18.06 Problem Set #1 Solutions

1a) (2 points) In order for the two vectors to be parallel we must have $(a, 4) = c(2, 5)$ for some constant c. Thus, we need $a = 2c$ and $4 = c5$. It follows that $c = \frac{4}{5}$ and $a = \frac{8}{5}$.

1b) (2 points) We look for those values of a for which the dot product, $(a, 2) \cdot (a, -2) = 0$. Since $(a, 2) \cdot (a, -2) = a^2 - 4$, we get $(a, 2) \cdot (a, -2) = 0$ if $a = 2$ or $a = -2$.

1c) (2 points) We compute

$$||(1, a, -3, 2)|| = \sqrt{(1, a, -3, 2) \cdot (1, a, -3, 2)} = \sqrt{1 + a^2 + 9 + 4} = \sqrt{14 + a^2}.$$

Hence $||(1, a, -3, 2)|| = 5$ when $a = \sqrt{11}$.

2 (6 points) Drawing all vectors as starting at the origin, we have the midpoint of the diagonal $v + w$ at the terminal point of the vector $\frac{1}{2}(v + w)$, while the midpoint of the diagonal $w - v$ is at $v + \frac{1}{2}(w - v)$. But

$$v + \frac{1}{2}(w - v) = v + \frac{1}{2}w - \frac{1}{2}v = \frac{1}{2}(v + w).$$

Hence the midpoints coincide and the diagonals bisect each other.

3a) (2 points)The matrix equation is

$$\begin{bmatrix} -3 & 4 \\ 6 & t \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ s \end{bmatrix}.$$

3b) (2 points) After elimination we have the augmented matrix

$$\begin{bmatrix} -3 & 4 & 8 \\ 0 & t+8 & s+16 \end{bmatrix}.$$

Therefore, the system will have exactly one solution for any values of t, s where $t + 8 \neq 0$ (this condition gives us a full set of 2 pivots).

3c) (2 points) Take $t = -8$ and let s be any value other than -16. Then we have an equation of the form $0 \cdot y = d$ where $d = s + 16 \neq 0$, and so there are no solutions.

3d) (2 points) Taking $t = -8$ and $s = -16$ gives $0 \cdot y = 0$, which is satified by any value of y; hence there are infinitely many solutions to the system.

3e) (2 points) In (b) two lines intersect in exactly one point. In (c) we have two parallel lines which never meet. In (d) we have two equations which give the same line, so any point on one line is automatically on the other.

4) (4 points) FALSE. Take

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Then

$$(A + B)^2 = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$$

while

$$A^2 + 2AB + B^2 = \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix}.$$

5) (4 points) Take

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$