1. A column of length $L$ and bending rigidity $EI$ is pin-connected at both ends and at the same time is connected at the lower end to a torsional spring (resisting rotation) having a spring constant of $K_T$ as shown in Figure 1.

(a) When the column is subjected to axial load $P$, show that the equation for critical load $P_{cr}$ for buckling is

$$\tan(kL) = \frac{kL}{1 + \frac{EI}{K_T}k^2} \quad \text{where} \quad k = \sqrt{\frac{P}{EI}} \quad (1)$$

(b) Instead of $P$, the column is under the action of uniformly distributed axial force per unit length $q$ (e.g. its weight) as shown in Figure 4-(b). Using the energy method, prove that $q_{cr}$ is given as follows. Assume the deflection curve is $y = A \sin(\frac{\pi x}{L})$.

$$q_{cr} = \frac{4K_T}{L^2} + \frac{2EI\pi^2}{L^4} \quad (2)$$

2. An overhanging beam ABC with rectangular cross section of height $h$ is heated to a temperature $T_1$ on the top and $T_2$ ($> T_1$) on the bottom (see Figure 2). The coefficient of thermal expansion of the material of the beam is $\alpha$. Assume the temperature within the thickness varies linearly. Young’s modulus of elasticity is $E$. Show that the vertical deflection $\delta_c$ at the free end $C$ of the overhang is

$$\delta_c = \frac{\alpha L}{2h} (T_2 - T_1)(L + a) \quad (3)$$

3. A rigid bar is supported by five columns of length $L$, cross sectional area $A_i$ and Young’s modulus of elasticity $E_i$ as shown in Figure 2. Considering tension and compression of the columns only, answer the following questions. (Assuming NO BUCKLING.)
(a) Express the total strain energy as a function of $v$, $\theta$ and $L$ under the action of moment $M$ at B. Here $v$ and $\theta$ are the vertical displacement and rotation of the rigid bar about point O, respectively.

(b) Show that the vertical displacement $v$ and rotation angle $\theta$ at O under the action of moment $M$ at B are

$$v = \frac{-b}{ac - \dot{\theta}^2} M$$

$$\theta = \frac{a}{ac - \dot{\theta}^2} M$$

where

$$a = \sum_{i=1}^{5} \frac{E_i A_i}{L}, \quad b = \sum_{i=1}^{5} E_i A_i |3 - i|.$$
\[ c = \sum_{i=1}^{5} E_i A_i (3 - i)^2 L \]

A diagram for Problem 3