Problem 1 - Palm 7.10

$$
\begin{aligned}
\frac{\Omega(s)}{\Omega_r(s)} &= \frac{K_I + K_p s}{Is^2 + (c + K_p)s + K_I} \\
\frac{\Omega(s)}{T_d(s)} &= \frac{-s}{Is^2 + (c + K_p)s + K_I} \\
\omega_n &= \sqrt{\frac{K_I}{I}} \\
2\zeta\omega_n &= \frac{c + K_p}{I}
\end{aligned}
$$

a)

$$
\begin{aligned}
\tau &= \frac{1}{\zeta\omega_n} \rightarrow \omega_n = \frac{1}{\tau\zeta} = 3.536 r/s \\
K_I &= \omega_n^2 I = 125 \\
K_p &= 2\zeta\omega_n I - c = 45
\end{aligned}
$$

b)

$$
\begin{aligned}
\omega_n &= 2.45\, r/s \\
K_I &= 62.5 \\
K_p &= 45
\end{aligned}
$$

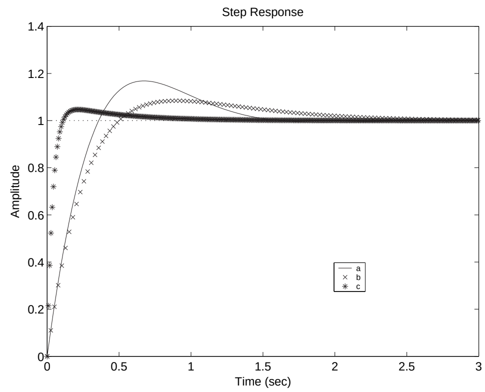
c) For $\tau = 0.4\, s$ the dominant pole must be at $s_1 = -2.5$, to get a 10x pole separation $s_2 = -250$, Thus

$$
\begin{aligned}
(s + 2.5)(s + 25) &= s^2 + \frac{c + K_p}{I} s + \frac{K_I}{I} \\
I(s + 2.5)(s + 25) &= Is^2 + (c + K_p)s + K_I \\
10s^2 + 275s + 625 &= 10s^2 + K_p + 5s + K_I \\
K_I &= 625 \\
K_p &= 270
\end{aligned}
$$

d)

$$
\begin{aligned}
\text{a) } \omega(t) &= -0.707e^{-2.5t} \sin(2.5t + 3\pi/4) + 1 \\
\text{b) } \omega(t) &= e^{-2.5t}(2t - 1) + 1 \\
\text{c) } \omega(t) &= 0.0889e^{-2.5t} - 1.0889e^{(} - 25t) + 1
\end{aligned}
$$

Maximum Overshoot: a) 1.17, b) 1.08, c) 1.05
10-90% Rise Time: a) 0.27, b) 0.35, c) 0.07



Problem 2 - Palm 7.11
The fastest but not necessarily the best way to solve this problem is to note on the time response in 7.10 that the maximum acceleration occurs at t=0. That means that using the initial value theorem and the transfer function we can get the maximum torque.

$$\frac{T(s)}{\Omega_r(s)} = \frac{IK_p s^2 + (K_I I + cK_p)s + K_I c}{Is^2 + (c + K_p)s + K_I}$$
$$T(0) = \lim_{s \to \infty} s * T(s) * u(s)$$
$$T(0) = K_p$$

T_{max}: a) 45, b) 45, c) 270
Problem 3 - Palm 7.17
I=10, c=5, $\tau = 0.1$
a) To solve this problem we need to use the final value theorem to determine the steady state errors. Looking first at P control:

$$\frac{e(s)}{\Theta(s)} = \frac{Is^2 + cs}{Is^2 + cs + K_p}$$
$$\frac{e(s)}{T_d(s)} = \frac{1}{Is^2 + cs + K_p}$$
$$e_{ss} = \lim_{s \to 0} \frac{Is^2 + cs}{Is^2 + cs + K_p} = 0$$
$$w_{ss} = \lim_{s \to 0} \frac{1}{Is^2 + cs + K_p} = \frac{1}{K_p}$$
$$K_p = 10$$
$$\tau = \frac{2I}{c} = 4$$

Thus we see that we cannot meet the time constant constraint with P only.

Looking at PD control:

$$\frac{e(s)}{\Omega(s)} = \frac{Is^2 + cs}{Is^2 + (c + K_d)s + K_p}$$

$$\frac{e(s)}{T_d(s)} = \frac{1}{Is^2 + (c + K_d)s + K_p}$$

$$e_{ss} = \lim_{s \to 0} \frac{Is^2 + cs}{Is^2 + (c + K_d)s + K_p} = 0$$

$$w_{ss} = \lim_{s \to 0} \frac{1}{Is^2 + (c + K_d)s + K_p} = \frac{1}{K_p}$$

$$K_p \geq 10$$

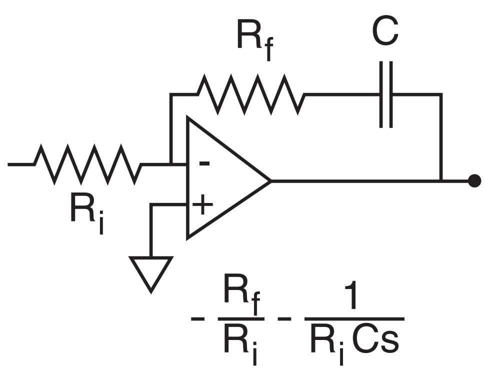
$$\tau = \frac{2I}{c + K_d} \to K_d = 195$$

$$\text{for } \tau = 1/\zeta\omega_n \quad \zeta \leq 1$$

$$K_p = \frac{1}{I}\left(\frac{c + K_d}{2\zeta}\right)^2 \geq \frac{1}{10}\left(\frac{200}{2}\right)^2 = 1000$$

b) With just P control we can not meet both the rise time and error criteria. With PD control one possible solution is to have $K_p = 1000$ and $K_d = 195$.

Problem 4 - Palm 7.26



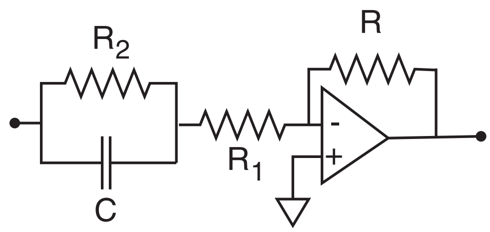
$$K_I = \frac{1}{CR_i} = 0.04 \to R_i = \frac{1}{1e^{-}6 * 0.04} = 25e^6\Omega$$

$$K_p = \frac{R_f}{R_i} = 2 \to R_f = 50e^6\Omega$$

Problem 5 - Palm 7.27

a)

$$K_P = \frac{R}{R_1 + R_2}$$



$$
\begin{aligned}
T_D &= R_2C = 1 \rightarrow\ R_2 = 1/1e-6 = 1e6\Omega \\
\alpha &= \frac{R_1}{R_1+R_2} \\
\omega_c &= \frac{1}{\alpha T_d} = 10 \rightarrow\ R_1 = \frac{R_2}{9} = 1/9e^6\Omega \\
R &= R_1 + R_2 = 10/9e6\Omega
\end{aligned}
$$

b)