Problem 1

a) This problem is a little tricky. To solve it correctly you need to replace the mass with an equivalent force (f_m) as shown in the figure.

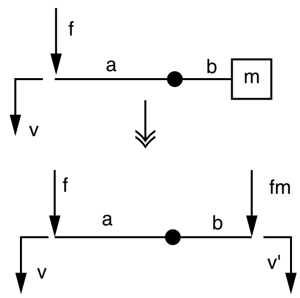

Figure 1: Free Body Diagram for 1-a

$$
\begin{aligned}
f_m &= m\dot{v}' \\
\frac{v}{a} &= -\frac{v'}{b} \\
v' &= -\frac{a}{b}v \\
f_m &= -m\frac{a}{b}\dot{v} \\
af &= -bf_m \\
f &= m\frac{a^2}{b^2}\dot{v} \\
m_{eq} &= m\frac{a^2}{b^2}
\end{aligned}
$$

b.) This problem should be solved using a free-body diagram. You should note that all of the masses travel at the same velocity, thus you only need a free body diagrams for one wheel and the combined masses.

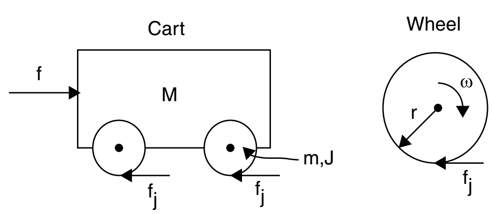

Figure 2: Free Body Diagram for 1-b

Note the force f_j, this force is not a friction force, although it is created by the friction between the tire and road surface, it is the force required to accelerate the rotational inertia of the wheel. From the figure,

$$\begin{aligned}
\text{On the cart } \Sigma F &= (M+4m)\ddot{x} = f - 4f_j \\
\text{On one wheel } \Sigma T &= J\dot{\omega} = f_j r \\
\text{No slip means } v &= \dot{x} = r\omega \Rightarrow \ \dot{\omega} = \frac{\ddot{x}}{r} \\
f_j &= \frac{J}{r^2}\ddot{x} \\
\left(M + 4m + 4\frac{J}{r^2}\right)\ddot{x} &= f \\
M_{eq} &= M + 4m + 4\frac{J}{r^2}
\end{aligned}$$

c.) Once again you should be solving this problem using free body diagrams. Note that the first pulley does not have any inertia. From the FBD,

$$\begin{aligned}
\text{Pulley \#1 } \Sigma T &= 0 = r1(f_1 - f_2) + T \\
\text{Pulley \#2 } \Sigma T &= J\dot{\omega'}
\end{aligned}$$

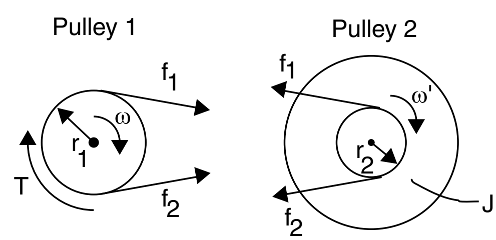

Figure 3: Free Body Diagram for 1-c

Problem 2

The first two figures for this problem are completely irrelevant to the actual solution, they are only there to show the thought process you would go through if you were solving this problem on your own.
a.) From the FDB,

$$\begin{aligned}
\Sigma F &= m_1\ddot{x} = f_g - f_k - f_b \\
\text{Where } f_g &= m_1 g \\
f_k &= kx \\
f_b &= b\dot{x} \\
m_1\ddot{x} &= m_1 g - b\dot{x} - kx
\end{aligned}$$

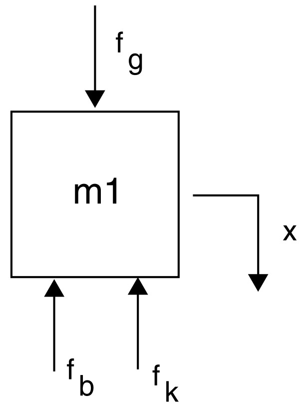

Figure 4: Free Body Diagram for 2

$$\begin{aligned}\ddot{(x)} + \frac{b}{m_1}\dot{x} + \frac{k}{m_1}x &= g \\ \text{with } x_0 &= 0,\ \dot{x}_0 = v_0\end{aligned}$$

b.) For the system to oscillate ζ must be less than 1.

$$\begin{aligned}\omega_n^2 &= \sqrt{\frac{k}{m}} \\ 2\zeta\omega_n &= \frac{b}{m} \\ \zeta &= \frac{1}{2}\frac{b}{m}\sqrt{\frac{m}{k}} = \frac{b}{2\sqrt{km}} \\ \zeta &< 1 \\ b &< 2\sqrt{km}\end{aligned}$$

c.) Since this is a pretty straight forward 2nd order system we can just add the forced and free responses

$$\begin{aligned}x(t) &= x_{free}(t) + x_{forced}(t) \\ x_{forced}(t) &= \frac{g}{k}\left[\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(t\omega_n\sqrt{1-\zeta^2}+\phi\right)+1\right] \\ x_{free}(t) &= \frac{v_0}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(t\omega_n\sqrt{1-\zeta^2}\right)\end{aligned}$$

For all of the circuit problems it is acceptable for $\frac{d}{dt}$ to be used rather than s.

