18.06 Problem Set #1

Due Wednesday, Feb.13

1) (a) Find all values of a so that the vectors $(a, 4)$ and $(2, 5)$ are parallel.

   (b) Find all values of a so that the vectors $(a, 2)$ and $(a, -2)$ are orthogonal.

   (c) Find all values of a so that the vector $(1, a, -3, 2)$ has length 5.

2) Suppose that v and w are non-parallel vectors in $\mathbb{R}^2$, with starting point at the origin. Show that the diagonals of the parallelogram determined by v and w bisect each other. (Hint: Use vectors to show that the midpoints of the two diagonals coincide.)

3) Consider the system of equations

$$\begin{aligned} -3x + 4y &= 8 \\ 6x + ty &= s \end{aligned}$$

   where t ans s are real numbers.
   (a) Write the matrix equation for this system.
   (b) Find values for s and t so that the system has exactly one solution.
   (c) Find values for s and t so that the system has no solutions.
   (d) Find values for s and t so that the system has infinitely many solutions.
   (e) Give a geometric interpretation of (b), (c), and (d).

4) True or False: If A and B are 2x2 matrices, then $(A + B)^2 = A^2 + 2AB + B^2$. If true give a brief explanation, if false give an example where the equality fails.

5) Write down a 3x3 matrix A so that if the vector $v = (x, y, z)$ in $\mathbb{R}^3$ is multiplied by A, the x and y coordinates of v are unchanged, but the z coordinate becomes zero.