Problem 1 - RLC circuit analysis

1.

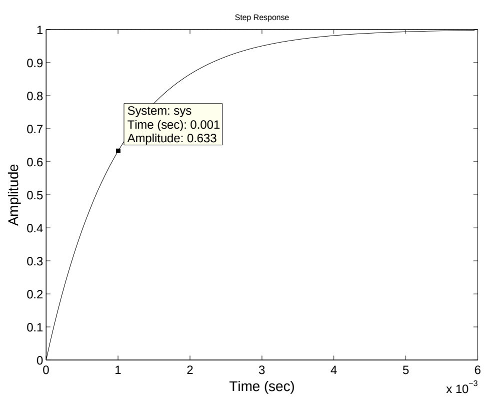
$$\frac{V_o}{V_i} = \frac{1}{LCs^2 + RCs + 1}$$

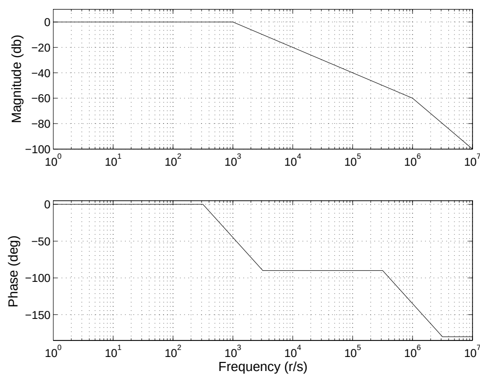
2.

$$\begin{aligned}
\omega_n &= 2 * \pi * 5000 = 31,400\, r/s = \frac{1}{\sqrt{LC}} \\
L &= \frac{1}{\omega_n^2 C} = 0.001\, H = 1\, mH \\
\frac{R}{L} &= 2\zeta\omega_n = 2 * 0.707 * 31,400 \\
R &= 44.4\, \Omega
\end{aligned}$$

3. There are no zeros, poles at roots of

$$\begin{aligned}
s^2 + \frac{1000}{0.001}s + \frac{1}{1e-6*1e-3} &= 0 \\
s_1 &\approx -1e6 \\
s_2 &\approx -1e3 \text{ dominant pole} \\
x_{ss} = 1
\end{aligned}$$





4.

5. $s = -1e3 = s_2$

6.

$$\text{For circuit with R\&C in } \| \quad \frac{v_o}{v_i} = \frac{R_2}{R_2LCs^2 + (R_1R_2C + L)s + R_1 + R_2}$$
$$\text{For circuit with R\&C in series } \quad \frac{v_o}{v_i} = \frac{R_2Cs + 1}{LCs^2 + (R_1 + R_2)Cs + 1}$$

Problem 2 - Op Amp analysis

1.

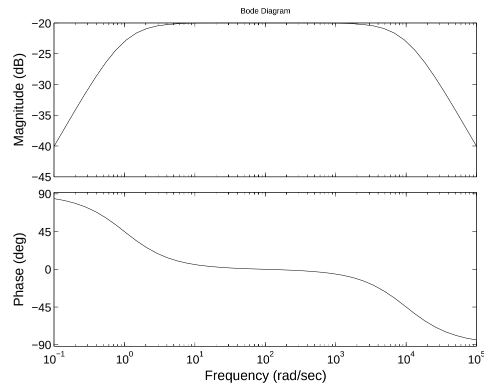
$$\frac{v_o}{v_i} = \frac{R_2C_1s}{(R_2C_2s + 1)(R_1C_1s + 1)}$$

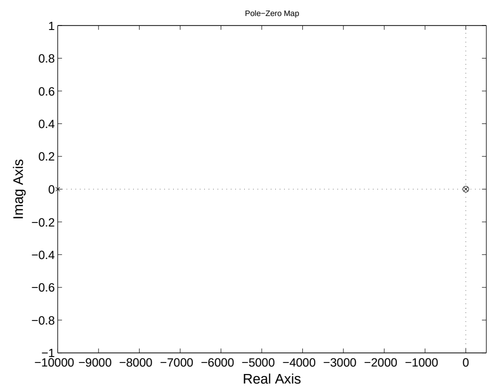
2.

$$M(\omega) = \frac{R_1C_1\omega}{\sqrt{(1 - R_1R_2C_1C_2\omega^2)^2 + ((R_1C_1 + R_2C_2)\omega)^2}}$$
$$\phi(\omega) = 90^\circ - \tan^{-1}\left(\frac{(R_1C_1 + R_2C_2)\omega}{(1 - R_1R_2C_1C_2\omega^2)}\right)$$

3.

