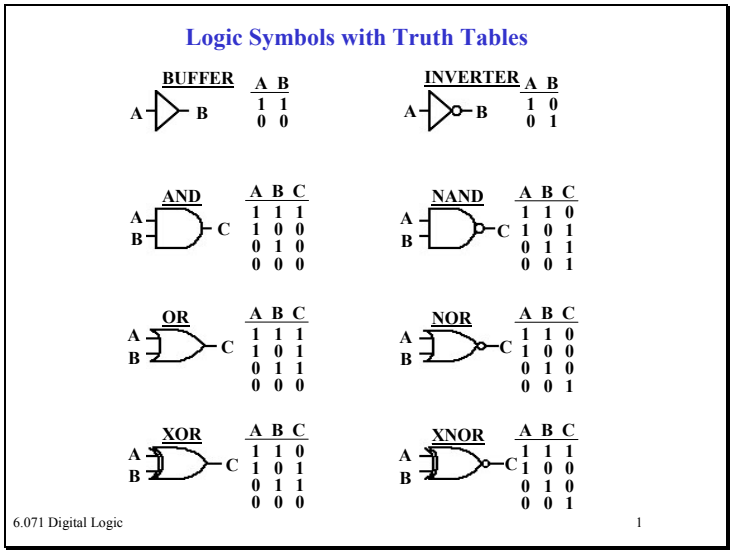
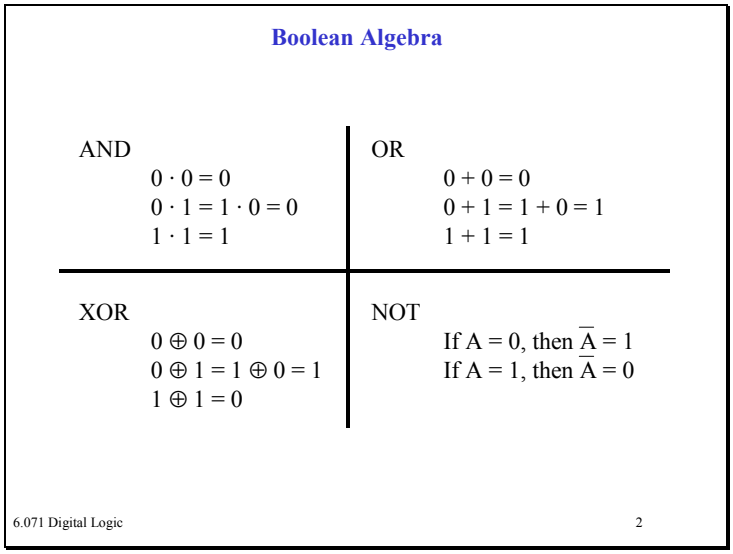
Slide 1



Digital logic can be described in terms of standard logic symbols and their corresponding truth tables. The electronics companies have made transistor based chips that carry out the function of each of these. The horizontal lines represent inputs or outputs (in the examples above read from left to right). The small circles at the outputs on the right correspond to an inverter (performs a logical NOT operation to the output).

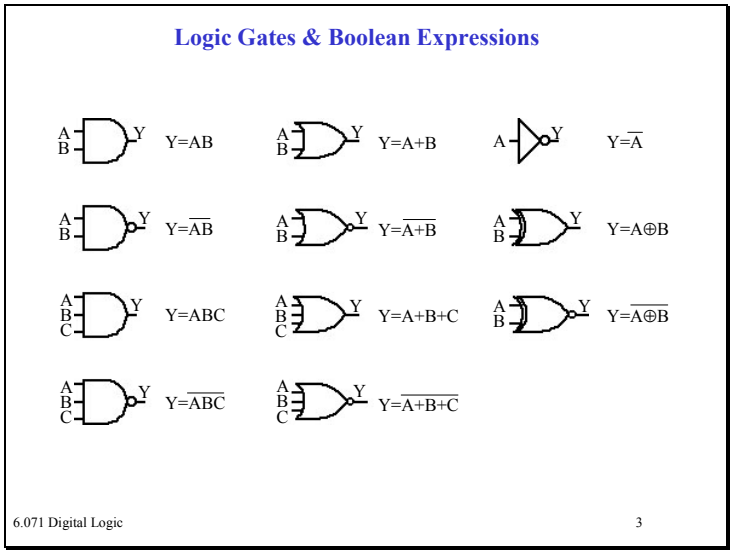
Slide 2



The action of logic circuits can be understood in terms of Boolean logic. We will typically use three elements of this. First you should recall that in our short hand notation 0 is FALSE and 1 is TRUE. The AND operation is indicated by a dot (which is often left out), and the logical table above looks familiar. The OR operation is indicated by a + sign, and the set of outcomes is mostly familiar, but notice that TRUE or TRUE is TRUE. The NOT operation is simply and inversion and is indicated by a bar over the state.

