Due:

1:00pm Mon. Feb. 11th
Note: No late prelabs will be accepted. Come to lab on time!

Required Reading

Palm, Section 1.7 Introduction to MATLAB®, pgs 32-39.

You will be using MATLAB® extensively throughout this course. For this labratory, you should familiarize yourself with the procedures for creating and entering vector and matrix data, the plot function, and how to create and run an M-File. MATLAB® is available on Athena and at the MECHENG clusters. Work completed on the MECHENG computers will be accessable from the computers in the labratory.

Introduction

In this lab, we measure the time constant of the spring-and-damper system sketched in Figure 1 by idealizing the system in the form shown in Figure 2.

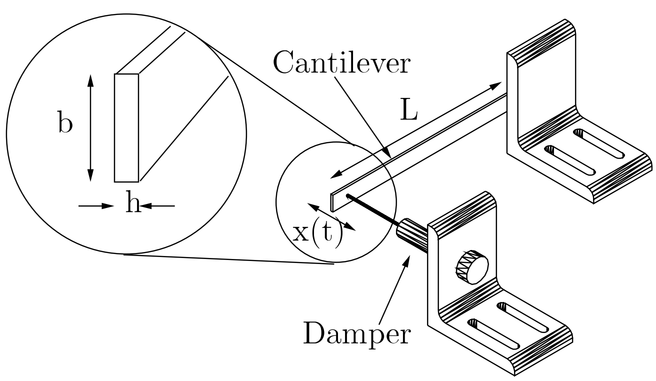

Figure 1: Picture of spring and dashpot system; L=205 mm, b=12.7 mm, h=1.27 mm.

We will model the spring-steel cantilever as a linear spring; that is, we assume that a plot of force vs. displacement is a straight line. Therefore, the

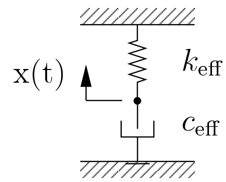

Figure 2: Idealization: spring and damper.

constitutive relationship for the "spring" has the form

$$f_k(t) = kx(t) \tag{1}$$

where

$$\begin{aligned} f_k(t) &= \text{force applied to spring in } x \text{ direction (N)} \\ k &= \text{the spring stiffness (N/m)} \\ x(t) &= \text{the displacement across the spring (m)} \end{aligned}$$

The stiffness k seen at the position L along a uniform cantilever beam can be derived from the bending beam equations as

$$k = \frac{3EI}{L^3} \tag{2}$$

where

$$\begin{aligned} E &= \text{Young's modulus (210 GPa for spring steel)} \\ I &= bh^3/12 \\ b &= \text{width of the beam's cross section} \\ h &= \text{height (thickness) of the beam's cross section} \\ L &= \text{length of the beam} \end{aligned}$$

This derivation can be found in our text book on page 248.

An air-pot attached to the end of the cantilever adds damping to the system. This air-pot—with simplifying assumptions—can be modeled as a pure damper where the damping force is proportional to velocity, so we write it's constitutive relationship as

$$f_c(t) = c\dot{x}(t) \tag{3}$$

where

$$\begin{aligned} f_c(t) &= \text{force applied to damper in the x direction (N)} \\ \dot{x}(t) &= \text{the velocity difference across the damper} \\ c &= \text{the damping coefficient} \end{aligned}$$

The spring and dashpot forces sum at the node joining them (be careful about signs as you go through this step!). Using the constitutive relationships for the spring and damper given by Equations (1) and (3), we can write a simple first-order model of the homogeneous response of our cantilever and air-pot system as

$$0 = c\dot{x}(t) + kx(t). \tag{4}$$

The response of this model to an initial unit displacement can be seen in Figure 3. The *time constant* τ is a frequently-used measure of this type of response. It is the time required for the output to transition through 63% of its total change to final value. Read pages 55-57 of our textbook for more discussion on the response of first-order systems.

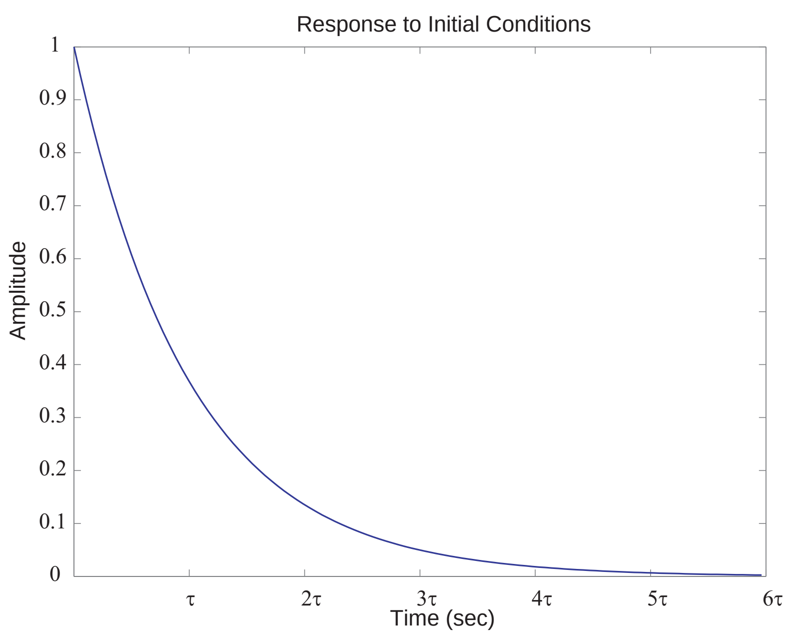

Figure 3: Response of a first-order system to an initial displacement.

Problems

1. Determine the numerical value of the spring constant k of the steel beam shown in Figure 1, at the position L where the air-pot is attached.

2. Derive the solution to Equation (4) if the system is given an initial displacement a_0 and released. The solution should be of the form

$$x(t) = Ae^{-t/\tau} \tag{5}$$

   where A is a constant to be determined from initial conditions, and τ is the *time constant*.

3. Suppose that we have measured the following output data from a spring-dashpot system, and that the dimensions of the spring steel cantilever beam are as sketched in Figure 1.

time (s)	x(t) (m)
0.05	0.603562
0.10	0.370003
0.15	0.237730
0.20	0.119463
0.25	0.097361
0.30	0.061777
0.35	0.067113
0.40	0.047411
0.45	0.018140
0.50	0.026348

   (a) Plot the data.

   (b) Fit the best exponential by "eyeballing" it. If you know of better methods use them appropriately.

   (c) Determine the damping constant c that best fits this data, given the spring-ratio calculated above.

Sample M-file for plotting

This file was generated using Matlab®

Note: % denote comments.

```
clear all;
% Deletes all of the variables from the Matlab® workspace
% In this case the ; suppresses printing the command to the
% Matlab® command line
close all;
% Deletes all currently displayed figures.
t=[0.05:0.05:0.5]
% Creates a vector t from 0.05 to 0.5 with step
% values of 0.05. t = [0.05 0.1 0.15 ... 0.5]
xt=[0.6, 0.37, 0.24, 0.12, 0.1, 0.06, 0.07, 0.05, 0.02, 0.03]
% Creates a 10 column vector xt with the entered values
% the comma is not required.
plot(t,xt)
% Creates an X-Y plot of t (x-axis) vs. xt (y-axis)
% Plot has many options type help plot for more details
xlabel('time (s)')
% Creates the label time (s) for the x-axis
% The hyphens are required but not displayed.
ylabel('X position (cm)')
% Creates the label X position (cm) for the y-axis
title('Position vs Time')
% Places the title Position vs Time at the top of the plot
```