4.





Problem 3

1.

$$\begin{aligned} T(s) &= \frac{K}{s^2 + 20s + K} \\ \omega_n &= \sqrt{K} \\ 2\zeta\omega_n &= 2\sqrt{K} = 20 \\ K &= 100 \end{aligned}$$

2.

$$\begin{aligned} \frac{V_{out}}{V_1} &= \frac{(s+3)(6s+1)}{(s+3)(6s+1) + (8s+7)} \\ \frac{V_{out}}{V_2} &= \frac{(6s+1)}{(s+3)(6s+1) + (8s+7)} \end{aligned}$$

3. Solve using superposition

$$V_{out}(6s^2 + 27s + 10) = V_1(6s^2 + 19s + 3) + V_2(6s + 1)$$

$$6\ddot{V}_{out} + 27\dot{V}_{out} + 10V_{out} = 6\ddot{V}_1 + 19\dot{V}_1 + 3V_1 + 6\dot{(V)}_2 + V_2$$

Problem 4
The transfer function for this system is

$$\begin{aligned} \frac{x(s)}{f(s)} &= \frac{1}{m_{eq}s^2 + cs + k} \\ \omega_n &= \sqrt{\frac{k}{m_{eq}}} \\ 2\zeta\omega_n &= \frac{c}{m_{eq}} \end{aligned}$$

1. From graph, we measure the following

$$\begin{aligned} T &\approx 1.0s \Rightarrow \omega_d = \frac{2\pi}{T} = 6.28r/s \\ \omega_d &= \omega_n\sqrt{1-\zeta^2} \\ M_p &\approx 100\frac{0.75-0.5}{0.5} = 50 \\ \zeta &= \frac{A}{\sqrt{\pi^2 + A^2}} = 0.215 \\ A &= ln\frac{100}{M_p} = 0.693 \\ \omega_n &= 6.43r/s \\ m_{eq} &= \frac{\omega_n^2}{k} = 4.8 \approx 5kg \\ c &= 2\zeta\omega_n m_{eq} = 13.8Ns/m \approx 14Ns/m \end{aligned}$$

   Alternately, you could determine ζ using the log decrement method.

2.

$$\begin{aligned} m_{eq} &= m + \frac{I}{r^2} \\ I &= 0.5\,kg\,m^2 \end{aligned}$$

Problem 5
The transfer function for this system is

$$\begin{aligned} \frac{\omega(s)}{\phi(s)} &= \frac{k}{Js^2 + cs + k} \\ \text{Thus } \omega_n &= \sqrt{\frac{k}{J}} \\ 2\zeta\omega_n &= \frac{c}{J} \end{aligned}$$

1. There are a couple of ways to solve this part of the problem. First, you can read $\omega_r = 9\, r/s$ and $M_p = 5\, dB$ from the bode plot and use the following relationships

$$\begin{aligned} M_p &= \frac{1}{2\zeta\sqrt{1-\zeta^2}} \\ \omega_r &= \omega_n\sqrt{1-2\zeta^2} \end{aligned}$$

   to find $\zeta \approx 0.3$ and $\omega_n \approx 10\, r/s$. Or you can read $\omega_n = 10\, r/s$ directly from the phase plot ($\theta = -90°$)

2. $k = 1500\, Nm/r$, $c = 90 Nms/r$

3.

$$\begin{aligned} \omega = 1.1\, r/s \;\; \theta(t) &\approx \sin(1.1t + 0) \\ \omega = 10\, r/s \;\; \theta(t) &\approx 1.58\sin(10t - \pi/2) \\ \omega = 20\, r/s \;\; \theta(t) &\approx 0.3\sin(20t - 2.75) \end{aligned}$$