We will also have occasional need for the EXCLUSIVE OR gate which is similar to the OR but is indicated by a + with a circle around it and TRUE EXCLUSIVE OR TRUE is FALSE.

Slide 3



We can rewrite the logic gate in terms of Boolean algebra. Notice that the AND and OR gates can be extended to beyond two inputs, in fact they can have any number.

Slide 4

Table of Logic Identities

1) $A+B = B+A$
2) $AB = BA$
3) $A+(B+C) = (A+B)+C$
4) $A(BC) = (AB)C$
5) $A(B+C) = AB+AC$
6) $(A+B)(C+D) = AC+AD+BC+BD$
7) $\overline{1} = 0$
8) $\overline{0} = 1$
9) $A \cdot 0 = 0$
10) $A \cdot 1 = A$
11) $A+0 = A$
12) $A+1 = 1$
13) $A+A = A$
14) $A\overline{A} = A$
15) $\overline{\overline{A}} = A$
16) $A+\overline{A} = 1$
17) $A\overline{A} = 0$
18) $\overline{A+B} = \overline{A}\,\overline{B}$
19) $\overline{AB} = \overline{A}+\overline{B}$
20) $A+\overline{A}B = A+B$
21) $\overline{A}+AB = \overline{A}+B$
22) $A \oplus B = \overline{A}B+A\overline{B} = (A+B)\overline{AB}$
23) $\overline{A \oplus B} = AB+\overline{A}\,\overline{B}$

6.071 Digital Logic 4

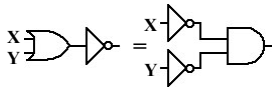
Boolean algebra is simple once you are used to it but takes some getting to know. The above sets of identities are straightforward to show. The first column you probably know (if you say them out in terms of TRUE and FALSE) and second column entries can be figured out and do not need to be memorized.

Slide 5

DeMorgan's Theorem

$\overline{(X+Y)} = \overline{X} \cdot \overline{Y}$

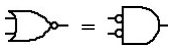
Not of the quantity, X OR Y is equal to
NOT X AND NOT Y



X	Y	X+Y	$\overline{X+Y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X} \cdot \overline{Y}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Compress to



6.071 Digital Logic 5

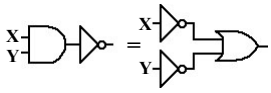
DeMorgan's theorem is probably the most important of the identities that are not immediately known. Here we show that it is true.
Notice that DeMorgan's theorem make concrete the concept that there are many ways of achieving the same truth table. In fact we will demonstrate latter that all logic can be created with NAND gates alone (though this is usually not the most convenient method). Also note that the bubble inverting the input or output of a device can take the place of an inverter.

Slide 6

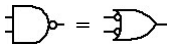
DeMorgan's Theorem 2

A second version is

$$\overline{X \cdot Y} = \overline{X} + \overline{Y}$$



or

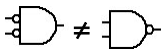


Another versions of DeMorgan's theorem, this time taking a AND type circuit to an OR. Since all logic could be created with NANDs and NANDs can be mapped into NORs then all logic could also be constructed solely out of NORs.

Slide 7

Problem:

Convince yourself that



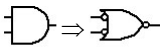
6.071 Digital Logic 7

Write out the truth table of this and convince yourself that you can not simply invert all inputs and outputs and have the same action.

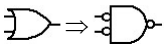
Slide 8

Bubble Pushing

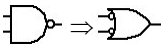
DeMorgan's Theorm says



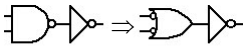
or



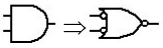
Note: as we quoted DeMorgan's Theorm, it said $\overline{A \cdot B} = \overline{A} + \overline{B}$



Now we add a NOT to each output:



or



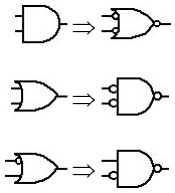
6.071 Digital Logic 8

Now we look at a much more general statement of De Morgan's theorem.

