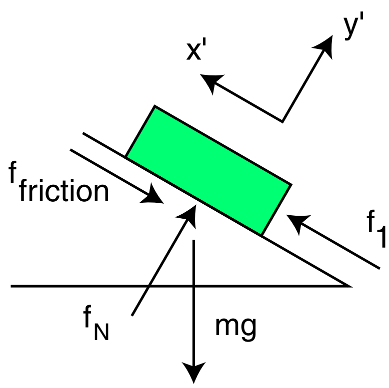
Problem 1 - Palm 1.6



a) Always begin a force balance with a FBD (free body diagram). If we assume $v > 0$, the dry friction will act in opposition to f_1 (as shown).

$$\begin{aligned} \Sigma F_{\acute{y}} &= 0 = f_n - mg\cos\phi \\ \Sigma F_{\acute{x}} &= m\ddot{x} = f_1 - mg\sin\phi - f_{\text{friction}} \\ m\ddot{x} &= f_1 - mg\sin\phi - \mu mg\cos\phi \\ \ddot{x} &= f_1/m - g(\sin\phi + \mu\cos\phi) \end{aligned}$$

b) m=2 kg; $\phi = 30°$; $v(0)$=3 m/s;μ=0.5.
i)

$$\begin{aligned} f_1 &= 50\text{N} \\ \ddot{x} &= \dot{v} = 50/2 - 9.8(\sin 30° + 0.5\cos 30°) \\ \dot{v} &= 25 - 9.8(0.5 + 0.433) = 15.86(m/s^2) \end{aligned}$$

To find the velocity we need to integrate $\dot{v}$ with respect to time

$$\begin{aligned} v(t) &= \int_0^t \dot{v}dt = \int_0^t 15.86dt = 15.86t + v(0) \\ v(t) &= 3 + 15.9t(m/s) \end{aligned}$$

ii)

$$f_1 = 5\text{N}$$

$$\begin{aligned} \ddot{x} &= \dot{v} = 5/2 - 9.8(\sin 30^\circ + 0.5\cos 30^\circ) \\ \dot{v} &= 2.5 - 9.8(0.5 + 0.433) = -6.64(m/s^2) \end{aligned}$$

Now, the acceleration is now negative, meaning the mass slows down. To find out whether the mass comes to rest or if it actually reversed direction and begins going downhill at some point, we can consider a new FBD where the friction force, f_{friction}, acts in the up hill direction:

$$\begin{aligned} \Sigma F_{\hat{x}} &= m\ddot{x} = f_1 - mg\sin\phi + f_{\text{friction}} \\ \ddot{x} &= f_1/m - g(\sin\phi - \mu\cos\phi) = 2.5 - 9.8(0.5 - 0.433) = 1.8(m/s^2) \end{aligned}$$

Thus, if the mass had a velocity in the downhill(negative x) direction, the acceleration would act to slow it down (since the acceleration is positive). The mass will not reverse direction, therefore; it will come to a rest.

Problem 2 - Palm 2.5

To simplify the problem let

$$f = (T - mg)$$

Thus the equation of motion for this system becomes;

$$m\dot{v} = T - mg - cv \Rightarrow \dot{v} + \frac{c}{m}v = \frac{f}{m}$$

The equation of motion is a first order ordinary differential equation (ODE). The general solution to a 1rst order ODE of the form $\dot{u} + p(t)u = f(t)$ and $u(t_0) = b$ is the following;

$$u(t) = Ke^{-P(t)} + e^{-P(t)}\int_{t_0}^{t} f(t)e^{P(t)}dt$$

where

$$\begin{aligned} K &= be^{P(t)} \\ P(t) &= \int_{t_0}^{t} p(t)dt \end{aligned}$$

