

Problem 1 - Palm 5.2

$q_i = \text{input}, h_1, h_2 = \text{outputs}$

$$q_1 = \text{Change of mass tank 1} = A_1 \dot{h}_1 \rho$$

$$q_2 = \text{Change of mass tank 2} = A_2 \dot{h}_2 \rho$$

$$q_{R1} = \frac{P_1}{R_1} = \frac{\rho g h_1}{R_1}$$

$$q_{R2} = \frac{P_2}{R_2} = \frac{\rho g h_2}{R_2}$$

Conservation of Mass

$$q_1 = q_i - q_{R1} = A_1 \dot{h}_1 \rho = q_i - \frac{\rho g h_1}{R_1}$$

$$A_1 \dot{h}_1 \rho + \frac{\rho g h_1}{R_1} = q_i$$

$$q_2 = q_{R1} - q_{R2} = A_2 \dot{h}_2 \rho = \frac{\rho g h_1}{R_1} - \frac{\rho g h_2}{R_2}$$

$$A_2 \dot{h}_2 + \frac{g}{R_2} h_2 = \frac{g}{R_1} h_1$$

Problem 2 - Palm 5.5

$$A = 20 \text{ ft}, \delta = 1.94 \text{ slugs/ft}^3$$

$$R_L = \frac{25000}{\text{ft s}}, \nu = 2 \times 10^{-5} \text{ lb} - \text{sec/ft}^2$$

1)

$$h_0 = 30 \text{ ft}$$

$$A \rho \dot{h} + \frac{\rho g h}{R_L} = 0$$

$$\tau = \frac{A \rho R_L}{\rho g} = \frac{A R_L}{g} = \frac{20 * 25000}{32.2} \frac{\text{ft}^2 \text{s}^2}{\text{ft s ft}} = 15528 \text{ s} = 4.31 \text{ hrs}$$

$$4\tau = 17.25 \text{ hrs}$$

2)

$$h_{ss} = \frac{\rho q_i R_L}{\rho g} = \frac{0.1 * 25000}{32.2} = 77.64 \text{ ft}$$

$$h(t) = 77.64(1 - e^{-\frac{t}{\tau}})$$

$$1 - e^{-\frac{t}{\tau}} = \frac{1}{3}$$

$$t(1/3) = -\tau \ln(2/3) = 1.75 \text{ hrs}$$

Problem 3 - Palm 5.24

Using conservation of energy

$$\begin{aligned}\frac{d}{dt}C_1T_1 &= q - q_1 = q - \frac{1}{R_1}(T_1 - T_2) \\ C_1\dot{T}_1 + \frac{1}{R_1}T_1 &= q + \frac{1}{R_1}T_2 \\ \frac{d}{dt}C_2T_2 &= q_1 - q_o = \frac{1}{R_1}(T_1 - T_2) - \frac{1}{R_2}(T_2 - T_o) \\ C_2\dot{T}_2 + T_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) &= \frac{1}{R_1}T_1 + \frac{1}{R_2}T_o\end{aligned}$$