18.06 Problem Set #4

Due Wednesday, Mar 20

1 Without computing A, find a basis (expain your answers) for each of the four subspaces associated with

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 8 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

2 a) Suppose the columns of A are unit vectors, all mutually perpendicular. What is $A^T A$?

b) Square matrices that satisfy the conditions in a) are called orthogonal matrices. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is a 2x2 orthogonal matrix for any real number θ.

c) Prove the set of 2x2 orthogonal matrices is not a vector space by giving counter examples.

d) Show that an $n \times n$ matrix A is orthogonal if and only if $A^T = A^{-1}$.

3 Let S be the vector space spanned by vectors $(1, 0, -2, 0)$, $(0, 2, 4, -1)$ and $(2, 2, 0, -1)$. Find $S^\perp, (S^\perp)^\perp$. What is the relationship between $(S^\perp)^\perp$ and S?

4 Find a basis for the vector subspace S, which is the intersection of U and V where U is the span of $\{(1, -2, 0, 3), (0, 1, 0, -1)\}$, and V is the span of $\{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0)\}$.

5 Find the projection of b onto the column space of A by solving $A^T A\hat{x} = A^T b$ and $p = A\hat{x}$.

a) $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 1 & 3 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$. b) $A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

6 Let A be an $m \times n$ matrix of rank n. Suppose that P is the projection matrix that projects onto the column space of A. What is the size of the matrix P? What is its rank? Explain.