j. The vector $\mathbf{a}$ specifies the decorrelating detector $d_{1,\text{dec}}(t)$ for the desired user. The M equations (2.6) can now be expressed in a concise vector equation,

$$\mathbf{Ra} = \mathbf{u_1} \tag{2.8a}$$

or,

$$\mathbf{a} = \mathbf{R}^{-1}\mathbf{u_1} \tag{2.8b}$$

where $\mathbf{u_1} = [1\ 0\ 0\ \cdots\ 0]^t$. For the synchronous CDMA communication system, the synchronous decorrelating detector has been shown to be the generalized maximum-likelihood detector for the white Gaussian noise channel [5] when no knowledge of the power is assumed.

2.2.4 MMSE Linear Detector

The minimum mean-square-error linear detector $d_{1,\text{mmse}}$ is defined as the signal $d_1(t)$ that minimizes the MSE

$$\text{MSE} = E\left[(\sqrt{P_1}b_1 - \int_0^T r(t)d_1(t)\,dt)^2\right]. \tag{2.9}$$

Let $d_{1,\text{mmse}}(t)$ be a linear combination of $s_1(t), s_2(t), \cdots, s_M(t)$, *i.e.*,

$$d_{1,\text{mmse}}(t) = \sum_{j=1}^{M} c_j s_j(t). \tag{2.10}$$

Equation (2.9) is then

$$\text{MSE} = P_1 - 2P_1(\mathbf{Ru_1})^t\mathbf{c} + \mathbf{c}^t\mathbf{RP}^2\mathbf{Rc} + \frac{N_0}{2}\mathbf{c}^t\mathbf{Rc} \tag{2.11}$$

where $\mathbf{P} = \text{diag}\{\sqrt{P_1}, \sqrt{P_2}, \cdots, \sqrt{P_M}\}$[‡] and $\mathbf{c}$ is a column vector whose jth component is c_j. The MMSE detector $d_{1,\text{mmse}}(t)$ is specified by $\mathbf{c}$. The vector $\mathbf{c}$ which minimizes (2.11) is obtained by differentiating (2.11) with respect to $\mathbf{c}$ and setting the derivative equal to zero

$$\mathbf{c} = P_1 \left(\mathbf{P}^2 \mathbf{R} + \frac{N_0}{2} \mathbf{I} \right)^{-1} \mathbf{u}_1. \tag{2.12}$$

The MMSE detector $\mathbf{c}$ can be calculated from (2.12) if the receiver knows *a priori* the signature waveform and power of each user. If the receiver does not have the necessary *a priori* knowledge, the MMSE detector waveform $d_{1,\text{mmse}}$ can be obtained adaptively by finding the waveform which minimizes the mean square error function over training sequences. The typical operation of the MMSE linear detector requires each transmitter to send a training sequence at start-up which the receiver uses for initial adaptation. After the training phase ends, adaptation during actual data transmission occurs in a decision-directed mode, *i.e.*, the estimates of symbols are used to minimize the mean square error function. While the decorrelating detector waveform can be also obtained adaptively, it can be done only by estimating unknown signature waveforms because there is no simple projection function which the decorrelating detector solely minimizes. One drawback of the MMSE detector is that any time there is a drastic change in the interference environment, the decision-directed adaptation becomes unreliable, the data transmission of the desired user must be temporarily suspended and a fresh training sequence must be used. This drawback has been shown to be avoidable in a recent paper in which the MMSE detector was obtained without the start-up training phase [10].

Before concluding the subsection, we will show that the MMSE detector converges to either the decorrelating detector or the matched filter detector in special cases.

Fact 2.1 The MMSE detector converges to the decorrelating detector when the noise level goes to zero.

[‡]By the notation $\text{diag}\{a_1, a_2, \cdots, a_M\}$ we mean an $M \times M$ diagonal matrix whose diagonal elements are $a_1, a_2, \cdots, a_M$.

Proof: From (2.12),

$$\lim_{N_0 \to 0} \mathbf{c} = P_1 \left(\mathbf{P}^2\mathbf{R}\right)^{-1} \mathbf{u}_1 = \mathbf{R}^{-1}\mathbf{u}_1 = \mathbf{a} \tag{2.13}$$

since $P_1\mathbf{P}^{-2}\mathbf{u}_1 = \mathbf{u}_1$. □

Fact 2.2 The MMSE detector converges to the matched filter detector when the relative power of interfering users to the desired users goes to zero.

Proof: From (2.12),

$$\lim_{\substack{P_j/P_1 \to 0 \\ j=2,\cdots,M}} \mathbf{c} = \left(\mathbf{u}_1\mathbf{u}_1{}^t\mathbf{R} + \frac{N_0}{2P_1}\mathbf{I}\right)^{-1} \mathbf{u}_1 = \frac{1}{1 + N_0/2P_1}\mathbf{u}_1 \tag{2.14}$$

since $\mathbf{u}_1\mathbf{u}_1{}^t\mathbf{R} + N_0/2P_1\mathbf{I}$ is an upper trianglular matrix, whose inverse is also an upper trianglular matrix [14]. The proof is completed by the invariance of the MMSE detector to positive scaling. □

2.2.5 Vector Space Representation

Roughly speaking, the matched filter detector maximizes the signal-to-noise ratio of the desired user while leaving the task of suppressing the interference to the power control method and a good signal correlation constellation design. The MMSE detector minimizes the combined effect of the interfering users and the noise while the decorrelating detector removes the effect of the interfering users at a cost of noise enhancement.

All three detectors, $d_{1,\mathrm{mtch}}(t)$, $d_{1,\mathrm{dec}}(t)$, and $d_{1,\mathrm{mmse}}$ are linear combinations of the signature waveforms $s_1(t), \cdots, s_M(t)$. If we denote V as the vector space of the signature waveforms, then the detectors are vectors in V. We will present three figures of the vector space V. In each figure, a detector and the signature waveforms are shown as vectors in the space.

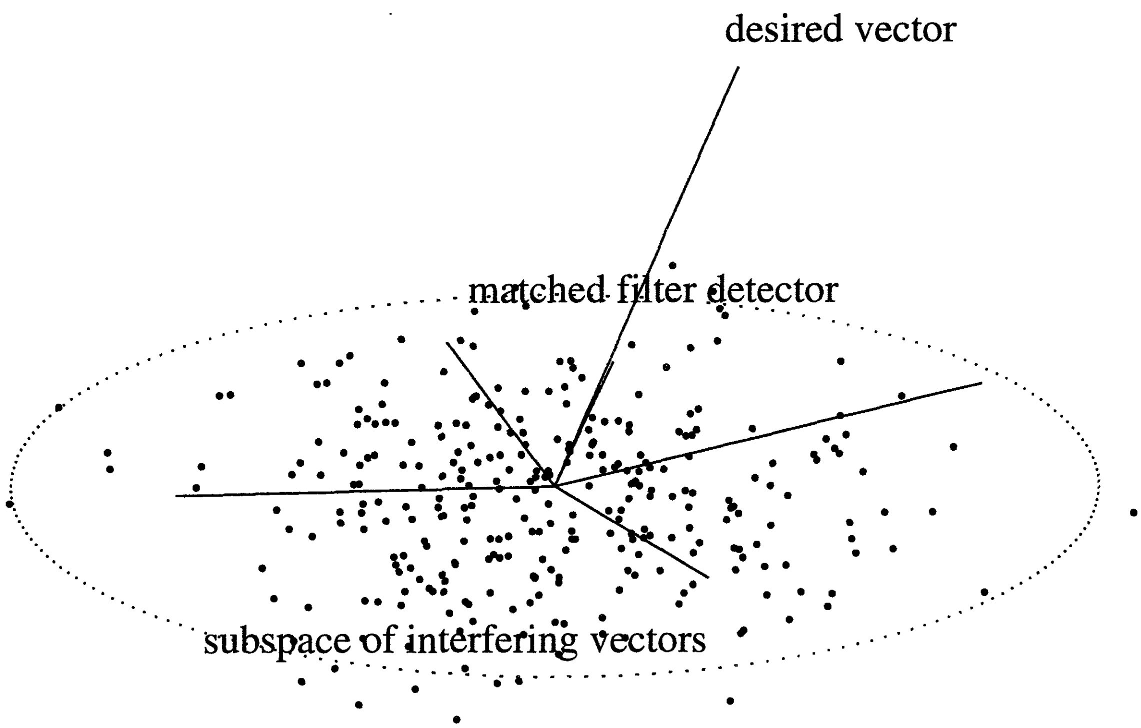


Figure 2-1: Vector space representation of the matched filter detector.