Slide 9

Bubble Pushing 2

1.) change AND to OR
or OR to AND.
2.) invert all inputs and outputs.



This does not sound as elegant or mathematical as DeMorgan's theorem, but it covers a much broader set of examples.

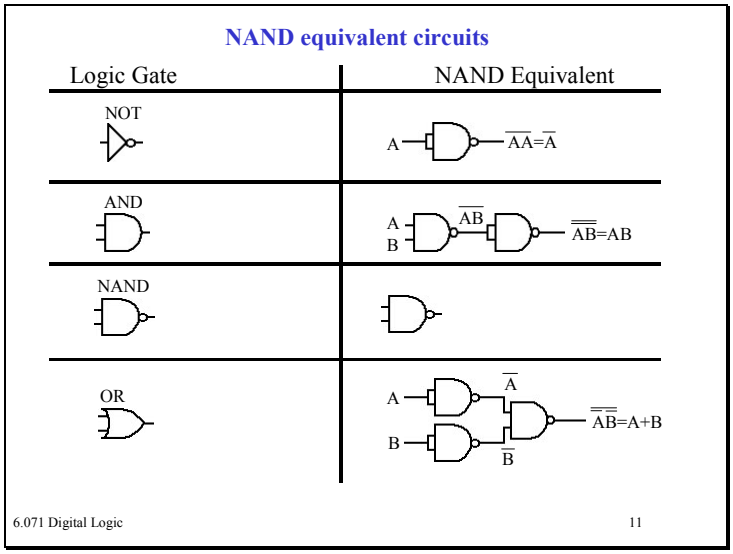
Slide 10

Generalized DeMorgan's Theorem

$$\overline{A \cdot B \cdot C} = \overline{A} + \overline{B} + \overline{C}$$
$$\overline{A + B + C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

The entire thing can be generalized to any number of inputs and always keeps the same structure.

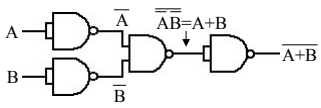
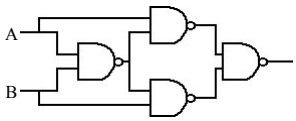
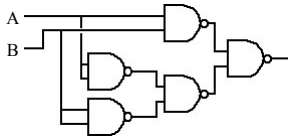
Slide 11



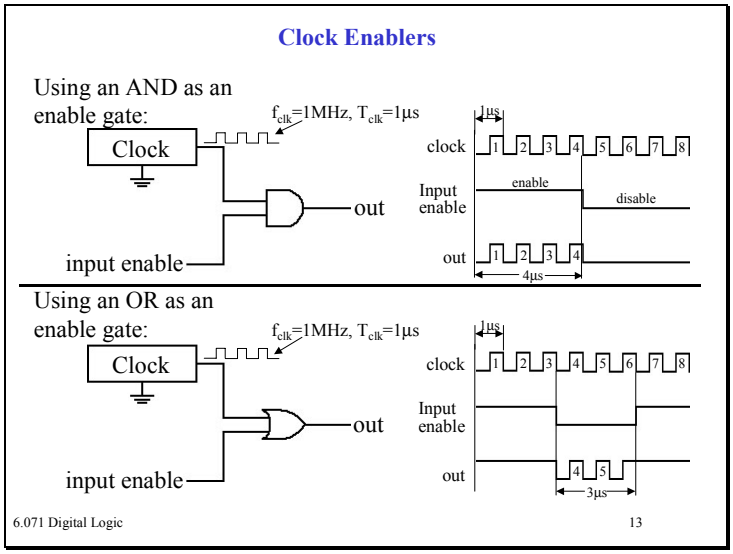
As we said, all logic can be written in terms of NANDs and here are some examples. Notice that in some cases the two inputs of the NAND are tied together to make an inverter.

