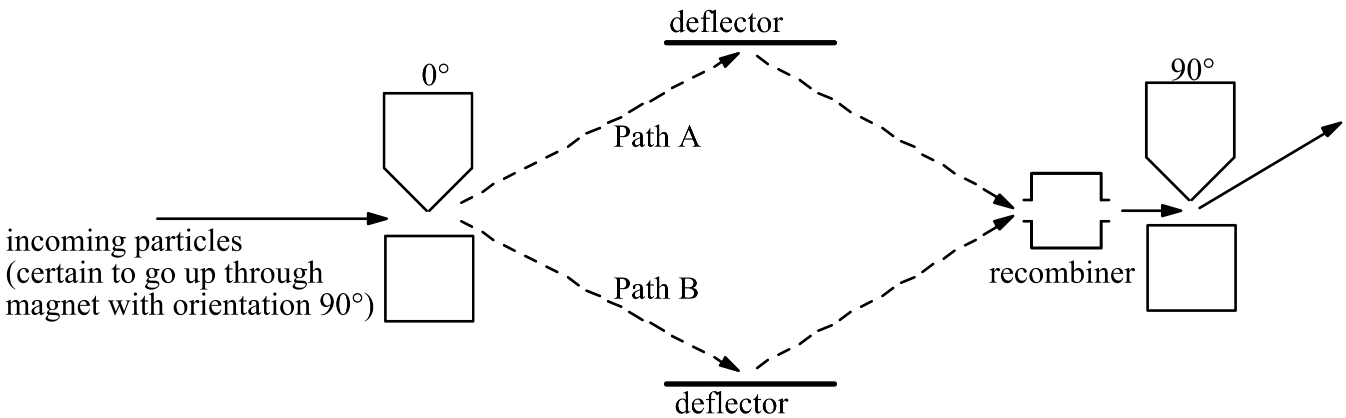
Handout #1: The Two-Path Experiment and Bell's Inequalities

I. The two-path experiment



Suppose we set up the two-path experiment as shown above, with the incoming particles prepared in such a way that they are certain to go up through a magnet with orientation 90°. When we put particle detectors in Paths A and B, we find that any incoming particle activates one or the other of them. Hence we conclude:

1. Every incoming particle follows either Path A or Path B.

Further, if we block off Path B—so that any particle that makes it through the second magnet must have followed Path A—we find that 50% of the particles entering the second magnet go up, and 50% go down. (We also find, if we rotate the second magnet so that it has orientation 0°, that every particle entering it goes up, as expected.) Hence we conclude:

2. Every particle that follows Path A has a 50% chance of going up through the second magnet.

Finally, if we block off Path A—so that any particle that makes it through the second magnet must have followed Path B—we again find that 50% of the particles entering the second magnet go up, and 50% go down. (We also find, if we rotate the second magnet so that it has orientation 0°, that every particle entering it goes down, as expected.) Hence we conclude:

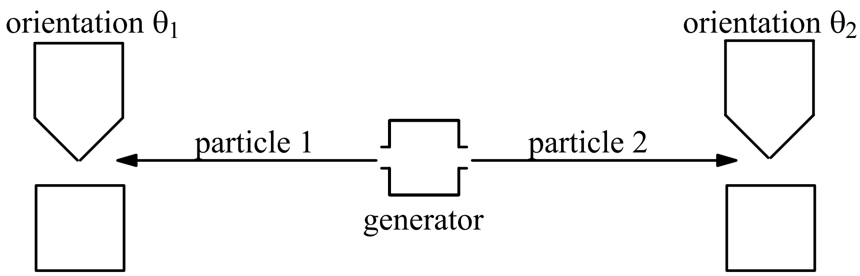
3. Every particle that follows Path B has a 50% chance of going up through the second magnet.

From 1, 2, and 3, it follows that

4. Every incoming particle has a 50% chance of going up through the second magnet.

But if we leave Paths A and B undisturbed—i.e., don't block them off, and don't put detectors in them—we observe what is depicted above: every incoming particle in fact goes up through the second magnet. Hence 4 is false, and even though it seems that we have excellent experimental confirmation for 1, 2, and 3, at least one of them must be given up.

II. Bell's Inequalities



In the experiment depicted above, the generator (which creates pairs of particles, sending one towards each magnet) can be set up in such a way that the two particles exhibit the following behavior: if $\theta_1 = \theta_2$, then either particle 1 goes up and particle 2 down, or particle 1 goes down and particle 2 up. Suppose you try to explain this behavior by means of the following hidden variables hypothesis: First, for each possible magnet orientation θ, each particle either has the property (up, θ)—in which case it will (with certainty) go up through a magnet with this orientation—or it has the property (down, θ)—in which case it will (with certainty) go down through a magnet with this orientation. Second, the particles are generated in such a way that, for any orientation θ, particle 1 has (up, θ) if and only if particle 2 has (down, θ).

Now suppose that we run the experiment many times, letting $\theta_1 = 0°$ or +120°, and letting $\theta_2 = 0°$ or -120°. Then quantum mechanics gives us the following *experimentally confirmed* probabilities:

θ_1	θ_2	Prob(one up, one down)
0°	0°	1
0°	-120°	.25
+120°	0°	.25
+120°	-120°	.25

What predictions does the hidden variables hypothesis make? That depends on the probabilities assigned to the various possible distributions of the relevant properties, which are these:

particle 1	particle 2	probability
(up, 0°),(up, +120°)	(down, 0°),(up, -120°)	p_1
(up, 0°),(up, +120°)	(down, 0°),(down, -120°)	p_2
(up, 0°),(down, +120°)	(down, 0°),(up, -120°)	p_3
(up, 0°),(down, +120°)	(down, 0°),(down, -120°)	p_4
(down, 0°),(up, +120°)	(up, 0°),(up, -120°)	p_5
(down, 0°),(up, +120°)	(up, 0°),(down, -120°)	p_6
(down, 0°),(down, +120°)	(up, 0°),(up, -120°)	p_7
(down, 0°),(down, +120°)	(up, 0°),(down, -120°)	p_8

Since these eight distributions are the only possible distributions, we must have $p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8 = 1$. Further, brief inspection shows us the the h.v. hypothesis yields the following table of probabilities:

θ_1	θ_2	Prob(one up, one down)
0°	0°	1
0°	-120°	$p_2+p_4+p_5+p_7$
+120°	0°	$p_1+p_2+p_7+p_8$
+120°	-120°	$p_2+p_3+p_6+p_7$

If the h.v. hypothesis is to yield the same predictions as quantum mechanics, then the two tables must be identical—in which case we must have

$p_2+p_4+p_5+p_7 = .25$

$p_1+p_2+p_7+p_8 = .25$

$p_2+p_3+p_6+p_7 = .25$

Adding these equations together, it follows that $p_1+3p_2+p_3+p_4+p_5+p_6+3p_7+p_8 = .75$. But this cannot be, since the left-hand side equals $1+2p_2+2p_7 \geq 1$. So the h.v. hypothesis we constructed to explain the observed perfect correlations (i.e., the fact that when $\theta_1 = \theta_2$, the two outcomes are always different) must be false.

A thorny question remains: How *else* can we explain these perfect correlations? Answer that, and you'll be famous.