Let V_I denote the interference subspace of V spanned by $\{s_2(t), \cdots, s_M(t)\}$. The interfering vectors in the three figures are $\sqrt{P_2}s_2(t), \cdots, \sqrt{P_M}s_M(t)$, and the desired vector is $\sqrt{P_1}s_1(t)$. The dots in the figures represent the white Gaussian noise. In Figure 2-2, the matched filter detector $d_{1,\mathrm{mtch}}(t)$ lies in parallel with the desired signature vector $\sqrt{P_1}s_1(t)$ regardless of the interference vectors. The matched filter detector is not changed when the interference vectors are changed in direction or in length. In Figure 2-3, the decorrelating detector is shown. The decorrelating detector lies in the direction which is orthogonal to the interfering subspace V_I. Changing lengths of the interfering vectors does not affect the decorrelating detector $d_{1,\mathrm{dec}}(t)$. Note that if the desired vector is orthogonal to the subspace of interfering vectors V_I, the decorrelating detector is the same as the matched filter detector. The direction of the decorrelating detector $d_{1,\mathrm{dec}}(t)$ has nothing to do with the desired vector $\sqrt{P_1}s_1(t)$. The MMSE detector is shown in figure 2-4. The direction of the MMSE detector depends on all relative factors, such as the lengths and directions of the desired and interfering vectors and the variance of the noise power. Note that in general, the

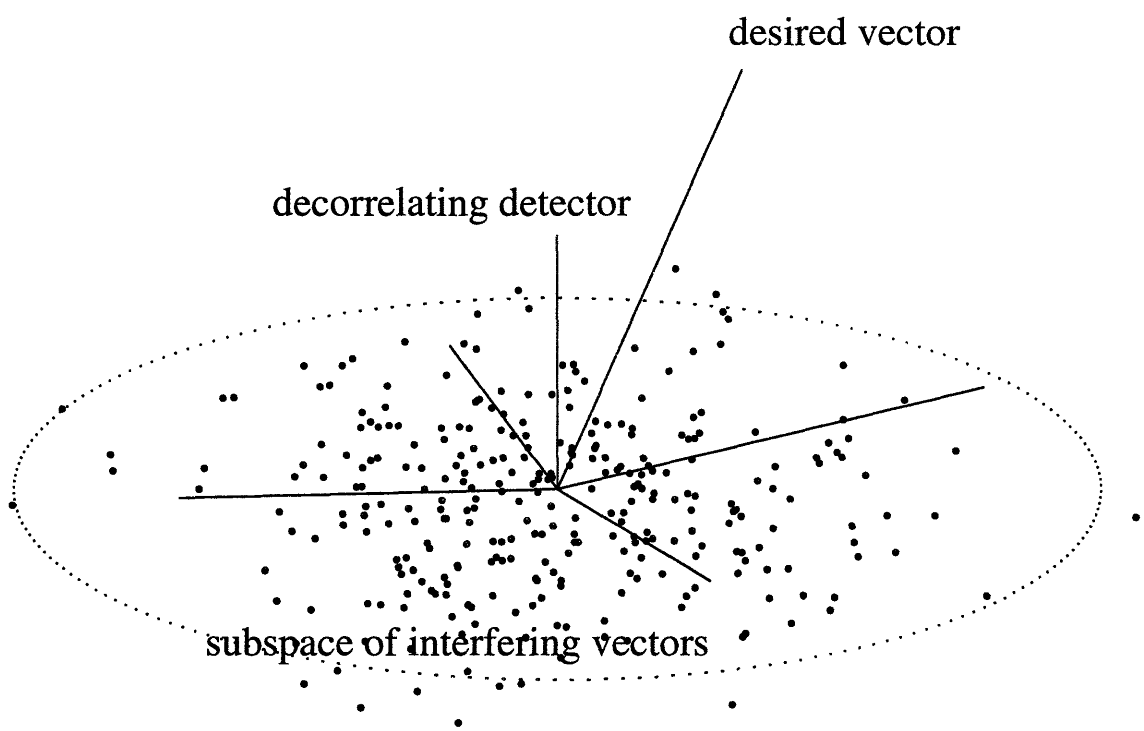


Figure 2-2: Vector space representation of the decorrelating detector.

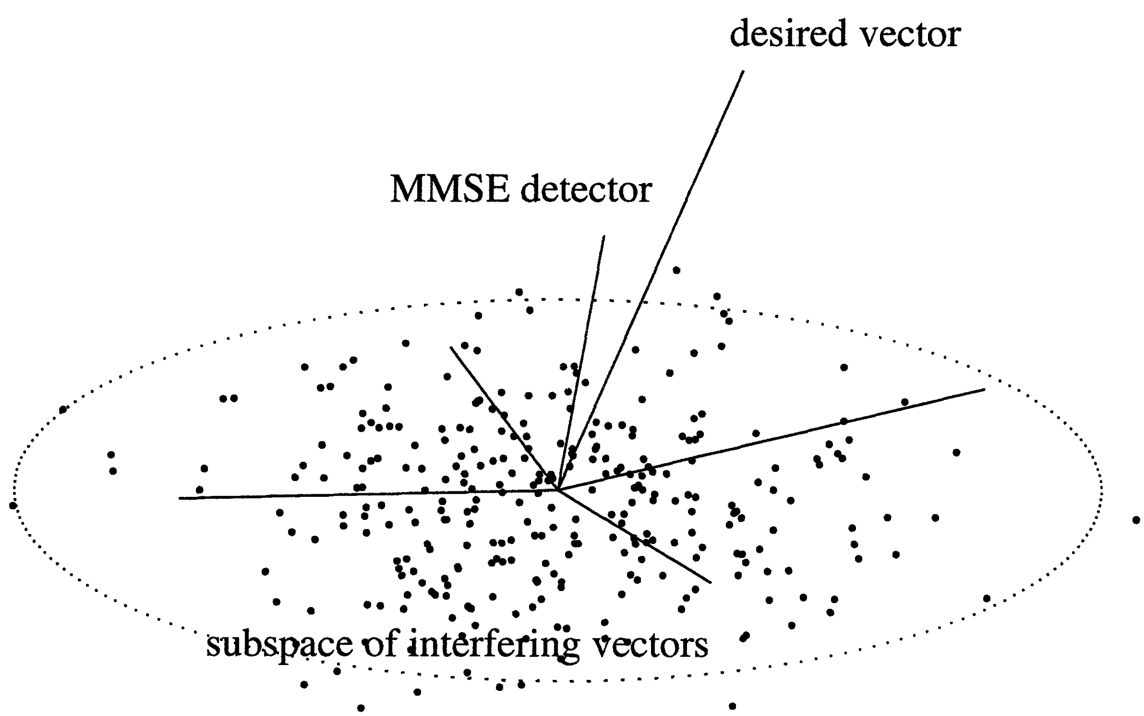


Figure 2-3: Vector space representation of the MMSE detector.

MMSE detector is neither orthogonal to the interfering subspace V_I nor in parallel with the desired vector in general.

2.3 Bit Error Rate

Various performance measures for CDMA receivers are proposed recently [3]-[6]. Perhaps the most relevant performance measure is the bit error rate (BER). Sometimes the interference transmissions contribute to the BER of a receiver (*e.g.*, the matched filter detector and the MMSE detector). For those receivers, the average BER over the possible interfering symbol combinations is the appropriate performance measure. In this section, we compare the average BER of three CDMA receivers.

2.3.1 General Case

The BER of the matched filter detector can be obtained from (2.4)†

$$y_{1,\mathrm{mtch}} \sim N\left(\sum_{j=1}^{M}\sqrt{P_j}b_j R\{j,1\},\ \frac{N_0}{2}\right). \tag{2.15}$$

Let us denote P and $\bar{P}$ as the bit error rate and the bit error rate averaged over the interfering bits, respectively. Then

$$P_{\mathrm{mtch}}(b_2,\cdots,b_M) = Q\left(\frac{\sqrt{P_1}+\sum_{j=2}^{M}\sqrt{P_j}b_j R\{j,1\}}{\sqrt{N_0/2}}\right) \tag{2.16}$$

†By $x \sim N\left(m,\sigma^2\right)$ we mean that x is a Gaussian random variable with mean m and variance σ^2, *i.e.*, one with probability density function

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{(x-m)^2/2\sigma^2\right\}.$$

where $Q(x) = (2\pi)^{-1/2} \int_x^\infty e^{-t^2/2}\, dt$. The average BER is, therefore, given by

$$\bar{P}_{\text{mtch}} = \frac{1}{2^{M-1}} \sum_{b_2,\cdots,b_M \in \{-1,1\}} P_{\text{mtch}}(b_2, \cdots, b_M). \tag{2.17}$$

The near-far problem that we discussed earlier is observed in (2.16) and (2.17). As $P_j/P_1 \to \infty$ for some j for which $R\{j,1\} \neq 0$, the average BER of the matched filter goes to 1/2. The power control method controls the P_j's so that the ratios P_j/P_1 remain equal to 1 for all j.

The BER of the decorrelating detector can be obtained from (2.3), (2.5), and (2.8). First,

$$\begin{aligned} y_{1,\text{dec}} &\sim N\left(\sqrt{P_1} b_1 \sum_{j=1}^{M} a_j R\{j,1\},\ \frac{N_0}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} a_i R\{i,j\} a_j\right) \\ &\sim N\left(\sqrt{P_1} b_1,\ \mathbf{u_1}^t \mathbf{R}^{-1} \mathbf{u_1} \frac{N_0}{2}\right). \end{aligned} \tag{2.18}$$

The BER is also the average BER for the decorrelating detector because the interfering transmission is completely suppressed (at the cost of noise enhancement). The noise enhancement property of the decorrelating detector is observed in (2.18) since $\mathbf{u_1}^t \mathbf{R}^{-1} \mathbf{u} \geq 1$ with the equality when $\mathbf{R}$ is the identity matrix (meaning that the signature waveforms are orthogonal). From (2.18),

$$\bar{P}_{\text{dec}} = Q\left(\sqrt{\frac{P_1}{\mathbf{u_1}^t \mathbf{R}^{-1} \mathbf{u_1} N_0/2}}\right). \tag{2.19}$$

The BER of the MMSE detector can be obtained from (2.3), (2.10), and (2.12). This time,

$$y_{1,\text{mmse}} \sim N\left(\sum_{i=1}^{M} \sum_{j=1}^{M} \sqrt{P_j} b_j R\{i,j\} c_i,\ \frac{N_0}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} c_i R\{i,j\} c_j\right). \tag{2.20}$$

Note the difference between (2.20) and (2.15). The MMSE detector $\mathbf{c}$ in (2.20) changes adaptively depending on the P_i's, $R\{i,j\}$'s, and N_0. This adaptive change prevents the second term from being relatively bigger than the first term. Now the BER and the average BER are given in (2.21).

$$P_{\text{mmse}}(b_2,\cdots,b_M) = Q\left(\frac{\sqrt{P_1}\sum_{j=1}^{M} c_j R\{j,1\} + \sum_{i=1}^{M}\sum_{j=2}^{M}\sqrt{P_j}b_j R\{i,j\}c_i}{\sqrt{\frac{N_0}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} c_i R\{i,j\}c_j}}\right) \tag{2.21a}$$

$$\bar{P}_{\text{mmse}} = \frac{1}{2^{M-1}} \sum_{b_2,\cdots,b_M\in\{-1,1\}} P_{\text{mmse}}(b_2,\cdots,b_M) \tag{2.21b}$$

2.3.2 Case of Two Users

The three equations of average BER in the previous subsection do not make it easy to decide which detector is better to use in certain environments. For example, in some situations the matched filter detector has a lower average BER than the decorrelating detector, but that is not obvious from the two related equations. The reason is that there is a number of variables that affect the performance of each detector. In this subsection, we will focus on the simple two user case, where one user is the desired user and the other is the interfering user. Reducing the number of interfering users to one makes the equations very simple while effects of interference can still be observed. We will express the average bit error rates of each detector as functions of the signal to noise ratios and the correlations between signature waveforms. We will also compare the three detectors in some special cases.

