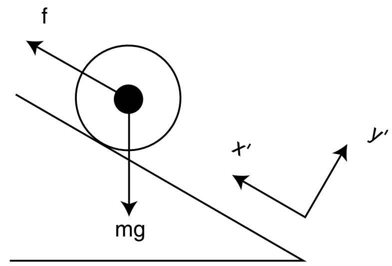
Problem 1 - Palm 1.11



Using the analysis of pp. 17-19, we can calculate a dynamically equivalent mass for the wheel with its inertia:

$$m_{equiv} = m_{wheel} + \frac{I_{wheel}}{R^2}$$

Then, we derive the equation of motion (using the FBD given in figures 1.4-3 and P1.11, balancing all forces on the body to determine the acceleration of the (equivalent) mass:

$$\begin{aligned} F &= m_{equiv} a = m_{equiv} \dot{v} \\ \Sigma F_x &= f - m_{wheel} g \sin\phi = m_{equiv} \dot{v} \end{aligned}$$

Assuming a "no slip" condition for the wheel, the translational and rotational velocities of the wheel will be related as:

$$\begin{aligned} v &= \omega R \\ \int \dot{v} dt &= \frac{1}{m_{equiv}} \int (f - m_{wheel} g \sin\phi)\, dt \\ v &= \frac{1}{m_{equiv}} (f - m_{wheel} g \sin\phi) \cdot t + c \end{aligned}$$

Since, $v(0) = 0$, the integration constant is zero: $c = 0$.

$$\begin{aligned} v &= \frac{1}{m_{equiv}} (f - m_{wheel} g \sin\phi) \cdot t \\ \omega &= \frac{v}{R} = \frac{1}{R m_{equiv}} (f - m_{wheel} g \sin\phi) \cdot t \end{aligned}$$

Now, plug in the values given in the problem statement:

$f = 400$ N, $m_{wheel} = 80$ kg, $R = 0.3$ m, $I = 3$ kg-m^2, $\phi = 25°$

$$m_{equiv} = 80 + 3/(0.3^2) = 113.3\text{kg}$$

After 60 s, we find the following values for the axle speed and rotational velocity:

$$\begin{aligned} v(60) &= \frac{1}{113.3}(400 - 80 \cdot 9.8 \cdot \sin 25°) \cdot 60 = 36.4\text{m/s} \\ \omega(60) &= \frac{1}{113.3 \cdot 0.3}(400 - 80 \cdot 9.8 \cdot \sin 25°) \cdot 60 = 121\text{rad/s} \end{aligned}$$

Problem 2 - Palm 4.9

For each system, we can derive equations of motion by creating a force balance ($F = ma$) for each "degree of freedom" (DOF) in the system. As we study a system, if a mass (or massless node) is free to move independently of all other previously identified DOF, then it is associated with a newly identified DOF. (These systems can be easily analyzed by inspection, but for more complicated cases, it may be necessary to balance foreces at each DOF.)

a) There are 2 DOF, x and y. At x, $f_x(t) + k(y - x) = 0$. (We don't really care about $f_x(t)$ in this problem, but we know this force is being applied, if there is an input, x.) At y, $-c\dot{y} - ky + kx = m\ddot{y}$ (Note you can chech if the signs are OK by testing each component on the left-hand side of the equation to see if it has the correct effect on the acceleration in y.) It turns out the equation at y is all we need here (since x is the input and y is the output, and we already have an equation that relates the tow directly), so we can reaffange the equation:

$$m\ddot{y} = c\dot{y} + ky = kx$$

b) Here, there are 2 DOF, x and y:
At x, $f_x(t) + k(y - x) + c(\dot{y} - \dot{x}) = 0$ (Again, we don't care about whatever $f_x(t)$ exists, since x is the input ... but we know some force does exist.)
At y, $kx - ky + c(\dot{x} - \dot{y}) = m\ddot{y}$

As with part (a), this second equation is all we actually need:

$$m\ddot{y} + c\dot{y} + ky = c\dot{x} + kx$$

c) This rotational system is completely analogous to the tanslational system from **(a)**:
At θ_{out}, $-k\theta_{out} + k\theta_{in} - c\dot{\theta}_{out} = I\ddot{\theta}_{out}$
So rearranging, we get:

$$I\ddot{\theta}_{out} + c\dot{\theta}_{out} + k\theta_{out} = k\theta_{in}$$

d) The system has 3 DOF, x,y, and z:
At the x node: $f_x(t) + k_1(y - x) = 0$
Just as in part a), we do not really need this eqn.
At node y: $m_1\ddot{y} = k_1(x - y) + k_2(z - y) - c_1\dot{y}$
At node z: $m_x\ddot{z} = k_2(y - z) - c_2z$
Putting these in the standard form:

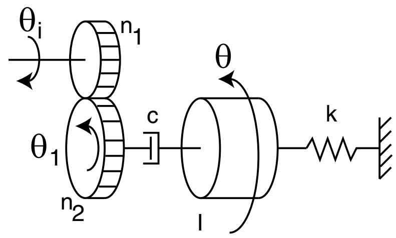
$$\begin{aligned} m_1\ddot{y} + c_1\dot{y} + (k_1 + k_2)y &= k_2z + k_1x \\ m_2\ddot{z} + c_2\dot{z} + k_2z &= k_2y \end{aligned}$$

e) The system has 3 DOF, θ_i, θ_1, and θ_2:
At the θ_i node: $T_{\theta_i} + k_1(\theta_1 - \theta_i) = 0$
At the θ_1 node: $I_1\ddot{\theta}_1 = k_1(\theta_i - \theta_1) + c(\dot{\theta}_2 - \dot{\theta}_1)$
At the θ_2 node: $I_1\ddot{\theta}_3 = -k_2\theta_2 + c(\dot{\theta}_1 - \dot{\theta}_2)$
Rearranging into the standard form:

$$\begin{aligned} I_1\ddot{\theta}_1 + c\dot{\theta}_1 + k_1\theta_1 &= k_1\theta_i + c\theta_2 \\ I_2\ddot{\theta}_1 + c\dot{\theta}_2 + k_2\theta_2 &= c\dot{\theta}_1 \end{aligned}$$

f) The system has 2 DOF, θ_i, and θ. To solve this problem it is helpful to define an additional variable θ_1, which tracks the motion of the second gear (see figure). Using the relationship from Table 4.3-1 can express θ_1 as a function of θ:

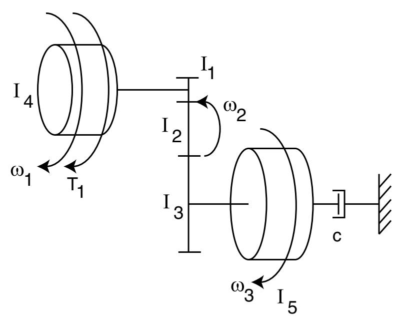
$$\begin{aligned} \frac{\theta_1}{\theta_i} &= \frac{n_1}{n_2} \Rightarrow \frac{\dot{\theta}_1}{\dot{\theta}_i} = \frac{n_1}{n_2} \\ \dot{\theta}_1 &= \dot{\theta}_i\frac{n_1}{n_2} \end{aligned}$$



Now at the θ node: $I\ddot{\theta} = c(\dot{\theta}_1 - \dot{\theta}) - k\theta$
Substituting for θ_1 and placing in standard form:

$$I\ddot{\theta} + c\dot{\theta} + k\theta = c\dot{\theta}_i \frac{n_1}{n_2}$$

Problem 3 - Palm 4.15



This problem ask us to collapse the gear train into the ω_1 frame. There are a number of methods to do this including doing force balances at each gear and using the the know relationships between the gear ratios to substitute for ω_2 and ω_3. A simpiler method would be to determine the equivelent interia and damping using the relationships in table 4.3-1 and collapse the system starting the in ω_3 frame. In the ω_3 frame:

$$\begin{aligned} I_{\omega 3} &= I_5 + I_3 \\ c_{\omega 3} &= c \end{aligned}$$

Next we need to find what the equivalents are in the ω_2 frame:

$$I_{e\omega 3} = I_{\omega 3}\left(\frac{\omega_3}{\omega_2}\right)^2 = I_{\omega 3}\left(\frac{1}{5}\right)^2$$

$$c_{e\omega 3} = c\left(\frac{\omega_3}{\omega_2}\right)^2 = c\left(\frac{1}{5}\right)^2$$

Now in the ω_2 frame:

$$\begin{aligned} I_{\omega 2} &= I_2 + I_{e\omega 3} = I_2 + I_{\omega 3}\left(\frac{1}{5}\right)^2 \\ c_{\omega 2} &= c_{e\omega 3} = c\left(\frac{1}{5}\right)^2 \end{aligned}$$

Getting the ω_1 equivalents:

$$\begin{aligned} I_{e\omega 1} &= I_{\omega 2}\left(\frac{\omega_2}{\omega_1}\right)^2 = \left(I_2 + I_{\omega 3}\left(\frac{1}{5}\right)^2\right)\left(\frac{1}{2}\right)^2 \\ c_{e\omega 1} &= c_{\omega 2}\left(\frac{\omega_2}{\omega_1}\right)^2 = c\left(\frac{1}{5}\right)^2\left(\frac{1}{2}\right)^2 \end{aligned}$$

Finally we can add all of the inertias in the ω_1 frame:

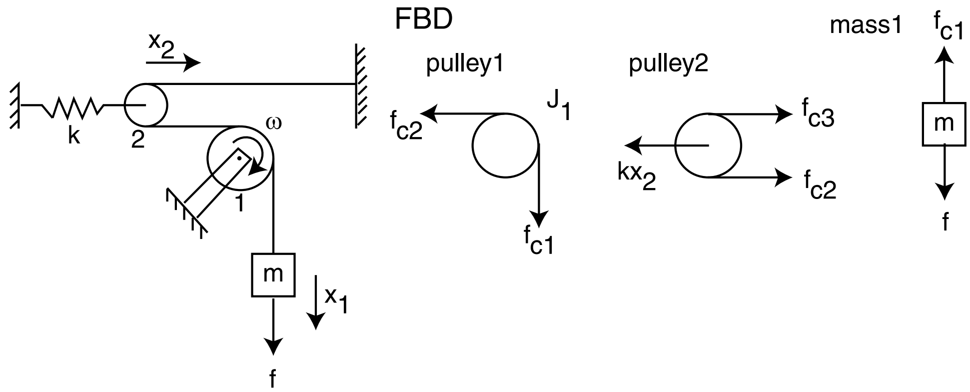
$$\begin{aligned} I_T &= I_4 + I_1 + \frac{1}{4}I_2 + \frac{1}{100}(I_3 + I_5) \\ c_T &= \frac{1}{100}c \end{aligned}$$

The equation of motion for this system is:

$$\begin{aligned} T_1 &= I_T\dot{\omega}_1 + c_T\omega_1 \\ T_1 &= \left(I_4 + I_1 + \frac{1}{4}I_2 + \frac{1}{100}(I_3 + I_5)\right)\dot{\omega}_1 + \frac{1}{100}c\omega_1 \\ T_1 &= 0.461\dot{\omega}_1 + 0.04\omega_1 \end{aligned}$$

Problem 4 - Palm 4.21

This solution assumes that there is no slip on either of the pulleys. To start with we note that $x_1 = 2x_2$. This system can be broken into 3 nodes centered at m_1, pulley 1, and pulley 2.



a) For part a, we are told that the mass and inertias of the pulleys are negligible thus summing the forces at each node results in the following:

$$\begin{aligned}
\text{At pulley 1: } I_1\dot{\omega}_1 &= 0 = \Sigma T_1 = R(f_{c1} - f_{c2}) \Rightarrow f_{c1} = f_{c2} \\
\text{At pulley 2: } I_2\dot{\omega}_2 &= 0 = \Sigma T_2 = r(f_{c2} - f_{c3}) \Rightarrow f_{c2} = f_{c3} \\
m_2\ddot{x}_2 &= 0 = \Sigma F = f_{c2} + f_{c3} - kx_2 \Rightarrow f_{c1} = f_{c2} = f_{c3} = \frac{kx_2}{2} \\
\text{At mass 1: } m_1\ddot{x}_2 &= \Sigma F = f - f_{c1} = f - \frac{kx_2}{2}
\end{aligned}$$

Where f_{c1} is the tension in the cable between pulley 1 and mass 1, f_{c2} is the tension in the cable between pulley 1 and pulley 2, and f_{c3} is the tension in the cable between pulley 1 and ground. Restating everything in terms of x_1 yields the following:

$$m_1\ddot{x}_1 + \frac{kx_1}{4} = f$$

b) In this part the inertia and mass of pulley 2 remain negligible but now pulley 1 has mass (since pulley 1 does not translate this does not enter our equations) and inertia. Summing the forces at the nodes now results in the following:

$$\begin{aligned}
\text{At pulley 1: } I_1\dot{\omega}_1 &= \Sigma T_1 = R(f_{c1} - f_{c2}) \\
\text{At pulley 2: } I_2\dot{\omega}_2 &= 0 = \Sigma T_2 = r(f_{c2} - f_{c3}) \Rightarrow f_{c2} = f_{c3} \\
m_2\ddot{x}_2 &= 0 = \Sigma F = f_{c2} + f_{c3} - kx_2 \Rightarrow f_{c2} = f_{c3} = \frac{kx_2}{2} \\
\text{At mass 1: } m_1\ddot{x}_2 &= \Sigma F = f - f_{c1}
\end{aligned}$$

$$\begin{aligned}
\text{Note: } R\omega_1 &= \dot{x}_1 \Rightarrow \dot{\omega}_1 = \frac{\ddot{x}_1}{R} \\
\text{Solving for } x_1,\ I_1\frac{\ddot{x}_1}{R} &= R\left(f - m_1\ddot{x}_1 - \frac{kx_1}{4}\right) \\
\left(\frac{I_1}{R^2} + m_1\right)\ddot{x}_1 + \frac{kx_1}{4} &= f
\end{aligned}$$