Problem 1 - Palm 3.38
To start with we need to choose our state variables. In this case, energy is stored in the inductor and the capacitor. Thus we would like to make them our state variables, ie. $x_1 = i_1$ and $x_2 = v_c$ where v_c is the voltage across the capacitor. The four following equations can be found using voltage, current, and component laws.

$$\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} + i_3 R \\ i_3 R &= v_2 + v_3 \\ i_1 &= i_2 + i_3 \\ v_c &= \frac{1}{C}\int i_2 dt \end{aligned}$$

Now we need to put these equations in state form.

$$\begin{aligned} L_1 \frac{dx_1}{dt} &= v_1 - i_3 R = v_1 - v_2 - v_c = v_1 - v_2 - x_2 \\ \frac{dv_c}{dt} &= \frac{i_2}{C} = \frac{1}{C}(i_1 - i_3) = \frac{1}{C}\left(x_1 - \frac{v_2}{R} - \frac{v_c}{R}\right) = \frac{1}{C}\left(x_1 - \frac{v_2}{R} - \frac{x_2}{R}\right) \end{aligned}$$

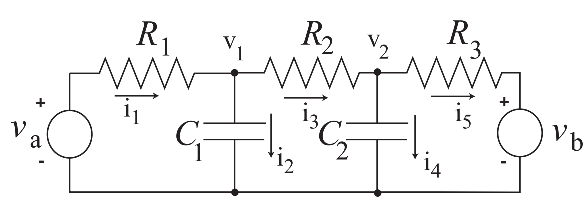
The outputs are expressed as

$$\begin{aligned} i_1 &= x_1 \\ i_2 &= \left(x_1 - \frac{v_2}{R} - \frac{x_2}{R}\right) \end{aligned}$$

Thus A, B, C, and D are

$$\begin{aligned} A &= \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \\ B &= \begin{bmatrix} 1/L & -1/L \\ 0 & -1/RC \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \\ 1 & -1/R \end{bmatrix} \\ D &= \begin{bmatrix} 0 & 0 \\ 0 & -1/R \end{bmatrix} \end{aligned}$$

Problem 2 - Palm 3.40 In this case our state variables are our outputs



$x_1 = v_1 = v_{c1}$ and $x_2 = v_2 = v_{c2}$. From the circuit,

$$\begin{aligned} v_a &= R_1 i_1 + v_1 \\ v_1 &= \frac{1}{C_1}\int i_2 dt \\ v_1 &= R_2 i_3 + v_2 \\ v_2 &= \frac{1}{C_2}\int i_4 dt \\ v_2 &= R_3 i_5 + v_b \\ i_1 &= i_2 + i_3 \\ i_3 &= i_4 + i_5 \end{aligned}$$

Solving for dv_1/dt and dv_2/dt

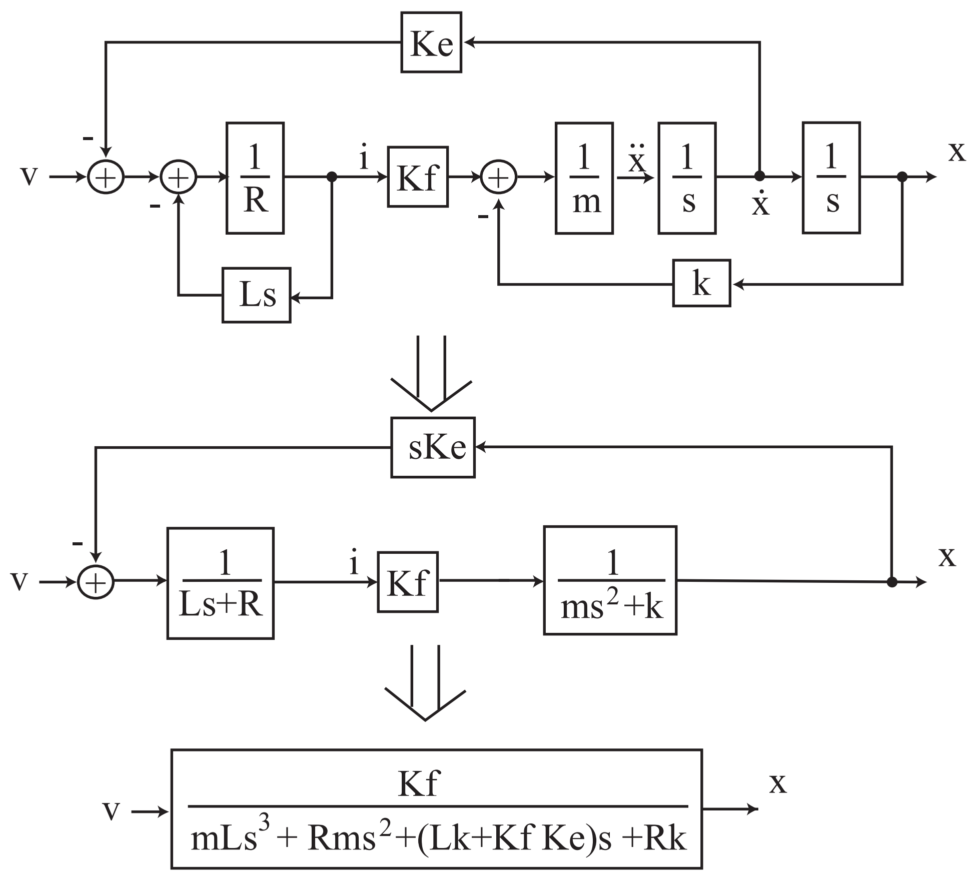
$$\begin{aligned} C_1\frac{dv_1}{dt} &= i_2 = i_1 - i_3 = \frac{1}{R_1}(v_a - v_1) - \frac{1}{R_2}(v_1 - v_2) \\ C_1\frac{dx_1}{dt} &= \frac{v_a}{R_1} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) x_1 + \frac{x_2}{R_2} \\ C_2\frac{dv_2}{dt} &= i_4 = i_3 - i_5 = \frac{1}{R_2}(v_1 - v_2) - \frac{1}{R_3}(v_2 - v_b) \\ C_2\frac{dx_2}{dt} &= \frac{x_1}{R_2} - \left(\frac{1}{R_2} + \frac{1}{R_3}\right) x_2 + \frac{x_b}{R_3} \end{aligned}$$

