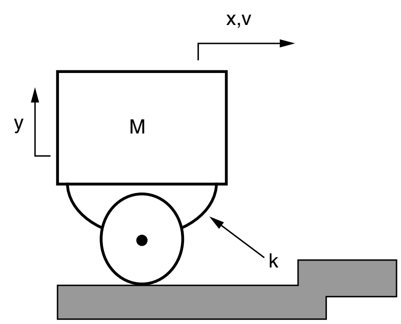

Figure 1: Fruit cart diagram

After spending the entire school year inside working hard at MIT, you decide that you really want to work outdoors over the summer so you take a job picking fruit in an orchard. The farm provides you with a small tractor and cart to haul the fruit from the orchard to the truck. Figure 1 illustrates the cart you are using to carry the fruit. The cart consist of a simple box to carry the load and a single axel with two wheels. The axel is attached to the box using a leaf spring (k). Now you run into a few problems with this cart while driving back and forth. The main problem is that you have to drive over a curb, illustrated as a step in the figure, and since you like to drive fast hitting this curb ends up bouncing half the fruit out of the cart. Since, you know all about dynamics now rather than slow down you decide to make a few improvements to the cart. You know that the cart masses about 450 kg fully loaded (you know this because you are paid by the mass of the fruit you manage to get back to the farm). You also measure the displacement of the spring to be 5 cm between loaded and free length.

1

You decide to model the cart as a planer model (ie. you ignore the cart dynamics into the page and collapse the two tires and spring into one) and ignore the mass and dynamics of the cart tires. Propose a model for this system, write the characteristic equation for this model, sketch the time response of this system to a step input to y carefully noting the period of oscillation.

2

To improve the cart you decide to add a damper to the suspension. So you head to the workshop grab a can of grease and a plunger and with a little work mount the simple damper to the cart as shown in Figure 2. After a quick test ride you measure the following response to a step input (Figure 3). From the data you are given in 1 and the figure determine the damping constant of your can of grease.

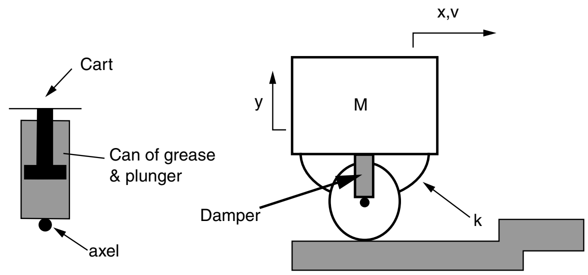

Figure 2: Your modification

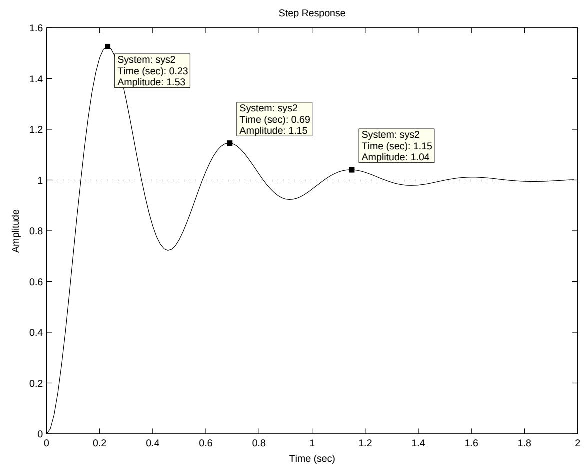

Figure 3: Measured response

3

Pleased with your success you call your friend Bob. Bob is a bit of a know-it-all and he tells you if you really want to make a good cart you need to include the suspension dynamics of the tire. He tells you that a tire really acts like a spring between the tire center of mass and the ground. Propose a model that includes both the mass of the tire and it's spring effect. Determine the equations of motion for this system and put them in the standard form.

