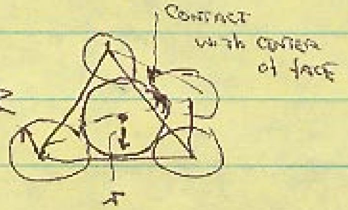
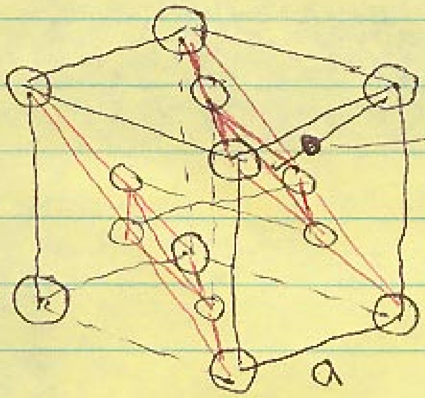
Sphere Poking out of the face of a Coordination Tetrahedron

Problem 7

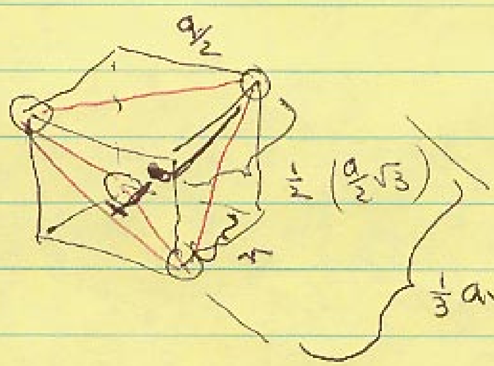




$$a\sqrt{2} = 4R$$

$$a = \frac{4}{\sqrt{2}} R$$

Looking at the sub-cube of edge $a/2$ that contains the tetrahedron



$$r = \frac{1}{3} a\sqrt{3} - \frac{1}{2}\left(\frac{a}{2}\sqrt{3}\right)$$

$$= \left(\frac{1}{3} - \frac{1}{4}\right) a\sqrt{3}$$

$$= \left(\frac{4}{12} - \frac{3}{12}\right) a\sqrt{3} = \frac{a\sqrt{3}}{12}$$

But $a = \frac{4}{\sqrt{2}} R$

$$\therefore r = \frac{4}{\sqrt{2}} R \, \sqrt{3}/12 \cdot 3$$

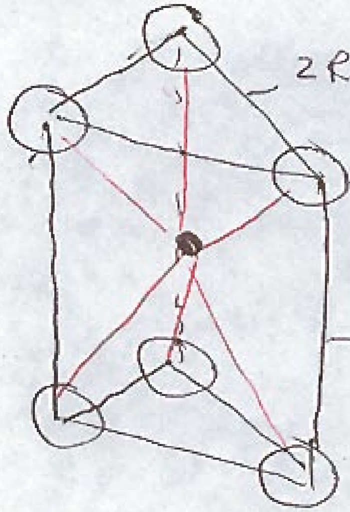
$$r = \frac{\sqrt{3}}{\sqrt{2}\, 3} R$$

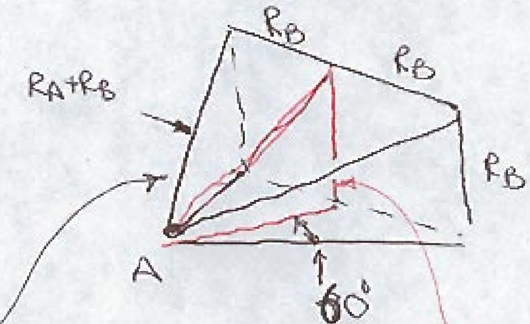
Sphere pokes out of face when

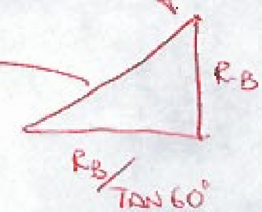
$$r/R = R_A/R_B = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{3}$$

This, I think, is greater than .225, the lower limit to R_A/R_B for tetrahedral coordination of B around A

Problem 6



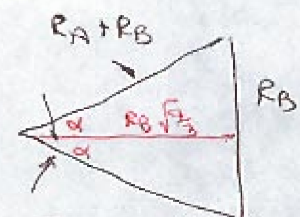




$$\left\{R_B^2 + \frac{R_B^2}{\tan^2 60°}\right\}^{\frac{1}{2}}$$

$$= R_B\left(1+\frac{1}{3}\right)^{\frac{1}{2}}$$

$$= R_B\sqrt{\frac{4}{3}}$$



$$\tan\alpha = \frac{R_B}{R_B\sqrt{\frac{4}{3}}} = \sqrt{\frac{3}{4}}$$

$$\alpha = 40.8934°$$

$$\frac{R_A+R_B}{R_B} = \frac{1}{\sin 40.8934°}$$

$$= 1.5275$$

$$\boxed{R_A/R_B = 0.5275 \to \infty}$$