Problem 1 & 2 - Palm 2.25 & 2.26

I have combined the solutions to the first two problems into one. For 2.25, you should find the roots of the characteristic equation (s_1 and s_2), the steady state response (x_{ss}), and the time to steady state (t_{ss}). For 2.26, you should find the time response ($x(t)$) using table 2.3-2. A quick note on finding x_{ss}: The proper way to find the steady state value of x would be the following

$$x_{ss} = lim_{x\to\infty} x(t)$$

Since this is tedious and requires that you have $x(t)$ formulated you can cheat a bit and simply look at the differential equation for the problem and throw out any time based terms.

$$\begin{aligned} m\ddot{x} + b\dot{x} + kx &= f \text{ becomes} \\ kx &= f \\ x_{ss} &= \frac{f}{k} \end{aligned}$$

This *usually* works but not always.

a)

$$\begin{aligned} s_1 &= -5,\ s_2 = -2,\ x_{ss} = \frac{4}{3},\ t_{ss} = 4\tau = -\frac{4}{s_2} = 2s \\ x(t) &= \frac{4}{3}\left(\frac{2}{3}e^{-5t} - \frac{5}{3}e^{-2t} + 1\right) \end{aligned}$$

b)

$$\begin{aligned} s_1 &= s_2 = -2,\ x_{ss} = \frac{72}{20},\ t_{ss} = 4\tau = -\frac{4}{s_2} = 2s \\ x(t) &= \frac{72}{20}\left[(-2t-1)e^{-2t} + 1\right] \end{aligned}$$

c)

$$\begin{aligned} s_{1,2} &= -2 \pm 5i,\ x_{ss} = \frac{95}{58},\ t_{ss} = 4\tau = -\frac{4}{a} = 2s \\ x(t) &= \frac{95}{58}\left[\frac{1}{5}\sqrt{\frac{58}{2}}e^{-2t}\sin(5t + 4.33) + 1\right] \end{aligned}$$

Note: For this problem you need to make sure to use the 2 argument tangent function.

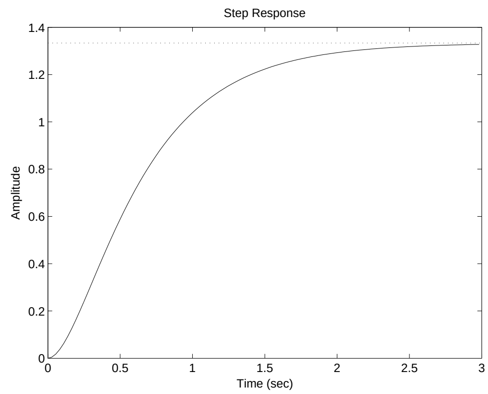

Figure 1: Step response for 2.26a

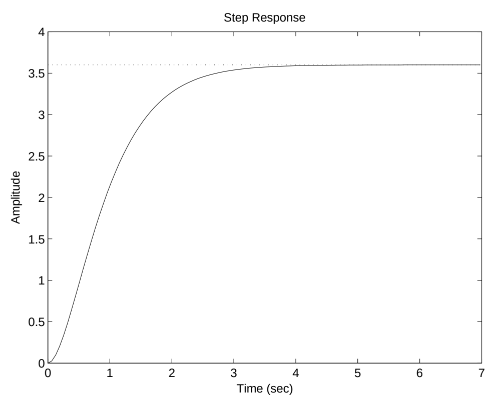

Figure 2: Step response for 2.26b

Problem 3 - Palm 4.26

The roots of the characteristic equation ($\ddot{x} + 4\dot{x} + 8x = 2$) for this problem are $s_{1,2} = -2 \pm 2i$.

$$
\begin{aligned}
\omega_n &= \sqrt{8} = 2\sqrt{2} \\
2\omega_n\zeta &= 4 \\
\zeta &= \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \\
x_{ss} &= \frac{2}{8} = \frac{1}{4} \\
P.O. &= 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 100e^{-\frac{\pi 0.707}{\sqrt{1-0.5}}} = 4.3\%
\end{aligned}
$$