Let us define the signal to noise ratio of the i th user;

$$\text{SNR}_i = \frac{P_i}{N_0/2} \tag{2.22}$$

The average BER of the matched filter detector can be derived from (2.16), as follows

$$\bar{P}_{\text{mtch}} = \frac{1}{2}Q\left(\sqrt{\text{SNR}_1} + \rho\sqrt{\text{SNR}_2}\right) + \frac{1}{2}Q\left(\sqrt{\text{SNR}_1} - \rho\sqrt{\text{SNR}_2}\right) \tag{2.23}$$

where $\rho = \int_0^T s_1(t)s_2(t)\,dt$.

The average BER of the decorrelating detector can be rewritten from (2.19), and is given by

$$\bar{P}_{\text{dec}} = Q\left(\sqrt{\text{SNR}_1(1-\rho^2)}\right). \tag{2.24}$$

The average BER of the MMSE detector can be derived from (2.21b) [7], and is given by

$$\begin{aligned} \bar{P}_{\text{mmse}} &= \frac{1}{2}Q\left(\frac{\sqrt{\text{SNR}_1}(1-\rho^2+\rho^2\frac{1}{1+\text{SNR}_2}) + \rho\sqrt{\text{SNR}_2}(\frac{1}{1+\text{SNR}_2})}{\sqrt{1-\rho^2+\rho^2\frac{1}{(1+\text{SNR}_2)^2}}}\right) \\ &\quad +\frac{1}{2}Q\left(\frac{\sqrt{\text{SNR}_1}(1-\rho^2+\rho^2\frac{1}{1+\text{SNR}_2}) - \rho\sqrt{\text{SNR}_2}(\frac{1}{1+\text{SNR}_2})}{\sqrt{1-\rho^2+\rho^2\frac{1}{(1+\text{SNR}_2)^2}}}\right). \end{aligned} \tag{2.25}$$

When $\rho \to 0$

The decorrelating detector and the MMSE detector converge to the matched filter,

$$\bar{P}_{\text{mtch}} = \bar{P}_{\text{dec}} = \bar{P}_{\text{mmse}} = Q\left(\sqrt{\text{SNR}_1}\right). \tag{2.26}$$

When $\rho \to 1$

The MMSE detector converges to the matched filter detector. The decorrelating detector, which is orthogonal to the interfering user, is now also orthogonal to the desired user because the signature waveforms of the two users are identical. Hence, the decorrelating detector rejects not only the interfering user but also the desired user.

$$\bar{P}_{\text{mtch}} = \bar{P}_{\text{mmse}} = \frac{1}{2}Q\left(\sqrt{\text{SNR}_1}+\sqrt{\text{SNR}_2}\right) + \frac{1}{2}Q\left(\sqrt{\text{SNR}_1}-\sqrt{\text{SNR}_2}\right) \tag{2.27a}$$

$$\bar{P}_{\text{dec}} = \frac{1}{2} \tag{2.27b}$$

When $\mathrm{SNR}_1 \to 0$

$$\bar{P}_{\text{mtch}} = \bar{P}_{\text{dec}} = \bar{P}_{\text{mmse}} = \frac{1}{2} \tag{2.28}$$

When $\mathrm{SNR}_1 \to \infty$

$$\bar{P}_{\text{mtch}} = \bar{P}_{\text{dec}} = \bar{P}_{\text{mmse}} = 0 \tag{2.29}$$

When $\mathrm{SNR}_2 \to 0$

As we have seen, the MMSE detector converges to the matched filter detector in this case. The decorrelating detector, however, remains unchanged as SNR_2 goes to zero. Thus, it suffers from unnecessary performance degradation. While the interference becomes negligibly small as SNR_2 goes to zero, the decorrelating detector remains orthogonal to the interfering user in order to reject interference completely at the cost of noise enhancement. We will discuss this shortcoming of the decorrelating detector in the next chapter.

$$\bar{P}_{\text{mtch}} = \bar{P}_{\text{mmse}} = Q\left(\sqrt{\mathrm{SNR}_1}\right) \tag{2.30a}$$

$$\bar{P}_{\text{dec}} = Q\left(\sqrt{\mathrm{SNR}_1(1-\rho^2)}\right) \tag{2.30b}$$

When $\mathrm{SNR}_2 \to \infty$

The MMSE detector converges to the decorrelating detector when SNR_2 becomes indefinitely large.

$$\bar{P}_{\text{mtch}} = \frac{1}{2} \tag{2.31a}$$

$$\bar{P}_{\text{dec}} = \bar{P}_{\text{mmse}} = Q\left(\sqrt{\mathrm{SNR}_1(1-\rho^2)}\right) \tag{2.31b}$$

Figure 2-4 shows the average BER of the decorrelating detector and the MMSE detector as a function of ρ. The performance of the decorrelating detector, drawn with the solid curve, remains unchanged when SNR_2 is changed. The average BER of the MMSE detector, drawn with the dash-dot curve, gets bigger as SNR_2 becomes larger. However, the dash-dot curve remains below the solid curve no matter how large SNR_2 is, telling us that the MMSE detector always performs better than the decorrelating detector for the two user case. The dash-dot curve converges to the solid curve as SNR_2 becomes indefinitely large. Changing SNR_1 affects the scale of the y axis, but the general trend is preserved.

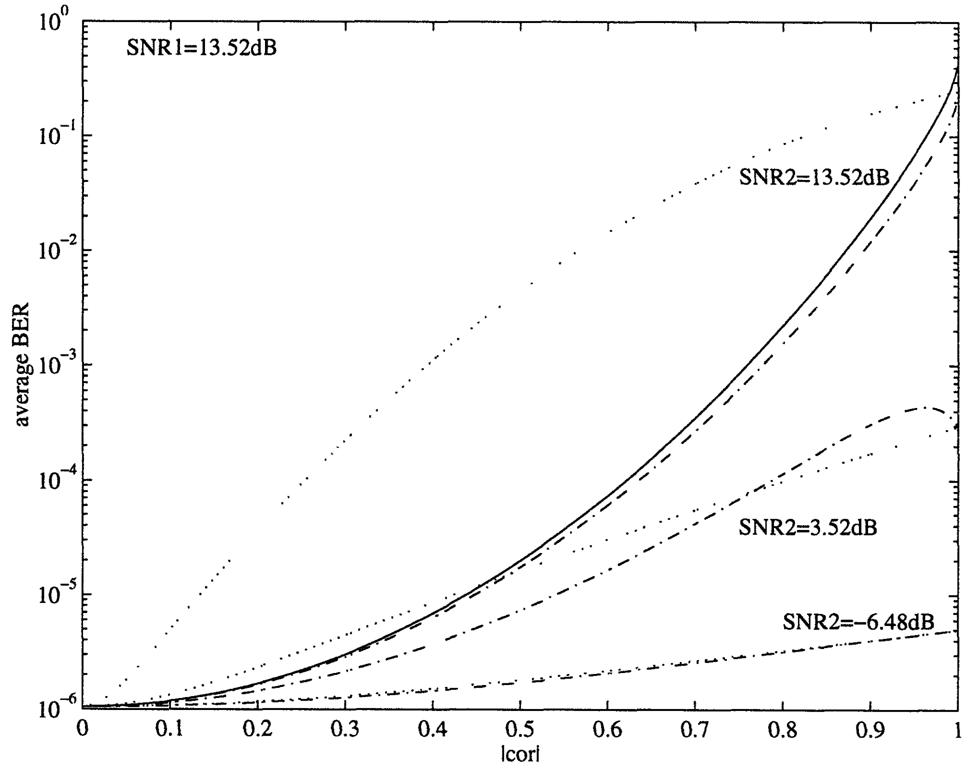


Figure 2-4: Comparison between the bit error rates of matched filter detector (dot curve), decorrelating detector (solid curve), and MMSE detector (dash-dot curve).

Chapter 3

Joint User Identification and Detection

3.1 Motivation

With FDMA (or TDMA), a multiple access channel is effectively partitioned into independent single-user sub-channels. Given a bandwidth B, the performance of the FDMA (or TDMA) system depends on how many sub-channels this bandwidth is partitioned into. Let's assume the given bandwidth B is used by M users. Each user will occupy B/M bandwidth, and the performance of the system, such as the quality of sound for a telephone network, depends on this B/M bandwidth of the sub-channel. The smaller the number of users is, the better the performance of the FDMA system gets.

Let's think of the following scenario. There are six persons, namely A1, A2, B1, B2, C1, C2. A1 is connected to A2 through a public radio telephone network. Similarly, B1 is connected to B2, and C1 to C2. Assume that the company which provides the service adopts the FDMA technique. Each user will then occupy one sixth of the given bandwidth B. However, it would be rare that all six users are talking at the same time. It is reasonable to say that some of them listen to the other persons at a given time, or that a pair might not speak for a duration. Assume a situation where only A1 and C1 are talking at a given time t. It would be nice if the FDMA system was able to assign half of the bandwidth B to each of A1 and C1, and nothing to the remaining users, A2, B1, B2, and C2 because we would get better performance out of it. However, this is hard for FDMA to do because the system would have to reassign the channels to a new set of *active* users whenever someone starts or stops talking, which happens very often. In practice, the FDMA system would assign one sixth of the bandwidth to each *potential* user regardless of which user is active. In this sense, the performance of the FDMA system is limited by the number of the potential users. When five are quiet and only B1 talks, the performance, or the quality of sound to B2, remains the same as when all six users are talking. In essence, FDMA assigns a sub-channel to each potential user no matter whether it is actively transmitting symbols or not and the performance does not improve as the number of active users decreases.