Slide 12

NAND equivalent circuits 2

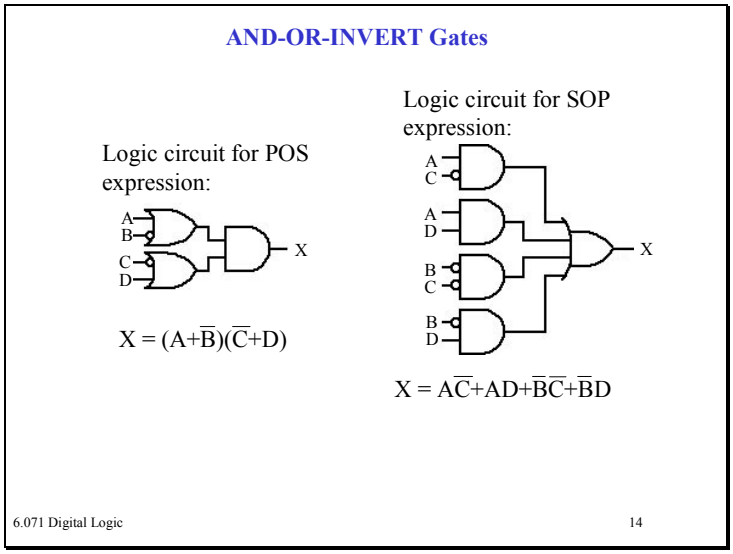
Logic Gate	NAND Equivalent
NOR	
XOR	
XNOR	

Slide 13



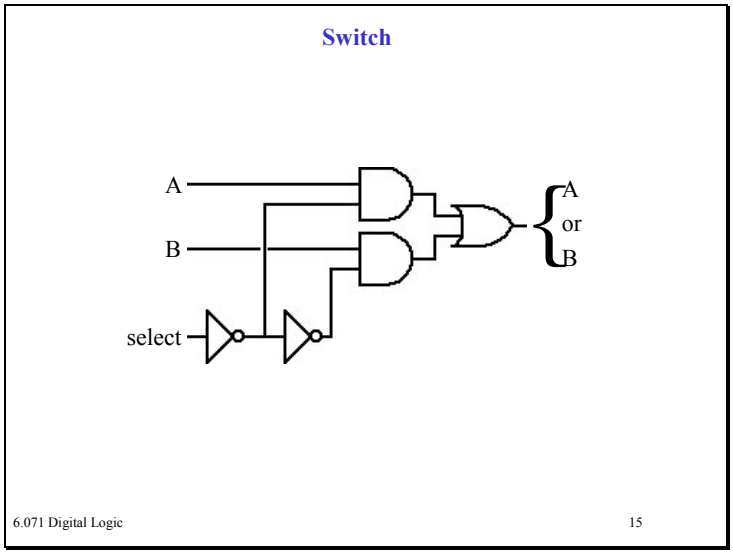
One of the many uses of digital logic is to enable a signal to be transmitted. In this case the clock is the signal and the AND or OR acts to control if it is transmitted. Notice the different actions and the output states when the device is disabled.

Slide 14



You can directly build digital circuits from Boolean logic. Two two circuits are the same, the left written as a product of sums and the right as a sum of products. There are also approaches to simplifying a network (including software packages).

Slide 15



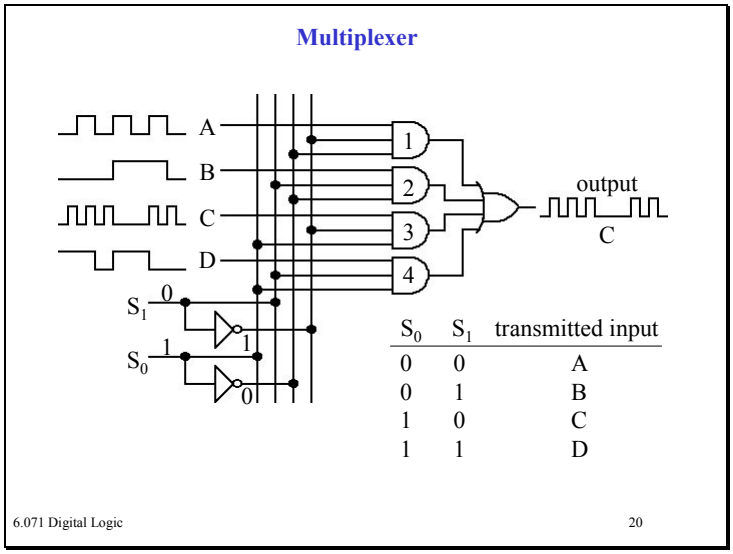
This shows the simple action of a multiplexer, it takes two inputs and switches the output between them.

