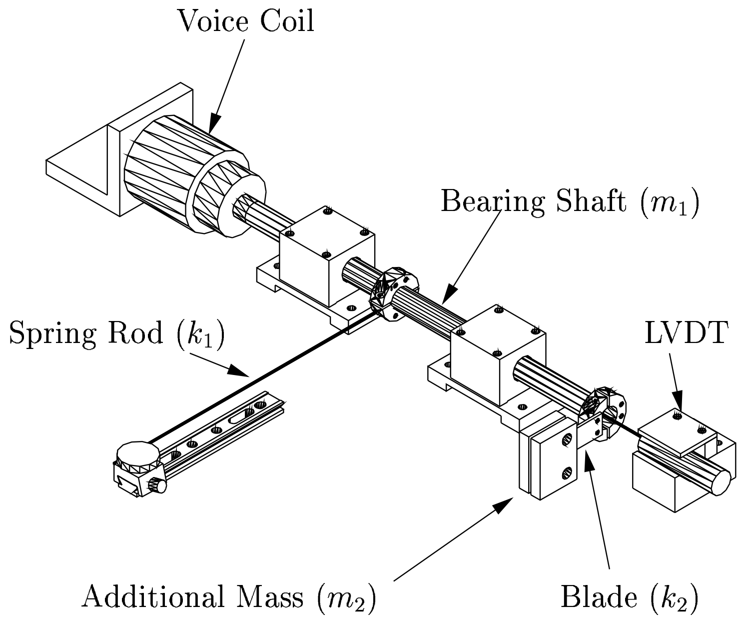
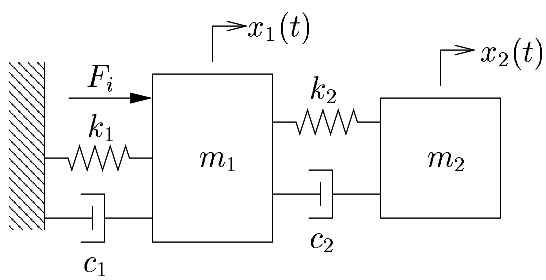
In this lab, we study the dynamics of a fourth-order mechanical system that we have constructed by coupling an additional spring and mass to the second-order system that we studied in Labs 3 and 4.



As in Lab 3, the bearing shaft (along with attached collars, LVDT core, voice coil, etc.) form a lumped mass (m_1), the slender spring rod behaves as a massless spring (k_1), and the voice coil serves as a force actuator as well as a damper (c_1). The additional mass (m_2) is attached to this system via a thin aluminum blade that can be reasonably modeled as a massless spring (k_2) with light damping (c_2). Therefore, our model of the system is:



The voice coil exerts a force F_i on the bearing shaft m_1, and the LVDT produces a voltage v_1 that is proportional to its displacement x_1.

1. As demonstrated by your instructors, measure the frequency response using the PC-based measurement system.

2. Have your lab instructor verify that the PC is disconnected from the power amplifier before turning on the function generator.

3. Use the function generator to drive the system at the first resonant frequency (the frequency corresponding to the lowest pair of complex poles). Estimate by eye the magnitude and phase of the response of the second mass relative to that of the first mass. You may find the strobe light helpful.

4. From the magnitude M_p of the peak response at the first resonance, estimate the damping associated with the first pair of complex poles. (Make the approximation that $\zeta^2 << 1$.)

5. Now drive the system at the second resonant frequency. Estimate by eye the magnitude and phase of the response of the second mass relative to that of the first mass.

6. Drive the system at the frequency of the complex zeros. Estimate by eye the magnitude and phase of the response of the second mass relative to that of the first mass. Based on what you observe, obtain a formula relating the zero frequency to the masses and stiffnesses of the components of the system.

7. We can write the transfer function relating the voltage v_1 produced by the LVDT to the voltage v produced by the function generator in the form
$$\frac{V_1(s)}{V(s)} = K\frac{s^2 + 2\zeta_z\omega_z s + \omega_z^2}{(s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2)(s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2)}$$
Determine each of the parameters in this transfer function. You should be able to determine many of them directly from the measurements you've taken so far. For the remaining parameters, adjust them by trial and error until the frequency response obtained from this transfer function matches the one you obtained by measurement.

8. If time permits, study the transient response of the system:
   (a) Plot the poles and zeros in the s plane.
   (b) Use your finger to provide disturbances at each of the two masses. What frequencies are present in the response? What determines how strongly each of the frequencies shows up?
   (c) How are the components of the time response related to the poles?
   (d) Can you come up with initial conditions that yield responses consisting of only one frequency?