

18.06 Problem Set #2

Due Wednesday, Feb 20

1) (5 points) Suppose A is 3 by 5, B is 5 by 3, C is 5 by 1, and D is 3 by 1. Which of these matrix operations are allowed, and what are the shape of the results?

BA , $A(B+C)$, ABD , $AC+BD$, $ABABD$

2) (5 points) Find the inverse (assuming they exist) of these block matrices:

$$\begin{bmatrix} I & 0 \\ C & -I \end{bmatrix}, \begin{bmatrix} A & B \\ 0 & D \end{bmatrix}, \begin{bmatrix} 0 & I \\ -I & D \end{bmatrix}.$$

3) (5 points) Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a , b , c , d to get $A=LU$ with four pivots.

4) (5 points) Which permutation matrix P makes PA upper triangular? Find permutations P_1, P_2 so that P_1AP_2 is lower triangular.

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 0 & 7 \\ 0 & 6 & 9 \end{bmatrix}.$$

5) (10 points) (a) Find missing entries:

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & 1 \\ -3 & -9 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 3 & 3 \\ * & -1 & -1 \\ * & * & -1 \end{bmatrix}.$$

(b) Use above to compute the inverse of $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 5 & -9 \\ 0 & 1 & -1 \end{bmatrix}$.