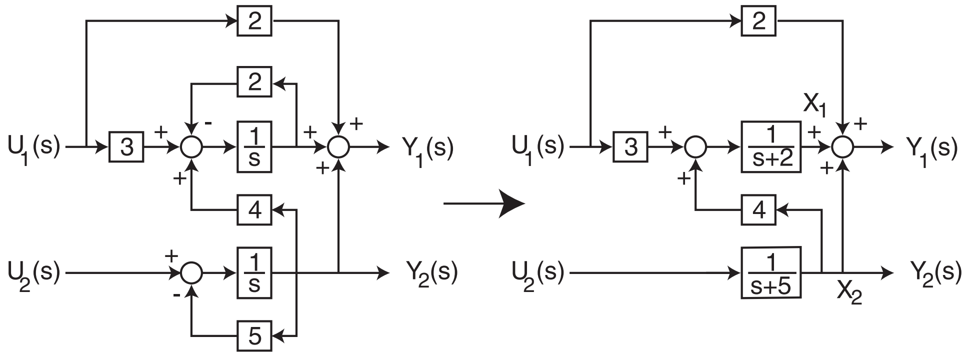
Problem 1 - Palm 3.45



a) The key to this problem is to choose the proper x_1 and x_2. In most cases, we would like to choose the state variables that represent the energy states of the system. In this case, we have two integrators. The output of these integrators are the natural states of this system. Thus, x_1 and x_2 are chosen to be the outputs of the integrators as noted in the figure. I have also chosen to simplify the block diagram but this is not necessary. Inspecting the block diagram yields the following:

$$
\begin{aligned}
x_1 &= \frac{1}{s+2}(4x_2 + 2u_1) \rightarrow \dot{x}_1 = -2x_1 + 4x_2 + 3u_1 \\
x_2 &= \frac{1}{s+5}(u_2) \rightarrow \dot{x}_2 = -5x_2 + u_2 \\
y_1 &= x_1 + x_2 + 2u_1 \\
y_2 &= x_2
\end{aligned}
$$

Putting this in State Space form

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -2 & 4 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\
A &= \begin{bmatrix} -2 & 4 \\ 0 & -5 \end{bmatrix} \\
B &= \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\
D &= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

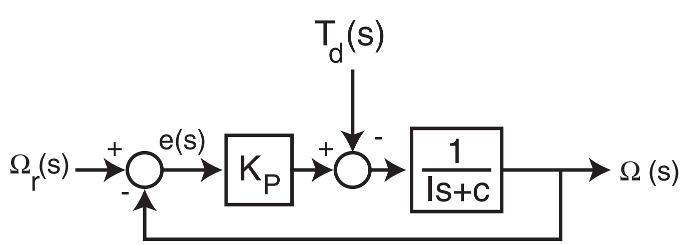
b) the four transfer functions we are trying to find are $\frac{y_1}{u_1}$, $\frac{y_1}{u_2}$, $\frac{y_2}{u_1}$, and $\frac{y_2}{u_2}$. It is possible to find the transfer function by matrix manipulation but I chose to solve by direct inspection.

$$\begin{aligned}
x_1(s) &= \frac{3}{s+2} \\
y_1(s) &= 2u_1 + x_1 = \left(2 + \frac{3}{s+2}\right) u_1 \\
\frac{y_1(s)}{u_1(s)} &= \frac{2s+7}{s+2} \\
x_2(s) &= \frac{1}{s+5} u_2(s) \\
y_1(s) &= x_2(s) + \frac{4}{s+2} x_2(s) = \left(\frac{1}{s+5} + \frac{4}{(s+2)(s+5)}\right) u_2(s) \\
\frac{y_1(s)}{u_2(s)} &= \frac{s+6}{(s+2)(s+5)} \\
\frac{y_2(s)}{u_1(s)} &= 0 \\
\frac{y_2(s)}{u_2(s)} &= \frac{1}{s+5}
\end{aligned}$$

c)

$$\begin{aligned}
y_1(s) &= \frac{y_1(s)}{u_1(s)} u_1(s) + \frac{y_1(s)}{u_2(s)} u_2(s) = \frac{2s+7}{s+2} u_1(s) + \frac{s+6}{(s+2)(s+5)} u_2(s) \\
(s+2)(s+5) y_1(s) &= (2s+7)(s+5) u_1(s) + (s+6) u_2(s) \\
(s^2+7s+10) y_1(s) &= (2s^2+17s+35) u_1(s) + (s+6) u_2(s) \\
\ddot{y}_1 + 7\dot{y}_1 + 10 y_1 &= 2\ddot{u}_1 + 17\dot{u}_1 + 35 u_1 + \dot{u}_2 + 6u_2 \\
y_2(s) &= \frac{y_2(s)}{u_2(s)} u_2(s) = \frac{1}{s+5} u_2(s) \\
(s+5) y_2(s) &= u_2(s) \\
\dot{y}_2 + 5 y_2 &= u_2
\end{aligned}$$

Problem 2 - Palm 7.8



I started this problem by writing the transfer functions for error for both inputs

$$\frac{e}{T_d} = \frac{1}{Is + (c + K_p)}$$

$$\frac{e}{\Omega_r} = \frac{Is+c}{Is+(c+K_p)}$$

a)

$$\begin{aligned}
e_{ss} &= \lim_{s \to 0} s\frac{Is+c}{Is+(c+K_p)}\frac{1}{s} = \frac{c}{c+K_p} \\
e_{ss} &= 0.1 \Rightarrow\ K_p = 45 \\
\tau &= \frac{I}{c+K_p} = 0.2\ s \\
e_{ss-T_d} &= \lim_{s \to \infty} s\frac{1}{Is+(c+K_p)}\frac{1}{s} = \frac{1}{c+K_p} = 0.02
\end{aligned}$$

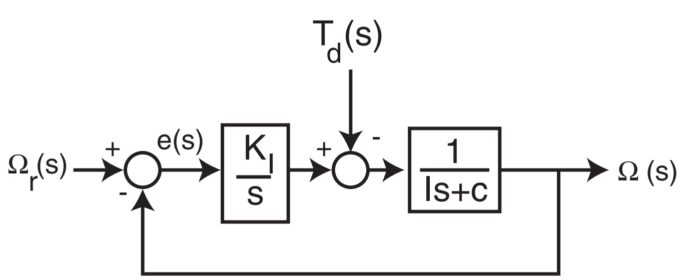
b)

$$\begin{aligned}
K_p &= 495 \\
\tau &= 0.02\ s \\
e_{ss-T_d} &= 0.002
\end{aligned}$$

c)

$$\begin{aligned}
\tau &= \frac{I}{c+K_p} = 0.1 \Rightarrow\ K_p = 95 \\
e_s s &= 0.05 \\
e_{ss-T_d} &= 0.01
\end{aligned}$$

Problem 3 - Palm 7.9



I started this problem by writing the transfer functions for error for both inputs

$$\begin{aligned}
\frac{e}{T_d} &= \frac{s}{Is^2+cs+K_I} \\
\frac{e}{\Omega_r} &= \frac{Is^2+cs}{Is^2+cs+K_I}
\end{aligned}$$

a)

$$\begin{aligned}
\omega_n &= \sqrt{\frac{K_I}{I}} \\
2\zeta\omega_n &= \frac{c}{I} \\
K_I &= \frac{c^2}{4\zeta^2 I} = 0.625 \\
\tau &= \frac{1}{\zeta\omega_n} = 4
\end{aligned}$$

b)

$$K_I = 2.5e - 4$$

$$\tau = 200$$