Massachusetts Institute of Technology
Physics Department

Physics 8.20 | IAP 2003
Introduction to Special Relativity | January 10, 2003

Assignment 2
Due January 17, 2003

Announcements

- Please remember to put your name at the top of your paper.
- Problem Sets can also be downloaded from http://web.mit.edu/8.20/
- Depending on our rate of progress, some of the problems at the end of the problem set may be delayed until Problem Set #3

Topics for this period

- The kinematic consequences of special relativity foundation for spacetime
- Velocity addition and other differential transformations
- Kinematics and "paradoxes"

Reading Assignment 2

- Resnick and Halliday, §2 and 3
- French §4 and §5 pages 125-134.
- Einstein §12 – 17 and Appendix 2

Problem Set 2

1. **A Lorentz transformation**

   The origins of two inertial frames, Σ and Σ', coincide at $t = t' = 0$. The origin of Σ' moves at speed $u = 6 \times 10^7$m/sec along the positive x axis in Σ. An event occurs at $x' = 2$m, $y' = 8$m, $z' = 3$m and $t' = 16$ sec as observed in Σ'. Where and when does this event occur in Σ?

2. **Lorentz transformations are rotations through imaginary angles**

   French §3, p. 87, Problem 3-9.

3. **Spacetime diagrams I**

   Once again: The origins of two inertial frames, Σ and Σ', coincide at $t = t' = 0$ and their axes are parallel. The origin of Σ' moves at speed u along the positive x axis in Σ.

   (a) Draw a figure from the point of view of an observer in the frame Σ showing the x' axis (ie. $t' = 0$) and the ct' axis (ie. $x' = 0$). Draw the figure to scale when $u = 0.5c$.

   (b) Now draw a figure describing the same situation from the point of view of an observer in Σ'. That is, plot the x and ct axes as observed from Σ'.

4. **Spacetime diagrams II**

   A space traveler sets off from the Earth at $v = 0.99c$ in a straight line toward a star 99 light years away. After 100 years (as measured on Earth) she quickly decelerates to $v = 0$ and explores the star for 30 years. Then she returns to earth by quickly reaccelerating to $v = 0.99c$, travelling for 100 years and decelerating as she approaches the Earth. Draw a spacetime diagram showing the space traveler's worldline from the point of view of an observer on the Earth. Measure time in years and distance in light years.

5. **Earth contraction**

   By what amount is the Earth shortened along its diameter (as measured by an observer at rest relative to the Sun) owing to its orbital motion around the Sun? [You can take the orbital velocity of the Earth to be 30 km/sec and its radius to be 6371 km.]

6. **How fast a ship?**

   A spaceship is moving at such a speed in the laboratory frame that its measured length is half its proper length. How fast is the spaceship moving relative to the laboratory frame?

7. Time dilation ≡ lifetime extension

If the mean lifetime of a muon when it is at rest is 2.2×10^{-6} seconds, calculate the average distance it will travel in vacuum before decay, if its velocity is a) $0.9c$, b) $0.99c$, c) $0.999c$.

8. To the galactic center

(a) Can a person travel (in principle) from the Earth to the galacti center (which is about 28,000 lyr distant) in a normal lifetime? Explain first using time dilation, then explain again using length contraction.

(b) What constant velocity would be needed to make the trip in 30 years?

9. String of lights across the desert I

A series of lights is arrayed in a straight line across the desert. Neighboring lights are separated by a distance d. They are set up to flash in sequence with an interval τ between neighboring lights (as measured in the rest frame of the lights). An observer, O, travels along the same line at a uniform speed v in the same direction of the wave of flashes.

(a) At what interval do the flashes occur in the rest frame of O?

(b) Suppose O travels in the direction opposite the wave. What is the interval in this case?

(c) For what choices of d, τ, and v do all flashes occur simultaneously in the rest frame of O?

10. String of lights across the desert II

In the previous problem you were asked for the times at which the flashed occured in the rest frame of O. Now consider what would be *seen* by the observer O:

Again let the straight line of lights be separated by d and flash in sequence with interval τ. Now compute the interval between the sequential flashes *as seen by* an observer travelling with uniform speed v along the direction of the flashes. [Hint: in this case you must consider not only the Lorentz transform of each flash event, but you must also consider how long it takes a flash to propagate to the observer O.] For what values of d, τ, and v will the flashes *appear to be* simulataneous to O?