Let's now turn our attention to CDMA systems. Depending on the kind of receiver the system uses, a CDMA system might not be limited by the number of potential users. For example, the performance of the matched filter receiver is not limited by the number of the potential users. Decrease in the number of active users brings decrease in interference. In turn, decrease in interference results in a better performance. In this sense, the matched filter receiver is limited by the number of active users, or by interference. The performance of the MMSE detector also depends on the number of active users.

How about the decorrelating detector? Is it limited by the number of the potential users or by the number of the active users? In this chapter, we will formulate and discuss this problem. We will show that the performance of the decorrelating detector is limited by the number of potential users. To improve the performance of the decorrelating detector as the number of active users decrease, we suggest an algorithm which first identifies the active users. By analyzing a simple case and presenting results of Monte-Carlo simulations, we show that out algorithm improves the performance of the decorrelating detector when the signal-to-noise ratios of the active interfering users are high.

3.2 Problem Formulation

Assume there are M potential users which are connected to the receiver, ready to send symbols. Only K users are actively sending symbols (active users), while the other $M-K$ users (inactive users) are not sending symbols, thus not using the channel. Although the base station knows all the signature waveforms of the M potential users because they are assigned by the base station, it does not know who are the K active users.

For this chapter, we will assume for simplicity of exposition that the users are synchronized. We will also assume side knowledge of the number K, *i.e.,* the base station knows the number K although it does not know who these users are. In practice, an estimate of K is often used. Sometimes, an upper bound for K is used when the side knowledge or an estimate of K cannot be obtained.

As in the previous chapter, the goal of the receiver is to estimate the transmission of the desired user (user 1) with a small bit error rate. We also assume that the potential users switch between active and inactive so frequently that any previous knowledge of the active users does not help the base station to decide who the current active users are (frequent switch assumption).

Assume that the M potential users are assigned M signature waveforms of $s_1(t)$, $s_2(t), \cdots, s_M(t)$. The assumed synchronous received signal during $[0, T]$ is given by

$$r(t) = \sum_{j=1}^{M} \sqrt{P_j} b_j s_j(t) + n(t) \qquad 0 \le t \le T. \tag{3.1}$$

The $M-K$ inactive users have zero signal power, *i.e.*, P_j's are equal to zero for $M-K$

inactive users. For this chapter, the signature waveforms are restricted to have the following form,

$$s_j(t) = \sum_{k=0}^{N-1} a_j[k]\psi(t - kT_c) \tag{3.2}$$

where $a_j[k] \in \{-1, 1\}$ is the k th element (or chip) of the signature sequence for the user j, $\psi(t)$ is the chip waveform, T_c is the chip interval, and $N = T/T_c$ is an integer called the processing gain. The chip waveform $\psi(t)$ in this chapter restricted to the pulse

$$\psi(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t \leq T_c \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

For different chip wave forms, discussions in the chapter can be applied. Note that $\int_{-\infty}^{\infty} s_j^2(t)\, dt = 1$.

3.3 Decorrelating Detector

In this section, we will show that knowing (or identifying correctly) the active users helps the decorrelating detector to perform better. Then we develop vector notations in order to express the decorrelating detector concisley for the next section.

3.3.1 Decorrelating Detector with Perfect Knowledge of Active Users

In Subsection 2.2.3, the receiver assumes that all M users are active users, and the receiver $d_1(t)$ is orthogonal to all the interfering signature waveforms. However, if $P_j = 0$, $d_1(t)$ does not have to be orthogonal to $s_j(t)$ because there is no interference from user j, i.e., $\sqrt{P_j}s_j(t) = 0$. Let's think of the case where there are three potential users whose signature waveforms are $s_1(t), s_2(t)$, and $s_3(t)$. As we know from Subsection 2.2.3, the decorrelating detector $d_1(t)$ is given by

$$d_1(t) = a_1 s_1(t) + a_2 s_2(t) + a_3 s_3(t). \tag{3.4a}$$

$$\mathbf{a} = \mathbf{R}^{-1}\mathbf{u}_1 \tag{3.4b}$$

$$R\{i,j\} = \int_0^T s_i(t)s_j(t)\,dt \quad \text{for} \quad i,j = 1,2,3. \tag{3.4c}$$

However, if user 3 is not active (i.e., $P_3 = 0$), the decorrelating detector can be

$$d_1(t) = a_1 s_1(t) + a_2 s_2(t) \tag{3.5a}$$

$$\mathbf{a} = \mathbf{R}^{-1}\mathbf{u}_1 \tag{3.5b}$$

$$R\{i,j\} = \int_0^T s_i(t)s_j(t)\,dt \quad \text{for} \quad i,j = 1,2. \tag{3.5c}$$

The difference between (3.4) and (3.5) is that user 3 (and its corresponding signature waveform $s_3(t)$) is simply ignored. The correlation matrix $\mathbf{R}$ in (3.5) is 2×2 rather than 3×3 as in (3.4). The question is, does $d_1(t)$ of (3.5) performs better than the $d_1(t)$ of (3.4)? The following *fact 3.1* is true when the correlation between any two potential user pair is same, i.e., $\int_{-\infty}^{\infty} s_i(t)s_j(t)\,dt = \rho$ for $i \neq j$

Fact 3.1 Let's assume that there are K active users out of M potential users. The K user decorrelating detector performs better than L user detector where $K < L \leq M$.

proof: The proof is given in Appendix A. □

The above fact tells us that knowing who the K active users are out of M known potential users improves the performance of the receiver when the correlations are equal between users. Figure 3-1 shows the average bit error rate of various detectors when there are two active users. Changing the number of active users does not affect the bit error rates of decorrelating detectors. The curves of the bit error rates of the MMSE detector and matched filter detector change as the number of active users changes, but the trend is preserved. In the graph, the correlations between potential user pairs are same, i.e., $\int_{-\infty}^{\infty} s_i(t)s_j(t)\,dt = \rho$ for $i \neq j$.

The bit error rate of the decorrelating detector without user identification with

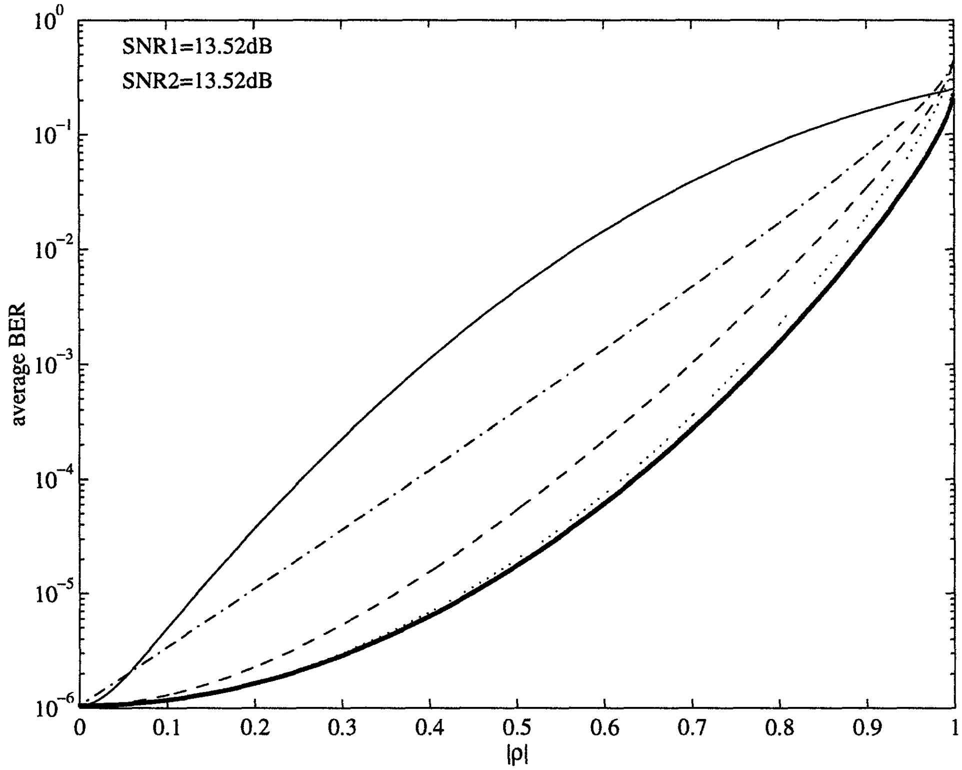


Figure 3-1: Comparison of average bit error rates of matched filter detector (thin solid curve), decorrelating detector with perfect user identification (dot curve), decorrelating detector without user identification (dash curve) with three potential users, decorrelating detector without user identification with infinite number of potential users (dash-dot curve), and MMSE detector (thick solid curve), case of two active users.

three potential users can be obtained from (2.19), and is given by

$$\bar{P}_{\text{dec}} = Q\left(\sqrt{\text{SNR}_1 \frac{(1-\rho)(1+2\rho)}{1+\rho}}\right). \tag{3.6}$$

The bit error rate of the decorrelating detector without user identification with infinite number of potential users can also be obtained from (2.19), and is given by

$$\bar{P}_{\text{dec}} = Q\left(\sqrt{\text{SNR}_1(1-\rho)}\right). \tag{3.7}$$

The bit error rates of the matched filter detector, the decorrelating detector with perfect user identification, the mmse detector in the graph are given in (2.23), (2.24), and (2.25), respectively.