Solution

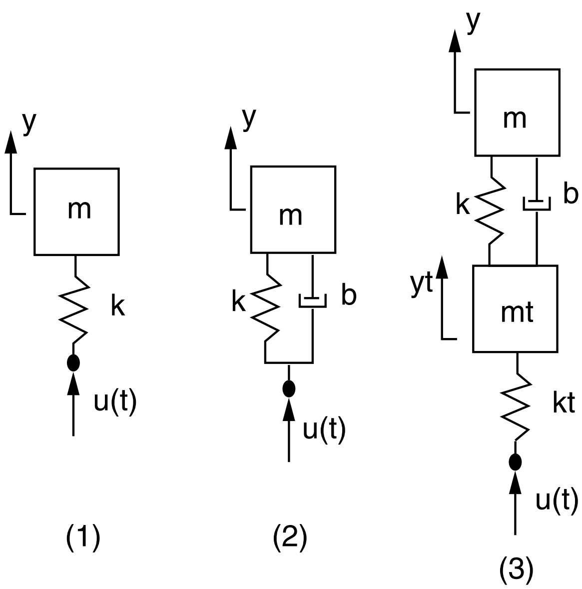

Figure 4: Models for parts 1, 2, and 3

1. The equation of motion for this system is

$$
\begin{aligned}
m\ddot{y} + ky &= f(t) \\
k &= \frac{force}{distance} = \frac{m * g}{0.05} = \frac{450 * 9.8}{0.05} = 88200\frac{N}{m} \\
f(t) &= k * u(t) \\
m\ddot{y} + km &= ku(t) \Rightarrow \ddot{y} + \frac{k}{m}y = \frac{k}{m}u(t) \\
s_{1,2} &= \pm\sqrt{\frac{k}{m}} = \omega_n = 14\frac{rad}{s} = 2.2hz \\
y(t) &= 1 + \sin\left(\omega_n t + 3\pi/2\right)
\end{aligned}
$$

Figure 5 show the step response of the system as modelled. **2.** This can be solved using either logarithmic decrement or overshoot.

$$
\begin{aligned}
T &= 0.23s \Rightarrow \omega_d = \frac{2\pi}{T} = 13.658\frac{rad}{s} \\
Mp &= 53\% \Rightarrow \zeta = \frac{A}{\sqrt{\pi^2 + A^2}} = \frac{0.634}{\sqrt{\pi^2 + 0.634^2}} = 0.2 \\
A &= \ln\frac{100}{Mp} \\
2\zeta\omega_n &= \frac{b}{m} \Rightarrow b = 2m\zeta\omega_n = 2520\frac{Ns}{m}
\end{aligned}
$$

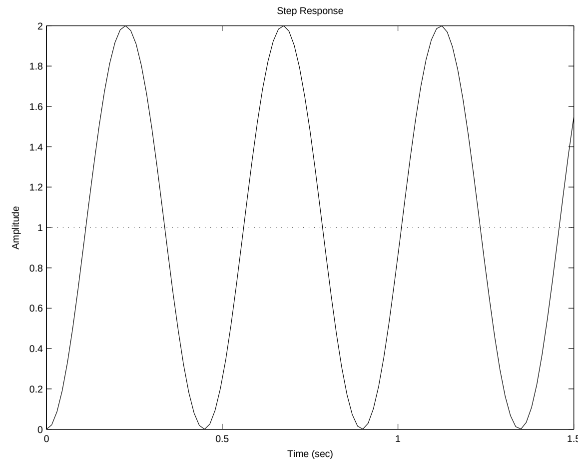

Figure 5: Step response without damper

ω_n can be found from ζ and ω_d or part 1

3. Doing a force balance at each mass, we find

$$
\begin{aligned}
m\ddot{y} &= k(y_t - y) + b(\dot{y}_t - \dot{y}) \\
m_t\ddot{y}_t &= k(y - y_t) + k_t(u(t) - y_t) + b(\dot{y} - \dot{y}_t) \\
\text{In standard form} & \\
m\ddot{y} + b\dot{y} + ky &= b\dot{y}_t + ky_t \\
m_t\ddot{y}_t + b\dot{y}_t + (k + k_t)y_t &= k_t u(t) + b\dot{y} + ky
\end{aligned}
$$