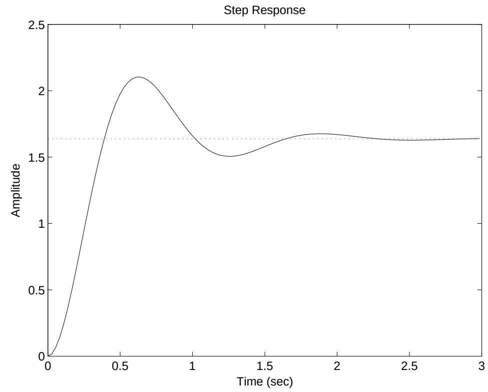

Figure 3: Step response for 2.26c

$$\begin{aligned}
\text{Overshoot} &= P.O. * x_{ss}/100 = 0.01 \\
\tau &= -\frac{1}{\zeta\omega_n} \Rightarrow t_{2\%} = 4\tau = 2s \\
t_p &= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d} = \frac{\pi}{2} = 1.57s \\
t_r &= \frac{2\pi - \phi}{\omega_n\sqrt{1-\zeta^2}} = \frac{2\pi - \frac{5}{4}\pi}{\omega_d} = \frac{1.25\pi}{2} = 1.179s \\
t_d &\approx \frac{1+0.7\zeta}{\omega_n} = \frac{1+0.7*0.707}{3.332} = 0.53s
\end{aligned}$$

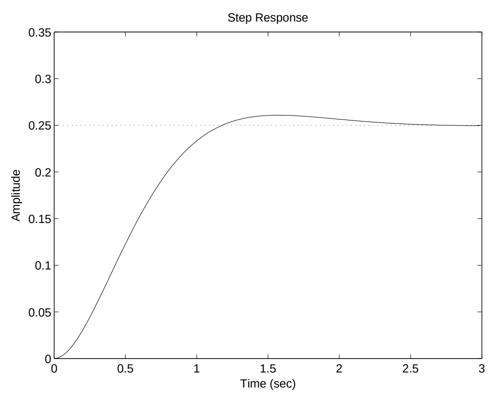

Figure 4: Step response for 4.26

Problem 4 - Palm 4.28

Given: $9\ddot{x} + c\dot{x} + 4x = f$ where $f = u(t)$. Percent overshoot 20% max., $t_{rise}100\% \leq 3s$
Solution:

$$
\begin{aligned}
\omega_n &= \sqrt{\frac{4}{9}} = \frac{2}{3} \\
M_p &= 20 = 100e^{1\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \ln 0.2 = -\frac{\pi\zeta}{\sqrt{1-\zeta^2}} \\
\zeta_{min} &= \left(\frac{1}{\frac{\pi^2}{(\ln 0.2)^2} + 1}\right)^{\frac{1}{2}} = 0.4559
\end{aligned}
$$

ζ must be greater than or equal to the that found above or the system overshoot (the dominant performance requirement) more than 20%.

$$
\begin{aligned}
t_{rise}100\% &= \frac{2\pi - \phi}{\omega_n\sqrt{1-\zeta^2}} = \frac{2\pi - 4.2389}{0.66\bar{6}\sqrt{1-0.4559^2}} = 3.44s \\
\phi &= \tan^-1\frac{\sqrt{1-\zeta^2}}{\zeta} + \pi = 4.2389
\end{aligned}
$$

Well clearly, we are not going to be able to achieve both requirements. This is because to increase the rise time we will have to decrease ζ or increase ω_n. Since we can not control ω_n for this system, that leaves only decreasing ζ, but this will increase the overshoot, which we are told is the dominant requirement. Given our minimum ζ we find:

$$
\frac{c}{9} = 2\omega_n\zeta \Rightarrow c = 18\left(\frac{2}{3}\right)0.4559 = 5.47
$$

You can also solve this problem a little less rigourously using values from Figure 4.4-10. Using this method:

$$
\begin{aligned}
\zeta &= 0.43 \\
c &= 5.16
\end{aligned}
$$

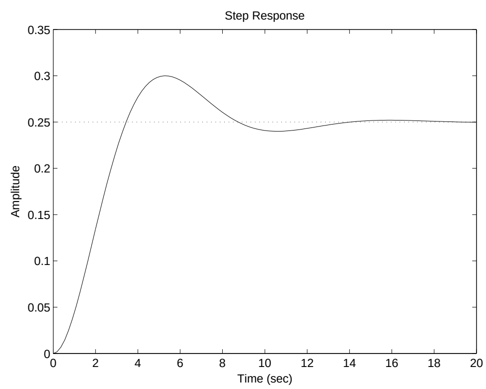

Figure 5: Step response for 4.28