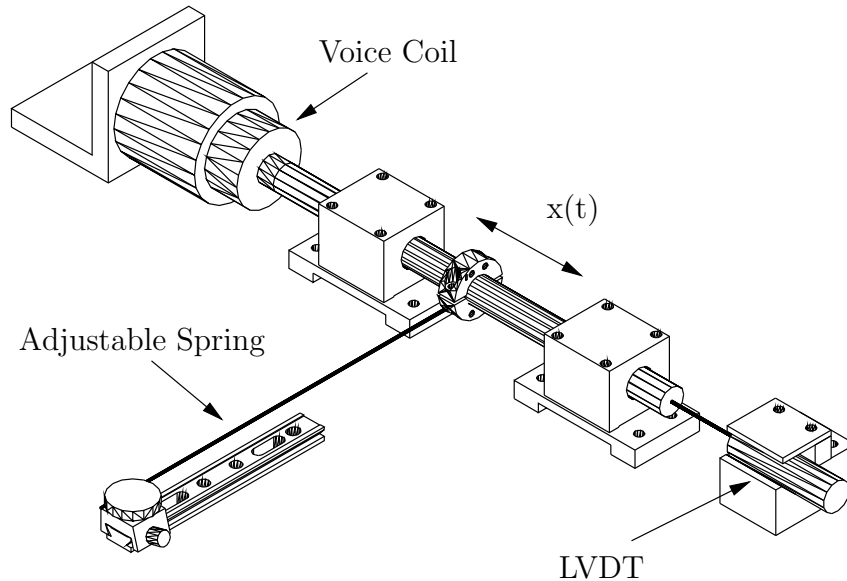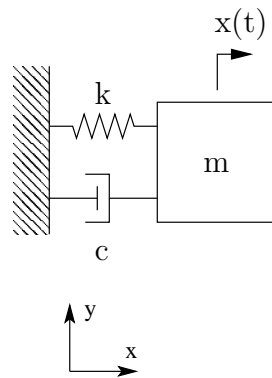In this lab, we will examine the dynamics of a second-order system composed of a spring, mass, and damper as shown below.

We model the bearing shaft (along with attached collars, LVDT core, voice coil, etc.) as a lumped mass of 0.85 kg, the slender rod as an "adjustable (linear) spring," and the voice coil as a viscous damper. Thus our system is modeled as shown below

The spring in this case is a beam with clamped-clamped end conditions. The stiffness of this beam at the point of attachment to the shaft can be computed from

$$k = \frac{12EI}{l^3} \tag{1}$$

where

$$
\begin{aligned}
E &= \text{modulus of elasticity} \\
I &= \pi r^4/4 \\
r &= \text{bearing shaft radius} \\
l &= \text{beam length}
\end{aligned}
$$

Our model has the form

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{2}$$

Solutions to this equation can be found in Table 2.3-1 of our text. As shown on pages 285-288 of our text, the system dynamics can be described graphically by plotting the system poles (or roots) in the complex plane as shown in the figure below.
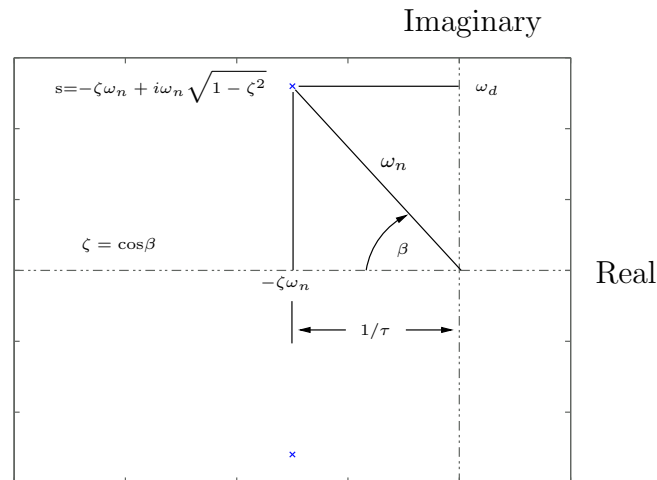
Problems

1. Given that our clamped-clamped beam has a diameter of 1.016 mm, a modulus of elasticity of 210 GPa, and a length that can be varied from about 50 to 160 mm, make a plot of stiffness as a function of length. Make sure that the plot is large and accurate so that you can read off values during lab.

2. Suppose that we've measured the system response shown in Figure 1.

    (a) Estimate ω_d and $\zeta\omega_n$.

(b) Plot the system poles in the complex plane and use your diagram to determine the value of ω_n.