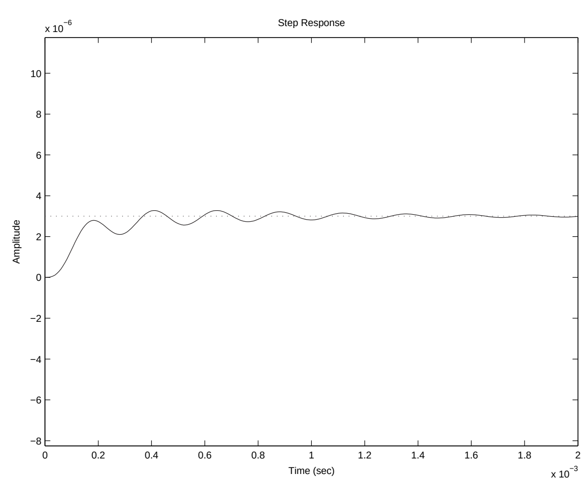
Thus A, B, C, and D are

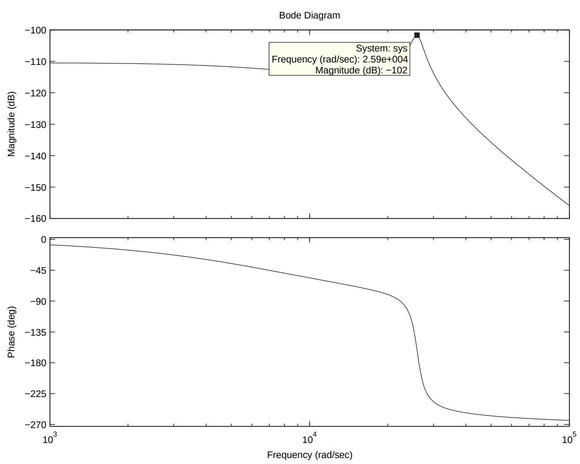
$$\begin{aligned} A &= \begin{bmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) & \frac{1}{R_2 C_1} \\ \frac{1}{C_2 R_2} & -\frac{1}{C_2}\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \\ B &= \begin{bmatrix} \frac{1}{R_1 C_1} & 0 \\ 0 & \frac{1}{R_3 C_2} \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D &= 0 \end{aligned}$$

Problem 3 - Palm 3.31
a) Roots $= -7.1e3,\ -1.45e3 \pm 2.65e4i$
b) $x_{ss} = 3x10^{-6}$, using dominant pole approximation (not really very good here) $\tau = 6.9x10^{-4}s$, and $t_{ss} = 4\tau = 2.756x10^{-3}s$
c)
Problem 4 - Palm 6.32
From the bode plot

$$\omega_p \approx 2.59e4\, rad/s$$
$$2.43e4 \geq \omega_b \leq 2.72e4\, rad/s$$


Ke
v
i
Kf
x
1/R
1/m
1/s
1/s
Ls
k
sKe
1/(Ls+R)
1/(ms^2+k)
Kf / (mLs^3 + Rms^2+(Lk+Kf Ke)s +Rk)


Step Response
x 10^-6
Amplitude
Time (sec)
x 10^-3
10
8
6
4
2
0
−2
−4
−6
−8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Bode Diagram
System: sys
Frequency (rad/sec): 2.59e+004
Magnitude (dB): −102
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)