**8.07 Homework 2**

**Problem 1.** Griffiths 1.48 (p.52).

**Problem 2**. Exercises with $\delta$-functions

(i) A charge $Q$ is spread uniformly over a spherical shell of radius $R$. Express the volume charge density using a delta function in spherical coordinates. Repeat for a ring of radius $R$ with charge $Q$ lying in the $xy$ plane.

(ii) Express a three dimensional delta function in cylindrical coordinates ($\rho, \phi$ and $z$).

(iii) A charge $\lambda$ per unit length is distributed uniformly over a cylindrical surface of radius $b$. Give the volume charge density using a delta function in cylindrical coordinates

(iv) What is $\nabla^2 \ln r$ in two dimensions ?

**Problem 3**. This problem is closely related to problem 1.60, p.56 of Griffiths. You will find useful hints there– but try without hints first!!. Show that:

(i) $\int_V \nabla\psi \, d^3x = \int_S \psi \, d\vec{a}$, where $S$ is the surface bounding the volume $V$. Show that as a consequence of this, $\int_S d\vec{a} = 0$ for a closed surface $S$.

(ii)$\int_V \nabla \times \vec{A} \, d^3x = \int_S d\vec{a} \times \vec{A}$, where $S$ is the surface bounding the volume $V$.

(iii) $\int_S \nabla\psi \times d\vec{a} = -\oint_\Gamma \psi \, d\vec{l}$, where $\Gamma$ is the boundary of the surface $S$.

(iv) For a closed surface $S$, one has $\int_S (\nabla \times \vec{A}) \cdot d\vec{a} = 0$.

**Problem 4**. Griffiths 1.61, p.57, parts (c), (d) and (e), only. In part (d) Griffiths is a bit unclear: take an arbitrary loop and draw the cone subtended by the loop and with apex at the origin.

**Problem 5.** Exercises: (i) Griffiths 2.2 (p. 61), (ii) Griffiths 2.5 (p.64)

**Problem 6**. (Challenging!) A direct calculation of $\vec{A}(\vec{x})$ given $\vec{B} = \vec{\nabla} \times \vec{A}$.

Prove that

$$\vec{A}(\vec{x}) = -\int_0^1 t \, dt \; \vec{x} \times \vec{B}(t\vec{x}),$$

by showing by explicit computation that the proposed $\vec{A}$ satisfies $\vec{\nabla} \times \vec{A} = \vec{B}$ provided that $\nabla \cdot \vec{B} = 0$. In order to solve this problem, it is useful to define $\vec{y} = t\vec{x}$, and then to prove that

$$\nabla_{\vec{x}} = t\nabla_{\vec{y}}, \quad \text{and} \quad \frac{d}{dt}\vec{B}(t\vec{x}) = \frac{1}{t}(\vec{x} \cdot \nabla_{\vec{x}})\vec{B}(t\vec{x}).$$

Final hint: try to turn the integrand into a total derivative.

**Problem 7.** Griffiths 2.12 (p.75). Do not compare with 2.8.

**Problem 8.** Griffiths 2.18 (p.75).