8.07 Homework 3

Problem 1. Griffiths 2.33 (p.95)

Problem 2. Griffiths 2.36 (p.101)

Problem 3. (Jackson 1.7) Two long cylindrical conductors of radii a_1 ande a_2 are parallel and separated by a distance d, which is large compared with either radius. Show that the capacitance per unit length is given approximately by

$$C \simeq \frac{\pi\,\epsilon_0}{\ln(d/a)}$$

where a is the geometrical mean of the two radii. What diameter wire in millimiters is needed for a two-wire transmission line with $C = 1.2 \times 10^{-11}$ F/m if the separation of the wires is 0.5 cm.?

Problem 4. (Challenging!). Griffiths 2.48, p.107. In part (e) one can guess a solution which must satisfy the stated condition that the field at the cathode is reduced to zero by the space charge.

Problem 5. Use virtual work to calculate the attractive force between conductors in the parallel plate capacitor (area A separation d). Do your virtual work computations in two ways:
(i) keeping fixed charges on the plates, and,
(ii) keeping a fixed voltage between the plates.

Problem 6. Use the identity

$$\phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{x}')}{R} d^3x' + \frac{1}{4\pi}\oint_S da' \Big[\frac{1}{R}\frac{\partial\phi}{\partial n'} - \phi\frac{\partial}{\partial n'}\Big(\frac{1}{R}\Big)\Big]\,.$$

discussed in lecture to prove the *mean value theorem*: In a charge-free region the value of the electrostatic potential at any point is equal to the average of the potential over the surface of any sphere centered at that point.

Problem 7. (Challenging!) In lecture we discussed relations of the form

$$V_i = \sum_{j=1}^{n} p_{ij} Q_j\,, \qquad Q_i = \sum_{j=1}^{n} C_{ij} V_j, \quad i, j = 1, 2, \ldots, n\,.$$

governing the potentials and charges of n conductors (with zero potential at infinity).
(i) Call C and p the matrices whose elements are C_{ij} and p_{ij} respectively. Show that C is the inverse of p. Prove that the matrix p must have an inverse.
(ii) Prove that $C_{ij} = C_{ji}$. (Hint: Green's theorem might be useful)
(iii) Consider a two-conductor configuration. Calculate the conventional capacity C in terms of C_{11}, C_{12}, C_{21} and C_{22}.
(iv) Consider two concentric conducting shells of radii a, b with $a < b$. Call the small shell conductor 1, and the larger shell conductor 2. Calculate the matrix of capacitances and the conventional capacity C. Verify that the formula you derived in (iii) holds.