Problem 1 - Palm 2.24

All of these problems are second order, thus the roots of the characteristic equations can be found using the quadratic formula;

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

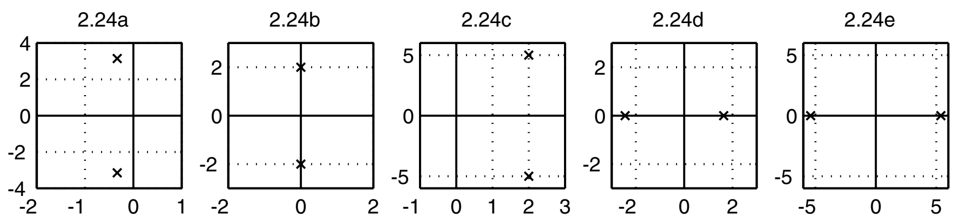

Figure 1: Pole plot for 2.24

a) $s_{1,2} = -0.3333 \pm 3.1447i$, Stable
b) $s_{1,2} = \pm 3.1447i$, marginally stable
c) $s_{1,2} = 2 \pm 5i$, unstable
d) $s_{1,2} = -2.4396,\ 1.6396$, unstable
e) $s_{1,2} = \pm 5.3853$, unstable

Problem 2 - Palm 2.22

a) $s_{1,2} = -5,\ -2$, no oscillation, response dominated by -2 pole thus $\tau_d = 0.5$, $t_{settle} = 4\tau = 2$ s.
b) $s_{1,2} = -2,\ -2$, repeated root, no oscillation, $\tau_d = 0.5$, $t_{settle} = 4\tau = 2$ s.
c) $s_{1,2} = -2 \pm 5i$, oscillates, $\tau = 0.5$, $t_{settle} = 4\tau = 2$ s, $\omega = 5\frac{rad}{s}$.

Problem 3 - Palm 2.15

While it is possible to solve this question using just the formulas in Table 2.3-1, I think it is valuable to see where those solutions were derived. In general the solution to any unforced 2nd order system $m\ddot{x} + c\dot{x} + kx = 0$ is:

$$x(t) = Ae^{s_1 t} + Be^{s_2 t}$$

where

$$s_{1,2} = \text{the roots of the characteristic equation}$$

A and B can be expressed in terms of the initial conditions of the system as follows:

$$\begin{aligned} x(0) &= x_0 = A + B \\ \dot{x}(0) &= \dot{x}_0 = As_1 + Bs_2 \\ A &= \frac{\dot{x}_0 - s_2 x_0}{s_1 - s_2} \\ B &= \frac{\dot{x}_0 - s_1 x_0}{s_2 - s_1} \end{aligned}$$

Without doing any additional work, the solution so far matches that for Case 1 (real distinct roots) in Table 2.3-1. The solution for Case 2 (real repeated roots) can be found in any differential equation textbook. The solution to Case 3 (complex conjugate pairs) is presented here since it is the most difficult and interesting. In the case of complex conjugate pairs, the solution to the characteristic equation is:

$$s_1 = a + bj \text{ and } s_2 = a - bj$$

Substituting these values into the general homogenous solution yields:

$$\begin{aligned} x(t) &= Ae^{(a+bj)t} + Be^{(a+bj)t} \\ &= e^{at}(Ae^{bjt} + Be^{-bjt}) \\ \text{Note: } e^{(a+bj)t} &= e^{at}e^{bjt} \end{aligned}$$

Substituting into our general expressions for A and B, we get:

$$\begin{aligned} A &= \frac{\dot{x}_0 - (a-bj)x_0}{a+bj-a+bj} = \frac{\dot{x}_0 - x_0(a-bj)}{2bj} \\ B &= \frac{\dot{x}_0 - (a+bj)x_0}{a-bj-a-bj} = -\frac{\dot{x}_0 - x_0(a+bj)}{2bj} \end{aligned}$$

Using Euler's Identity for complex exponentials, we get

$$\begin{aligned} e^{bjt} &= \cos bt + j \sin bt \\ e^{-bjt} &= \cos -bt + j \sin -bt = \cos bt - j \sin bt \\ \text{Note: } \cos -bt &= \cos bt \\ \sin -bt &= -\sin bt \end{aligned}$$

Combining the equations above yields:

$$\begin{aligned} x(t) &= e^{at}\left(\frac{\dot{x}_0 - x_0(a-bj)}{2bj}(\cos bt + j\sin bt) - \frac{\dot{x}_0 - x_0(a+bj)}{2bj}(\cos bt - j\sin bt)\right) \\ &= e^{at}\left(\frac{\dot{x}_0 - x_0(a-bj) - \dot{x}_0 - x_0(a+bj)}{2bj}\cos bt + \frac{\dot{x}_0 - x_0(a-bj) + \dot{x}_0 - x_0(a+bj)}{2b}\sin bt\right) \\ &= e^{at}(x_0 \cos bt + \frac{\dot{x}_0 - x_0 a}{b}\sin bt) \end{aligned}$$

