18.06 Problem Set 5

Due Wednesday, April 3.

Problem 1. Find the least squares parabola for the data points $\{(1,2),(2,5),(3,7),(4,1)\}$.

Problem 2. The following table gives (in U.S. dollars) the estimated public expenditures on education, per inhabitant, in the indicated years in Europe and North America.

Year	Europe	North America
1970	90	317
1975	197	474
1980	335	816
1985	394	1101

(a) Find the best fit line that give expenditures as a function of time (take 1970 as time zero) for Europe. Do the same for North America.

(b) Estimate the per inhabitant expenditures for Europe in 2000.

(c) Does one expect the difference between per inhabitant expenditures in Europe and North America to increase or decrease after 1985? Why?

Problem 3. Let $S = \{\mathbf{v_1}, \ldots \mathbf{v_n}\}$ be an orthogonal set. Show that S is linearly independent.

Problem 4.

(a) Let A and B be $n \times n$ orthogonal matrices. Show that AB is an orthogonal matrix.

(b) Show that if A is an orthogonal matrix, then $det(A)$ is either 1 or -1.

Problem 5 Construct an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by {(3,0,0,0), (0,1,2,1), (0,-1,3,2)}.

Problem 6 Show that the Vandermonde determinant,

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b).$$