22a 16-19

Go to the manufacturer's web site to obtain a datasheet of their product. Please follow these steps:

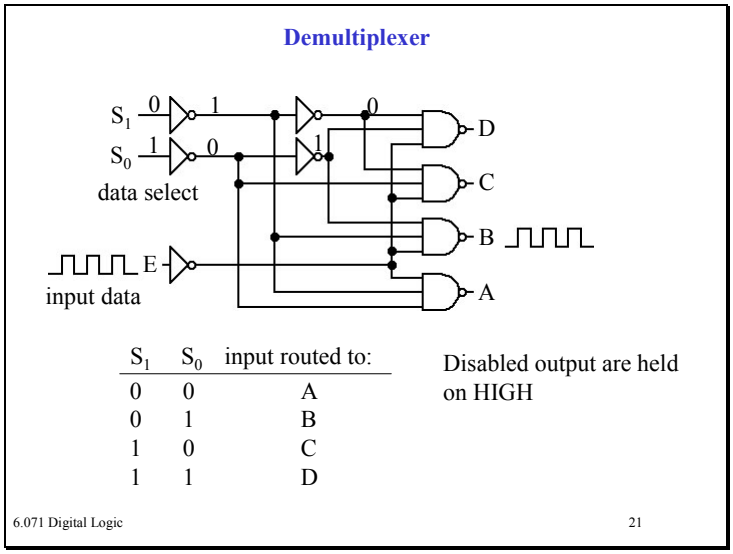
1. Go to the web site for Fairchild Semiconductor: http://www.fairchildsemi.com/
2. View the conditions of use for the web site by following the link on the home page called Site Terms & Conditions, or by following this link: http://www.fairchildsemi.com/legal/index.html
3. Return to the home page.
4. In the search box, enter the product number DM74LS157 or DM74LS158 into the search box, select "Product Folders and Datasheets" and click "go". You want the datasheet for Quad 2-Line to 1-Line Data Selectors/Multiplexers.
5. You will be presented with several options (download PDF, email for example). Select how you would like to receive this datasheet.

Slide 20



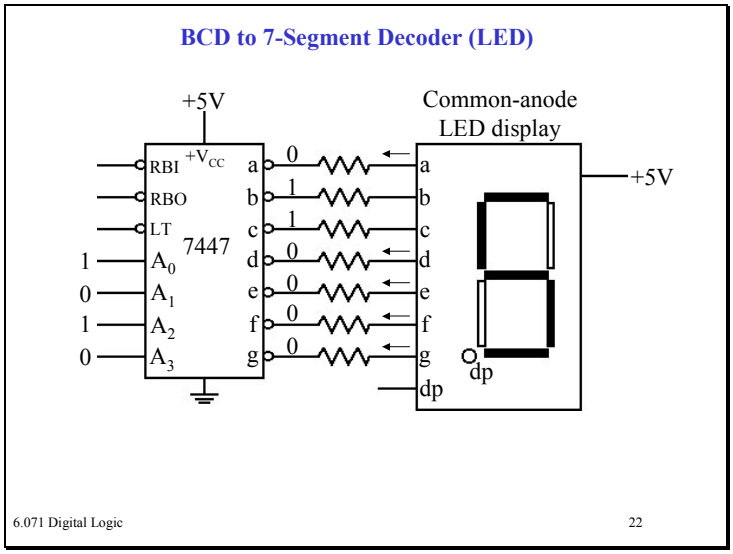
The multiplexer can be expanded to many more lines. Notice that in this case each AND gate has been expanded to three inputs so that the full coding can appear at each. We would need to add one more input to each AND for every power of two increase in the number of inputs. How do you build the same circuit using the ANDs only as enables?

Slide 21



Of course the opposite action can also be implemented. A demultiplexer sends one signal to one of many lines.

Slide 22



Another complex chip, in this case designed to control an LED numeric display.

22a 23-26

Go to the manufacturer's web site to obtain a datasheet of their product. Please follow these steps:

1. Go to the web site for Fairchild Semiconductor: http://www.fairchildsemi.com/
2. View the conditions of use for the web site by following the link on the home page called Site Terms & Conditions, or by following this link: http://www.fairchildsemi.com/legal/index.html
3. Return to the home page.
4. In the search box, enter the product number DM7446A or DM7447A into the search box, select "Product Folders and Datasheets" and click "go". You want the datasheet for BCD to 7-Segment Decoders/Drivers.
5. You will be presented with several options (download PDF, email for example). Select how you would like to receive this datasheet.