3.3.2 Decorrelating Detector with Vector Notations

In this subsection, we develop some vector notations in order to express the decorrelating detector more concisely. We will define $\mathbf{r}$ and $\mathbf{s}_j$ whose kth elements are given by

$$r\{k\} = \int_{kT_c - T_c}^{kT_c} r(t)\sqrt{N}\psi(t - kT_c + T_c)\, dt,$$

$$s_j\{k\} = \int_{kT_c - T_c}^{kT_c} s_j(t)\sqrt{N}\psi(t - kT_c + T_c)\, dt.$$

Note $R\{i,j\} = \int_0^T s_i(t)s_j(t)\, dt = \mathbf{s}_i^t\mathbf{s}_j$.

Instead of the continuous received waveform $r(t)$ and the signature waveforms $s_i(t)$'s, we will utilize the vectors $\mathbf{r}$ and $\mathbf{s}_i$ to represent the received signal

$$\mathbf{r} = \sum_{j=1}^{M} \sqrt{P_j} b_j \mathbf{s}_j + \mathbf{n} \tag{3.8}$$

where the kth element of the vector $\mathbf{n}$ is defined as

$$n\{k\} = \int_{kT_c - T_c}^{kT_c} n(t)\sqrt{N}\psi(t - kT_c + T_c)\, dt. \tag{3.9}$$

Define a matrix $\mathbf{S}$ whose i th column is $\mathbf{s}_i$. Note that the correlation matrix is given by $\mathbf{R} = \mathbf{S}^t\mathbf{S}$. Now the received vector $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{SPb} + \mathbf{n} \tag{3.10}$$

where the kth element of $\mathbf{b}$ is b_k. The decorrelating detector $\mathbf{d}_1$ is the vector which is orthogonal to all interfering signature vector $\mathbf{s}_2, \cdots, \mathbf{s}_M$, *i.e.*,

$$\begin{aligned} \mathbf{d}_1^t \mathbf{s}_2 &= 0 \\ &\vdots \\ \mathbf{d}_1^t \mathbf{s}_M &= 0. \end{aligned} \tag{3.11a}$$

We also assume that

$$\mathbf{d}_1^t \mathbf{s}_1 = 1. \tag{3.11b}$$

Equation (3.11) is equivalent to (3.12)

$$\mathbf{d}_1^t \mathbf{S} = \mathbf{u}_1{}^t. \tag{3.12}$$

The explicit expression of $\mathbf{d}_1$ is

$$\mathbf{d}_1 = \mathbf{S}\left(\mathbf{S}^t\mathbf{S}\right)^{-1}\mathbf{u}_1 = \mathbf{SR}^{-1}\mathbf{u}_1. \tag{3.13}$$

Now the decorrelating detector output is given by

$$y_1 = \mathbf{d}_1^t \mathbf{r} \tag{3.14a}$$

$$\quad = \mathbf{u}_1{}^t\mathbf{Pb} + w \tag{3.14b}$$

$$= \sqrt{P_1} b_1 + w \tag{3.14c}$$

where $w = \mathbf{d}^t \mathbf{n}$. Note that

$$y_1 \sim N\left(\sqrt{P_1} b_1, \frac{N_0}{2} \mathbf{u_1}^t \mathbf{R}^{-1} \mathbf{u_1}\right) \tag{3.15}$$

Note also that y_1 is an unbiased estimate of $\sqrt{P_1} b_1$. The decorrelating detector for user j, $\mathbf{d}_j$ can be found using the same method. If we define a matrix $\mathbf{D} = \mathbf{S}\mathbf{R}^{-1}$, then $\mathbf{d}_j$ is given by the jth column of the matrix $\mathbf{D}$.

3.4 User Identification and Detection

In this section, we describe receivers which jointly identify the K active users and their transmissions from the received waveform. The receivers proposed in the section operate by first to determining which $M - K$ users have zero power and then detecting the symbols of the active users using K-user decorrelating detector. With the assumption that the users are synchronous and that the users switch frequently between active and inactive (frequent switch assumption), all we can use to identify the K active users are the received vector $\mathbf{r}$, the known M potential user signature vectors $\mathbf{s}_i$'s and the knowledge that there are K active users.

3.4.1 User Identification by Minimizing Noise Power

The first algorithm that we propose identifies the active users by minimizing the noise power. The idea of minimizing the noise power comes from the maximum-likelihood estimation of a Gaussian random variable. The Gaussian random vector $\mathbf{r}$ has a probability density function $f_{\mathbf{r}}(\mathbf{r}; \mathbf{P}, \mathbf{b})$

$$f_{\mathbf{r}}(\mathbf{r}; \mathbf{P}, \mathbf{b}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{(\mathbf{r} - \mathbf{SPb})^t (\mathbf{r} - \mathbf{SPb})}{2\sigma^2}\right) \tag{3.16}$$

where $\sigma = \sqrt{N_0/2}$.

The maximum-likelihood estimate of $\mathbf{b}$ maximizes the probability density function f over $\mathbf{b}$ given the knowledge that there are only K active users. While $\mathbf{b}$ is a random vector, $\mathbf{P}$ is an unknown power matrix. Without knowing what $\mathbf{P}$ is, maximizing the likelihood function f is ill-defined. Therefore, we will use the idea of the generalized maximum likelihood estimation, in which the likelihood function is maximized over random variable(s) and unknown parameter(s). In this problem, the generalized maximum-likelihood user identifier (GML identifier) maximizes f over $\mathbf{b}$ and $\mathbf{P}$. We will assume that user 1 is always active because this is the desired user.

$$\begin{aligned}
\hat{\mathbf{b}} &= \operatorname{sgn}\left(\underset{\mathbf{b}\in\{-1,1\}^M}{\operatorname{argmax}} \max_{\substack{P_1>0,\, P_i\geq 0\\ i=2,\cdots,M \\ M-K \text{ of } P_i\text{'s are } 0}} f_{\mathbf{r}}(\mathbf{r};\mathbf{P},\mathbf{b}) \right) && \text{(3.17a)}\\
&= \operatorname{sgn}\left(\underset{\mathbf{b}\in\{-1,1\}^M}{\operatorname{argmax}} \max_{\substack{P_1>0,\, P_i\geq 0\\ i=2,\cdots,M \\ M-K \text{ of } P_i\text{'s are } 0}} \log_e f_{\mathbf{r}}(\mathbf{r};\mathbf{P},\mathbf{b}) \right) && \text{(3.17b)}\\
&= \operatorname{sgn}\left(\underset{\mathbf{b}\in\{-1,1\}^M}{\operatorname{arg\,min}} \min_{\substack{P_1>0,\, P_i\geq 0\\ i=2,\ldots,M \\ M-K \text{ of } P_i\text{'s are } 0}} (\mathbf{r}-\mathbf{SPb})^t(\mathbf{r}-\mathbf{SPb}) \right) && \text{(3.17c)}
\end{aligned}$$

$\mathbf{P}$ and $\mathbf{b}$ can be combined to make (3.17c) one vector optimization problem. Define $\mathbf{x} = \mathbf{Pb}$. Then

$$\hat{\mathbf{b}} = \operatorname{sgn}\left(\underset{\substack{\mathbf{x}\in\mathbb{R}^M,\, x\{1\}\neq 0 \\ M-K \text{ of } x_i\text{'s are } 0}}{\operatorname{argmax}} \left(2\mathbf{r}^t\mathbf{Sx} - \mathbf{x}^t\mathbf{Rx}\right) \right). \tag{3.18}$$

Using the sub-vector and the sub-matrix notation described in Appendix B, we can rewrite (3.18) as

$$\hat{\mathbf{b}} = \operatorname{sgn}\left(\underset{\substack{\mathbf{x}\in\mathbb{R}^M \\ i=1,2,\cdots,\binom{M-1}{K-1}}}{\arg\max} \left(2(\mathbf{r}^t\mathbf{S})^K_{a_i}\mathbf{x}^K_{a_i} - \left(\mathbf{x}^K_{a_i}\right)^t \mathbf{R}^K_{a_i}\mathbf{x}^K_{a_i}\right) \right) \tag{3.19}$$

If there is more than one $\mathbf{x}$ which maximizes (3.19), the $\mathbf{x}$ with the smallest $||\mathbf{x}||$ is chosen. This assumption gives $\mathbf{x}$ with $M-K$ zero elements, which is the same $\mathbf{x}$ as in (3.18). It is possible to show that the $\mathbf{x}_{a_i}^K$ which maximizes the argument $2(\mathbf{r}^t\mathbf{S})_{a_i}^K\mathbf{x}_{a_i}^K - \left(\mathbf{x}_{a_i}^K\right)^t \mathbf{R}_{a_i}^K\mathbf{x}_{a_i}^K$ is $\mathbf{x}_{a_i}^K = \left(\mathbf{R}_{a_i}^K\right)^{-1}(\mathbf{S}^t\mathbf{r})_{a_i}^K$.

Algorithmic Procedure of (3.19)

[i] Choose i which maximizes $(\mathbf{r}^t\mathbf{S})_{a_i}^K\left(\mathbf{R}_{a_i}^K\right)^{-1}(\mathbf{S}^t\mathbf{r})_{a_i}^K$.

[ii] From the i obtained in [i], identify the active users. If $j \in a_i$, then user j is among the active users.

3.4.2 User Identification by Using the Decorrelating Detector

The GML identifier suffers from high computational complexity when M and K are large. The number of the combinations over which the GML identifier has to evaluate $(\mathbf{r}^t\mathbf{S})_{a_i}^K\left(\mathbf{R}_{a_i}^K\right)^{-1}(\mathbf{S}^t\mathbf{r})_{a_i}^K$ is $\binom{M-1}{K-1}$.

