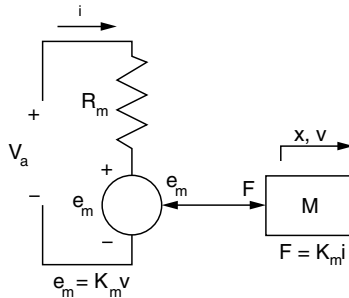


Figure 1: Schematic of Lab setup

For Lab 4, you will be examining the step response of the spring/mass/damper system studied in Lab 3. For this lab, we need to generate a new system model since we are using the voice coil motor to excite the system.

Figure 2: Circuit of voice coil motor driven by a voltage (V_a)

In general, dc motors (a voice coil motor is a single phase dc motor that produces linear motion) produce a force that is proportional to the current driven through the motor

$$F_m = K_m i \quad (1)$$

where K_m is the motor force constant with units of N/A for linear systems. A dc motor can be driven either by a current or voltage source. For this lab you will be driving the system with a voltage source (the power amplifier). The circuit diagram for this system is shown in Figure 2. In this circuit, V_a is the voltage from the power amplifier, R_m is the electrical resistance of the motor coils, and x is motor displacement (For our purposes, we can ignore the coil inductance and capacitance). The actuator introduces a new variable e_m called the back electromotive force (back emf). e_m is the voltage

that is generated in the motor coils when the coils are moved through a magnetic field. In fact it can be shown that

$$e_m = K_m \dot{x} \quad (2)$$

$$\text{Units: } [V] \equiv \frac{[J]}{[C]} = \frac{[J/s]}{[C/s]} = \frac{[W]}{[A]} = \frac{[N \cdot m]}{[A \cdot s]}$$

Summing the voltages around the circuit yields

$$V_a = iR_m + e_m = iR_m + K_m \dot{x} \quad (3)$$

Solving equation 3 for i

$$i = \frac{V_a - K_m \dot{x}}{R_m} \quad (4)$$

Substituting i into equation 1 to determine the force out of the voice coil motor

$$F_m = K_m i = K_m \frac{V_a - K_m \dot{x}}{R} = \frac{K_m}{R} V_a - \frac{K_m^2}{R} \dot{x} \quad (5)$$

As you can see, the force from the motor contains both a term proportional to the system input V_a and the negative of a term proportional to the velocity of the coil/shaft system. The K_m^2/R term can now be directly interpreted as an equivalent damping constant c with units of Ns/m .

As you saw in Lab 3, the back emf is not the only source of damping in the voice coil motor. Another source of damping is the currents flowing in the aluminum cup supporting the motor coils. Since aluminium is conductive, the same physics (Faraday's law of induction) that apply to moving the copper motor coils through a magnetic field apply to the aluminium cup. Moving the cup through the motor's magnetic field causes a voltage potential in the cup. Since the cup is a continuous part, the voltage induces a current (typically termed eddy current) to flow around the cup. The induced current then interacts with the magnetic field to create a force on the cup (Ampere's law). It is possible to analytically calculate a damping constant associated with the eddy currents in the cup but for present purposes we will use the damping constant measured in lab ($c_{i_{eddy}} = 5 \text{ Ns/m}$).

Including the eddy current damping into our motor model yields the following

$$F_m = \frac{K_m}{R} V_a - \left(\frac{K_m^2}{R} + c_{i_{eddy}} \right) \dot{x} \quad (6)$$

Note that because the amplifier acts as *voltage* source, the damping in the system is the same as with the actuator coil shorted.

Problems

1. Given the motor force characteristics (equation 6), derive the differential equation relating the voltage input from the signal generator v_s to the shaft position x for this system.
2. It is often difficult to measure the 2% settling time of a system in the lab. A more commonly used metric is the 5% settling time. Determine the relationship between τ and the 5% settling time (ie. $t_s(5\%) = a\tau$ where a is a constant).
3. Given that

$$\begin{aligned}K_m &= 7.1 \frac{[N]}{[A]} \\R &= 5.5 [\Omega] \\m &= 0.85 [kg] \\k &= \frac{0.132 [N]}{l^3 [m]}\end{aligned}$$

Analytically calculate the step response for spring lengths of 50, 100, and 150 mm and give us an expression for their time functions (ie. determine $x(t)$ for each spring length). Use MATLAB to plot each of these time responses.

4. For each response in 3, graphically determine t_r , t_p , M_p , and $t_s(5\%)$.
5. Calculate t_r , t_p , M_p , and $t_s(5\%)$ directly from the differential equation using the formulas in Palm section 4.4. Compare these to your graphically determined values.