(c) Given that the mass is 0.85 kg, compute the stiffness and damping parameters for this second-order model.
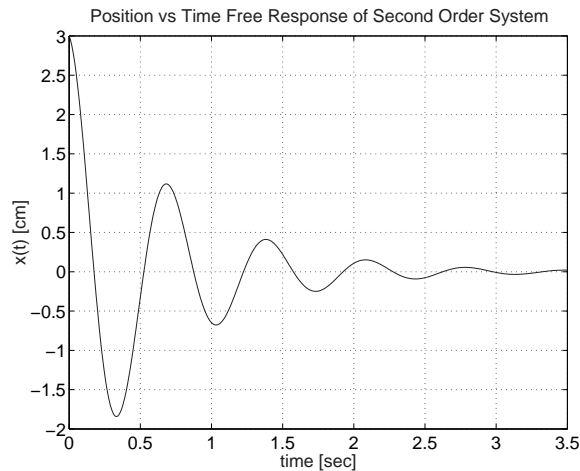

Figure 1: Second order system response

3. Make an accurate s-plane plot showing how the poles will move as the length of the spring rod is varied gradually from 50 to 160 mm if

   (a) the voice-coil circuit is closed and we measure $c = 14$ N·s/m

   (b) the voice-coil circuit is open and we measure $c = 6$ N·s/m

4. Suppose that the system is overdamped with poles at $s = -3$ and $s = -13$ rad/sec. Derive expressions for the response to

   (a) an initial displacement x_0 with zero initial velocity

   (b) an initial velocity $\dot{x}_0$ with zero initial displacement

   For both cases make an accurate sketch of the contribution due to each pole as well as the total response.

5. We often wish to design a system to move from an initial to a final position in minimum time. During lab, you will be able to run the system with the voice-coil circuit open or closed and with the length of

the spring rod anywhere from 50 to 160 mm. If the system is released with zero velocity from a displacement of 10 mm, determine the configuration of the system that minimizes the time that it takes for the system to

(a) settle within 0.5 mm of equilibrium with no overshoot

(b) settle within ± 1.0 mm of equilibrium if overshoot is acceptable

Use the values of c given in the previous problem and indicate where the optimum designs lie on your s-plane plot. Be sure to explain how you found the optimum designs.

Matlab Hints

In questions 3 and 4, you are asked to generate a number of plots. While you are not required to generate these using Matlab, all of these plots may be created quickly and easily using .m files. The following functions may be of use:

hold on holds the current plot and all axis properties so that subsequent graphing commands add to the existing graph.

hold off returns to the default mode whereby PLOT commands erase the previous plots and reset all axis properties before drawing new plots.

roots(c) computes the roots of the polynomial whose coefficients are the elements of the vector c. Example:

         C=[1 2 2]
         A=roots(C)

Returns

         A= -1+1i
            -1-1i

imag(c) returns the imaginary component of a complex number c.

real(c) returns the real component of a complex number c.

length(c) returns the length of a vector c.

You may also find it useful to begin using *for* loops. Here is an .m file that uses a *for* loop as well as the other commands.

```
%sample.m
a=1;
b=2;
c=[0:1:10];
% Creates a vector from 1-10
hold on;
for i=1:length(c)
%Begins a for loop whose index starts at 1 and counts up to the size
%of c.  length(c)=11 for this example

    d=[a b c(i)]%Forms d using the ith element of c
    f=roots(d); %in this case 1rst element of c is 0
    x=real(f);  %and the 11th element is 10
    y=imag(f);  %d=as²+bs+c
    plot(x,y,'x')


end
hold off
```

Appendix: How an LVDT works

In this lab, we'll use a linear variable differential transformer to measure the shaft's displacement. It is an electromechanical transducer that produces a voltage proportional to the core's displacement. It consists of (1) a movable magnetic core, (2) primary coils, and (3) secondary coils, as shown in the figure below.

Primary Coil

Magnetic Core
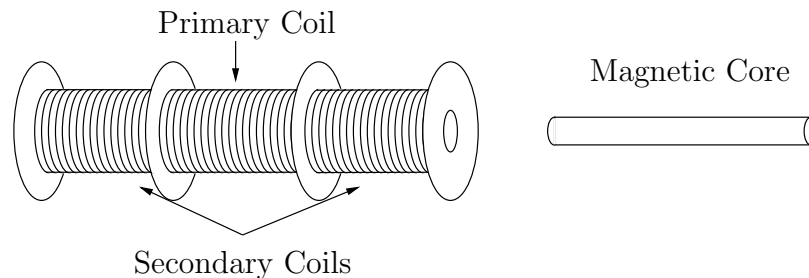
Secondary Coils

Application of an AC voltage to the primary coil induces voltages in the two secondary coils. The voltages are opposite in polarity and proportional to the area of overlap between the magnetized core and the secondary-coils. When the core is centered, the voltages in the secondary coils are equal in magnitude so they cancel out.

Figure 2 depicts a displaced magnetized core. The voltage in the top coil increases (and the voltage in the bottom coil decreases) in proportion to the displacement $x(t)$, so that V_{out} is proportional to $x(t)$.
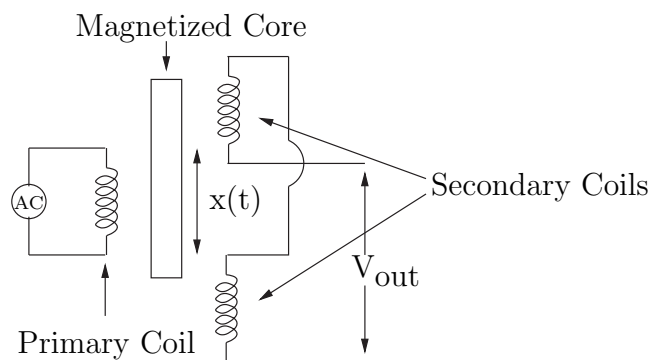
Magnetized Core

Secondary Coils

AC

x(t)

V_{out}

Primary Coil

Figure 2: LVDT circuit