In this section, a computationally less-intensive two step decorrelating detector is proposed. While the GML identifier applies $\binom{M-1}{K-1}$ K-user decorrelating detectors to the received vector, the two step decorrelating detector applies one M-user decorrelating detector and one K-user decorrelating detector to the received vector. The first step is to apply the M-user decorrelating detector. This first step is used to identify the active K users. The second step is to apply the K-user decorrelating detector using the K users identified in the first step.

From (3.15) and the argument following it, the vector $\mathbf{y}$ whose jth component is the output of the jth user decorrelating detector is

$$\mathbf{y} = \mathbf{D}^t\mathbf{r} \tag{3.20a}$$

$$= \mathbf{P}\mathbf{b} + \mathbf{D}^t\mathbf{n} \tag{3.20b}$$

$$E(\mathbf{y}) = \mathbf{Pb} \tag{3.20c}$$

As we can see from (3.20), $\mathbf{y}$ is an unbiased estimate of the power multiplied by the signal bits. Since the signal bits are either 1 or -1, we may decompose $\mathbf{y}$ into the estimate of the signal powers and the estimate of the signal bits by

$$\hat{\mathbf{b}} = \mathrm{sgn}(\mathbf{y}) \tag{3.21}$$

$$\hat{\mathbf{P}} = \mathbf{I} \cdot \mathrm{abs}(\mathbf{y}) \tag{3.22}$$

where $\mathrm{abs}(\mathbf{x}) = [\mathrm{abs}(x\{1\}), \mathrm{abs}(x\{2\}), \cdots, \mathrm{abs}(x\{M\})]^t$. Note that $\hat{\mathbf{P}}$ is now a biased estimate of $\mathbf{P}$. It can be easily shown that

$$E\left(\hat{P}\{i,i\}\right) > P\{i,i\} \tag{3.23}$$

However, it can also be shown that

$$\lim_{\mathrm{SNR}_i \to \infty} E\left(\hat{P}\{i,i\}\right) = P\{i,i\} \tag{3.24}$$

Therefore, in high signal to noise ratio area, $\mathrm{abs}(\mathbf{y})$ is an asymptotically unbiased estimate of the signal powers. The first step of the two step decorrelating detector finds the K biggest elements of $\mathrm{abs}(\mathbf{\Gamma}^{-1}\mathbf{y})$ and identifies them as the K active users.

Algorithmic Procedure for the Two Step Decorrelating Detector

[i] Apply the M user decorrelating detector $\mathbf{D}$ to $\mathbf{r}$.

[ii] Find the biggest K elements of $\mathbf{y} = \mathrm{abs}(\mathbf{D}^t\mathbf{r})$ and identify them as the K active users.

[iii] Construct the sub-matrix of $\mathbf{D}$, $\mathbf{D}_s$, by removing $M-K$ elements of $\mathbf{D}$ which are identified as inactive users.

[iv] Apply the smaller K user decorrelating detector $\mathbf{D}_s^t$ to $\mathbf{r}$.

3.5 Analysis of the Two Step Decorrelating Detector

Both the GML identifier and the two step decorrelating detector identify the K active users first. After identifying the K active users, both of them apply the K-user synchronous decorrelating detector to the received vector $\mathbf{r}$. Therefore, the only difference between the two algorithms is how they identify the K active users. If the identified users are the actual active users, then the second step is the usual K user decorrelating detector. If the identified users are not the actual active users, the second step is not the right K user decorrelating detector. We will refer to this as the wrong user set decorrelating detector. The number of wrong user set decorrelating detectors could be very large. For example, if $M = 10$, and $K = 3$, the number of possible wrong user set decorrelating detectors is 35. This large number of combinations makes the analysis of the detector very hard. To make the analysis feasible, we will focus on the $K=2, M=3$ case. In this case, the number of possible wrong user decorrelating detectors is just one, simplifying the analysis. We also note that after a very long but tedious manipulation, it is possible to show that when there are two active users out of three potential users and the correlations between users are equal for any pair signature vectors, namely ρ, the two detectors are actualll idnetical. In more general cases, the two step decorrelating detector is generally not the same as the GML identifier. It is possible to get the analytic expression of the probability of choosing the wrong user pair for the two step decorrelating detector.

$$\begin{aligned}\Pr(\text{choosing the wrong user pair}) &= Q\left(\sqrt{\tfrac{(1-\rho)\mathrm{SNR}_I}{2}}\right) + Q\left(\sqrt{\tfrac{(1+2\rho)(1-\rho)\mathrm{SNR}_I}{2}}\right) \\ &\quad -2Q\left(\sqrt{\tfrac{(1-\rho)\mathrm{SNR}_I}{2}}\right) \cdot Q\left(\sqrt{\tfrac{(1+2\rho)(1-\rho)\mathrm{SNR}_I}{2}}\right) \qquad (3.25)\end{aligned}$$

where SNR_I is the signal to noise ratio of the interfering user. The derivation of (3.25) is given in Appendix C. Note that the probability depends only on the signal to noise ratio of the interfering user and the correlation ρ.

Figure 3-2 shows the probability of choosing a wrong user pair as a function of the correlation ρ for various interfering signal to noise ratios. As the signal to noise ratio of the interfering user becomes large, the probability of choosing the wrong user becomes small. The interference from the high power user is rejected completely at the second step decorrelating detector once the right active user is identified. Therefore, the probability of error converges to the active two user decorrelating detector as the interfering signal to noise ratio becomes higher.

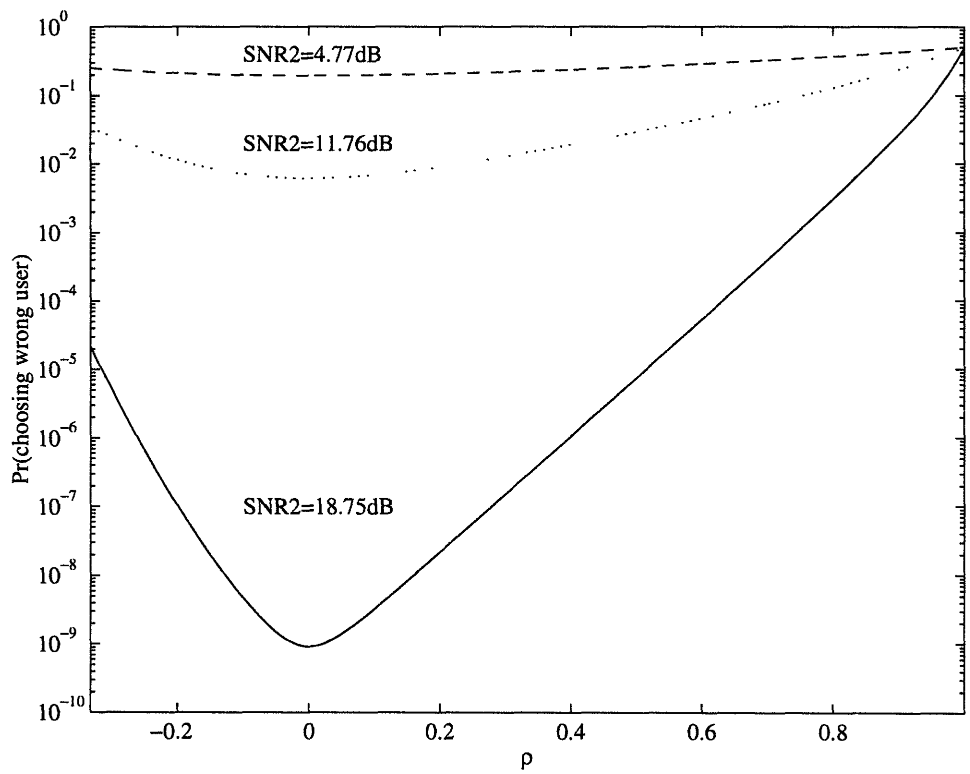


Figure 3-2: Probability of choosing a wrong user pair, when $K = 2, M = 3$.

Figure 3-3 and Figure 3-4 are based on Monte-Carlo simulations. Both figures depict the case of two active users out of three potential users. The correlation between the signature vectors is -1/3 in Figure 3-3 while it is equal to 27/31 in Figure 3-4. In both figures we can see that the performance of the two step decorrelating detector converges to the performance of the active user decorrelating detector as the

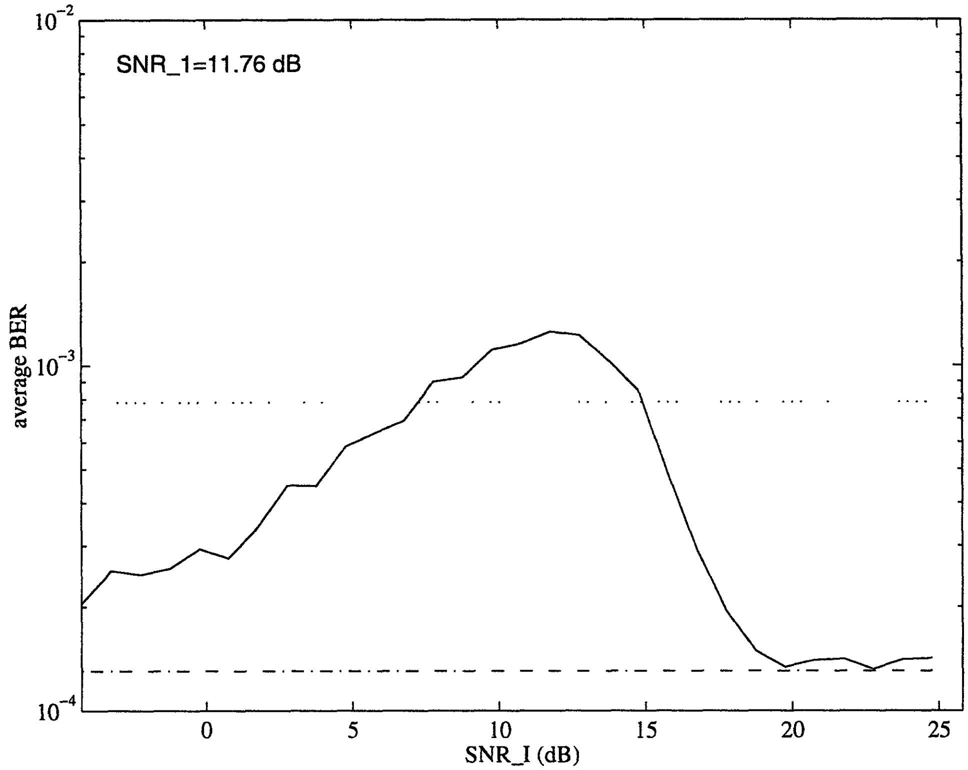


Figure 3-3: Comparison between the bit error rate of the two step detector (solid curve) with the decorrelating detector without user identification (dot line) and the decorrelating detector with perfect user identification (dash-dot line), when $\rho = -1/3$.