11. Events in two different frames I

Two events, A and B, are observed in two different inertial frame, Σ, and Σ'. Frame Σ' moves along the x axis in Σ. Event A occurs at the spacetime origin in both frames,

$$x_A = y_A = z_A = ct_A = 0$$
$$x'_A = y'_A = z'_A = ct'_A = 0$$

Event B occurs at

$$x_B = 10, \quad y_B = z_B = 0, \quad ct_B = 2$$

(all distances are in meters).

The two events occur *simultaneously* in frame Σ'.

(a) Find the velocity of Σ' with respect to Σ.

(b) What is the space separation of the two events in the frame Σ'?

(c) What is the smallest spatial separation between the two events in any inertial frame?

12. Events in two different frames II

Two events, A and B, are observed in two different inertial frame, Σ, and Σ'. Event A occurs at the spacetime origin in both frames,

$$x_A = y_A = z_A = ct_A = 0$$
$$x'_A = y'_A = z'_A = ct'_A = 0$$

Event B occurs at

$$x_B = 8, \quad y_B = z_B = 0, \quad ct_B = 10$$

(all distances are in meters) as observed in Σ.

The two events occur *at the same point* in frame Σ'.

(a) Find the velocity of Σ' with respect to Σ.

(b) What is the time separation of the two events in the frame Σ'?

(c) What is the shortest time separation between the two events in any inertial frame?

13. Events in two different frames III

Two events, A and B, are observed in two different inertial frame, Σ, and Σ'. Event A occurs at the spacetime origin in both frames,

$$x_A = y_A = z_A = ct_A = 0$$
$$x'_A = y'_A = z'_A = ct'_A = 0$$

Event B occurs at

$$x_B = 2, \quad y_B = z_B = 0, \quad ct_B = 3$$

in Σ, and at

$$x'_B = 3, \quad y'_B = z'_B = 0$$

(all distances are in meters) in Σ'.

(a) What time does event B occur in Σ'?

(b) What is the relative velocity of Σ' relative to Σ?

14. The travels of elementary particles reconsidered

Return to Problem 2 on Problem Set 1 and recalculate the speed of the particles given their lifetimes and pathlengths. The problem is reproduced below for your convenience:

> Elementary particles have very short average lifetimes (at least measured on our perceptual scale). In experiments at accelerators the particles are produced and then detected at points away from the point of production. If the rules of Newtonian physics were correct for particles travelling at great speeds, the particles would have to travel much faster than the speed of light in order to go so far before they decay.
>
> Assuming that the particle in question lives for the average lifetime of that species, compute its speed using Newtonian mechanics:
>
> (a) A "muon" has a lifetime of 2.2×10^{-6} sec. It is observed 5 kilometers away from its point of production.
>
> (b) A "Lambda" hyperon has a lifetime of 2.6×10^{-10} sec. It is observed 10.5 meters away from its point of production.
>
> (c) A "B^0 meson" has lifetime 1.5×10^{-12} sec. It is observed 3 millimeters away from its point of production.

15. Invariance of the interval

(a) Show that the square of the spacetime interval (Δs^2) associated with two events (assumed to occur in the $x - x'$ axis) is invariant under a Lorentz transformation. That is, show that

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

(b) For each of the following pairs of events, say whether

i. there is a frame where they are simultaneous, or

ii. there is a frame where they occur at the same place, or

iii. neither

$$\begin{array}{lll}
\text{I} & x_A = 3, y_A = 4, z_A = 0, & ct_A = 2 \\
 & x_B = 1, y_B = 2, z_B = 0, & ct_B = 1 \\[1em]
\text{II} & x_A = 6, y_A = 7, z_A = 2, & ct_A = 8 \\
 & x_B = 3, y_B = 3, z_B = 2, & ct_B = 3 \\[1em]
\text{III} & x_A = 2, y_A = 4, z_A = 1, & ct_A = 2 \\
 & x_B = 3, y_B = 3, z_B = 2, & ct_B = 4
\end{array}$$

16. The expanding universe

Resnick and Halliday §2, page 87, Problems 54 and 55.

17. Perpendicular velocities

Resnick and Halliday §2, page 87, Problem 61.

18. Transforming angles I

A particle moves with speed u in the $x - y$ plane, making an angle θ with respect to the x-axis in frame Σ. The origin of Σ moves to the right (along the positive x' axis) in the frame Σ' with speed v. What speed u' and angle θ' will the particle appear to have to an observer in Σ'?

19. Transforming angles II

A right triangular plate is at rest in the frame Σ. Its legs are places on the x and y axes and its hypoteneuse makes an angle θ with respect to the x-axis. The origin of Σ moves to the right (along the positive x' axis) in the frame Σ' with speed v. What are the angles of the triangle as measured in Σ'?