Matching our system to the general solution, when $h(0) = v(0) = 0$, yields the following;

$$\begin{aligned}
P(t) &= \int_0^t \frac{c}{m} dt = \frac{c}{m} t \\
b &= 0 \Rightarrow K = 0 \\
v(t) &= e^{-\frac{c}{m}t} \int_0^t \frac{f}{m} e^{\frac{c}{m}t} dt = e^{-\frac{c}{m}t} \left(\frac{f}{m}\right) \frac{m}{c} \left. e^{\frac{c}{m}t} \right|_0^t \\
&= e^{-\frac{c}{m}t} \frac{f}{c} \left(e^{\frac{c}{m}t} - 1\right) = \frac{f}{c} \left(1 - e^{-\frac{c}{m}t}\right)
\end{aligned}$$

The height of the rocket, $h(t)$, is just the time integral of the velocity.

$$\begin{aligned}
h(t) &= \int v(t) dt = \int_0^t \frac{f}{c} (1 - e^{-\frac{c}{m}t}) dt \\
&= \frac{f}{c} \left. \left(t + \frac{m}{c} e^{-\frac{c}{m}t}\right) \right|_0^t = \frac{f}{c} \left(t + \frac{m}{c} \left(e^{-\frac{c}{m}t} - 1\right)\right)
\end{aligned}$$

Alternately we can solve this problem using Laplace Transforms. For the laplace transform method, we need to restate the equation of motion to account for the unit step input $(u(t))$:

$$m\dot{v} = T - mg - cv \Rightarrow m\dot{v} + cv = fu(t) \text{ for } t > 0$$

Taking the Laplace transform of this system:

$$\begin{aligned}
L(m\dot{v} + cv) &= v(s)(ms + c) \\
L(fu(t)) &= f \frac{1}{s} \\
v(s)(ms + c) &= f \frac{1}{s} \\
v(s) &= f \left(\frac{1}{s(ms + c)}\right)
\end{aligned}$$

By partial fraction expansion:

$$v(s) = f \left(\frac{a}{s} - \frac{b}{ms + c}\right) = f \left(\frac{1}{cs} - \frac{m}{c} \frac{1}{ms + c}\right) = \frac{f}{c} \left(\frac{1}{s} - \frac{1}{s + \frac{c}{m}}\right)$$

Taking the inverse Laplace transform to find $v(t)$:

$$\begin{aligned} L^{-1}(v(s)) &= v(t) = \frac{f}{c}(u(t) - e^{-\frac{c}{m}t}) \\ \text{Thus for t} > 0 \ v(t) &= \frac{f}{c}(1 - e^{-\frac{c}{m}t}) \end{aligned}$$

Problem 3 - Palm 2.6

$$\begin{aligned} 2\dot{v} &= 900 - 8v \Rightarrow \dot{v} + 4v = 450 \\ v(0) &= 0 \end{aligned}$$

Once again the is a 1rst order ODE. Using the general solution to a 1rst order ODE yields the following

$$\begin{aligned} P(t) &= \int_0^t 4dt = 4t \\ b &= 0 \Rightarrow K = 0 \\ v(t) &= e^{-4t} \int_0^t 450 e^{4t} dt = e^{-4t} \left(\frac{450}{4} e^{4t} \right) \Big|_0^t \\ &= \frac{450}{4} e^{-4t} \left(e^{4t} - 1 \right) = \frac{450}{4} \left(1 - e^{-4t} \right) \end{aligned}$$

Alternately, we can solve the problem using Laplace transforms.

$$\begin{aligned} 2\dot{v} + 8v &= 900u(t) \\ L(2\dot{v} + 8v) &= v(s)(2s + 8) = 2v(s)(s + 4) \\ L(900u(t)) &= \frac{900}{s} \\ v(s) &= 450 \left(\frac{1}{s(s+4)} \right) = 450 \left(\frac{1}{4s} - \frac{1}{4(s+4)} \right) \\ L^{-1}(v(s)) &= 450 L^{-1} \left(\frac{1}{4s} - \frac{1}{4(s+4)} \right) = \frac{450}{4}(u(t) - e^{-4t}) \\ v(t) &= \frac{450}{4}(1 - e^{-4t}) \ t > 0 \end{aligned}$$