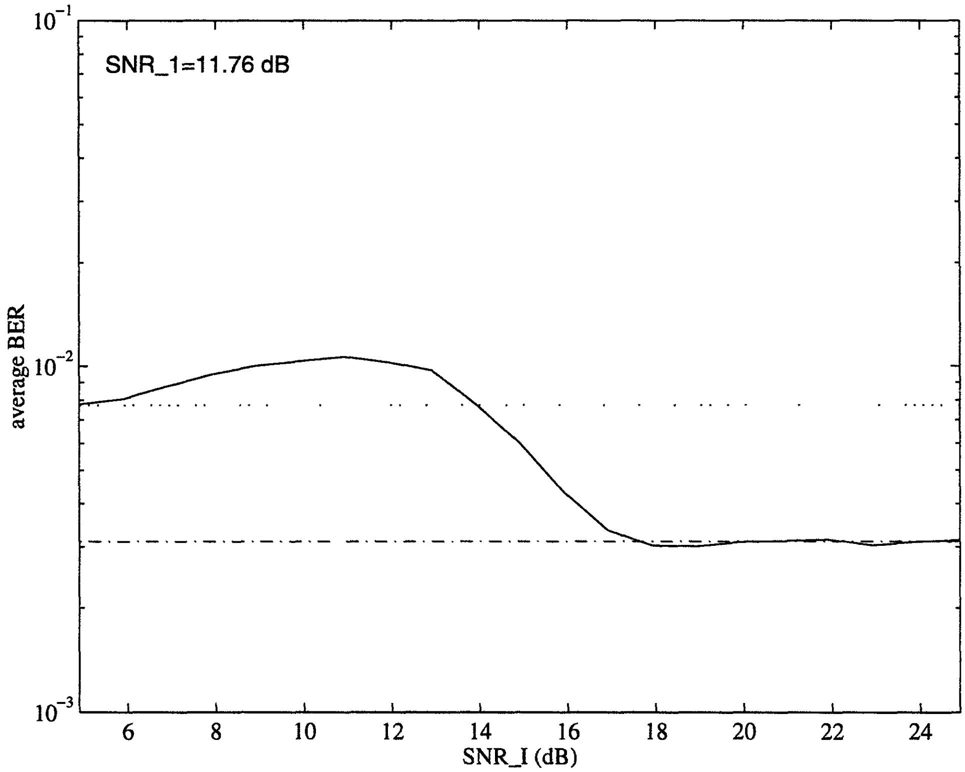


Figure 3-4: Comparison between the bit error rate of the two step detector (solid curve) with the decorrelating detector without user identification (dot line) and the active user decorrelating detector (dash-dot line), when $\rho = 27/31$.

SNR of the interfering user becomes bigger. That is because as SNR_I becomes bigger and bigger, the first step of the two step decorrelating detector identifies the active interfering user with less probability of error. In the range of $\text{SNR}_I = [7.5 \quad 15]$ in Figure 3-3 and $\text{SNR}_I = [5 \quad 14.5]$ in Figure 3-4, however, the probability of choosing wrong user is relatively big and that worsens the overall BER of the two step decorrelating detector. In these ranges, the performance of the two step decorrelating detector is even worse than that of the potential user decorrelating detector which does not identify the active user. In the low SNR_I region, the performance of the two step decorrelating detector converges again to the active user decorrelating detector. The reason is not because the first step identifies the active user well, but mostly because the second step is a two user decorrelating detector instead of three. In this region, even if the first step chooses the wrong user as the active user, the interference from the interfering user is so small that its effect is negligible. This can be easily understood by looking at the following situation. Assume that there are three potential users. The receiver assumes that there are two active users, but there is actually only the desired user. After the first step, the two step decorrelating detector chooses a non-existing user as an active interfering user. However, the two step decorrelating detector performs better than the potential user decorrelating detector because it is a two user decorrelating detector instead of a three user one.

Figure 3-5 shows the performance of the two step decorrelating detector compared with the three potential user decorrelating detector and the two active user decorrelating detector in a power controlled situation, *i.e.,* when $\text{SNR}_1 = \text{SNR}_I$. The graph is shown as a function of SNR. We can see that in the power controlled situation the two step decorrelating detector does not perform better than the potential user decorrelating detector in high SNR situations. The reason is that the ABER of the potential user decorrelating detector decreases faster than the probability of choosing the wrong user for the two step decorrelating detector.

Although the performance of the two step decorrelating detector is better as the

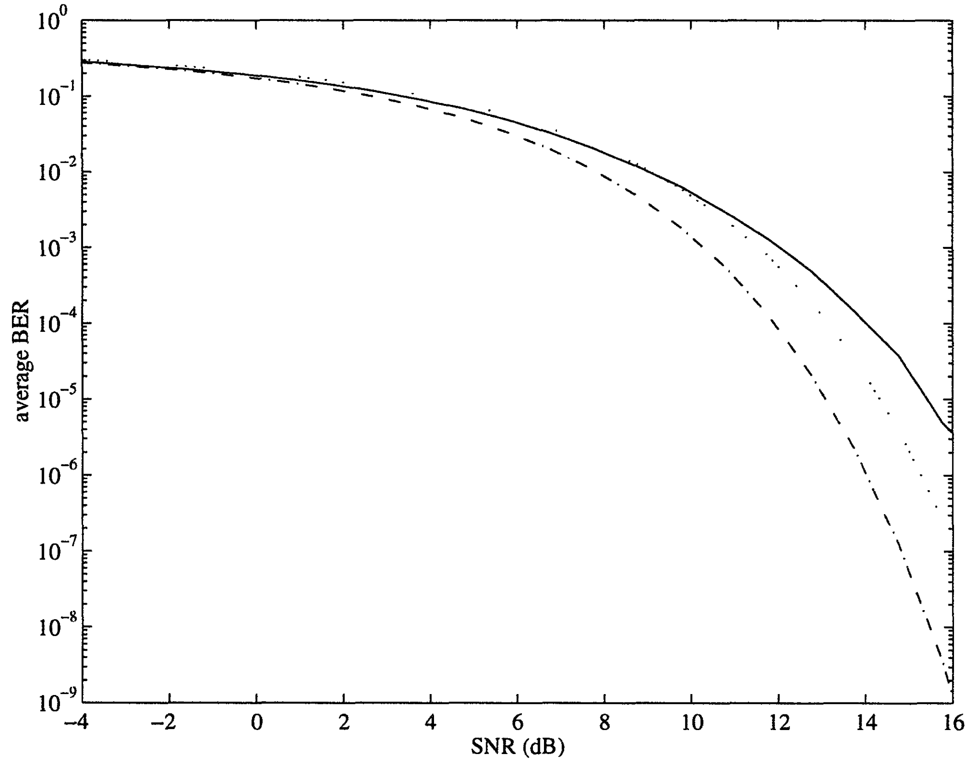


Figure 3-5: Comparison between the bit error rate of the two step detector (solid curve) with the decorrelating detector without user identification (dot curve) and the active user decorrelating detector (dash-dot curve), in the power controlled case, when $\rho = -1/3$.

SNR becomes bigger, its performance does not improve as fast as the decorrelating detector without user identification, thus making the two step decorrelating detector unattractive in the power controlled case. Simulation shows that this is true for different K and M values. Overall, in low desired signal power and high interfering signal power case the two step decorrelating detector achieves increased performance over the decorrelating detector without user identification.

Chapter 4

Conclusion

In this thesis we have considered the problem of detecting transmissions in CDMA-based wireless communication system.

Three previously proposed receivers were described and compared in the thesis. The matched filter detector is the maximum-likelihood detector assuming that the signal is embedded in a white Gaussian noise, but big interfering signals can cause the near-far problem, thus requiring power control to achieve reliable communication. The decorrelating detector removes the interference by making the detector waveform orthogonal to the interfering signature waveforms. The MMSE detector adaptively changes the detector waveform to minimize the mean square error. Through an analysis, we discussed how changes in the power of the desired user, the interfering users, and noise can effect the MMSE detector waveform.

The orthogonality property of the decorrelating detector does not allow it to take advantage of a decrease in interference. Especially when there is a small number of active users out of a large number of potential users, the decorrelating detector without user identification suffers from unnecessary performance degradation. In order to avoid this degradation, we have suggested the two step decorrelating detector which first identifies the active users from the received waveform. The two step decorrelating detector is shown by Monte-Carlo simulation and analysis to perform better than the decorrelating detector without user identification when the SNR of the interfering users is high relative to the SNR of the desired user.

4.1 Future Work

In this thesis, we have suggested the two step decorrelating detector which shows an improved performance over the simple decorrelating detector when there is a small number of strong interfering users. We assumed symbol synchronous detection throughout the analysis. The effect of the asynchronism has to be addressed. Moreover, our analysis on the two step decorrelating detector was limited to the case of two active users out of three potential users. We also could find another algorithms which identify the active users better than the two step decorrelating detector without much increase in complexity. In addition, we made the frequent switch assumption so that no previous knowledge about the active users is relevant to identifying current active users. New receiver structures, such as the MMSE detector, could be developed if this assumption is removed.

