18.06 Problem Set 3

Due Wednesday, March 13.

Problem 1. (5) Which of the following sets are bases for $\mathbb{R}^3$?

(a) {(1,2,0), (0,1,-1)}

(b) {(1,1,-1), (2,3,4), (4,1,-1), (0,1,-1)}

(c) {(3,2,2), (-1,2,1), (0,1,0)}

(d) {(1,0,0), (0,2,-1), (3,4,1), (0,1,0)}

Problem 2. (5)

(a) Let M_2 be the vector space of 2×2 matrices. Determine whether or not the following set is a basis for M_2: $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$.

(b) Find a basis for the subspace of M_2 consisting of symmetric matrices.

Problem 3. (8) Find the dimensions of the following subspaces of $\mathbb{R}^4$:

(a) All vectors of the form (a, b, c, d) with $d = a + b$.

(b) All vectors of the form (a, b, c, d) with $c = a - b$ and $d = a + b$.

(c) All vectors of the form (a, b, c, d) with $a = b$.

(d) All vectors of the form $(a + c, a - b, b + c, -a + b)$.

Problem 4. (6) Find a basis for each of the four subspaces associated to the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

Problem 5. (6) Suppose that $\{v_1, v_2, \dots, v_n\}$ is a basis for $\mathbb{R}^n$. Show that if A is an $n \times n$ invertible matrix, then $\{Av_1, Av_2, \dots, Av_n\}$ is also a basis for $\mathbb{R}^n$.