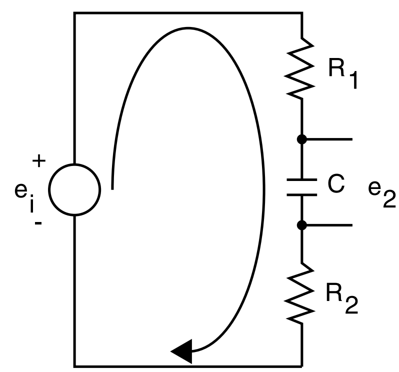

Figure 5: Circuit for P3

Problem 3 - Palm 3.4

This is a fairly straight forward problem. We are interested in the voltage across the capacitor e_2, to solve this we need to sum the voltages around the loop and then get the nodal currents.

$$\begin{aligned}
\Sigma V &= e_1 - e_{R1} - e_c - e_{R2} \\
\text{Where } e_{R1} &= i_{R1} R_1 \\
e_c &= \frac{i_c}{sC} \\
e_{R2} &= i_{R2} R_2 \\
\text{currents at node 1 } i_{R1} - i_c &= 0 \\
\text{currents at node 2 } i_c - i_{R2} &= 0 \\
\text{thus } i &= i_c = i_{R2} = i_{R1} \\
\text{a little algebra yields } i &= \frac{sC}{sC(R_1 + R_2) + 1} e_1 \\
\text{substituting into } e_2 \ \frac{e_2}{e_1} &= \frac{1}{sC(R_1 + R_2) + 1}
\end{aligned}$$

Problem 4 - Palm 3.5

This circuit has three voltage loops, we need to work with only two of them.

$$\begin{aligned}
\Sigma V_1 &= 0 = e_i - e_{R2} \\
\Sigma V_2 &= 0 = e_{R2} - e_{R1} - e_c \\
\text{Alternately } \Sigma V_3 &= 0 = e_i - e_{R1} - e_c \\
\text{Where } e_c &= e_2 = \frac{i_c}{sC} \\
e_{R1} &= i_{R1} R_1 \\
e_{R2} &= i_{R2} R_2
\end{aligned}$$

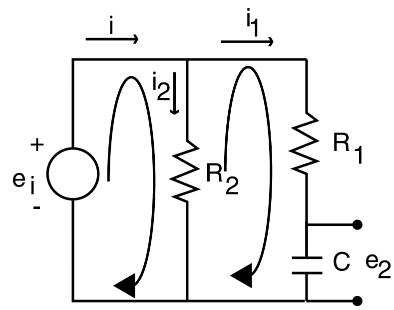

Figure 6: Circuit for P4

$$\begin{aligned}
\text{Note } i_{R1} &= i_c = i1 \\
\text{Solving for } i_1 \; i_1\left(R_1 + \frac{1}{sC}\right) &= e_i \\
i_1 &= \frac{sC}{sCR_1+1}e_i \\
\text{Substituting to find } e_2 \; \frac{e_2}{e_i} &= \frac{1}{sCR_1+1}
\end{aligned}$$

Problem 5 - Palm 3.6

This circuit has three voltage loops, we only need two to solve the problem.

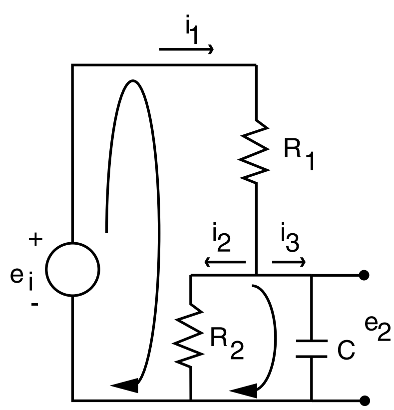

Figure 7: Circuit for P5

$$\begin{aligned}
\Sigma V_1 &= 0 = e_i - e_{R1} - e_c \\
\Sigma V_2 &= 0 = e_{R2} - e_c \\
\text{Where } e_c &= e_2 = \frac{i_c}{sC}
\end{aligned}$$

$$
\begin{aligned}
e_{R1} &= i_{R1}R_1 \\
e_{R2} &= i_{R2}R_2 \\
\text{Summing currents } i_{R1} &= i_{R2} + i_c \\
\text{Solving for } i_{R2}\ i_{R2}R_2 &= \frac{i_c}{sC} \\
i_{R2} &= \frac{i_c}{sCR_2} \\
\text{Solving for } i_{R1}\ i_{R1} &= \left(1 + \frac{1}{sCR_2}\right) i_c \\
\text{Solving for } i_c\ e_i &= R_1\left(1 + \frac{1}{sCR_2}\right) i_c + \frac{i_c}{sC} \\
i_c &= \frac{sC}{sCR_1 + \frac{R_1+R_2}{R_2}} e_i \\
\text{Solving for } e_2\ \frac{e_2}{e_i} &= \frac{1}{sCR_1 + \frac{R_1+R_2}{R_2}}
\end{aligned}
$$