Since velocity is the derivative of position with respect to time

$$\begin{aligned} x(t) &= \int_0^t v(t)\,dt = \frac{450}{4}\int_0^t (1-e^{-4t})\,dt \\ &= \frac{450}{4}\,(t+\frac{1}{4}e^{-4t})\Big|_0^t \\ &= \frac{450}{4}\left(t+\frac{1}{4}e^{-4t}-\frac{1}{4}\right) \end{aligned}$$

We want to find the time at which x = 2500 m, thus

$$\begin{aligned} 2500 &= \frac{450}{4}\left(t+\frac{1}{4}e^{-4t}-\frac{1}{4}\right) \\ t+\frac{1}{4}e^{-4t} &= \frac{200}{9}+0.25 \simeq 22.47 \end{aligned}$$

this equantion can be solved either graphically or by newtonian iteration. More simply one can take advantage of the fact that an exponential decays to 2% of it's value after 4τ. For this case τ=0.25 which means that the exponential term is largely irrelevent past 1 second. Thus $t = 22.47$.

Problems 4-5 Palm 2.9

I have combined the solutions for problems 4-5. For problem 4 you are only required to determine the steady-state response (v_{ss}) and the time to 98% of v_{ss}. Problem 5 requires you to determine the time response of the system and to plot it using MATLAB®. Only one sample script for plotting is presented.

a) $2\dot{v}+v=10f(t)$; $f(t)=u(t)$; $v(0)=0$;
Since $v(0)=0$ we know the response of this system consist only of the forced response. Using Table 2.2-1 in the text we find that for first order systems in the form $m\dot{v}+cv=f$:

$$\begin{aligned} v(t) &= \frac{f}{c}(1-e^{-\frac{t}{\tau}}) \\ v_{ss} &= \frac{f}{c} \\ v(4\tau) &= 0.98v_{ss} \\ \tau &= \frac{m}{c} = \text{time constant} \end{aligned}$$

for our system $m = 2$, $c = 1$, and $f = 10$. Thus

$$\begin{aligned} \tau &= 2s \\ v(t) &= 10(1 - e^{-\frac{t}{2}}) \\ t_{98\%} &= 4 * 2 = 8s \\ v_{ss} &= 10 \end{aligned}$$

One way to plot this response using MATLAB® is the following:

```
clear all; close all;
%clears workspace of old variables and plot.
m=2; c=1; f=10;
%Creates variables m, c, and f with listed values
tau=m/c;
%Creates and calculates variable tau.
t=[0:0.01:10]
%Creates a vector from 1-10 s with 1000 points.
%Simulated a little beyond 98% time of 8 seconds for completeness
v=(f/c)*(1-exp(-t/tau));
%Calculates a vector v.
plot(t,v)
%Creates and X-Y plot with t on x-axis and v on y.
xlabel('Time (s)'); ylabel('v'); title('time vs. v');
%Creates title and labels for the x and y axis.
```

b) For this part the equation of motion and the forcing function are identical. The only difference is that we now have and initial condition $v(0) = 5$. We solve this with the concept of superposition. That is we can add the solution for the forced response with the solution for a free response (the response for a system with initial conditions but not forcing function) to get the total response. From table 2.2-1 we find that the free resoponse is the following;

$$\begin{aligned} v(t) &= v(0)e^{-\frac{t}{\tau}} \\ \tau &= m/c \end{aligned}$$

Combining the free response with the forced response results in;

$$v(t) = v(0)e^{-\frac{t}{\tau}} + \frac{f}{c}(1 - e^{-\frac{t}{\tau}})$$

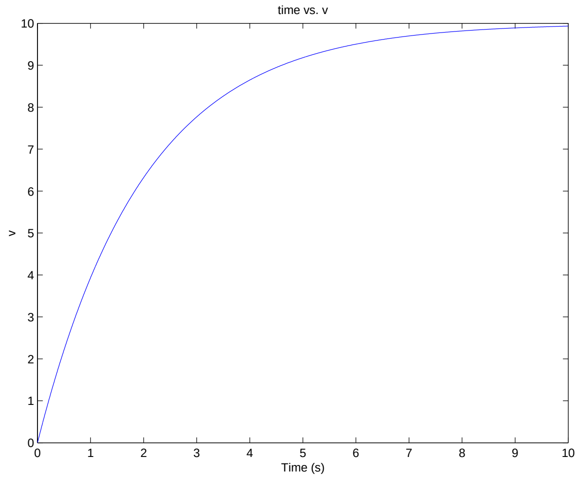


Figure 1: Response for 2.9-a

Since τ for the free and forced response are equal and $v_{ss} = 0$ for the free system:

$$\begin{aligned} t_{98\%} &= 4 * 2 = 8s \\ v_{ss} &= 10 \end{aligned}$$

c) This is identical to part a except $f(t) = 20u(t)$. Thus;

$$\begin{aligned} t_{98\%} &= 4 * 2 = 8s \\ v_{ss} &= 200 \end{aligned}$$

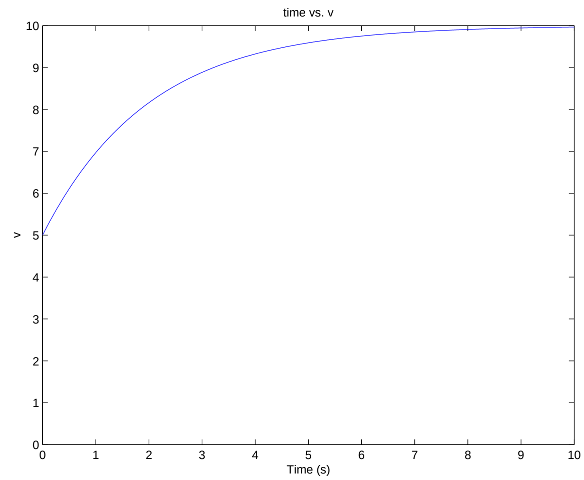


Figure 2: Response for 2.9-b

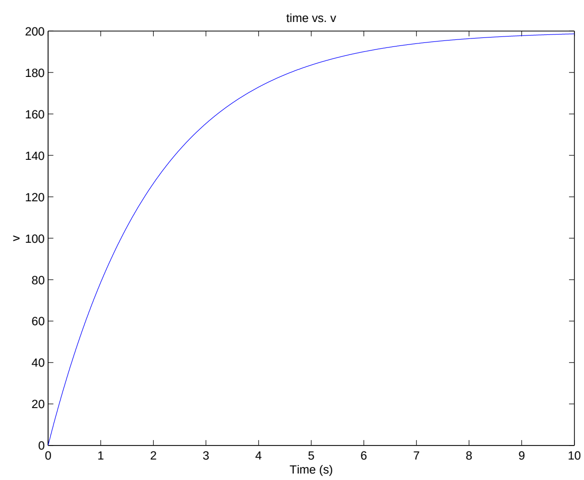


Figure 3: Response for 2.9-c

Problem 6 - Palm 1.15

a) $(-3+5i)(-6+7i) = 18 - 21i - 30i - 35 = -17 - 51i$
b)

$$\frac{-3+5i}{-6+7i} = \frac{(-3+5i)(-6-7i)}{(-6+7i)(-6-7i)} = \frac{18+21i-30i+35}{36+49} = 0.6235 - 0.106i$$

Problem 7 - Palm 1.16

a) $x = -10 - 5i = 11.2e^{-2.68i} = 11.2e^{(2\pi-2.68)i} = 11.2e^{3.61i}$
Note: The 'atan2' function in MATLAB® returns the (correct) 4 quadrant angle. Drawing the location of the number on the complex plane should eliminate ambiguity about the angle of the point in polar coordinates. The angle is **NOT** equal to 26.6° (which is in the first quadrant); it is 180° off from this at −153.4°.
b) $x = 6e^{8i} = 6e^{(8-2\pi)i} = 6\cos(8) + i\sin(8) = -0.87 + 5.9i$
$r = 6$, $\phi = 8 - 2\pi = 1.72\ rad = 98.4°$
Again, drawing the location of the point may be helpful.