Using the trigonometric identity:

$$\begin{aligned} A\cos bt + B\sin bt &= \sqrt{A^2+B^2}\sin(bt+\phi) \\ \text{where } \phi &= \tan^{-1}\frac{A}{B} \end{aligned}$$

We can show that

$$\begin{aligned} x(t) &= e^{at}\sqrt{x_0^2 + \frac{(\dot{x} - x_0 a)^2}{b^2}}\sin(bt+\phi) \\ \phi &= \tan^{-1}\frac{x_0 b}{\dot{x}_0 - a x_0} \end{aligned}$$

This expression is equivalent to that given in Table 2.3-1. The slightly different expression due to fact that I do not assume that a is a negative number.

For all sections $x_0 = 0$ and $\dot{x}_0 = 1$

a) $s_{1,2} = -2 \pm 2i$, complex conjugate pair

$$x(t) = e^{-2t}\sin 2t$$

b) $s_{1,2} = -6, -2$, Real distinct roots

$$x(t) = 0.25e^{-2t} - 0.25e^{-6t}$$

c) $s_{1,2} = -2, -2$, Repeated roots

$$x(t) = te^{-2t}$$

Problem 4 - Palm 4.29

This is a bit of a trick question since you need more information to determine k and c. Specifically, you need to know the period of the oscillation. Nonetheless, we can determine ζ using the Logarithmic decrement:

$$\begin{aligned}
\delta &= \frac{1}{n}\ln\frac{B_i}{B_{i+n}} \\
\zeta &= \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}} \\
\delta &= \frac{1}{30}\ln\frac{1}{0.5} = 0.0536 \\
\zeta &= \frac{0.0536}{\sqrt{(2\pi)^2+0.0536^2}} = 0.0085
\end{aligned}$$

If we had the period P, we could calculate k and c using the following relationships:

$$\begin{aligned}
k &= m\omega_n^2 = \frac{m\omega_d^2}{1-\zeta^2} = \frac{m(2\pi/P)}{1-\zeta^2} \\
\zeta &= \frac{c}{2\sqrt{mk}}
\end{aligned}$$

Problem 5 - Palm example4.3-3

The characteristic equation for this problem is:

$$\begin{aligned}
I_e\ddot{\theta} + c_e\dot{\theta} + k_e\theta &= 0 \\
\text{where} & \\
I_e &= I_m + I_s + \frac{m_pR^2}{2} + m_rR^2 \\
k_e &= kR^2
\end{aligned}$$

This means that:

$$\omega_n^2 = \frac{k_e}{I_e} = \frac{kR^2}{I_m + I_s + \frac{m_pR^2}{2} + m_rR^2}$$

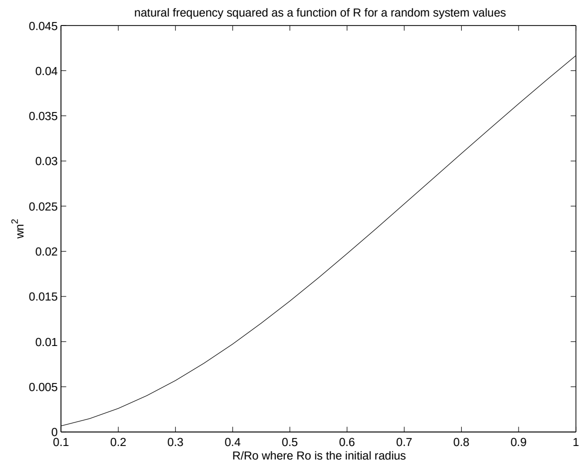

Figure 2: Natural frequency vs R

It is a little difficult to see since both the numerator and denominator contain R, but we can see that as R drops the denominator converges to a positive real value, while the numerator converges to zero. This that the system natural frequency drops as R drops. Figure illustrates how the natural frequency drops as R gets smaller for this system with some set of values for I, R, m, and k.

Problem 6 - Palm 1.21

a) ≫(-3+5i)*(-6+7i)
ans=-17-51i
b) ≫(-3+5i)/(-6+7i)
ans=0.625-0.1059i
c)≫3*i/2
ans=0+1.5i
d) ≫3/(2i)
ans=0-1.5i

Problem 7 - Palm 1.22

a)≫x=-5-7i;y=6+2i
≫x+y
ans=1-5i
b) ≫x*y

ans=-16-52i
c) $\gg$x/y
ans=-1.1-0.8i