Problem 1

$$
\begin{aligned}
T(s) &= \frac{k}{ms^2+cs+k} = \frac{100}{s^2+10s+100} \\
\omega_n &= \sqrt{\frac{k}{m}} = 10\frac{r}{s} \\
\zeta &= \frac{10}{2\omega_n} = 0.5 \\
\omega_r &= \omega_n\sqrt{1-\zeta^2} = 7.07\frac{r}{s} \\
M_p &= \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.155 \\
M(\omega_b) &= 0.707M_p = 0.816 \Rightarrow\ \omega_b = 0 \text{ and } 11.7\frac{r}{s}
\end{aligned}
$$

Problem 2

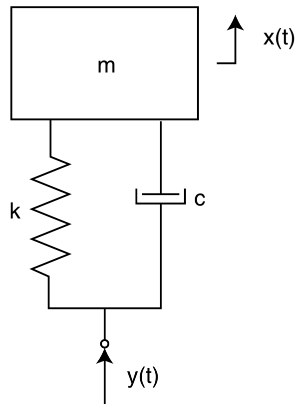

Figure 1: Model for P-2

For a quarter car model, we will assume that the mass of the car is evenly distributed between the wheels. Thus,

$$
m = \frac{m_c}{4} = \frac{w_c}{4g} = \frac{3000}{32.2*4}\frac{lb}{ft/s} = 24.84\,slugs
$$

The transfer function for this system

$$
\frac{x}{y} = \frac{bs+k}{ms^2+bs+k} = \frac{300s+3000}{25s^2+300s+3000}
$$

To get the frequency of the input, we need to convert the distance (30 ft) and speed into time

$$\begin{aligned} T &= \frac{d}{v} = \frac{30}{v} \frac{ft * hr}{mi} \frac{1}{5280} \frac{mi}{ft} \frac{3600}{1} \frac{s}{hr} \\ \omega_f &= \frac{2\pi}{T} = \frac{1}{30} \frac{5280}{3600} 2\pi v \\ \omega_f(30\, mph) &= 9.2 \frac{r}{s} \\ \omega_f(60\, mph) &= 18.43 \frac{r}{s} \end{aligned}$$

Note for this system

$$\begin{aligned} \omega_n &= \sqrt{\frac{3000}{25}} = 10.9 \frac{r}{s} \\ \zeta &= \frac{300/25}{2 * 10.9} = 0.55 \end{aligned}$$

Using the eqns from chapt 6.5,

$$\begin{aligned} x &= y \sqrt{\frac{1 + 4\zeta^2 r^2}{(1 - r^2)^2 + 4\zeta^2 r^2}} \\ F_t &= r^2 k x \\ r &= \frac{\omega_f}{\omega_n} \end{aligned}$$

For 30 mph

$$r = 0.838,\ x = 0.07\, ft,\ F_t = 148\, lb$$

For 60 mph

$$r = 0.1.67,\ x = 0.04\, ft,\ F_t = 341\, lb$$

Problem 3

Given:

$$m = 1500\, kg,\ k = 20000\, \frac{N}{m},\ \zeta = 0.04,\ y = 0.01\, m$$

Using the eqns from the chapter

$$\begin{aligned} r &= \frac{\omega_r}{\omega_n} = \frac{\omega_n \sqrt{1 - 2\zeta^2}}{\omega_n} = \frac{\sqrt{1 - 2\zeta^2}}{1} = 0.998 \\ F_t &= r^2 k y \sqrt{\frac{4\zeta^2 r^2 + 1}{(1 - r^2)^2 + 4\zeta^2 r^2}} = 2500N \end{aligned}$$

Problem 4

You can solve this problem a number of ways, I have chosen to leave the equations in terms of there original variables but you can certainly determine ζ and r for this problem.

a)

$$
\begin{aligned}
m\ddot{x} &= c(\dot{y}-\dot{x}) - kx \\
m\ddot{x} + c\dot{x} + kx &= c\dot{y} \\
\frac{X}{Y} &= \frac{sc}{ms^2+cs+k} \\
M(\omega) &= \frac{c\omega}{\sqrt{(k-\omega^2 m)^2 + c^2\omega^2}} \\
X &= \frac{c\omega}{\sqrt{(k-\omega^2 m)^2 + c^2\omega^2}} Y = \frac{2\zeta r}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} Y
\end{aligned}
$$

b) For this part of the problem you only need to realize that the force transmitted to the base is a function of the size of the displacement X and the spring constant k

$$
\begin{aligned}
F &= kX \\
F &= \frac{kc\omega}{\sqrt{(k-\omega^2 m)^2 + c^2\omega^2}} Y \\
F &= \frac{2\zeta r k}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} Y
\end{aligned}
$$

Problem 5

a) Matlab® script.

```
close all;clear all;
kt=0.04, c=7e-5, l=2e-3, il=4e-5, ke=0.04, r=0.6, im=2e-5;
i=il+im;
ov=tf([kt],conv([i c],[l r])+[0 0 ke*kt])
ot=tf([lr],conv([i c],[l r])+[0 0 ke*kt])
figure(1)
bode(ov,1 1000)
figure(2)
bode(ot,1,1000)
```

Neither system has a resonant peak and both systems have a bandwidth of 0-50 r/s.

b) The response to this input can be written as follows.

$$\omega(t) = 10\omega_{ss} + 2M(130)\sin(130t + \phi(130))$$

We can read all this data from the bode plot.

$$\omega_{ss} = M(0) = 27.7db = 24.27$$

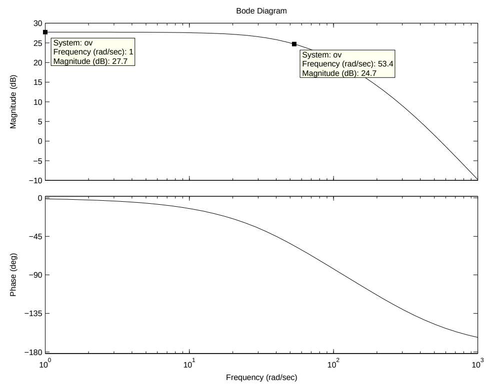

Figure 2: Frequency response for voltage input

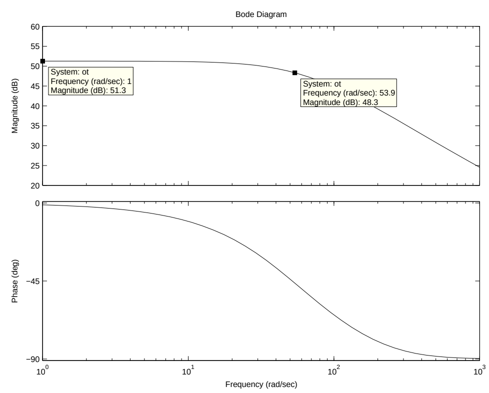

Figure 3: Frequency response for disturbance torque input

$$
\begin{aligned}
M(130) &= 18.5db = 8.41 \\
\phi(130) &= -94.7^\circ = -1.653rad \\
\omega(t) &= 242.7 + 16.8\sin(130t - 1.653)
\end{aligned}
$$