8.07 Homework 5

Problem 1. An azimuthally symmetric charge distribution produces a potential $V(r, \theta)$. On the z axis ($\theta = 0$) the potential is given by

$$V(r, 0) = V_0 \left(1 - \frac{r^2 - a^2}{r\sqrt{r^2 + a^2}}\right), \quad r > a.$$

Find the leading two terms for the potential $V(r, \theta)$ when $r >> a$.

Problem 2. A charge q is put at a distance z from the center of a grounded sphere of radius a (smaller than z). Expand the potential due to the charge in terms of basic axial solutions. Expand the potential due to the charge distribution on the sphere in terms of basic solutions with undetermined coefficients. Fix the value of these coefficients using the relevant boundary condition. Show that the potential due to the sphere can be thought of as the potential of a point image charge of the expected strength and position.

Problem 3. Griffiths 3.20 (p.145)

Problem 4. Griffiths 3.22 (p.145)

Problem 5. Griffiths 3.23 (p.145)

Problem 6. Based on Jackson 3.3 (challenging!)

A thin, flat, conducting circular disk of radius R is located in the x–y plane with its center at the origin, and is mantained at a fixed potential V. With the information that the charge density on a disk at fixed potential is proportional to $(R^2 - \rho^2)^{-1/2}$, where ρ is the distance out from the center of the disk, show that for $r > R$

$$V(r, \theta) = \frac{2V}{\pi} \frac{R}{r} \sum_{l=0}^{\infty} \frac{(-1)^l}{2l + 1} \left(\frac{R}{r}\right)^{2l} P_{2l}(\cos\theta)\,.$$

Calculate the capacitance of the disk.

Problem 7. For a dipole of dipole moment $\vec{p}$ located at the origin, the potential is

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{x}}{|\vec{x}|^3}$$

Compute the electric field from $\vec{E} = -\nabla V$, and obtain the result in equation (3.104), p.155 of Griffiths. Show also that this result is equivalent to the result quoted in equation (3.103), p.153.

Problem 8. Griffiths 3.41 (p.156).

Problem 9. Tensors under rotations – Exercises.

(i) Consider a tensor M_{ij}. Show that $\det(M)$ is a scalar under rotations.

(ii) Consider an antisymmetric tensor A_{ij}. Show that it can be written as $A_{ij} = \epsilon_{ijk} v_k$ by giving an explicit expression for v_k. Show that v_k is a pseudo-vector.