8.07 Fall 2001 – Test 1

One hour and a half. Closed book and notes – Formula sheet allowed.

Problem 1. (30 points) Three exercises:

(i) (10 points) A ring centered around the z-axis and parallel to the xy-plane has a charge Q uniformly distributed. Any point on it is a distance r_0 away from the origin, and any line joining the origin with the ring makes an angle θ_0 with the z-axis. Express the charge *density* $\rho(\vec{x})$ using delta functions in spherical coordinates.

(ii) (10 points) Derive, using index notation a formula for $\nabla \times (\psi \mathbf{F})$, where ψ is a scalar function and $\mathbf{F}$ a vector function. Present your answer without index notation.

(iii) (10 points) Let T_{ij} be a pseudo-tensor of rank two (under rotations). What kind of object is $\delta_{ij} T_{ij}$. Prove your assertion.

Problem 2. (20 points) Leading term of a potential.

Find the leading term for the far away potential $V(r, \theta)$ for a charge configuration consisting of four charges $(q, -q, -q, +q)$ lying on the z-axis and located at $z = +3a, +a, -a, -3a$, respectively.

Problem 3. (25 points) Superconducting sphere in a constant magnetic field.

(i) (5 points) Consider a magnetic dipole moment of magnitude m placed at the origin and whose direction is aligned with the positive z-axis. Give the expression for the magnetic induction $\mathbf{B}$ using spherical coordinates (forget the delta function piece!!!)

(ii) (5 points) Express a constant magnetic field $\mathbf{B}_0 = B_0\,\hat{\mathbf{z}}$ which points in the z-direction using spherical coordinates.

A superconducting sphere of radius a is inmersed inside a uniform external magnetic field configuration with $\mathbf{B}_0 = B_0\,\hat{\mathbf{z}}$. Inside a superconductor the magnetic induction must vanish. We would like to find the magnetic induction $\mathbf{B}$ outside the sphere. In fact, you will show that the magnetic field outside the sphere can be represented as the sum of the field of a magnetic dipole centered at the origin and the external field $\mathbf{B}_0$.

(iii) (10 points) Consider the relevant boundary conditions and determine the magnetic moment of the required dipole. Give the expression of the exterior magnetic field $\mathbf{B}$.

(iv) (5 points) Calculate the surface current density $\mathbf{K}$ on the surface of the sphere.

Problem 4. (25 points) Images for a dipole outside a conducting sphere.

Consider a grounded conducting sphere of radius a centered at the origin. Outside the sphere, on the y axis and with coordinate y there is an ideal point electric dipole with dipole moment p pointing in the positive y-direction. Find the set of images that should be placed inside the sphere in order to solve for the potential everywhere outside the sphere. Give their position, direction, etc. and write all values in terms of p, a and y.

Hints: An ideal point dipole with dipole moment of magnitude p can be modeled as a pair of charges with charge $+q$ and $-q$ separated a distance d, with $p = qd$ in the limit when the separation d goes to zero and the charge q goes to infinity, keeping the product a constant. Use such a model for the dipole outside the sphere, calculate the images, and take the limits to interpret.