Appendix A

Proof of Fact 3.1

Define $\mathbf{R}_M$ as an $M \times M$ matrix which has the following form.

$$\mathbf{R}_M = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & & \vdots \\ \vdots & & & \vdots \\ \vdots & & 1 & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix} \tag{A.1}$$

For convenience, we will denote the inverse of $\mathbf{R}_M$ as $\mathbf{B}_M$, i.e.,

$$\mathbf{B}_M = (\mathbf{R}_M)^{-1}. \tag{A.2}$$

From (2.19), the average BER of the decorrelating detector is given by

$$\bar{P}_{\text{dec}} = Q\left(\sqrt{\frac{P_1}{B_M\{1,1\}N_0/2}}\right) \tag{A.3}$$

Now we have to show that $B_M\{1,1\} \geq B_K\{1,1\}$ when $M > K$ to complete the proof.

From Cramer's rule,

$$B_M\{1,1\} = \frac{\text{Det}\,(\mathbf{R}_{M-1})}{\text{Det}\,(\mathbf{R}_M)} \tag{A.4}$$

The relation between the Det $(\mathbf{R}_M)$ and Det $(\mathbf{R}_{M-1})$ is given by

$$\text{Det}\,(\mathbf{R}_1) = 1 \tag{A.5a}$$

$$\text{Det}\,(\mathbf{R}_M) = \text{Det}\,(\mathbf{R}_{M-1}) - (M-1)\rho^2(1-\rho)^{M-2} \tag{A.5b}$$

The explicit expression of Det $(\mathbf{R}_M)$ can be found by solving (A.5).

$$\text{Det}\,(\mathbf{R}_M) = M\rho(1-\rho)^{M-1} + (1-\rho)^M \tag{A.6}$$

After a tedious but straightforward manipulation, the following result can be obtained.

$$B_M\{1,1\} - B_{M-1}\{1,1\} = \rho^2(1-\rho) \geq 0 \tag{A.7}$$

Now the proof is completed because

$$B_M\{1,1\} \geq B_{M-1}\{1,1\} \cdots \geq B_K\{1,1\} \tag{A.8}$$

$\square$.

Appendix B

Sub-Matrix Notation

Let $\Omega = \{1, 2, 3, \cdots, M\}$. Let $S = \{A \subset \Omega \mid n(A) = K\}$ where $n(A)$ is the number of elements of the set A and K is an given integer. The elements of S are $a_1, a_2, \cdots, a_{\binom{M}{K}}$ and they are numbered so that $a_1 = \{1, 2, \cdots, K\}, a_2 = \{1, 2, \cdots, K-1, K+1\}$ and so on. For example, for $\Omega = \{1, 2, 3, 4\}$ and $K = 3$, $a_1 = \{1, 2, 3\}, a_2 = \{1, 2, 4\}, a_3 = \{1, 3, 4\}, a_4 = \{2, 3, 4\}$.

A sub-vector of a vector $\mathbf{b}$ is defined as the vector which can be obtained by removing some elements from $\mathbf{b}$. Let $\mathbf{x}$ be an $M \times 1$ column vector. A $K \times 1$ sub-vector of $\mathbf{x}$ is denoted as $\mathbf{x}_{a_i}^K$, where a_i is an element of S. The sub-vector $\mathbf{x}_{a_i}^K$ is made by removing $M - K$ elements from $\mathbf{x}$. Those elements removed are specified by a_i. If an integer $j \in \Omega$ is also in $\Omega - a_i$, the j th element if $\mathbf{x}$ is removed from $\mathbf{x}$ to produce $\mathbf{x}_{a_i}^K$. The same column elimination applies to a row vector.

Example A.1 Let $\mathbf{x} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7]^t$ and $K = 4$. Since $M = 7$, $\Omega = \{1, 2, 3, 4, 5, 6, 7\}$. $\mathbf{x}_{a_{11}}^K = [x_1\ x_3\ x_4\ x_5]^t$ because $a_{11} = \{1, 3, 4, 5\}$, and consequently, $\Omega - a_{11} = \{2, 6, 7\}$.

A sub-matrix of a matrix $\mathbf{B}$ is defined as the matrix which can be obtained by removing columns and/or rows of $\mathbf{B}$. If $\mathbf{B}$ is a symmetric matrix and if a sub-matrix is made by removing the same rows and columns of $\mathbf{B}$, the sub-matrix is called a symmetric sub-matrix of $\mathbf{B}$. Let $\mathbf{C}$ is an $M \times M$ symmetric matrix. A $K \times K$ symmetric sub-matrix of $\mathbf{C}$ is denoted as $\mathbf{C}_{a_i}^K$, where a_i is an element of S. The symmetric sub-matrix $\mathbf{C}_{a_i}^K$ is made from $\mathbf{C}$ by removing $M-K$ rows and $M-K$ columns. Those rows and columns removed are specified by a_i, or more exactly, by $\Omega - a_i$. If an integer $j \in \Omega$ is also in $\Omega - a_i$, the j th row and column are removed from $\mathbf{C}$ to produce $\mathbf{C}_{a_i}^K$.

Example A.2 Let $\mathbf{C}$ is a 5×5 matrix and $K=3$. Since $M=5$, $\Omega = \{1,2,3,4,5\}$. $\mathbf{C}_{a_5}^K$ is made by removing the second and fourth rows and columns from $\mathbf{C}$ because $a_5 = \{1,3,5\}$, and consequently, $\Omega - a_5 = \{2,4\}$.

Appendix C

Derivation of (3.25)

Without loss of generality, we assume that the active users are user 1 and user 2, *i.e.*,

$$\mathbf{r} = \sqrt{P_1} b_1 \mathbf{s}_1 + \sqrt{P_2} b_2 \mathbf{s}_2 + \mathbf{n}.$$

The outputs of the first step decorrelating detector are

$$\begin{aligned} y_1 &= \mathbf{d}_1^t \mathbf{r} = \sqrt{P_1} b_1 + \mathbf{d}_1^t \mathbf{n} \\ y_2 &= \mathbf{d}_2^t \mathbf{r} = \sqrt{P_2} b_2 + \mathbf{d}_2^t \mathbf{n} \\ y_3 &= \mathbf{d}_3^t \mathbf{r} = \mathbf{d}_3^t \mathbf{n}. \end{aligned}$$

If $|y_2| \leq |y_3|$, the first step of the two step decorrelating detector chooses the wrong user pair. Note that $|y_2| \leq |y_3|$ is equivalent to $y_2^2 \leq y_3^2$. Therefore, the condition of choosing the wrong user pair is given by

$$y_2^2 - y_3^2 = \left(\sqrt{P_2} b_2 + \mathbf{d}_2^t \mathbf{n}\right)^2 - \left(\mathbf{d}_3^t \mathbf{n}\right)^2 \leq 0.$$

By solving the above equation, we get

$$\left(\mathbf{d}_2^t - \mathbf{d}_3^t\right) \mathbf{n} \leq \sqrt{P_2} b_2 \leq \left(\mathbf{d}_2^t + \mathbf{d}_3^t\right) \mathbf{n}$$

or,

$$\left(\mathbf{d}_2^t + \mathbf{d}_3^t\right)\mathbf{n} \le \sqrt{P_2}b_2 \le \left(\mathbf{d}_2^t - \mathbf{d}_3^t\right)\mathbf{n}.$$

For notational convenience, let t_1 denote $(\mathbf{d}_2^t - \mathbf{d}_3^t)\,\mathbf{n}$ and t_2 denote $(\mathbf{d}_2^t + \mathbf{d}_3^t)\,\mathbf{n}$. Then

$$t_1 \sim N\left(0,\ \frac{2\sigma^2}{1-\rho}\right)$$

$$t_2 \sim N\left(0,\ \frac{2\sigma^2}{1+\rho-2\rho^2}\right)$$

where $\sigma = \sqrt{N_0/2}$. Note that t_1 and t_2 are independent because they are Gaussian random variables and $E\left(t_1 t_2\right) = 0$. Without loss of generality, we assume that $b_2 = 1$. Then,

$$\begin{aligned}\Pr\left(t_1 \le \sqrt{P_2} \le t_2\right) &= \Pr\left(t_1 \le \sqrt{P_2}\right)\Pr\left(t_2 \ge \sqrt{P_2}\right)\\ &= \left(1 - Q\left(\frac{\sqrt{(1-\rho)P_2/2}}{\sigma}\right)\right)\cdot Q\left(\frac{\sqrt{(1+\rho-2\rho^2)P_2/2}}{\sigma}\right)\end{aligned}$$

and

$$\begin{aligned}\Pr\left(t_2 \le \sqrt{P_2} \le t_1\right) &= \Pr\left(t_2 \le \sqrt{P_2}\right)\Pr\left(t_1 \ge \sqrt{P_2}\right)\\ &= \left(1 - Q\left(\frac{\sqrt{(1+\rho-2\rho^2)P_2/2}}{\sigma}\right)\right)\cdot Q\left(\frac{\sqrt{(1-\rho)P_2/2}}{\sigma}\right).\end{aligned}$$

Then the probability of choosing the wrong user pair is given by

$$\begin{aligned}\text{P(choosing wrong user pair)} &= \Pr\left(t_1 \le \sqrt{P_2} \le t_2\right) + \Pr\left(t_2 \le \sqrt{P_2} \le t_1\right)\\ &= Q\left(\sqrt{\frac{(1-\rho)\text{SNR}_I}{2}}\right) + Q\left(\sqrt{\frac{(1+\rho-2\rho^2)\text{SNR}_I}{2}}\right)\\ &\quad -2Q\left(\sqrt{\frac{(1-\rho)\text{SNR}_I}{2}}\right)\cdot Q\left(\sqrt{\frac{(1+\rho-2\rho^2)\text{SNR}_I}{2}}\right).\end{aligned}$$

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