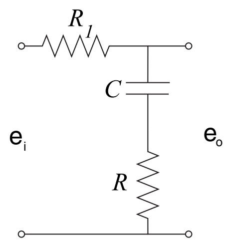


Figure P3.15

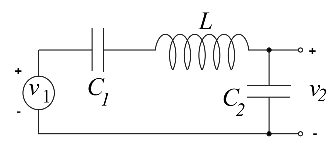


Figure P3.18

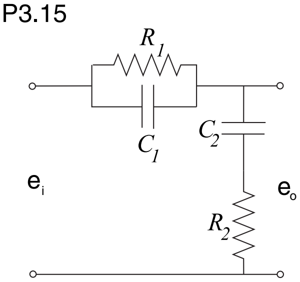


Figure P3.20

Problem 1 - Palm 3.15

To start with the impedance equivalent of various circuit components is as follows:

$$\begin{aligned}\text{Capacitor } Z_c &= \frac{1}{Cs}\\ \text{Inductor } Z_i &= Ls\\ \text{Resistor } Z_r &= R\end{aligned}$$

For this circuit we have the following relationships:

$$\begin{aligned}e_i &= (Z_{R1} + Z_c + Z_R)i\\ e_o &= (Z_c + Z_R)i\end{aligned}$$

Substituting and solving for i yields,

$$\frac{e_o}{e_i} = \frac{Z_c + Z_R}{Z_{R1} + Z_c + Z_R}$$

Substituting in for the impedances yields,

$$\begin{aligned}\frac{e_o}{e_i} &= \frac{\frac{1}{Cs} + R}{R1 + \frac{1}{Cs} + R}\\ \frac{e_o}{e_i} &= \frac{RCs + 1}{(R_1 + R)Cs + 1}\end{aligned}$$

Problem 2 - Palm 3.18

The following two relationships can be found from the circuit:

$$\begin{aligned} v_1 &= (Z_{c1} + Z_L + Z_{c2})i \\ v_2 &= Z_{c2} i \end{aligned}$$

Substituting and solving for i yields,

$$\frac{v_2}{v_1} = \frac{Z_{c2}}{Z_{c1} + Z_L + Z_{c2}}$$

Substituting for the impedances yields

$$\begin{aligned} \frac{v_2}{v_1} &= \frac{\frac{1}{C_2 s}}{\frac{1}{C_1 s} + Ls + \frac{1}{C_2 s}} \\ \frac{v_2}{v_1} &= \frac{C_1 s}{s(C_1 C_2 L s^2 + (C_1 + C_2))} \\ \frac{v_2}{v_1} &= \frac{C_1}{C_1 C_2 L s^2 + (C_1 + C_2)} \end{aligned}$$

Problem 3 - Palm 3.20
The circuit yields the following relationships

$$\begin{aligned} e_i &= (Z_{eq} + Z_{c2} + Z_{R2})i \\ e_o &= (Z_{c2} + Z_{R2})i \\ \text{where } Z_{eq} &= \left(\frac{1}{Z_{R1}} + \frac{1}{Z_{c1}}\right)^{-1} \\ Z_{eq} &= \frac{R_1}{R_1 C_1 s + 1} \end{aligned}$$

Substituting and solving for i yields,

$$\frac{e_o}{e_i} = \frac{Z_{c2} + Z_{R2}}{Z_{c2} + Z_{eq} + Z_{R2}}$$

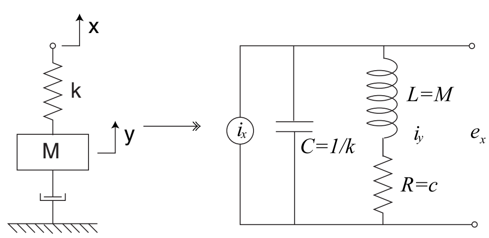
Substituting for the impedances yields

$$\begin{aligned} \frac{e_o}{e_i} &= \frac{\frac{1}{C_2 s} + R_2}{\frac{R_1}{R_1 C_1 s + 1} + \frac{1}{C_2 s} + R2} \\ \frac{e_o}{e_i} &= \frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{C_1 C_2 R_1 R_2 s^2 + (R_1 C_1 + R_1 C_2 + C_2 R_2)s + 1} \end{aligned}$$

Problem 4 - Palm 4.9-a Not Required
Even though Prof. Gossard removed this problem from the problem set and did not lecture on the impedance method for mechanical systems, I though you might like to see how to solve this problem using impedance. Feel free to completely ignore this problem if you want.
To start with we should list what the equivalents are between mechanical and electrical systems.

Element type	Electrical		Mechanical	
	Name	Impedance	Name	Impedance
effort source	voltage	e or v	force	f
flow source	current	i	velocity	v
flow storage	capacitor	$\frac{1}{Cs}$	spring	$\frac{k}{s}$
effort storage	inductor	Ls	mass	ms
dissipator	resistor	R	damper	b or c

I should note, the terminology used here is my own and may not match that presented by Prof. Gossard or found in other literature. In addition, the equivalents here are only the most commonly used, it is possible to define alternative equivalents.
The easiest way to solve this problem using impedance, is to determine an electrical circuit that is analogous to the mechanical system shown. First let us note the variables of interest x and y. These do not match exactly any of our define variables. They are in fact the integrals of our flow variables such that $x = \frac{v_x}{s}$ and $y = \frac{v_y}{s}$. Since the impedance method prefers to work with forces and velocities, it is much easier to find the relationship $\frac{v_y}{v_x}$ than directly solving $\frac{y}{x}$ but we are in luck because these to quantities are equivalent. To determine our circuit equivalent, we note that the force applied to spring and mass due to x are equal (ie. they share a common effort). Secondly, we note that the mass and the damper share at least one common velocity (ie. they share a common flow).



The solving the circuit

$$e_x = Z_{eq} i_x = Z_{eq}\dot{x}$$

$$Z_{eq} = \left(\frac{1}{Z_y} + \frac{1}{Z_c}\right)^{-1}$$

$$Z_y = Ls + R = Ms + c$$

$$Z_c = \frac{k}{s}$$

$$Z_{eq} = \frac{k(Ms+c)}{Ms^2 + bs + k}$$

$$i_y = \dot{y} = \frac{e_x}{Z_y}$$

$$\frac{y}{x} = \frac{\dot{y}}{\dot{x}} = \frac{i_y}{i_x} = \frac{k}{Ms^2 + bs + k}$$